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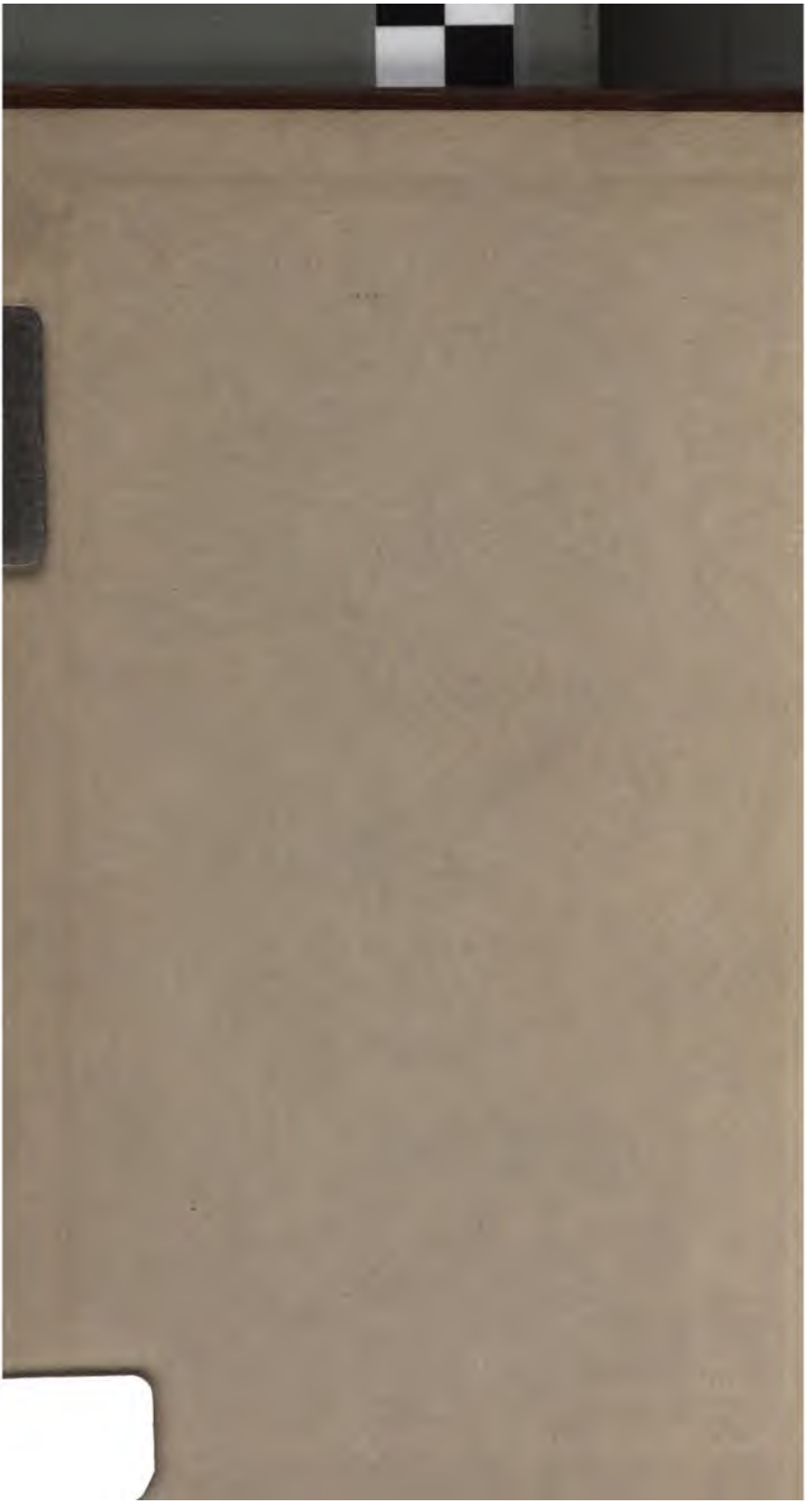

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REPORTS OF THE PROCEEDINGS

OF THE SOCIETY

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXIV. NOVEMBER 13, 1903. No. 1

Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

G. Bigourdan, Observatoire, Paris ;
G. W. Hough, Director of the Dearborn Observatory, Evanston, Ill., U.S.A. ;
W. J. Hussey, Lick Observatory, Mount Hamilton, Cal., U.S.A. ; and
Max Wolf, F.R.A.S., Astrophysikalisches Observatorium, Königstuhl, Heidelberg, Germany,

were balloted for and duly elected Associates of the Society.

The Rev. Edmund Goetz, S.J., Director of the Astronomical and Meteorological Observatory, Bulawayo, Rhodesia, South Africa ;
A. E. Hodgson, Natal Observatory, Durban, Natal ;
H. E. Zufur Jung, G.C.B., Military Minister to His Highness the Nizam, Hyderabad, India ;
Percy Lankester, Highwood House, Kingston Hill, Surrey ;
Thomas Robson, B.A., 14 King's Road, Doncaster ; and
Benjamin Spencer Wolfe, M.A., Victoria College, Jersey,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Henry John Wolverton Brennand, B.A., M.B., Ch.M., F.C.S., Government Statistician's Department, and 203 Macquarie Street, Sydney, New South Wales (proposed by G. H. Knibbs) ;

Albert Edward Garrett, F.R.G.S., Science Lecturer, Lothair Road, Finsbury Park, N. (proposed by E Reeves) ;

Joseph M. Harvey, 77 Clousdale Road, Upper Tooting, S (proposed by Sir R. S. Ball) ;

John George Hatchard, Mechanical Engineer, C.S.A. Rai Works, Pretoria, Transvaal, South Africa (propose A. Fowler) ;

The Rev. Gustavo Heredia, S.J., Stonyhurst College Obs tory, Blackburn, Lancashire (proposed by Rev. Sidgreaves) ;

Herbert Kitchin, Electrical Contractor, Marshall Sq Johannesburg, Transvaal, South Africa (proposed T. A. Innes) ;

George Aimer Russell, M.A., B.Sc., 29 Glebe Road, marnock, Scotland (proposed by R. Copeland) ; and

Frank Herbert Shaw, Burlington House, Huddersfield posed by T. K. Mellor).

One hundred and eighty-eight presents were announc having been received since the last meeting, including, am others :—

J. C. Clancey, Aid to Land-surveying, 2nd edition Calculating Tables, part 1, 4th edition ; Gavin J. Burns, showing proper motions of 2,500 stars ; H. Fritsche, Atla Erdmagnetismus, presented by the authors ; Galileo, Edizio Nazionale, vol. 13, presented by the Italian Governm Galileo, the private life of ; Mary Somerville, personal re tions of, by her daughter, presented by Mrs. McCance ; Schroeter, Eigenbewegung von Sternen in der Zone $+65°$ zu presented by the Christiania Observatory ; A. R. Wallace, Place in the Universe, presented by the author ; Paris, C Astrophotographique International ; Catalogue photograp Tome I. ; Carte photographique (18 charts), presented b French Government ; Potsdam Observatory, Photograp Himmelskarte, Band 3, presented by the Observatory.

Photographic portrait of Sir J. F. W. Herschel, present Sir W. Herschel, Bart. ; Spectroheliograph of the Sun, pre by the Solar Physics Observatory ; photographs of the spe of lightning, presented by Dr. Lockyer ; Negatives Sun, showing great sun-spots, presented by Mr. Newl photographs (negatives) showing absorption of the light of by comet Borrelly, presented by Dr. Max Wolf.

On the Systematic Proper Motions of Bright Stars relatively to Faint Stars in the Oxford Zones (+25° to +31°). By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In *Monthly Notices*, vol. lxiii. pp. 56–71, a preliminary discussion is given of the systematic proper motions in R.A. of bright stars relatively to faint, as determined from the Oxford photographs about 27° declination. The systematic proper motions were deduced by an *indirect* process as follows: Comparison of measures on plates taken at Oxford about 1894, with Cambridge meridian observations made about 1879, gives the sum of two quantities—viz. proper motion for fifteen years + Cambridge magnitude equation. Again, comparison of measures on plates taken at Oxford about 1899 with the Cambridge meridian observations gives

proper motion for 20 years + Cambridge magnitude equation.

Assuming the magnitude equation to be the same in both cases, we get by subtraction the systematic proper motions for five years.

2. Such a process has defects which are only too obvious and the results can only be put forward provisionally. I hope that before long we may undertake a direct determination of the relative proper motions in at least a part of our zone. We hope to finish the measures of our 1,180 plates prescribed in the programme before the end of the present year. When this is done it will be about ten years since the first of them was taken. We might therefore with advantage repeat the early plates, and measure the relative star places again after this interval of ten years. This proceeding could be restricted, in the first instance, to the odd zones +25°, +27°, +29°, +31°, so that we need only repeat four-sevenths of the work, and would be a general check on the previous measures, besides furnishing relative proper motions for ten years. As a general check it would not be much more laborious than the comparison of the measures on overlapping plates, for the numerical work necessary for this comparison, though straightforward, is considerable in amount. (See *Monthly Notices*, vol. lxi. p. 422.) As determining proper motions it would give greater completeness to the survey. The following remark is made by Herr E. Anding in reviewing Professor Newcomb's "Stars: a Study of the Universe" (*V. J. S. Ast. Ges.* 37. Jahr, iv. Heft, p. 333):—

"Schon S. 14 heisst es, der Bau des Universums sei das grösste Problem, das sich der Menschengeist jemals gestellt habe. Solchen Aeusserungen begegnet man nicht selten in der Literatur, häufiger in der letzten Zeit als früher. Doch hat Ref.

noch niemals Gründe gefunden, warum diese 3*n* Anfangsc
stanten wichtiger sein sollen, als die 3*n* Geschwindigkeit
Nicht einmal ein heuristisches Moment lässt sich gelt
machen : denn man wird die 3*n* Coordinaten gar nicht best
men können, ohne dass man zugleich die Geschwindigkei
einführt."

To determine the places of a number of stars at any epoch
after all, only half an investigation ; we must also determ
their proper motions, at least approximately, if we require t
positions at other epochs.

3. Meanwhile it seems desirable to complete the for
indirect investigation by treating the declinations in the s
way as the R.A.'s, as promised in § 32 of the paper alre
quoted ; and this is the object of the present paper. I shall
give for the *y* coordinate tables precisely similar to those form
given for the *x* coordinate.

4. It seems unnecessary to repeat Tables I. and II., wh
will be found on p. 58 of vol. lxiii. Table I. gives the m
date of the groups and the number of plates in each. Bri
there are about sixty "early" plates and sixty "late" plate
each of the zones +26°, +27°, +29° ; and the mean differe
of date is about five years. Table II. shows the numbers of s
of various magnitudes on these plates.

5. In Table III. we have what was called in the earlier p
the "raw material" for the discussion. The letters E an
are used to denote "early" and "late" plates.

TABLE III. (*y*.)

Mean y Residual (*Oxford – Cambridge*) *for each group in units of* 0″·3.

Zone +26°.

Octant of R.A. h h	Mag. 6·0. E.	L.	Mag. 7·5. E.	L.	Mag. 8·5. E.	L.	Mag. 9·0. E.	L.	Mag. E.
0– 3	– 5	– 4	–32	–17	–10	– 6	0	+ 4	+15
3– 6	– 9	+ 3	–30	– 9	+ 3	– 4	– 1	+ 4	+ 2
6– 9	–53	+ 3	– 5	– 7	+ 2	– 1	0	– 1	+ 6
9–12	–27	–17	–30	–19	+ 1	+ 8	– 4	– 9	+ 7
12–15	– 7	– 3	–12	– 1	– 4	– 6	– 1	+ 5	+ 6
15–18	–40	0	+30	–16	+ 2	– 1	+13	0	– 8
18–21	– 4	+ 4	– 4	– 5	0	– 2	– 1	– 3	+ 1
21–24	–13	–25	–22	– 7	– 4	–16	+ 2	+15	+ 6
Mean	–19·8	– 4·9	13·1	–10·1	– 1·3	– 3·5	+ 1·0	+ 1·9	+ 4·

Zone + 27°.

Octant of R.A. h	h	Mag. 6·0. R.	L.	Mag. 7·5. R.	L.	Mag. 8·5. R.	L.	Mag. 9·0. R.	L.	Mag. 9·4. R.	L.
0	3	°(−33)	−15	°(−25)	+ 7	°(− 7)	− 5	°(0)	+ 7	°(+13)	+12
3	6	−12	+ 5	−13	−11	+ 1	− 3	+ 3	0	+ 2	+ 6
6	9	− 32	33	− 9	− 8	0	+ 1	+ 2	+ 3	+ 6	+12
9	12	40	−15	0	−11	+ 5	− 2	+18	+ 1	+10	+ 2
12	15	− 9	−20	− 8	− 4	0	− 6	− 2	0	+10	+13
15	18	−13	+ 8	3	− 7	0	− 3	+ 4	+ 2	+ 7	+ 3
18	21	+13	−11	0	− 5	− 5	− 4	+ 2	− 3	0	0
21	24	7	+ 3	9	−13	0	− 7	− 1	+ 4	+11	+ 4
Mean		16·6	9·8	8·4	− 6·5	− 0·8	− 3·6	+ 3·3	+ 1·8	+7·4	+ 6·5

Zone + 29°.

Octant of R.A. h	h	Mag. 6·0. R.	L.	Mag. 7·5. R.	L.	Mag. 8·5. R.	L.	Mag. 9·0. R.	L.	Mag. 9·4. R.	L.
0	3	−60	43	−19	− 2	− 4	− 4	+ 1	+ 3	+11	+10
3	6	12	− 9	− 6	2	− 2	− 9	+ 4	− 4	+ 5	+12
6	9	− 6	− 55	4	− 7	− 7	− 6	− 1	0	+16	+ 7
9	12	5	− 5	7	− 5	− 3	− 2	− 2	+ 1	+11	+ 2
12	15	− 1	− 37	− 3	−10	7	2	1	7	− 8	+43
15	18	− 5	− 19	− 6	−25	5	3	0	− 1	− 7	+ 2
18	21	− 1	− 20	1	9	5	− 15	− 3	− 1	− 2	− 2
21	24	− 18	10	9	25	− 4	− 9	3	8	− 6	+ 7
Mean		12·0	13·8	− 5·4	9·4	− 3·9	− 5·8	− 0·6	1·9	+ 8·3	+10·1

6. To follow the course of the previous paper we first set down as a preliminary result the differences between the means of the eight octants as they stand. These do not give us comparable proper motions, because the intervals, although all approximately five years, differ sensibly among themselves; but we get a sufficient test of the homogeneity of our material.

TABLE IV. (*y*.)

Approximate Mean P.M.'s in Decl. for five years in units of 0″·03.

		Magnitude. 6·5	7·5	8·5	9·0.	9·4.
Decl.	+ 26	+ 14·9	+ 3·0	− 2·2	+ 0·9	− 2·1
	+ 27	− 6·8	+ 1·9	2·8	− 1·5	0·9
	+ 29	1·8	4·0	1·9	1·3	− 1·8
Mean		− 6·6	+ 0·3	2·3	− 0·6	0·4

* The mean of results for zones + 26° and + 29° has been substituted.

7. These numbers differ in a marked manner from the c
responding numbers for R.A., in which there was a disti
progression. It seems worth repeating the former table
comparison :

TABLE IV. (*x*.)

Approximate Mean P.M.'s in R.A. for five years in units of 0″·03 *or* 0ˢ·00

	Magnitude.				
	6·0	7·5.	8·5.	9·0.	
Decl. + 26°	− 18·6	− 8·7	+ 1·5	− 0·3	+
,, + 27°	− 6·9	− 7·2	+ 0·2	+ 0·9	+
,, + 29°	− 15·1	− 4·8	− 3·5	− 1·1	+
Mean	− 13·5	− 6·9	− 0·6	− 0·2	+

In Table IV. (*x*) there was a sufficient accordance betw
the numbers for the three years to encourage us to believe t
we were dealing with some systematic phenomenon ; but
accordance is not reproduced in Table IV. (*y*.) Indeed,
results for zone + 26° and zone + 29° are almost precisely
variance ; while zone + 27° falls in between the two, as is sh
by its agreement with the means. We are driven to infer t
the systematic proper motions, in declination at any rate,
very small compared with those which do not follow
systematic law of drift ; or possibly that the accidental er
of measurement are larger than we should naturally exp
And in the light of these new results we naturally regard
apparent accordance of Table IV. (*x*) with more caution.

8. We proceed nevertheless to combine the three zones.
may still hope to diminish the "accidental errors" by so do
though the whole work is put on a more tentative b
Table VI. (*y*) has been formed in the same way as Table V
the former paper. We may first repeat Table V., as it is requ
for the formation of Table VII. It shows the number of pl
for each octant of R.A. and the mean interval in years.

T

Mean R

(The

Octant of R.A. h h	Interval in Years.	6·0			7·5		
		E.	L.	L—E.	E.	L.	I
0 − 3	5·2	− 41·7	− 27·3	+ 14·4	− 23·3	− 5·6	+
3 − 6	5·2	− 11·3	− 0·8	+ 10·5	− 15·2	− 7·1	+
6 − 9	4·9	− 27·6	− 26·8	+ 0·8	− 6·8	− 7·3	−
9 − 12	5·3	− 16·2	− 8·6	+ 7·6	− 15·3	− 7·4	+
12 − 15	4·5	− 3·6	− 13·3	− 9·7	− 5·9	− 3·1	+
15 − 18	4·6	− 4·5	+ 7·8	+ 12·3	+ 6·2	− 14·2	−
18 − 21	4·1	+ 1·4	+ 3·6	+ 2·2	− 1·9	− 5·9	−
21 − 24	4·0	− 14·1	− 1·5	+ 12·6	− 13·1	− 14·5	−

TABLE V.

Octant of R.A.	0^h–3^h	3^h–6^h	6^h–9^h	9^h–12^h	12^h–15^h	15^h–18^h	18^h–21^h	21^h–24^h
Num. E plates	12	23	29	13	29	19	33	38
Num. L. plates	16	27	24	38	23	35	23	12
Mean interval	5·2	5·2	4·9	5·3	4·5	4·6	4·1	4·0

TABLE VII. (*y*.)

Deduced Relative Motions in each Octant.

(The unit is 0″·03.)

Octant. h h	Mag. 6·0.	Mag. 7·5.	Mag. 8·5.	Mag. 9·0.	Mag. 9·4.	Gradients. Bright Stars.	Mean.	Faint Stars.
0– 3	+2·8	+3·4	+0·2	+0·6	−0·6	−0·4	+1·4	+0·9
3– 6	+2·0	+1·6	−1·2	−0·5	+0·7	+0·3	+1·0	−2·1
6– 9	+0·2	−0·1	−0·1	0·0	−0·3	+0·2	+0·1	+0·2
9–12	+1·4	+1·5	−0·1	0·0	−1·4	−0·1	+0·9	+1·4
12–15	−2·2	+0·6	−0·1	+0·6	+0·2	−1·9	−0·5	−0·3
15–18	+2·7	−4·4	+0·1	−0·3	−0·4	+4·7	−0·3	+0·6
18–21	+0·5	−1·0	−0·5	−0·3	−0·6	+1·0	+0·1	+0·1
21–24	+3·2	−0·4	−1·2	+1·0	−0·7	+2·4	+0·8	−0·6
Mean	+1·33	+0·15	−0·36	+0·14	−0·39	+0·78	+0·44	+0·03

9. The differences between early and late plates have then been divided by the mean intervals shown in Table V., so as to get the annual P.M.'s., shown in Table VII., still expressed in units of 0″·03. The "gradients" in the last three columns have been formed as follows: Subtracting from the mean of the results for magnitudes 6·0 and 7·5 the mean of the three results for 8·5, 9·0, and 9·4, and dividing the difference by 2·2 (the difference of the mean magnitudes), we get the mean relative P.M. for one magnitude, which is called the mean gradient. But

VI. (*y*.)

for all Zones.

is 0″·03.)

8·5			9·0			9·4		
E.	L.	L–E.	E.	L.	L–E.	E.	L.	L–E.
−6·0	−4·8	+1·2	+0·7	+3·8	+3·1	+12·3	+9·3	−3·0
+0·8	−5·5	−6·3	+2·3	−0·3	−2·6	+ 2·7	+6·1	+3·4
−1·8	−2·1	−0·3	+0·7	+0·5	−0·2	+ 9·1	+7·7	−1·4
+2·4	+1·7	−0·7	−1·2	−1·4	−0·2	+ 9·4	+2·2	−7·2
−5·3	−5·7	−0·4	−1·1	+1·8	+2·9	+ 7·8	+8·9	+1·1
−2·9	−2·3	+0·6	+2·4	+1·1	−1·3	+ 5·4	+3·5	−1·9
−5·2	−5·3	−2·1	−1·4	−2·6	−1·2	+ 1·3	−1·0	−2·3
−3·2	−8·1	−4·9	−1·0	+2·9	+3·9	+ 7·1	+4·3	−2·8

as the gradient may not be uniform two other columns h been formed, that for "bright" stars by comparing the res for 6·0 and 7·5 and dividing by 1·5, and that for "faint" s by comparing 8·5 and 9·4 and dividing by 0·9. The accide errors of these columns are, of course, much larger than that the mean.

10. The question raised in the previous paper was: How can these relative proper motions of bright and faint stars referred to the Sun's motion through space? The expressi for the parallactic motion are

$$\mu_\alpha = +(\mathrm{V}\cos\mathrm{D}\sec\delta)\times\varpi\times\sin(\alpha-\mathrm{A})$$
$$\mu_\delta = +(\mathrm{V}\cos\mathrm{D}\sec\delta)\times\varpi\times\mathrm{K},$$

where

$$\mathrm{K} = \sin\delta\cos\delta\cos(\alpha-\mathrm{A})-\cos^2\delta\tan\mathrm{D},$$

V being the Sun's velocity, A and D the R.A. and Decl. of point towards which the Sun is moving, and ϖ the ann parallax of a star whose R.A. and Decl. are α and δ.

11. Assuming V cos D sec δ to be constant for the resu now under discussion, the values of (V cos D sec δ) $\times\varpi$ w found in the previous paper (p. 68) by dividing μ_α by sin $(\alpha-$ They can now be found independently from the P.M.'s in de nation by dividing μ_δ by K, and a comparison of the resu will tell us how far we are right in ascribing these relative P.M to the Sun's motion in space. The comparison is shown Table VIII.

TABLE VIII.

Octant. h h	μ_α	sin $(\alpha-\mathrm{A})$.	$\mu_\alpha/\sin\alpha-\mathrm{A}$	μ_δ.	K.	μ_δ
0– 3	−2·1	+·9	−2·3	+1·4	−·6	−
3– 6	−0·1	+·4	−0·2	+1·0	−·8	−
6– 9	−0·8	−·4	+2·0	+0·1	−·8	−
9–12	−0·1	−·9	+0·1	+0·9	−·6	−
12–15	−0·7	−·9	+0·8	−0·5	−·3	+
15–18	+0·5	−·4	−1·1	−0·3	(−·1)	(+
18–21	−0·3*	+·4	−0·8*	+0·1	(−·1)	(−
21–24	−0·8	+·9	−0·9	+0·8	−·3	−

12. The values of sin $(\alpha-\mathrm{A})$ and K are only approximate, a for the groups 15^h-18^h and 18^h-21^h the value of K cannot trusted as a divisor. (An accurate value might just possibly ha a small positive sign.) If we exclude these groups and arran the others according to the value of $\mu_\alpha/\sin(\alpha-\mathrm{A})$ we get t following quantities, which should agree :—

* There is unfortunately a slight error in Table VII. of the previous pap which is here corrected. The consequent errata are given at end of t paper.

Octant.	0h–3h	21h–24h	3h–6h	9h–12h	12h–15h	15h–18h
$\mu_\alpha/\sin(a-A)$	−2·3	−0·9	−0·2	+0·1	+0·8	+2·0
μ_δ/K ...	−2·3	−2·7	−1·3	−1·5	+1·7	−0·1

13. Considering the large accidental error which obviously affects the results, the accordance is not unsatisfactory. If we accept it as showing that we are getting some indication, however rough, of the relative parallaxes of bright and faint stars in the different octants, then we may combine the two sets of results to improve this determination. Let us change the sign of μ_α when $\sin(a-A)$ is negative, and of μ_δ when K is negative, and add the two together, dividing by the numerical sum of the factors, as in Table IX. There is now less reason for excluding the groups bracketed.

TABLE IX.

Octant. h h	$\pm\mu_\alpha \pm \mu_\delta$.	$\pm\sin(a-A) \pm K$.	Ratio $= \varpi V \cos D \sec \delta$.	Galactic Latitude. °
0– 3	−3·5	1·5	−2·3	−34
3– 6	−1·1	1·2	−0·9	−13
6– 9	+0·7	1·2	+0·6	+22
9–12	−0·8	1·5	−0·5	+61
12–15	+1·2	1·2	+1·0	+80
15–18	−0·2	0·5	−0·4	+40
18–21	−0·4	0·5	−0·8	+ 2
21–24	−1·6	1·2	−1·3	−25

14. The approximate galactic latitude of the group has been inserted, as in Table XI. of the former paper (*Monthly Notices*, lxiii. p. 68). If we arrange the results simply according to latitude we get the following series, the results of the former paper, obtained from R.A. alone, being added for comparison :—

Deduced Value of ϖ V *cos* D *sec* δ *for different Galactic Latitudes.*

Gal. Lat.	+2°	−13°	+22°	−25°	−34°	+40°	+61°	+80°
R.A. and Decl.	−0·8	−0·9	+0·6	−1·3	−2·3	−0·4	−0·5	+1·0
R.A. alone	−0·8	−0·2	+2·0	−0·9	−2·3	−1·1	+0·1	+0·8

15. It does not, therefore, seem impossible that there may be some reversal, over a limited range of magnitudes, of the law that parallax and brightness diminish together, which is specially marked at about 35° from the galaxy. At any rate a case has been made out for further inquiry.

---

16. We will now consider the results obtained from another point of view, which depends, however, on an assumption which may not be true.

Can we assume that the Cambridge magnitude equation is th same for all R.A.'s?

This assumption has usually been made, but it must at an rate be carefully examined, and recent work (see *Obs.* xxvi. p. 210 seems to show that it is sometimes, perhaps always, erroneous But by adopting it for an independent investigation of th quantities under discussion we shall throw some light on th validity of the assumption itself. The use to be made of it wil now be stated.

17. Consider for simplicity two stars only, a bright B and faint F. About 1879 these two stars were observed at Cam bridge, and taking B as correct, the observed R.A. of F was to large, by M, say. About 1894 these stars were photographed a Oxford. F had meanwhile been moving with reference to B an had increased its relative R.A. by 15P, say, where P is the rela tive P.M. per year. Hence we should find, on comparing th Oxford photograph with the Cambridge places,

$$M_1 = M + 15P,$$

as the Cambridge magnitude equation, instead of M, as befor Again, other photographs were taken at Oxford about 189 Had they been of the same actual stars we could have compare them directly with those of 1894, and avoided using the Cam bridge observations as an intermediary ; but they were of differe regions, and we thus find for the Cambridge magnitude equati

$$M_2 = M' + 20P'.$$

Now in what precedes it has been assumed that in the same pa of the sky (*i.e.* in each separate octant) $M = M'$ and $P = P'$, f the mean of a large number of stars. The former assumption probably correct ; but the latter is more doubtful, as we notic in § 7 of this paper. Still we have hitherto been working on th assumption ; and from it we have deduced, by subtracting t above equations,

$$5P = M_2 - M_1,$$

and if we substitute this value of P in either of the above equ tions we get

$$M = 4M_1 - 3M_2.$$

18. Now, if we could assume that the value of M should the same for all R.A.'s we could get for it by combining all t octants a much better value, M_0, than the value deducible fr a single octant alone. Using this mean value we could th find P for each octant from the equations

$$35P = (M_1 + M_2) - 2M_0 \quad \ldots \quad \ldots$$

and a comparison of the results with those already found fr the equations

$$5P = M_2 - M_1 \quad \ldots \quad \ldots \quad \ldots$$

would give a valuable check upon them. The same would, of course, hold good if we in any way knew the value of M_0 in equation (*a*) *independently* of the present observations. But to substitute the value determined from the present observations, viz.

$$M = 4M_1 - 3M_2 \quad \ldots \quad \ldots \quad \ldots \quad (c)$$

simply reduces equation (*a*) to identity with (*b*).

19. Hence we cannot get any value for the P.M.'s independent of that already found unless we can find some value for M different from that directly furnished by equation (*c*); though if M may be regarded as the same for all octants, its mean value, derived from the mean of eight equations like (*c*), would practically satisfy requirements. Let us in the first place see whether it seems likely that M *is* constant in value; and first consider the results for the *y* coordinate, collected in the present paper.

20. Recurring to Table VI. (*y*), the letters E and L correspond to what we have called M_1 and M_2. Now from (*c*)

$$\begin{aligned} M &= M_2 - 4(M_2 - M_1) \\ &= L - 4(L - E). \end{aligned}$$

We can thus form M very readily by subtracting four times the column (L−E) from the column preceding it. The factor four should not be absolutely constant: it is really the ratio of the interval between the Cambridge observations and the Oxford L plates to the interval between L and E. The former may be taken as 20; the latter is given in Table VI. The results are shown in Table X. (*y*) for each magnitude; and in the column "Mean Gradient" a result is found, as before, by subtracting the mean of the last three columns from the mean for 6·0 and 7·5, and dividing by 2·2.

TABLE X. (*y*.)

Values of M *deduced from Table VI.* (*y*) *in units of* 0″·03.

Octant of R.A. h h	Factor.	6·0	7·5	8·3	9·0	9·4	Mean Gradient.
0– 3	3·8	−82·0	−72·9	− 9·4	− 8·0	+20·7	−35·7
3– 6	3·8	−40·7	−37·9	+18·4	+ 9·6	− 6·8	−21·0
6– 9	4·1	−30·1	− 5·2	− 0·9	+ 1·3	+13·4	−10·0
9–12	3·8	−37·5	−37·4	+ 4·4	− 0·6	+29·6	−22·0
12–15	4·4	+29·4	−15·4	− 4·0	−11·0	+ 4·1	+ 4·8
15–18	4·4	−46·3	+75·6	− 4·9	+ 6·8	+11·9	+ 4·5
18–21	4·9	− 7·2	+13·7	+ 5·0	+ 3·3	+10·3	− 1·3
21–24	5·0	−64·5	− 7·5	+16·4	−16·6	+18·3	−19·1
Mean	4·3	−34·9	−10·9	+ 3·1	− 1·9	+12·7	−12·5

21. Now a mere glance at this table is sufficient to show tha the values of M indicated are far too large and irregular. Take for instance, the column headed "Mean Gradient," which repre sents the mean magnitude equation *per magnitude* in declinatio at the epoch 1875·0, if the systematic P.M.'s we are dealing wit are treated as accurate and carried back twenty years; and it value in the first octant is found to be $-35{\cdot}7 \times 0''{\cdot}03 = -1''{\cdot}07$ while in the 5th and 6th it is about $+0''{\cdot}14$. This range i quite inadmissible. Even the mean value $-12{\cdot}5$ or $-0''{\cdot}37$ i too large, being more than double the quantity found, fo instance, by Auwers (*Ast. Nach.* No. 3844, p. 72).

22. It would, therefore, appear that our results do not trul represent *systematic* proper motions. Either the individua proper motions are too large compared with any systematic drif to satisfactorily nullify one another in our means or there i some unknown source of accidental error. It may be that th magnitude equation varies with R.A., but it certainly does nc vary as much as would be indicated by Table X. (*y*), and w have no independent determination of its variation.

23. But it is still of interest to find what systematic P.M. would be given by the assumption that M_0 was the same for a R.A.'s, from the equation (*a*) of § 18. In this method w practically cease to differentiate E and L plates. We take th mean of both, and find how much M has strayed from its origin value M_0 in each octant owing to systematic P.M. during th interval of about 17·5 years between the Cambridge observatio and the Oxford plates.

24. We must, however, decide what value to use for M_0. I Auwers's paper above referred to (*Ast. Nach.* No. 3844, p. 7 the values of M_0 referred to magnitude 9·0 as zero, and expresse in our units of $0''{\cdot}03$, would be

TABLE XI. (*y*.)

Values of M_0.

	6·0	7·5	8·5	9·0	9·4	Mean Gr
Auwers	−12·3	−9·7	−4·0	0·0	+ 3·2	− 4·9
Table X. (*y*) (Means)	−33·0	−9·0	+5·0	0·0	+14·6	−12·5

In order to render the new results as much as possible indepe dent of the old we will adopt the results given by Auwers they stand, and find the values of P for each octant from equ tion (*a*) of § 18, where for M_1 and M_2 we take simply t E and L of Table VI. (*y*.) The results are given in Table XII. (and the results of Table VII. (*y*) are added for comparison.

TABLE XII. (*y*.)

...tematic P.M.'s in Declination deduced from the 17½ years' interval between Cambridge Meridian Observations and Oxford Plates, assuming Auwers's determination of the Magnitude Equation to hold good in all R.A.'s.

...tant R.A.	Mag. 6·0. New Det.	Mag. 6·0. Old (Table VII.)	Mag. 7·5. New	Mag. 7·5. Old	Mag. 8·5. New	Mag. 8·5. Old	Mag. 9·0. New	Mag. 9·0. Old	Mag. 9·4. New	Mag. 9·4. Old	Mean Gradient. New Det.	Mean Gradient. Old (Table VII.)
h												
– 3	−1·3	+2·8	−0·3	+3·4	−0·1	+0·2	+0·1	+0·6	+0·4	−0·6	−0·4	+1·4
– 6	+0·4	+2·0	−0·1	+1·6	+0·1	−1·2	+0·1	−0·5	+0·1	+0·7	0·0	+1·0
– 9	−0·9	+0·2	+0·2	−0·1	+0·1	−0·1	0·0	0·0	+0·3	−0·3	−0·2	+0·1
– 12	0·0	+1·4	−0·1	+1·5	+0·3	−0·1	−0·1	0·0	+0·2	−1·4	−0·1	+0·9
– 15	+0·2	−2·2	+0·3	+0·6	−0·1	−0·1	0·0	+0·6	+0·3	+0·2	+0·1	−0·5
– 18	+0·8	+2·7	+0·3	−4·4	+0·1	+0·1	+0·1	−0·3	+0·1	−0·4	+0·2	−0·3
– 21	+0·8	+0·5	+0·3	−1·0	0·0	−0·5	−0·1	−0·3	−0·2	−0·6	+0·3	+0·1
– 24	+0·3	+3·2	−0·2	−0·4	−0·1	−1·2	+0·1	+1·0	+0·1	−0·7	0·0	+0·8

25. The last pair of columns alone is sufficient to show that there is no correspondence between the new determination of systematic proper motions and those of Table VII. : either one or the other set is quite untrustworthy. We can improve the accordance a little by assuming a different value for the original magnitude equation of the Cambridge observations, which means simply adding a constant to the column headed "New Determination;" but the improvement is not sufficient to give at length.

26. Before proceeding to examine what values the new determination would give to the relative parallax of bright and faint stars, as in Table VIII., we will form the corresponding results for R.A. on the new plan. The following tables, X. (*x*), XI. (*x*), and XII. (*x*), correspond precisely to those given above for (*y*), and it is perhaps unnecessary to repeat the explanations. They are derived from Table VI. on p. 62 of *Monthly Notices*, vol. lxiii., in exactly the same way as the tables for (*y*) are derived from Table VI. (*y*) of the present paper.

TABLE X. (*x*).

Values of M *deduced from Table VI.* ("*Monthly Notices*," *vol. lxiii. p.* 62) *in units of* 0″·03.

Distance of R.A.	Factor.	6·0	7·5	8·5	9·0	9·4	Mean Gradient.
h h							
0– 3	3·8	+165·7	+146·7	+57·1	+34·8	−45·0	+63·9
3– 6	3·8	+ 25·5	+ 13·0	−14·1	−11·3	−34·1	+17·8
6– 9	4·1	+ 21·6	+ 76·2	+ 1·6	−16·0	−63·7	+34·1
9–12	3·8	+ 74·3	− 0·7	+23·9	−19·3	−22·6	+19·5
12–15	4·4	+ 53·6	+ 70·1	+22·0	+12·7	−61·4	+32·2
15–18	4·4	+ 21·3	+ 17·0	+ 3·5	−27·9	+ 5·1	+11·7
18–21	4·9	+ 68·4	+ 48·4	+46·7	−11·2	−20·7	+24·3
21–24	5·0	+103·8	+ 51·8	+13·9	+19·6	−34·3	+35·5
Mean	...	+ 66·8	+ 52·8	+19·3	− 2·3	−34·6	+29·9

TABLE XI. (x.)

Values of M_0.

	6·0.	7·5.	8·5.	9·0.	9·4.	Mean Grad'
Auwers	+46·7	+35·9	+19·1	0·0	−20·0	+18·9
Table X. (x) Means	+69·1	+55·1	+21·6	0·0	−32·3	+29·9

TABLE XII. (x).

Systematic P.M.'s in R.A. deduced from the 17½ years' interval between Cape Meridian Observations and Oxford Plates, assuming Auwers's determination Magnitude Equation to hold good in all R.A.'s.

Octant of R A.	Mag. 6·0. New.	Old.	Mag. 7·5. New.	Old.	Mag. 8·5. New.	Old.	Mag. 9·0. New.	Old.	Mag. 9·4. New.	Old.	Mean (New. Det^n.	T
h h												
0− 3	+0·7	−6·3	+0·9	−5·5	+0·1	−2·1	−0·5	−2 6	−0·5	+0·9	+0·5	
3− 6	−0·5	+0·7	−0·9	+0·4	−0·3	+1·7	−0·2	+0·4	−0·8	+0·1	−0·1	
6− 9	−0·1	+1·4	−0·2	−2·5	−0·1	+0·9	0·0	+0·9	−0·6	+1·9	0·0	
9−12	−0·5	−2·1	·0·5	+1·6	−0·2	−0·4	−0·5	+0·6	−0·3	−0·1	−0·1	
12−15	−0·7	−1·1	−0·3	−2·2	−0·2	−0·4	−0·2	−1·0	−1·1	+1·3	0·0	
15−18	+0·1	+1·5	−0·5	+0·6	−0·4	+0·5	0·0	+1·5	−0·5	−1·9	0·0	
18−21	−0·1	−1·3	−0·1	−0·8	0·0	−1·4	+0·1	+0·7	−0·4	−0·3	0 0	
21−24	+0·3	−2·9	+0·1	−0·8	+0·4	+0·7	+0·1	−1·0	−0·5	+0·3	+0·1	

27. We may now proceed to repeat Table VIII. of present paper with the new values of μ_α and μ_δ given Tables XII. (y) and XII. (x).

TABLE XIII.

Corresponding to Table VIII., with the New Values of μ_α *and* μ_δ.

Octant. h h	μ_α.	sin (α−A).	μ_α/sin(α−A).	μ_δ.	K.	μ_δ/K.	$\pm\mu_\alpha \pm\mu_\delta$. $\pm\sin(\alpha-A)\pm1$
0− 3	+0·5	+·9	+0·6	−0·4	−·6	+0·7	+0·6
3− 6	−0·1	+·4	·0·2	0·0	−·8	0·0	−0·1
6− 9	0·0	−·4	0·0	−0·2	−·8	+0·3	+0·2
9−12	−0·1	−·9	+0·1	−0·1	−·6	+0·2	+0·1
12−15	0 0	−·9	0·0	+0·1	−·3	−0·3	−0·1
15−18	0·0	−·4	0·0	+0·2	(−·1)	(−2·0)	−0·4
18−21	0·0	+·4	0·0	+0·3	(−·1)	(−3·0)	−0·6
21−24	+0·1	+·9	+0·1	0·0	−·3	0·0	+0·1

28. The last two columns give the equivalent of Table and we may now compare the values of $\varpi V \cos D \sec\delta$ different galactic latitudes (as we did in § 14) given by the methods.

TABLE XIV.

Octant.	18^h–21^h	3^h–6^h	6^h–9^h	21^h–24^h	0^h–3^h	15^h–18	9^h–12^h	12^h–15^h
Gal. Lat.	$+2°$	$-13°$	$+22°$	$-25°$	$-34°$	$+40°$	$+61°$	$+80°$
Old	... $-0''.8$	$-0''.9$	$+0''.6$	$-1''.3$	$-2''.3$	$-0''.4$	$-0''.5$	$+1''.0$
New	... $-0''.6$	$-0''.1$	$+0''.2$	$+0''.1$	$+0''.6$	$-0''.4$	$+0''.1$	$-0''.1$

29. It cannot be said that any conclusion emerges at present from such figures; the "new" and the "old" are almost irreconcilable. If the longer interval available for the "new" entitles them to preference, then we must frankly admit that the systematic proper motions found in the former paper (*Monthly Notices*, vol. lxiii. p. 56) and in the early part of this, and any conclusions based on them, are spurious. On the other hand we must remember that the "new" determinations are based on the assumptions—

(*a*) That the magnitude equation is sensibly the same for all R.A.'s.

(*b*) That Auwers's determination of it (in *Ast. Nach.* 3844) is correct.

Neither of these assumptions may be true, but they would have to be seriously wrong to remove the discordances made apparent in Table XIV.

30. It is easy to show the extent to which they must be in error by a graphical method. Let us suppose, as in §§ 17–19, that we are concerned with three definite epochs—1879·0, when the Cambridge observations were made; 1894·0, when the E plates were taken at Oxford; and 1899·0, when the L plates were taken—and for simplicity consider only the quantity which we have called the "mean gradient," which represents the difference *per magnitude* in units of $0''.03$.

Let us plot the value of this quantity as ordinate against the date as abscissa. Then, if assumptions (*a*) and (*b*) of the last paragraph are correct, and our determinations of systematic proper motion in the different octants absolutely trustworthy, we should have a diagram like figure (1).

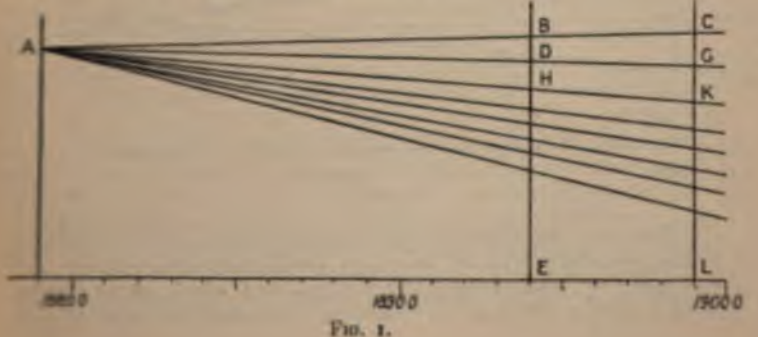

FIG. 1.

Representing the E value of the mean gradient in any give
octant by the point B, and the L value by the point C, then th
straight line BC, produced backwards, should pass through th
point A, representing Auwers's mean gradient for 1879·0. An
for other octants we should have lines ADG, AHK, &c., a
passing through the point A.

31. If assumption (*b*) of § 29 is incorrect, but (*a*) still hold
then the various lines would still pass through a point on th
line OA, though not the point A. Finally, if assumption (*a*)
incorrect, the eight lines will cut OA in eight different points
but unless the variation of magnitude equation with R.A.
comparable with its mean value, these eight points will be
distances from A small compared with AO.

32. It will be found on consideration of the formation of th
last columns of Table X. (*y*) in § 20, and of Table X. (*x*) in § 2
that the quantities therein given represent the values of O
for the different octants. The arithmetical process used corr
sponds exactly to the production of lines joining points, such
C and B, backwards to cut the line OA.

33. To complete the diagram we want the slopes of the line
and it is easily seen that these are given by the "old" system
systematic proper motions, as shown in the columns μ_α ar
μ_δ of Table VIII. in § 11. Hence we construct figures (2) an
(3), which correspond to R.A. and declination.

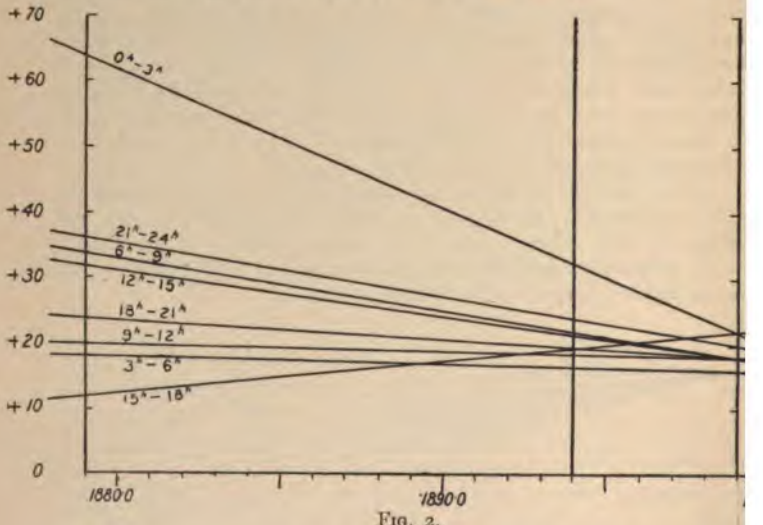

FIG. 2.

The lines are sometimes so close together that it is difficult t
distinguish them, and the following small table, which gives th
material for constructing the figures, may assist us to follo
them. Under OA is given (omitting decimals) the value of th

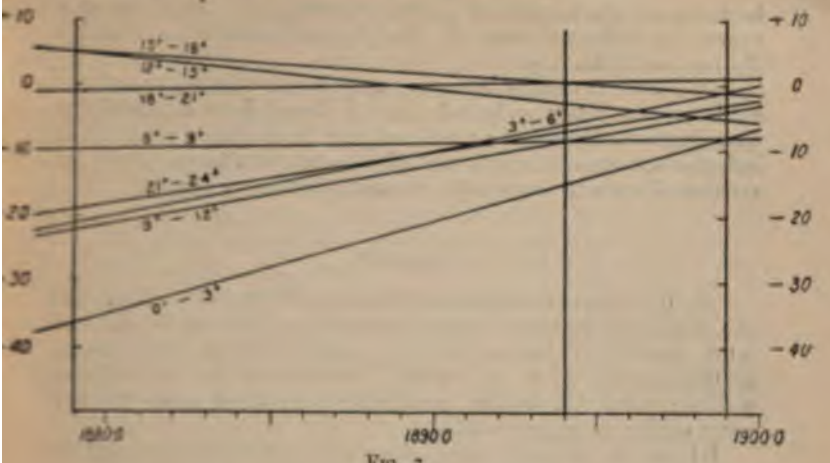

FIG. 3.

mean gradient in 1879·0, as shown in Tables X. (x) and X. (y). Adding twenty years' proper motions, as shown in the columns $20\mu_x$ and $20\mu_y$ (got from Table VIII.), we find the values for epoch 1899·0 as shown in the columns headed LC.

TABLE XV.

Ordinal. h h	Fig. (1). OA.	Fig. (1). 20μx.	Fig. (1). LC.	Fig. (2). OA.	Fig. (2). 20μy.	Fig. (2). LC.
0 – 3	+64	−42	+22	−36	+28	−8
3 – 6	+18	− 2	+16	−21	+20	−1
6 – 9	+34	−16	+18	−10	+ 2	−8
9 – 12	+20	− 2	+18	−22	+18	−4
12 – 15	+32	−14	+18	+ 5	−10	−5
15 – 18	+12	+10	+22	+ 5	− 6	−1
18 – 21	+24	− 6	+18	− 1	+ 2	+1
21 – 24	+36	−16	+20	−19	+16	−3

34. It is apparent either from the figures or from Table XV. that the lines do not tend to converge to a point on the line for 1879·0. There is rather an apparent tendency to converge to a point on the line for 1899·0; but this is somewhat spurious. Producing the lines 20 years into the future it will be seen that the convergence is really to the epoch 1894–1899 generally, and is a natural effect of the admixture of "accidental" errors of some kind. These "accidental" errors may quite reasonably be assigned to real individual proper motions of the stars; but it is of course a possibility, which must be faced, that they are due to errors of measurement. Against this latter alternative we may,

however, set the number of plates measured, and the accordan shown by different zones in the former paper (§ 9, *Month Notices*, vol. lxiii. p. 60).

35. But it is clearly better to show at once the nature of t evidence previously collected ; and I should here acknowled that Professor Seeliger has pointed out (*Ast. Nach.* 3865) fr independent considerations that there must be a large element accidental error in the results given in the former paper.

Summary.

36. (*a*) In a previous paper (*Monthly Notices*, vol. lxiii. p. the relative systematic proper motions in R.A. of bright a faint stars in an interval of five years, as found from pho graphic plates taken at Oxford, were found by an indir method, using Cambridge meridian observations made twe years earlier as an intermediary.

(*b*) In the present paper the same method is extended proper motions in declination.

(*c*) Attributing the proper motions so deduced to the par lactic effect of the Sun's motion in space, and determini thence the relative parallax of bright and faint stars in differ octants of the R.A. circuit from the two components, a f agreement was obtained, which suggested some reality for determination (see § 15).

(*d*) But on using a different method for testing the resu obtained they were not confirmed. This different met depends on one or more assumptions (see § 29), but it is likely that these assumptions are so incorrect that the for method can still be accepted.

(*e*) It must, therefore, be admitted that either the interva five years is not sufficiently long, in spite of the number of st measured, to enable these relative proper motions to be de mined, or that the systematic proper motions are so small c pared with intrinsic proper motions that we require a m larger number of stars to get trustworthy mean values.

Errata in former paper (*Monthly Notices LXIII., pp.* 61, *&c.*).

Page 61. Last line but two: col. 2 *for* $-0\cdot9$ *read* $-1\cdot3$; col. 7 *for* $-$ *read* $-0\cdot3$; and col. 8 *for* $-0\cdot23$ *read* $-0\cdot33$.
,, 61. Last line: col. 2 *for* $-1\cdot21$ *read* $-1\cdot26$; col. 7 *for* $-0\cdot05$ $-0\cdot08$; and col. 8 *for* $-0\cdot55$ *read* $-0\cdot56$.
,, 62. Table VIII. first line: *for* $-1\cdot21$ *read* $-1\cdot26$; *for* $+0\cdot44$ $+0\cdot39$.
,, 68. Line 13 (18^h-21^h) *for* $-0\cdot2$ *read* $-0\,8$; *for* $-0\cdot6$ *read* $-0\cdot8$.
,, 68 Line 20: *for* $-0\cdot6$ *read* $-0\cdot8$.
,, 68. Last line but three: *for* $a-2m$ *read* $a-2m$.

Preliminary Note on a Method of Photographing the Moon with Surrounding Stars. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. If the Moon and surrounding stars can be photographed simultaneously, we can measure the place of the Moon in the same way as that of a planet, comet, or other object, by reference to the surrounding stars; and such observations will be specially valuable near new Moon, when meridian observations are difficult or impossible. It was to obtain observations of the Moon in the first and last quarters that Airy set up the altazimuth at Greenwich in 1847; and unfortunately the instrument did not satisfactorily solve the problem. The present Astronomer Royal has recently set up a larger and steadier instrument with the same end in view, and already something is known of the success which is likely to attend the experiment. But even if early promise is fulfilled, there is ample room for an alternative photographic method.

2. It has for some time been in my mind to adapt for this purpose the method of observation suggested by Captain Hills, R.E., for obtaining terrestrial longitudes. (See *Mem. R.A.S.*, liii. p. 117.) With a fixed camera he takes snapshots of the Moon at times carefully recorded; and *at other times*, when the Moon has passed out of the field (or before it has entered it), a bright star or a planet is photographed on the same plate—several short trails being taken at recorded times. The method is applicable as it stands to the determination of the Moon's place, the disadvantages, however, being:

(*a*) Errors in recording the separate times of exposure enter directly into the inferred R.A. of the Moon.

(*b*) We can only use bright stars or planets, and must thus have rather a large field, or wait long intervals to secure them; perhaps both. The former condition means that the scale of the photograph will in general be small; the second introduces dangers of the camera being moved, or refraction changing considerably in the interval.

3. The second disadvantage can be considerably reduced by the device of a moving plate, moving at the same rate as the stars, and thus allowing faint stars to be impressed by accumulated exposure, which is not possible with trails. The position of the plate can be recorded by photographing on it a fixed point of light, the image of which is cut off automatically by the clock at regular intervals.

4. But a simpler expedient recently occurred to me as follows:—Point an ordinary photographic equatorial to the Moon and its surrounding stars and prepare in the usual way to take a photograph of the region, by an exposure of (say) one minute or of several minutes even, guiding on a star, so that stars of ninth

or tenth magnitude at least would in the ordinary course (*i.e.* if the Moon were not there) be impressed. But cover up the Moon by an opaque screen, C D E F, which is a rectangle of sides, C D, something more than the Moon's diameter, and C E double C D. Let there be a narrow slit, A B, across the middle, and during the exposure let the screen be slowly drawn across the Moon's image in the direction X Y. Then successive portions of the Moon are exposed for short periods determined by the width of the slit and the rate of travel. By selecting these elements properly we can get good definition on the Moon. The image will, of course, be a distorted image—an ellipse instead of a circle—and in measuring different craters on the Moon allowance must be made for the difference in epoch of exposure. The star images are produced by

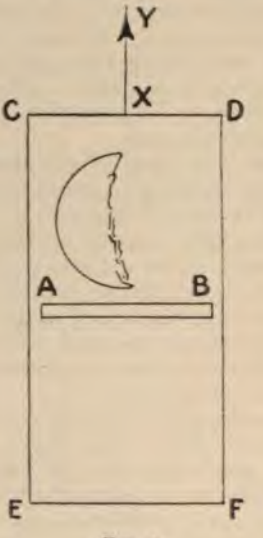

FIG. 1.

the summation of a series of exposures, each of which has a corresponding portion of the Moon ; and theoretically we should measure *every* portion of the Moon to get an accurate correspondence with the stars ; but in practice it will probably be found that sufficient accuracy is obtained by measuring a few well-selected craters. There is in any case an obvious gain over any method which gives one short exposure to the whole Moon and a longer exposure to the stars, leaving us all the uncertainties involved in finding the relation between the two exposures.

5. It will be seen that this method overcomes at once the two chief difficulties :

(*a*) The brightness of the Moon.

(*b*) Its rapid motion among the stars.

6. Several trials of the method have been made at Oxford with very promising results. The first was on October 26 about 6 P.M., when the Moon was about six days old: R.A. $19^h\ 0^m$, decl. $-17°\ 30'$; Z.D. about 70°. An exposure of three minutes was given. The screen was attached to a fixed object by a string, and thus the motion of the telescope automatically drew the screen across. On development the plate was, of course, found to be much fogged by the moonlight, but the following stars were easily recognised upon it.

	Zone.	No.	Mag.		Zone.	No.	Mag.
Schönfeld ...	$-19°$	5312	6·0	Schönfeld ...	$-19°$	5281	8·5
" ...	−19	5273	6·1	" ...	−19	5323	8·5
" ...	−18	5206	6·5	" ...	−19	5304	8·8
" ...	−19	5275	7·0	" ...	−19	5324	9·0
" ...	−19	5317	7·4	" ...	−18	5222	9·0
" ...	−18	5219	7·8	" ...	−18	5211	9·1

And there may be others also. Many stars are, of course, lost by the screen, and those in zone $-17°$ were not looked for.

7. Other trials on October 28, October 30, and November 6 were made with shorter exposures as the brightness of the Moon increased, and ninth magnitude stars were not always shown, though there was no difficulty in getting eighth magnitude. These preliminary trials show that in the first and last quarters, which are the most important, we can get plenty of stars.

8. The screen may be trailed in any direction. The convenience of moving it in declination is that the whole length of path is approximately constant and equal to the Moon's diameter. But in some cases it might be more convenient to trail it in R.A. For instance, this method might be used in place of that of Captain Hills described in § 2, if a moving plate be provided. Point a *fixed* telescope to the region round the Moon, and let the plate move so as to follow the stars. Cover up the Moon with a *fixed* screen having a slit in it; the Moon's image will transit the slit along with the plate and give the desired result.

9. The method has the drawback common to all photographic methods, that it requires finer weather than eye observations. We can observe the Moon through cloud, but we can scarcely photograph faint stars. For this reason, if a successful attempt is to be made to get a good series of observations of the Moon near new Moon, photographs should be taken at more than one observatory.

10. We may hope by experiment to improve the details; but when such good results were obtained on the very first trial, it seems doubtful whether there is anything essential to be added. One small point may be noticed. To check the uniformity of motion of the slit, it is advisable to close the exposing shutter for a few seconds once or twice during the exposure, recording the

times carefully. Also care should be taken to provide that som portion of the Moon shall be shining through the slit during th whole of the exposure. It is, of course, not necessary to phot graph the whole visible Moon.

11. The present note is written with the object of attracti attention to the method, which, it is hoped, will be tried other observatories, especially those in fine climates. The ne apparatus required is practically negligible. A good series photographs of the Moon of this kind, taken day by day fro last quarter to first quarter in lunations extending over a yea say, would be of immense value. By combination of observatori in different parts of the world a very complete series might secured.

12. Attention may also be drawn to the note by Mr. B. Wade in the present number, on observations of the Moon as possible check on longitude observations.

To state his point in rather different words : if we have t approximately equal arcs A B, C D on the Earth's surface, it well known that we may test their equality in two ways.

(*a*) By direct (geodetic) measurement of the arcs A P B a C Q D.

(*b*) By astronomical observations of the angle between verticals at A and B, using stars as an intermediary ; and simi observations of the angle between the verticals at C and D.

Mr. Wade points out that there is a third method, *independ* of these two, viz. :

(*c*) Observation of the angles subtended at the Moon by chord A B and the chord C D.

13. These observations are comparatively rough owing to great distance of the Moon ; but they may serve as a use check, particularly if sufficient pains are spent on the lu

observations, *e.g.* by numerous repetitions. If photographs of the Moon among the stars were assiduously taken over long periods at numerous observatories, it does not seem impossible that we might obtain ultimately information as to the figure of the Earth comparable in value with that derived from telegraphic longitudes. For at present the latter are only made at rare intervals; and they are affected by some unknown, though small, sources of error, especially when submarine cables are employed. For instance, there is as yet no means of checking the fundamental assumption which enters into all telegraphic longitudes, that the time of transmission of an electric signal from A to B is the same as the time of return from B to A. The error of this assumption is no doubt a fraction of the whole time of transmission, but the latter may be as much as 0ˢ·3 across the Atlantic, and there is room for an error of 0ˢ·05 in the assumption of equal velocity to and fro. Good lunar observations systematically compared ought to be capable of easily checking such quantities as this.

Errors in the Moon's Tabular Longitude as affecting the comparison of the Greenwich Meridian Observations from 1750 *with Theory.* By P. H. Cowell.

(*Communicated by the Astronomer Royal.*)

I preface the account of my investigations with some extracts from the researches of various lunar theorists. I also exhibit a comparison between Airy's expression for the tabular longitude of the Moon and Hansen's. Had I realised when I made a certain analysis described in this paper that the errors of Airy's formula would produce a large systematic effect, as I endeavour to show in this paper that they do, my analysis would have taken a slightly different form. However, I now publish such results as I have obtained, and also the evidence on which I base the conclusion that the longitudes from 1750 to 1851 should be compared with a better formula than Airy's. I may also add that the bulk of the labour of this paper consisted in arranging the errors for analysis, and that this arrangement will enormously simplify the labour of correcting Airy's tabular places. The small fraction of the computations that represent the analysis after the arrangement was complete will in the near future be revised, but the provisional results are now published.

The arrangement which I refer to above as having formed the bulk of the labour is not described in this paper, nor are the details of the analysis. Complete information will be given with the corrected results.

The terms with coefficients over 0″·3, of which it is necessary to take account in the longitude of the Moon due to causes other

than the direct action of the Sun are approximately (1) terms
to planetary action, calculated by Radau (*Ann. de l'Observat*
de Paris, Mémoires, vol. xxi.)

$$
\begin{aligned}
&- 0''.860 \sin (V-E)\\
&+ 0.283 \sin 2(V-E)\\
&- 0.681 \sin (g+2\varpi+3V-5E)\\
&- 0.348 \sin (2V-3E+85^\circ)\\
&+ 14.42 \sin (A+30^\circ) \text{ where } A = 18V-16E-g\\
&+ 0.806 \sin (A+30^\circ-g)\\
&+ 0.756 \sin (A+30^\circ+g)\\
&- 0.422 \sin (2M-E+49^\circ)\\
&+ 0.646 \sin (E-J)\\
&- 0.881 \sin (g+2\varpi-2J)\\
&+ 0.316 \sin (g+2\varpi-3J+7^\circ)
\end{aligned}
$$

to which may be added Newcomb's empirical alteration
Venus term

$$-15''.5 \cos A(1+2e \cos g)$$

the second factor being introduced because $-15''.5 \cos A$
correction to the fundamental argument or mean anomaly.
effect which is just sensible is also produced by this term on
evection and variation ; (2) terms due to the figure of the E
calculated by Hill (*Astron. Papers Amer. Eph.* vol. iii.)

$$
\begin{aligned}
&+7''.67 \sin \Omega\\
&+1.039 \sin \Omega \cos g\\
&+0.391 \sin (2g+2\varpi-\Omega);
\end{aligned}
$$

(3) nutation, where two terms of Hansen are sensibly correc

$$
\begin{aligned}
&-17''.332 \sin \Omega\\
&- 1.254 \sin 2\odot
\end{aligned}
$$

Now the meridian observations at Greenwich since 175
into four groups, of which the first two slightly overlap.

(1) 1750–1851, tabular places according to Airy's formu
(2) 1847–1861, tabular places according to Hansen's ta
with an error of sign corrected.

(3) 1862–1882, tabular places according to Hansen's t
unaltered.

(4) 1883 onwards, tabular places according to Han
tables, with an error of sign corrected and Newcomb's correc
introduced.

Before coming to the solar terms I set down here the corrections required by Airy and Hansen's tabular longitude to reduce them to the above formulæ, terms less than $0''.3$ being omitted.

Corrections common to all four periods:

$$-0''.681 \sin (g+2\varpi+3V-5E)$$
$$-0.348 \sin (2V-3E+85°)$$
$$-0.881 \sin (g+2\varpi-2J)$$
$$+0.316 \sin (g+2\varpi-3J+7°)$$

Additional corrections required in the first period:

$$+14''.42 \sin (A+30°)$$
$$+ 1.562 \sin (A+30°) \cos g$$
$$- 0.422 \sin (2M-E+49°)$$
$$-15.5 \cos A (1+2e \cos g)$$
$$+ 0.45 \sin \Omega$$
$$+ 1.039 \sin \quad \cos g$$

It will be noticed that Airy gives a figure of the Earth term omitted by Hansen, and that in the coefficient of sin Ω Airy makes two errors that nearly cancel, the term occurring both in figure of Earth and nutation.

Additional corrections required by all tabular longitudes based on Hansen's tables:

$$-0''.92 \sin (A+30°)$$
$$+0.63 \cos \Omega$$
$$+0.445 \sin \Omega \cos g$$
$$+0.391 \sin (2g+2\varpi-\Omega)$$

Further additional corrections required in the period 1847–1882:

$$[1+2e \cos g] [-21''.47 \sin (8V-13E+274° 14') - 1''.14 - 29''.17t - 3''.76 t^2 - 15''.5 \cos A]$$

and in the period 1862–1882 a still further correction

$$-0''.62 \sin (2g-4g'+2\omega-4\omega')$$

Of the term $+0''.63 \cos \Omega$ it may be mentioned that it is inserted to reduce Hansen's figure of the Earth terms to pure theory in accordance with Hill's calculations. Airy found $-1''.06 \cos \Omega$ from the observations 1750–1851, and Hansen has apparently adopted the term with its coefficient reduced. It

will be seen in the latter part of this paper that a coefficient of the principal elliptic term [illegible] is obtained, whereas Airy has obtained [illegible]. This paper sufficiently explains why Airy's value is wrong. I however tried to find a similar explanation of this new term, and I have not succeeded: so that at the present moment I do not know whether the term really exists or not.

In the following table of some terms in the Moon's longitude with coefficients of [illegible] the *first* column gives a reference number, the *second* column classifies the term with reference to the discussion below in cases when Airy's coefficient appears to be [illegible] in error, the *third*, *fourth*, and *fifth* columns exhibit the arguments as sums of multiples of g, g', $\ldots$: D, g, F, D, g, $\ldots$ respectively. The first method is Hansen's: its convenience is that the approximate period of the term is known from inspection: the second method is that used by Airy, Delaunay, Brown, and others: its convenience is that it gives the "characteristic" of the coefficient. The third method bears reference to the present paper. The *sixth* column gives the coefficient according to Newcomb's transformation of Hansen's theory (*Astron. Papers of Amer. Eph.* vol. i.). These coefficients correspond to the eccentricity of Hansen's tables, and to a solar parallax of 8″.848, which is not the parallax of Hansen's tables. Newcomb's comparison with Delaunay shows that these coefficients are subject to small errors only. Hansen's tables differ from Hansen's theory very slightly in quantities seldom exceeding [illegible]. The *seventh* column gives Airy's coefficients. Airy accepted Damoiseau's values of the arguments, but revised his coefficients. His action has been most unfortunate, as Damoiseau's coefficients were much superior to those that Airy substituted, and in particular Damoiseau's coefficient of the principal elliptic term was practically correct. The *eighth* column gives estimates of the correct value of two coefficients, and the consequent change in a third. The *ninth* column gives the correction to Airy's coefficient.

Table I

Ref. No.	Class	Argument expressed in Multiples of									Coefficient in Hansen's Theory modified by Newcomb.	Coefficient in Airy's Reductions.	Apparent make true Value.	Correction required by Airy.
		p'	z'	w	w'	D	g	g'	F	D	z'	w	w				
1	A	1	...	..	..	0,	1.	0, 0		0,	1	...	...	+ 22640″15	+ 22641″6	+ 22639″6	− 2·0
2	C	2	...	...	...	0,	2.	0, 0		0,	2	...	...	+ 769·06	+ 769·5	...	0·4
3	C	3	..	...	...	0,	3.	0, 0		0,	3	...	...	+ 36·13	+ 36·7	...	0·6
4	...	4	...	...	...	0,	4.	0, 0		0,	4	...	...	+ 1·94	+ 2·0	...	...
5	...	3.	1	...	...	0,	− 3.	1, 0		1.	4.	1,	1	+ 0·55	+ 0·4	...	...
6	C	2.	1	...	...	0,	2,	− 1. 0		1.	− 3,	− 1.	1	+ 7·67	+ 7·3	...	+ 0·4
7	C	1.	1	...	...	0,	1,	− 1, 0		1.	− 2,	− 1.	1	+ 109·92	+ 111·1	...	− 1·2
8	B	0,	− 1	...	...	0.	0,	1, 0		1.	− 1,	− 1,	1	+ 669·85	+ 668·6	...	+ 1·2
9	...	1,	1	...	...	0,	1.	1, 0		1.	0,	− 1,	1	+ 148·02	+ 148·1	...	...
10	...	2.	1	...	...	0,	2.	1. 0		1.	1.	− 1,	1	+ 9·72	+ 9·6	...	...
11	...	3.	1	...	..	0,	3.	− 1. 0		1,	2.	1,	1	+ 0·67	+ 0·4	...	...
12	...	1.	2	...	...	0,	− 1,	− 2, 0		2,	3.	− 2.	2	+ 1·18	+ 1·2	...	...
13	C	0,	− 2	...	...	0,	0,	2, 0		2,	2.	− 2.	2	+ 7·51	+ 7·9	...	− 0·4
14	B	1.	2	...	...	0,	1,	2, 0		2.	1.	2.	2	+ 2·59	+ 2·1	...	+ 0·5
15	A	1,	0,	2,	− 2	2,	− 1.	2, 0		0,	1,	2,	− 2	− 2·54	− 2·0	...	− 0·5
16	B	0,	− 1.	2,	− 2	2.	2,	1, 0		1,	− 1,	1.	− 1	+ 2·52	1·4	...	+ 3·9
17	...	1.	1,	2,	2	2,	1,	1. 0		1.	0,	1,	− 1	− 28·56	− 28·8	...	...
18	B	2.	− 1,	2,	− 2	2.	0,	1. 0		1.	1.	1.	− 1	− 24·45	·· 23·6	...	− 0·8
19	...	3.	1,	2.	2	2.	1.	1, 0		1.	2	1,	− 1	− 2·93	− 2·9	...	...
20	...	− 2.	− 2,	2,	− 2	2,	− 4.	0, 0		2,	− 4,	0,	0	+ 0·95	+ 0·9	...	...

Ref. No.	Class.	Argument expressed in Multiples of										Coefficient in Hansen's Theory modified by Newcomb.	Coefficient in Airy's Reductions	Approximate true Value.	Correction required by Airy.
		g	g'	ω	ω'	D	g	g'	F	D	g	ω	ω'				
21	C	−1,	−2,	2,	−2	2,	−3,	0,	0	2,	−3,	0,	0	+ 13″·19	+ 12″·8	... ″	+0″·4
22	C	0,	−2,	2,	−2	2,	−2,	0,	0	2,	−2,	0,	0	+ 211·71	+ 212·4	...	0·7
23	B	1,	−2,	2,	−2	2,	−1,	0,	0	2,	−1,	0,	0	+ 4586·56	+ 4585·6	+ 4586·4	+0·8
24	C	2,	−2,	2,	−2	2,	0,	0,	0	2,	0,	0,	0	+ 2369·75	+ 2370·3	...	−0·5
25	...	3,	−2,	2,	−2	2,	1,	0,	0	2,	1,	0,	0	+ 191·95	+ 192·1	...	...
26	...	4,	−2,	2,	−2	2,	2,	0,	0	2,	2,	0,	0	+ 14·38	+ 14·1	...	...
27	...	5,	−2,	2,	−2	2,	3,	0,	0	2,	3,	0,	0	+ 1·06	+ 0·8	...	...
28	...	−1,	−3	2,	−2	2,	−3,	−1,	0	3,	−4,	−1,	1	+ 0·48	+ 0·3	...	...
29	C	0,	−3,	2,	−2	2,	−2,	−1,	0	3,	−3,	−1,	1	+ 8·66	+ 7·8	...	+0·9
30	C	1,	−3,	2,	−2	2,	−1,	−1,	0	3,	−2,	−1,	1	+ 206·46	+ 209·7	...	−3·2
31	B	2,	−3,	2,	−2	2,	0,	−1,	0	3,	−1,	−1,	1	+ 165·52	+ 165·9	...	−0·4
32	C	3,	−3,	2,	−2	2,	1,	−1,	0	3,	0,	−1,	1	+ 14·60	+ 14·0	...	+0·6
33	B	4,	−3,	2,	−2	2,	2,	−1,	0	3,	1,	−1,	1	+ 1·18	+ 0·6	...	+0·6
34	...	1,	−4,	2,	−2	2,	−1,	−2,	0	4,	−3,	−2,	2	+ 7·44	+ 7·5	...	...
35	...	2,	−4,	2,	−2	2,	0,	−2,	0	4,	−2,	−2,	2	+ 8·13	+ 7·8	...	...
36	B	3,	−4,	2,	−2	2,	1,	−2,	0	4,	−1,	−2,	2	+ 0·76	+ 0·3	...	+0·5
37	C	2,	1,	2	...	0,	0,	1,	2	−1,	3,	3,	−1	+ 0·42	...	...	+0·4
38	...	...	...	2	...	0,	−2,	0,	2	0,	0,	2,	0	+ 1·09	+ 1·1	...	...
39	A	1,	0,	2	...	0,	−1,	0,	2	0,	1,	2,	0	− 39·58	− 37·2	...	−2·4
40	C	2,	0,	2	...	0,	0,	0,	2	0,	2,	2,	0	− 411·60	− 411·0	...	−0·6

Ref. No.	Name	Arguments expressed in Multiples of												Inequality in Hansen's Theory made that by Newcomb.	Inequality in Airy's Reduction.	Approximate mean Value.	Correction to be applied by Airy.
		$\frac{1}{2}\Sigma$	Σ	$\frac{1}{2}\Sigma'$	Σ'	$\frac{1}{2}\Sigma''$	Σ''	$\frac{1}{2}\Sigma'''$	Σ'''	$\frac{1}{2}\Sigma''''$	Σ''''						
42		4	8	0	0	2	2	0	0	0	0	+	4·00	4·1	...	...	
43	C	1	3	0	0	1	2	1	2	0	0	+	0·40	...	...	...	+ 0·4
44	...	0	3	0	2	0	2	1	3	1	3		2·15	2·3	...	...	...
45	...	2	2	0	2	0	2	0	2	0	2	+	0·43	0·5	...	...	...
46	...	1	2	0	2	0	2	1	2	0	2	+	6·36	6·2	...	...	...
47	...	0	2	0	2	0	2	2	2	0	2		55·25	54·9	...	...	...
48	C	2	2	0	2	0	2	2	4	0	2	+	0·56	0·2	...	...	+ 0·4
49	...	0	1	0	2	0	2	1	1	1	1	+	1·55	1·5	...	...	...
50	C	0	0	1	1	0	1	0	0	1	1	+	1·33	0·5	...	...	+ 0·8
51	A	1	0	1	1	0	1	0	1	1	1	+	18·09	17·2	...	...	+ 0·9
52	...	2	0	1	1	0	1	0	2	1	1	+	1·27	1·0	...	...	...
53	C	1	1	1	1	0	0	0	2	0	0	−	1·78	0·8	...	...	− 1·0
54	H	0	1	1	1	0	0	1	1	0	0	−	18·70	18·0	...	...	− 0·7
55	C	1	1	1	1	0	0	0	1	0	0		125·43	122·1	124·8	...	- 2·7
56	...	2	1	1	1	0	0	0	1	1	1	−	8·48	8·2	...	...	...
57	...	3	1	1	1	0	0	1	1	2	2	−	0·59	0·4	...	...	...
58	...	1	2	1	1	0	1	0	2	1	1	−	0·60	0·4	...	...	...
59	C	1	3	1	3	0	0	1	3	1	2	−	1·22	0·6	...	...	− 0·6
60	...	2	3	1	3	0	0	1	3	1	2	−	3·23	3·0	...	...	...
61	C	3	3	1	3	0	0	1	3	1	3	+	0·41	0·9	...	...	− 0·5

and b were corrected for planetary terms, and also for the solar terms Nos. 15, 39, 51 in the preceding list. Newcomb's empirical term was not inserted. There remained in the corrected values of a and b large accidental errors, and also some signs of periodicity with considerable coefficients. The accidental errors I attribute to the errors of Airy's formula for the tabular longitude, which, as the preceding table shows, are considerable. The periodicity cannot be an unknown term in the Moon's motion, as it would have appeared in 1883–1898; it is not likely to be due to systematic errors of observation, for the period is an unlikely one to arise from this cause. I attribute it to the terms marked B in the foregoing table. Possibly also the small term in the observations 1883–1898 arises in a similar manner.

It will be seen that terms of class A can be corrected for, in the values of a and b, after solution. For example, take term No. 39: here Airy's tabular place requires a correction $-2''{\cdot}38 \sin(g+2\omega)$; during a period of analysis in which g, it will be remembered, completes fifteen revolutions, 2ω moves 136°. If its variations, $\pm 68°$ on each side of its mean value, could be ignored, the correction to a would be $-2''{\cdot}38 \sin 2\omega$, and to b $-2''{\cdot}38 \cos 2\omega$; the coefficient has, however, been reduced to $-1''{\cdot}87$ as an expression of the fact that the centre of gravity of an arc of a circle is not on its circumference.

The effect of terms of class C is, I believe, purely accidental. As about 120 observations were usually to be found in a period of analysis, and as a and b are determined from these observations, with weights $\cos^2 g$ and $\sin^2 g$ respectively, the accidental errors of a and b on this account should be about one-eighth of the accidental errors of the tabular place. The last column of the table suggests $4''{\cdot}4$ as the probable error of a tabular place, and therefore $0''{\cdot}55$ as the probable error of a and b for one period. The effect of terms of class B would be accidental were it not for the fact that the observations are systematically distributed with regard to D, the age of the Moon. Let us take term No. 16 as an illustration. The formula for a is

$$a = \frac{\Sigma \text{ error} \times \cos g}{\Sigma \cos^2 g}$$

hence the systematic effect of a correction

$$+3''{\cdot}92 \sin(\mathrm{D} - g + \omega - \omega')$$

in the tabular place will be (over and above a considerable addition to the accidental error) approximately the mean value of

$$3''{\cdot}92 \sin(\mathrm{D} + \omega - \omega')$$

or

$$3''{\cdot}92 \sin(\omega - \omega') \times \text{mean value of } \cos \mathrm{D}$$
$$+3''{\cdot}92 \cos(\omega - \omega') \times \text{mean value of } \sin \mathrm{D}$$

As a rough estimate, I should equate this to

$$3''{\cdot}92 \sin(\omega - \omega') \times -\tfrac{6}{7}$$
$$+3''{\cdot}92 \cos(\omega - \omega') \times +\tfrac{1}{10}$$

but it is clear that a very considerable periodicity may easily result.

It is not surprising, therefore, that I have not obtained as yet any additional light on the suspected periodic term in the 1883–1898 observations. In fact, the individual values of a and b are worthless, except as confirming the above conclusions, and they should be grouped eight at a time, eight periods of analysis being approximately the period of $\omega - \omega'$, the argument of the above inequality.

Turning now to term 54 in the above list, reasoning similar to the above indicates a correction to a equal to $-0''{\cdot}7 \times$ mean value of $\sin D = -0''{\cdot}07$ nearly, and a correction to b equal to $+0''{\cdot}7 \times$ mean value of $\cos D$, or $-0''{\cdot}31$ nearly. This latter quantity is introduced into the estimate of the principal elliptic coefficient made later in this paper.

The following table gives the corrected values of a, b for the means of eight periods, which are approximately equivalent to nine years. The values here given are corrected for solar terms 15, 39, 51, and also for five planetary terms (or figure of Earth) of analogous form; that is to say, the arguments differ from g by angles of very long period.

Period.	a.	b.	Period.	a.	b.
1–8	$+0''{\cdot}41$	$+1''{\cdot}49$	49–56	$-0''{\cdot}61$	$+2''{\cdot}60$
5–12	$+0{\cdot}18$	$+1{\cdot}84$	53–60	$-0{\cdot}19$	$+2{\cdot}68$
9–16	$-0{\cdot}14$	$+1{\cdot}63$	57–64	$-0{\cdot}15$	$+2{\cdot}44$
13–20	$-0{\cdot}63$	$+2{\cdot}00$	61–68	$-0{\cdot}26$	$+2{\cdot}10$
17–24	$-0{\cdot}38$	$+2{\cdot}60$	65–72	$+0{\cdot}09$	$+2{\cdot}22$
21–28	$-0{\cdot}34$	$+2{\cdot}49$	69–76	$-0{\cdot}28$	$+2{\cdot}60$
25–32	$-0{\cdot}45$	$+2{\cdot}27$	73–80	$+0{\cdot}24$	$+2{\cdot}86$
29–36	$-0{\cdot}73$	$+2{\cdot}40$	77–84	$+0{\cdot}77$	$+2{\cdot}05$
33–40	$-0{\cdot}39$	$+2{\cdot}41$	81–88	$+0{\cdot}19$	$+2{\cdot}00$
37–44	$-0{\cdot}58$	$+1{\cdot}48$			
41–48	$-0{\cdot}45$	$+1{\cdot}95$	Mean	$-0{\cdot}21$	$+2{\cdot}22$
45–52	$-0{\cdot}72$	$+2{\cdot}61$			

From these figures I draw the following conclusions:

α. A correction $-2''{\cdot}22$ is apparently required by the principal elliptic coefficient, but this requires modification by about $+2''{\cdot}19$ on account of terms Nos. 54 and 56 of the first table. This gives the true coefficient as $22639''{\cdot}57$, while the R.A.'s 1883–1898 gave $22639''{\cdot}65$. Newcomb has given $22639''{\cdot}58$.

ζ. Damoiseau's mean anomaly for 1800 requires an increase

of about 2″, and its mean motion is nearly correct if we ignor long-period corrections (among them Newcomb's empiric correction) and secular terms.

η. The correction a in 1891 is $+1''{\cdot}41$, which becom $+2''{\cdot}16$ if Damoiseau's value for the mean anomaly is use This is quite inconsistent with the above values of a, except c the assumption that there is a long-period term or secul correction required.

Newcomb's empirical *Venus* term supplies about the rig amount of curvature, its values in 1750, 1800, 1850 and 18 being

$$+0''{\cdot}86,\ +1''{\cdot}70,\ +0''{\cdot}58,\ -0''{\cdot}96$$

making the value of a fairly constant and equal to $+1''$ implying that Damoiseau's value of the mean anomaly is t large by 11″, or that the Hansen–Newcomb value requires correction

$$+4''{\cdot}6-14''{\cdot}0t+4''{\cdot}7t^2$$

reckoning t in centuries from 1800, a formula whose values a probably within 5″ of the truth from 1750 to 1900.

I have shown in the *Observatory Magazine* for 1903 Nove ber that the consequence of assuming that no long-period ter exist other than those known from gravitational theory is th the secular term in the mean anomaly must be halved.

θ. My last conclusion is that it would be well worth while correct Airy's tabular places for most of the larger inequaliti In Table I. a list of 47 inequalities of $0''{\cdot}4$ or greater is giv (which can be reduced to 22 by omitting inequalities of l than $0''{\cdot}7$, though I do not think that this is desirable). The s of the coefficients without regard to sign is 46″, and the probal error of a tabular place can hardly be less than $4''{\cdot}4$. T probable error of a discordance between theory and observati is probably capable therefore of being divided by 3, or in otl words imperfections in the tabular places have hitherto redu the weight of this century of observations to one-ninth of t value they might have. At the same time it would be wo while to similarly correct the comparison with Hansen parallactic inequality, planetary terms, and correction to t principal elliptic inequality and value of the mean anoma the corrections being adjusted so that the tabular places may as nearly continuous as possible.

Finally I give here the separate values of c, a, and b for e period, the a's and b's being corrected for the effect of terms class A, Nos. 15, 39, and 51, and planetary terms belonging the same class.

None of the numerical conclusions of this paper are ba upon these separate values, but only on the means of eig The separate values are given here as evidence of the necess for revising Airy's formulæ.

Period.	No. of Obs.	c	a Corrected.	b Corrected.	Period.	No. of Obs.	c	a Corrected.	b Corrected.
		″	″	″			″	″	″
1	115	+3·38	+3·49	+1·94	40	131	−6·08	+1·09	+1·02
2	141	+2·64	+0·98	+2·59	41	127	−1·04	−0·78	+1·93
3	148	+4·02	−0·76	+0·98	42	101	−2·42	−2·25	+2·20
4	128	+3·27	−1·38	+4·05	43	127	−0·99	−1·15	+2·87
5	125	+2·60	−0·39	+0·53	44	112	−1·41	−2·70	+1·04
6	125	+0·60	−0·31	−0·03	45	145	−0·75	−1·02	+1·82
7	123	+1·29	+0·24	+0·77	46	165	−0·27	+0·93	+2·19
8	152	+1·40	+1·45	+1·07	47	158	−1·26	+3·77	+1·42
9	120	+1·68	+0·64	+3·44	48	110	−1·33	−0·38	+2·18
10	72	+0·25	+2·10	+4·41	49	119	−0·05	−1·99	+4·67
11	61	+2·50	+0·54	+2·15	50	127	−0·92	−1·61	+3·70
12	91	+2·44	−2·80	+2·38	51	132	−0·78	−3·12	+3·24
13	42	−1·79	−1·57	−2·07	52	112	−1·00	−2·34	+1·65
14	115	−0·86	−0·95	−0·45	53	117	−2·95	+0·94	+1·76
15	129	+1·11	−0·13	−0·04	54	118	−0·56	+1·03	+1·44
16	123	−1·14	+1·08	+3·24	55	119	−0·86	+1·47	+0·90
17	129	−3·72	+0·49	+4·21	56	128	−1·99	+0·77	+3·47
18	121	−4·62	−1·54	+4·01	57	87	+0·07	+0·37	+5·55
19	111	−0·26	−1·38	+3·14	58	109	+1·55	−2·21	+4·47
20	82	−0·16	−1·04	+3·95	59	93	+2·66	−2·50	+3·45
21	125	−1·98	−0·60	+0·24	60	105	+1·54	−1·40	+0·38
22	100	−4·25	−0·94	−0·01	61	109	+2·77	+0·59	+0·68
23	96	−4·34	+1·22	+2·09	62	118	+2·77	−0·12	+1·10
24	142	−3·90	+0·73	+3·15	63	123	+0·64	+1·92	+1·73
25	129	−5·61	−1·50	+6·34	64	126	+0·76	+2·12	+2·20
26	63	−2·58	−1·75	+3·10	65	112	+1·02	−1·34	+4·56
27	114	−3·21	+1·83	+3·21	66	84	+1·14	−1·30	+4·64
28	111	−1·68	−1·72	+1·78	67	126	+0·74	−2·65	+1·66
29	136	−3·12	−0·59	+0·84	68	117	+0·80	−1·31	+0·24
30	110	−2·84	+0·59	−0·82	69	112	+1·53	+0·45	+0·20
31	129	−2·82	+0·35	+1·42	70	92	−0·09	+0·81	+0·28
32	124	−3·73	−0·85	+2·27	71	104	+0·68	+3·32	+1·94
33	127	−4·49	+0·09	+3·78	72	102	+2·15	+2·72	+4·26
34	125	−4·91	−1·09	+7·09	73	81	+3·95	−0·37	+3·43
35	126	−4·27	−2·65	+3·24	74	103	+0·90	−1·70	+5·24
36	105	−3·13	−1·73	+1·37	75	115	+2·25	−1·83	+3·37
37	117	−3·08	+0·89	−0·70	76	92	+1·81	−1·14	+2·11
38	111	−4·19	−0·27	+0·38	77	100	+1·09	+1·67	+1·51
39	127	−6·21	+0·51	+3·07	78	104	+1·65	+2·42	+0·85

Period.	No. of Obs.	c	a Corrected.	b Corrected.	Period.	No. of Obs.	c	a Corrected.	b Corrected.
		"	"	"			"	"	"
79	112	+1·64	+3·05	+2·24	85	125	−1·04	+0·91	−0·44
80	111	+1·20	−0·41	+4·13	86	120	−0·62	+1·10	+2·77
81	122	+0·30	+0·85	+3·46	87	136	−2·53	+0·55	+1·90
82	99	−0·40	−1·42	+3·91	88	123	−2·91	−0·25	+4·14
83	132	−1·00	−0·96	+0·86	89	123	−3·88	−2·72	+3·47
84	105	−2·69	+0·76	−0·56					

On the Large Sun-spots of 1903 *October* 4–18 *and October* 25–*November* 6, *and the Associated Magnetic Disturbances.*

(*Communicated by the Astronomer Royal.*)

As two striking instances have occurred in less than three weeks of a marked magnetic disturbance simultaneous with the appearance of a great group of spots near the centre of the Sun's disc, some particulars of both phenomena may be of interest, though there has not been time as yet to do more than measure and reduce two or three of the photographs taken of the Sun, and though the record is imperfect until the arrival of photographs from India and Mauritius to supplement the Greenwich series.

The two groups of spots in question were both in the southern hemisphere, and both very nearly in the same latitude. The first to appear was very much the larger, and may be distinguished as the Great Southern Group; whilst the other may be termed the Second or Smaller Southern Group.

The Great Southern Group was first seen on the east limb on October 4, when a regular spot of no great area came into view. The rest of the group was not completely seen until October 5, the above-mentioned leader being separated by nearly 3° of heliographic longitude from the next portion of the group. The entire length of the group as measured on October 9 was more than 17°, and its greatest breadth in heliographic latitude was 6°·7. Its total area on this day was 2080, as expressed in the usual unit—millionths of the Sun's visible hemisphere.

The heliographic latitude of the centre of the principal spot was −21°. The heliographic longitude of the same point was 205°·5. The positions of this point relative to the central meridian of the Sun, at the times when the magnetic disturbance began and ended, and when its most marked phase began and ended, were as follows :—

Magnetic Disturbance.	Greenwich Civil Time. d h	Longitude of C.M.	Longitude of Spot from C.M.
		°	°
Disturbance begun	Oct. 12 18	188·2	17·3 W.
Most marked phase begun ...	12 20	187·1	18·4
Most marked phase ended ...	13 3	183·2	22 3
Disturbance ended	13 4	182·6	22·9

The longitude of the central meridian of the Sun at noon on October 10 was 217°·8; on October 11, 204°·6; on October 12, 191°·5. The leader spot of the group reached the central meridian about October 10^d 18^h; the centre of the principal spot about October 11^d 8^h; the last spots of the group passing about October 12^d 8^h; Greenwich Civil Time being used throughout.

The group underwent great and rapid changes of form; its chief characteristics being the number and complexity of the bright bridges which invaded it. Thus on October 9 a very broad bright bridge entered the principal spot from the west, and stretched out in an undulating line towards the east, almost obliterating the umbra, and dividing the spot itself into two elongated and parallel sections. The after-part of the group was even more completely divided by bright material, crossing it in both the east and west and the north and south directions, so that it formed four distinct spots. By October 12 the group had very greatly changed its appearance; the chief bridge in the principal spot now entering it from the south, and cutting it across its length, and not, as before, in the direction of its length.

The group passed out of view at the west limb on October 17 and 18.

It returned again to the east limb on November 2, as a much broken and extended stream, divided into four distinct sections. Of these the two preceding sections broke up, like an exploding shell, into a great number of small faint spots; and the two following sections coalesced to make one long and nearly continuous stream.

Although October 4 was the first appearance of the group, a short-lived spot marked the place where the great group was afterwards to appear, from September 10 to September 16. This spot, however, closed up and disappeared some days before reaching the west limb.

The Second or Smaller Southern Group appeared on the east limb on October 25. It consisted of a nearly continuous stream, which gradually broke up in its passage across the disc. The leading spot of the group reached the central meridian October 31, 10^h, and the entire group, which was about 10° of heliographic longitude in length, had passed by November 1, 4^h. The principal spot of the group was in heliographic latitude −20°·0, and longitude 296°·0. The position of its centre relative

to the central meridian of the Sun at the times when the magnetic disturbance began and ended, and when its most marked phases occurred, were as follows: the longitude of the central meridian at mean noon being 314°·0 on October 30, 300°·8 on October 31, and 287°·7 on November 1.

Magnetic Storm.	Greenwich Civil Time. d h m	Longitude of C.M. °	Longitude of Spot from C.M. °
Disturbance began	Oct. 31 6	304·1	8·1 E
Violent movements began ...	13	300·3	4·3
Most westerly declination ...	15 40	298·8	2·8
Most easterly declination ...	19	297·0	1·0
Violent movements ceased ...	23	294·8	1·2 W.
Disturbance ended	Nov. 1 5	291·5	4·5

The area of the group on October 31 was about 670; less than one-third that of the Great Southern Group.

The group as it passed towards the west limb took an arched form, convex to the solar equator, and the following portion of the group gradually closed up.

The group reached the west limb on November 6.

A third group of spots—one in the Northern hemisphere—was seen from October 2 to October 14, and returned to the east limb on October 29. It consisted at its first appearance of a large regular spot, followed by a stream of small spots. On its return it consisted chiefly of a pair of large regular spots. The mean longitude of the group on October 31 was 232°; its latitude 17½° N.; its area 698. Its centre passed the central meridian about November 5, 18^h. It reached the west limb on November 11. A recognisable, but quite minor, magnetic disturbance took place on November 5.

The Magnetic Disturbances of 1903 *October* 12–13 *and October* 31–*November* 1.—The disturbance of October 12–13, although not falling within the category of "great" disturbance, presented many interesting and characteristic features. The limit of the disturbance period in the magnetic elements was comprised between October 12^d 18^h and 13^d 4^h, and the amplitude of declination disturbance amounted to 35′. The movements in Earth currents were especially marked between October 12^d 20^h and 13^d 3^h. The magnetic changes in this disturbance were of a bolder and more active character than those recorded in any similar disturbance during the preceding five years.

The disturbance of October 31–November 1 was of a far more important type. Commencing with characteristic suddenness at October 31^d 6^h, it speedily developed a phenomenal degree of activity, and from October 31^d 13^h to 23^h the movements of all the magnets were extremely violent; changes exceeding one degree in arc in declination being frequently shown

in very short periods of time. The spots of light of the horizontal force and vertical force magnets passed beyond the range of the recording barrels for many hours during the afternoon and evening, and for shorter periods at other parts of the day, but little loss will be incurred on this account, as eye observations were made in great detail. Active disturbance ceased about November 1^d 5^h, the movements gradually subsiding after midnight. The declination magnet reached its extreme westerly position at October 31^d 15^h 40^m and its most easterly position at 31^d 19^h, the extent of change exceeding 2°.

This disturbance is greater than any recorded at the Royal Observatory since the great disturbance of 1882 November 17, which it closely resembled in many respects.

Royal Observatory, Greenwich:
1903 *November* 12.

Observations of Mars in 1903. By the Rev. T. E. R. Phillips, M.A.

The planet *Mars* was in opposition on 1903 March 28. His maximum diameter scarcely reached 14″·6, but the apparition was nevertheless a very favourable one for an examination of the planet's N. hemisphere. The latitude of the centre of the disc was +22°·6 at the time of opposition, and increased to over 25° by the middle of May. The weather, too, was on the whole very propitious, and the seeing conditions, despite some bad periods, frequently excellent. Between the middle of February and the end of May I was able to observe the planet on no fewer than sixty-six occasions. Thirty-two whole-disc drawings were made, besides drawings of special features. The instrument employed was a 9¼-inch silvered-glass reflector (mirror by With), with powers between 217 and 450.

The following is a description of the results obtained, the letters and numbers referring to the accompanying chart:—

The Maria.—The boundaries of the *Maria* were very well defined, and, in some cases, bordered by bright regions. As usual the *Maria* were by no means uniform in depth of tone, lighter and darker patches being frequently seen. The *Syrtis Major* (H) especially contained brighter areas—*Œnotria* (α′), *Iapygia* (β)—in its S. portion, but was very dark towards its N. extremity. The *Syrtis Minor* (G), *Sinus Sabæus* (I), *Margaritifer Sinus* (A), *Aurora Sinus* (D), *Mare Acidalium* (B), and *Baltia* (C) were all noted as dark. *Mare Sirenum* (E), doubtless in consequence of its proximity to the limb, appeared less dark than usual. *Atlantis* separating it from *Mare Cimmerium* (F) was not

observed, though *Hesperia* (ϕ) was clearly seen. The large dusky area in the neighbourhood of the *Casius* (44) was very prominent, and contained two large dark spots or condensations.

The "Canals."—Several canals were seen with great distinctness. I have no doubt whatever as to the actual existence of many of them, though it is quite probable that at closer quarters they would appear much less regular. The theory that their appearance results from the unconscious joining together by the eye of projections from the *Maria* and dark spots does not satisfy the observations. The soft delicate lines or narrow streaks were frequently seen with great distinctness when no such prominent features were visible to account for them.

The interesting experiments described by Mr. E. W. Maunder and Mr. J. E. Evans in their paper published in *Monthly Notices* for 1903 June seem to me on careful consideration to suggest only a partial explanation of the canal system. They illustrate well what is very probably true—viz. that many of the reported canals are not really straight lines, but the impression produced on the retina by irregular markings, too faint and intricate to be clearly grasped. On the other hand there are canals so distinct and plain, under conditions of the very finest definition, as to preclude the idea that they are in any way subjective or imaginary.

A careful and systematic scrutiny of the planet reveals the noteworthy fact that the so-called "canal system" includes features of widely different nature and appearance. Dark and broadish bands, soft and delicate streaks with ill-defined condensations and spots, fine narrow dark lines, and the dusky edges of faint shadings and half-tones, are all termed "canals." It seems to me, therefore, that one hypothesis will neither explain them all, nor establish the contention that the canals generally are illusory.

The following is a list and description of the canals seen :

Agathodæmon (11). Dark and obvious.

Alcyonius (41). Not a separate canal, only the edge of dark shading of *Utopia*.

Amenthes (43). Distinct dusky line or narrow streak.

Astaboras (51). Glimpsed sometimes, but difficult.

Astusapes (50). Rather difficult.

Boreas (29). Rather dark. Very well seen on several occasions.

Boreosyrtis (49). Very dark and conspicuous. The region between this and the *Nilosyrtis* is lightly shaded.

Callirhoe (5). Very well seen as a soft streaky line.

Casius (44). Very large dark shading. In no sense canal form.

Cephissus (42).* Broad and dark.

* By an error both the *Cephissus* and the *Marsyas* are numbered (42)

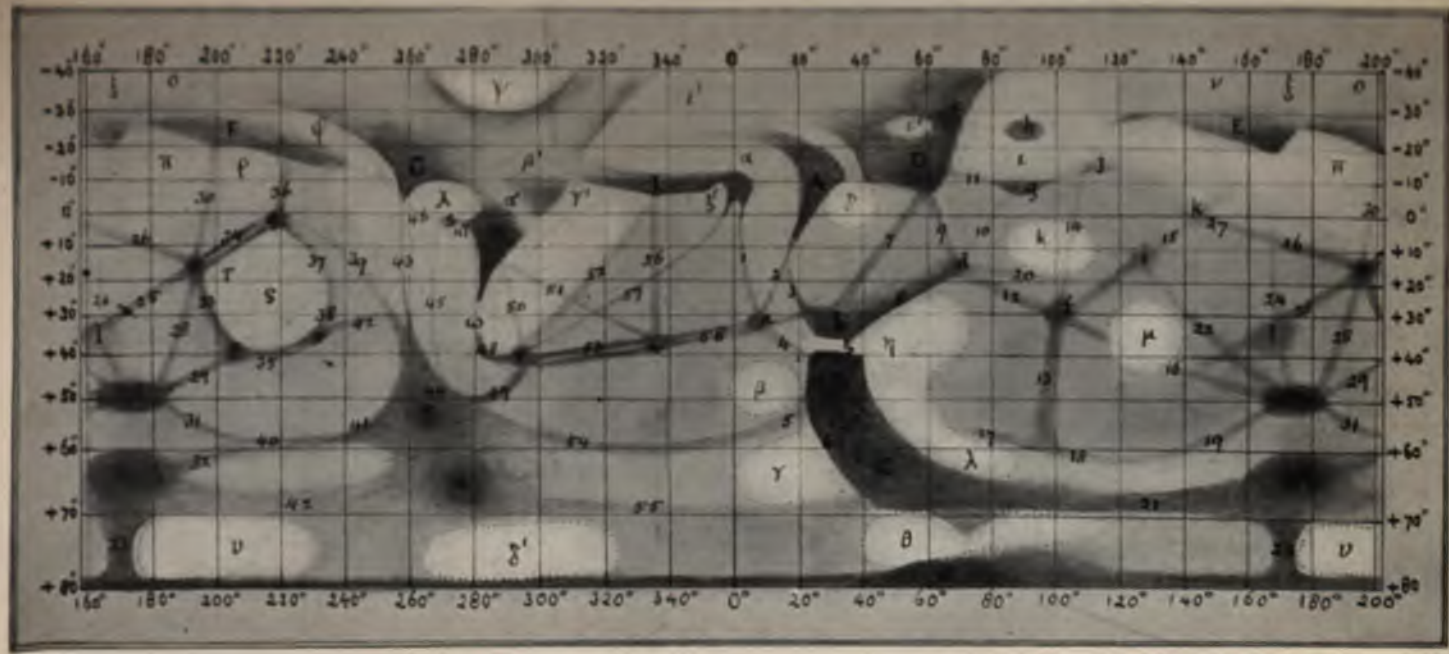

CHART OF MARKINGS ON MARS.

SEEN AT CROYDON ABOUT THE OPPOSITION OF 1903

REV T. E. R. PHILLIPS.

Nepenthes (47). Short and dark, swelling out into *Mœris L.*

Nilokeras (8). Quite dark and sharply bounded on N. edge. Soft and diffuse to S. In previous apparitions this canal has appeared "anomalously" double.

Nilosyrtis (48). Dark curved line, frequently seeming separated from the tip of *Syrtis Major.*

Nilus (12). Rather conspicuous, but soft and streaky.

Orcus (26). Faint and difficult.

Oxus (2). Narrow and sharp near *Margaritifer Sinus*, but becoming more diffuse towards *Siloe Fons.*

Pactolus (39). Seen frequently as a curved narrow streak connecting *Pambotis L.* with the *Casius.*

Pallas (46). Probably illusory. Only the edge of shading round *Libya.*

Phison (52). Appeared to be the dusky edge of orange tone of disc.

Phlegethon (16). Interrupted by white area central at about $\omega = 128°$, $\phi = +35°$.

Pierius (54). Dark and conspicuous streak.

Protonilus (53). Very dark and prominent. Appeared distinctly double on May 12 with Dr. Kibbler's 12½-inch reflector at Stamford Hill, and also on May 14 with my own instrument.

Pyriphlegethon (22). Broad and diffuse.

Sitacus (57). Very narrow, dark, curved line. Note on May 14: "Wonderfully distinct this apparition."

Styx (33). Dark and sharply defined on its W. side by the bright region of *Elysium.* Soft and diffuse on E. side. All the region included by the *Hades*, *Boreas*, and *Styx* appeared at times lightly shaded.

Tanais (17). Only a trace seen. The brightness of *Nerigos* (λ) seemed to unite with that of the region W. of *Achillis Pons* and obliterate completely the p. end of *Tanais.*

Thoth (45). Fine narrow line.

Titan (24). Soft, diffuse, and difficult. Not traced beyond the *Orcus.*

Uranius (20). Seen a few times, but rather ill-defined.

A broad streak (21) was seen connecting *Arsenius L.* (*n*) with the shaded region of *Baltia* (C.).

Another rather faint dusky streak (55) was found running from about $\omega = 290° + 70°$ to the E. side of *Baltia* (C.).

On March 10 and 11, and again on April 17, there appeared to be a "new" canal running from the bend of *Nilosyrtis* ($\omega = 232°$, $\phi = +38°$) towards the *Syrtis Minor.* Subsequently this feature could not be recovered, though on April 19 the *Thoth* was seen with great distinctness.

The "Lakes."—The following were observed:

Arsenius L. (*n*). Very large and dark spot about 14° N. of *Propontis.*

Colce Palus (*v*). Very conspicuous spot at end of *Nilosyrtis*.

Cepais Palus (*t*). Very dark and conspicuous spot.

Hecates L. (*p*). A conspicuous object.

Hyperboreus L. (*e*). Very dark and large swelling on band surrounding polar cap.

Ismenius L. (*w*). Well defined and conspicuous spot. Appeared double on May 12 and 14, when the *Protonilus* was also doubled.

Lunæ L. (*d*). Fairly dark and conspicuous.

Mareotis L. (*f*) Quite a dark spot on *Ceraunius* at meeting of this canal with *Nilus*, *Phlegethon*, and *Gigas*.

Maris L. (*s*). Dark swelling on *Nepenthes*.

Morpheos L. (*r*). Not very large, but easily seen. At junction of *Chaos*, *Hyblæus*, and *Marsyas*.

Nectaris Fons (*c*). Very dark spot seen on May 7, W. of *Protei Regio* (*e*), near the mouth of *Nectar*.

Niliacus L. (*b*). Sharply bounded by *Achillis Pons*, but soft and diffuse to S.

Nodus Gordii (*i*). A dusky, ill-defined shading about the junction of *Pyriphlegethon* and *Gigas*.

Pambotis L. (*q*). Seen well at times at junction of *Cyclops* and *Cerberus*.

Phœnicis L. (*j*). Dusky spot seen a few times near extremity of *Pyriphlegethon*.

Phrygius L. (*k*). Faint dusky spot at the junction of *Gigas* and *Eumenides*. This appears identical with the *Nodus Gordii* of Lowell.

Propontis (*m*). Much elongated and very dark spot. Appeared almost rectangular at times.

Siloe Fons (*a*). Rather small, but well defined.

Solis L. (*h*). Dark and very well seen, despite its proximity to the limb.

Tithonius L. (*g*). A dark and conspicuous marking.

Tricium Charontis (*o*). Not quite so large as at some oppositions, but very conspicuous.

A large, ill-defined dusky shading (*l*) was seen about the junction of *Erebus* and *Titan* ($\omega = 168°$, $\phi = +35°$).

Another dark and large spot (*u*) was noted at about $\omega = 277°$, $\phi = +66°$. Is this a new feature?

White Regions and Bright Spots.—These furnished the special feature of *Mars* during the past apparition. As usual, some of the borders of the *Maria* and regions near the limb appeared intensely white. Thus *Elysium* (ζ), *Libya* (χ), *Hellas* (ψ), the region of *Syrtis Major* and *Pharos Insula* (γ'), *Edom Promontorium* (ζ'), *Deucalionis Regio* (α), the S. part of *Chryse* ($\tilde{\epsilon}$), *Thaumasia* (ι), and the regions of *Phæthontis* (ν), *Electris* (ξ), *Eridania* (σ), and *Noachis* (ϵ'), foreshortened at the S. limb, all appeared very bright; *Zephyria* (π) and *Æolis* (ρ) also appeared so at the end of May; but the features demanding special notice

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March 7. "There appears to be much fog round the N. Pole, and I don't think the snow itself was glimpsed at all."

March 8. "Large patch at N. Pole, but the snow could be seen at times flashing out with dazzling whiteness."

March 10. "The fog (?) at the N. Pole appears to be diminishing."

March 14. "All trace of fog seems to have left the polar regions. The snow cap was seen now and then in moments of superb definition to be quite definite and bounded by a very narrow dark line, with a somewhat similar but less bright region adjoining it at about $\omega = 220°$."

Of the three bright areas in the N. polar regions, that just mentioned (ν) was the first to be observed; subsequent observations seemed to place its centre at a little over 200° of longitude. It was frequently a conspicuous object, and towards the end of April was found to be sharply bounded at its p. side by the canal *Magnes*.

On March 31 I noticed an intensely bright streak (θ) severing *Baltia* from the dark shading round the polar snow. On April 29 and May 1 the p. end of this region was comparable with the cap itself in brightness, but *f* this bright nucleus there seemed to be a dusky projection from *Baltia* extending towards *Hyperboreus L.* This projection, however, was not subsequently observed.

A third very brilliant region (δ') was detected on May 14 adjoining the polar snow cap. It was again seen on May 15 and 19, and the longitude found to be about 295°.

After the disappearance of the fog or mist in the early part of March the snow cap was seen to be bounded by an intensely dark line. This line was quite narrow in most longitudes, but at the end of March it was found greatly swollen at about $\omega = 60°$. This swelling, which on April 24 seemed to indent the snow cap, is doubtless identical with *Hyperboreus L.* of Schiaparelli. It was intensely dark against the snow, but appeared to shade off gradually in an *s f* direction.

At the time of opposition the season of the planet's N. hemisphere was a little past midsummer, and the polar cap was quite small. It seemed to diminish but very slowly subsequently, the minimum diameter (observed in May) being estimated at about 10° or 12°. The snow at times shone with a slightly bluish tinge.

To sum up the results of my observations, a careful and systematic scrutiny of the planet during the past four apparitions has revealed to me the following facts:

(1) Changes, partly due to seasonal influences and the appearance of clouds or mists, and partly real, unquestionably occur from time to time in the details of the surface configuration.

(2) The main results of Professor Schiaparelli's work are imperishable and beyond question. During recent years some

observers have given to the so-called "canals" a hardnes and an artificiality which they do not possess, with the resul that discredit has been brought upon the whole canal systen No doubt the time has come when a distinction must be mad between what is real on *Mars* and what is subjective or illusory but of the substantial accuracy and truthfulness (as a basis o which to work) of the planet's configuration as charted by th great Italian in 1877 and subsequent years, there is, in my min no doubt.

(3) Contrast, as has been so ably pointed out by M. E. M Antoniadi, is doubtless accountable for very many of the extr ordinary appearances observed on the planet. Not a few of th canals are now seen to be the intensified edges of faint ton in accordance with the late Mr. Green's suggestion, whi M. Antoniadi's explanation of the phenomena of gemination due to the same effect of contrast appears both simple and sati factory.

Observations of White Spots on Saturn in 1903.

By A. Stanley Williams.

The visibility of a bright spot on *Saturn* was announced Professor E. E. Barnard on June 24 of the present ye Unfortunately I had just left England for Ireland, so th although Professor Kreutz kindly sent me a postcard announc the discovery, it was only on my return home towards the end July that I heard of the existence of the spot, and was able make any observations of the planet. At that time the proba position of the spot detected by Barnard was quite unknown, that I confined my attention simply to observing the transits any spots that might happen to be visible, purposely avoid any attempt at comparing the observations, or to identify th with the spot referred to, in order that the observations mi not be biassed thereby.

The observations recorded below all relate to white spo situated in a bright zone lying to the north of a broad dark b on the north side of the bright equatorial zone. The for bright zone is hereafter termed the N. temperate zone. broad dark band between it and the equatorial zone has b called the N. equatorial belt It is actually coarsely dou consisting of two dark bands separated by an ill-defined ligh interval, not so bright or definite as either the equatorial zon the N. temperate zone. The telescope used was a $6\frac{1}{2}$-inch re tor, a power of 225 being always employed. At first g

* I have called the spots *white* spots for distinction, but they have us appeared to me to be distinctly yellowish.

difficulty was experienced in observing the transits of the spots with any satisfaction owing to the observer being out of practice, and consequently the earlier observations are usually less satisfactory than the later ones. The weather, moreover, proved most unfavourable, and this, combined with the low altitude of the planet, formed a serious obstacle to obtaining satisfactory results. The observations that I have been able to obtain are therefore comparatively few in number, and not so accurate as I could have wished. For this reason I have not attempted to make any discussion of the results, or any identification of the spots observed, of which, however, it is evident that there must be several. Since, however, these observations may be of value in conjunction with those of other observers for determining the rotation period of the surface material in this latitude of *Saturn*, the full details of them are given below :—

1903 July 24, $11^h\ 34^m$ (G.M.T.) White spot transited. Weight (on a scale ranging from 1, bad, to 5, good)=1. Time very uncertain owing to definition being very unsteady, though the spot itself was certain. When first seen, $\frac{1}{4}^h$ earlier, it was some distance E. of the central meridian of the disc. Once the spot seemed to be double east and west.

July 31, $10^h\ 50^m$. A plain white spot visible about $\frac{1}{2}^h$ before transit. The following *estimated transits** were observed :— $13^h\ 33^m - 12^m = 13^h\ 21^m$; $13^h\ 41^m - 20^m = 13^h\ 21^m$. Both were rough estimates owing to the image being dim from cloud.

August 3, $11^h\ 18^m$. A plain, round, well-defined spot transited. W. = 2. The following estimated transits were also observed :—$11^h\ 07^m + 23^m = 11^h\ 30^m$ (good) ; $11^h\ 13^m + 07^m = 11^h\ 20^m$ (good) ; $11^h\ 29^m - 12^m = 11^h\ 17^m$ (fair) ; $11^h\ 37^m - 22^m = 11^h\ 15^m$ (good).

August 13, $10^h\ 24^m$. Small, rather well-defined white spot estimated 10^m past transit, which therefore occurred at $10^h\ 14^m$.

August 22, $11^h\ 03^m$. Small, rather faint, inconspicuous spot transited. W.=1. It seemed, however, to be pretty well defined. The following estimated transits were also made:— $10^h\ 55^m + 8^m = 11^h\ 03^m$; $11^h\ 13^m - 7^m = 11^h\ 05^m$; $11^h\ 16^m - 12^m = 11^h\ 04^m$; $11^h\ 18^m - 15^m = 11^h\ 03^m$ (widely past transit).

September 1, $9^h\ 20^m$. Very bright and plain spot transited. W. = 1, very rough owing to poor seeing. The following estimated transits were also observed :—$9^h\ 13^m + 12^m = 9^h\ 25^m$; $9^h\ 29^m - 7^m = 9^h\ 22^m$ (distinctly past transit, but only slightly so) ; $9^h\ 34^m - 10^m = 9^h\ 24^m$. Note added September 2 : "For a long distance preceding this spot the N. temp. zone was very bright and luminous looking."

September 3, $8^h\ 47^m$. A remarkably plain, bright, and definite spot transited. Definition good, but cloud came up, so that the above time is only very approximate.

September 5, $9^h\ 43^m$. A rather well-defined spot estimated

* *See Monthly Notices*, vol. liv. p. 298.

20^m past * transit, which therefore occurred at $9^h\ 23^m$. Following this spot the N. equat. belt seemed broader than usual, so as to encroach on the N. temp. zone, which latter was narrow and faint, with one or two small faint spots following the central meridian. On the same date the following estimated transits were made of a small and difficult spot :—$10^h\ 26^m + 30^m = 10^h\ 56^m$; $10^h\ 46^m + 25^m = 11^h\ 12^m$ (very rough); $10^h\ 52^m + 17^m = 11^h\ 09^m$ (very rough).

September 7. The following estimated transits of a rather bright and not difficult spot were observed :—$8^h\ 46^m - 15^m = 8^h\ 31^m$ (fair obs.); $8^h\ 54^m - 25^m = 8^h\ 29^m$ (rough estimate owing to frequent clouds).

September 9, $9^h\ 08^m$. An apparently pretty plain spot transited, but seeing very indifferent. W. = 1. Same date, $10^h\ 41^m$. A well-defined and fairly bright spot transited. W. = 2. The following estimated transits were also observed :—$10^h\ 32^m + 10^m = 10^h\ 42^m$ (good); $10^h\ 52^m$, the spot seemed distinctly past transit, but seeing very confused. The spot appeared to be rather a bright one for a *Saturn* spot.

September 13, $8^h\ 50^m$. A bright, plain white spot transited. W. = 2. The following estimated transits were also observed :—$8^h\ 34^m + 13^m = 8^h\ 47^m$ (good); $9^h\ 04^m - 20^m = 8^h\ 44^m$ (good). Same date, $10^h\ 24^m$; a white spot transited. W. = 2. Spot difficult to observe owing to indifferent seeing. It was strongly suspected to be double east and west. The following estimated transits were also observed :—$10^h\ 08^m + 16^m = 10^h\ 24^m$ (poor obs.); $10^h\ 11^m + 12^m = 10^h\ 23^m$ (fair); $10^h\ 17^m + 5^m = 10^h\ 22^m$ (fair); $10^h\ 33^m - 8^m = 10^h\ 25^m$ (poor estimate).†

September 14, $9^h\ 59^m$. White spot estimated 12^m past transit, which therefore occurred at $9^h\ 47^m$. Seeing very poor and spot difficult to observe.

September 17, $10^h\ 07^m$. A rather small but bright, well-defined, and nearly round spot transited. W. = 2. The following estimated transits were also made :—$9^h\ 57^m + 10^m = 10^h\ 07^m$ (good); $10^h\ 14^m - 8^m = 10^h\ 06^m$ (good).

September 22. The following estimated transits were observed of a rather bright and pretty large spot, which was not at all difficult :—$9^h\ 16^m\ 10^m = 9^h\ 06^m$ (good); $9^h\ 22^m - 15^m = 9^h\ 07^m$ (indifferent); $9^h\ 24^m - 20^m = 9^h\ 04^m$ (fair). At $9^h\ 36^m$ another white spot transited. W. = 1. This spot was very small and inconspicuous, and much more difficult than the previous one, though it seemed to be well defined. The following estimated transits were also observed :—$9^h\ 22^m + 10^m = 9^h\ 32^m$ (rough); $9^h\ 24^m + 12^m = 9^h\ 36^m$ (fair).

* The observation is actually entered as 20^m *before* transit, but it is pretty certain that the spot must have been past transit from the note that follows.

† The two observations of September 13 are not quite independent, as the observer had a rough recollection of his observations of September 9, when probably the same spots were observed, assuming a rotation period of about $10^h\ 38^m$.

September 23, $9^h\ 21^m$. Small, faint, inconspicuous spot transited. W. = 1. Some very sharp views of *Saturn* were obtained on this night, but there were no *conspicuous* white spots in sight. The N. temp. zone seemed to be very narrow in this part of the planet.

In the following table I have brought together all the observed transits of spots in the N. temp. zone. The concluded times of transit are what seem to me to be the most probable times according to the foregoing observations. The relative degrees of accuracy are indicated in the third column. With reference to observations of this kind, made under such unfavourable conditions, it should be mentioned that it is very difficult for an observer to avoid being biassed in his subsequent estimates by his first view of a spot. I have tried hard to avoid being thus biassed, but cannot hope to have escaped this influence altogether. The trouble is that in trying to struggle against the influence of such a first impression one is apt to err in the opposite direction.

Concluded Transits of White Spots on Saturn.

Date.	G.M.T. of Transit. h m	Weight.	Notes.
1903 July 24	11 34	1	Perhaps double.
31	13 21	1	Plain spot.
Aug. 3	11 19	4	Plain, round, well defined.
13	10 14	1	Small, rather well defined.
22	11 03·5	2	Faint, inconspicuous.
Sept. 1	9 22	2	Very bright and plain.
3	8 47	1	Remarkably plain and bright.
5	9 23	1	Rather well defined.
5	11 05	1	Small, difficult.
7	8 30	1	Rather bright, not difficult.
9	9 08	1	
9	10 41	3	Rather bright.
13	8 48	3	Plain and bright.
13	10 24	3	Suspected double.
14	9 47	1	
17	10 07	3	Small, bright, nearly round, well defined.
22	9 06	1	Pretty large, rather bright.
22	9 35	2	Very small.
23	9 21	1	Small and faint.

It remains to add a few words on the appearance of the spots. Generally speaking, the N. temperate zone was extremely brilliant, but it was very irregularly bright. In some parts of the

planet it was exceedingly bright and luminous looking, whilst in other places it was comparatively dull and lustreless. The spots constituted the brightest regions of this irregularly luminous zone. They were frequently indefinite, particularly in an east and west direction, so that it was often difficult to locate their position satisfactorily, or even to fix upon anything sufficiently definite to observe the transit of. They also sometimes gave the impression of being double in an east and west direction, though it was difficult to be sure of this in the unsteady definition almost always prevailing at so low an altitude. The unsteady seeing formed, I think, the chief obstacle to making good observations. Under more favourable conditions I believe the spots would have been plain and easy objects. Too much value should not be placed upon the descriptions of the spots in the notes as their visibility would no doubt be greatly affected by the unfavourable conditions of the seeing, so that the same spot might be described as being plain and relatively bright on one night, and only as faint and inconspicuous on another night without any real change in brightness or appearance being necessarily implied thereby, the difference arising merely from the circumstance that on one of the nights the bad seeing did not permit the spot to be seen so clearly as on the other.

In general appearance these N. temperate spots had considerable resemblance to the white spots which I observed about ten years ago in the equatorial zone,* but the latter spots were probably not quite so bright and luminous looking as the former. During the course of the recent observations I have on several occasions examined the equatorial zone. It appeared remarkably white, whiter than the N. temperate zone, and there were faint but distinct indications of brighter spots in it, though compared with the last-mentioned zone it was very uniform in brightness. Owing to the unfavourable position of *Saturn* no systematic observation of these equatorial spots was attempted. On August 13, however, a faint spot was observed to transit at $10^h\ 43^m$. Also on August 21 a comparatively bright and conspicuous spot was visible in the equatorial zone. At $10^h\ 0^m$ was estimated to be 13^m before transit, but owing to cloud the transit could not be directly observed. The spot, which was well defined, seemed to be divided into two by a narrow, faint dark belt in the equatorial zone, probably the same belt as that which used to be known as the "equatorial mottled belt."†

* See *Monthly Notices*, vol. liv. p. 309, and vol. lv. p. 361.

† During the past two or three months the equatorial spots have certainly been decidedly fainter and more difficult than the N. temp. spots.

20 *Hove Park Villas, Hove:*
1903 *October* 13.

Observations of Borrelly's Comet (c 1903) made at the Natal Observatory, Durban.

(Communicated by E. Nevill.)

The following observations were made by Mr. Rendell by means of a cross-bar micrometer with the equatorial refractor, aperture 8 inches, focal length 10 feet, magnifying power 50.

Date. 1903	Greenwich Mean Time.	Apparent Difference Comet—Star. R.A.	N.P.D.	Comet's Approx. Hour-angle East.	No. of Comparisons.	Comparison Star.
	h m s	m s	′ ″	h m		
June 24	9 3 53	+0 39·32	−5 2·1	4 35	7	*a*
25	9 5 11	+0 12·87	−5 54·6	4 29	9	*b*
27	8 53 28	+2 1·11	−2 36·0	4 30	5	*c*
28	9 0 25	+0 7·10	+2 15·7	4 18	5	*d*
July 2	9 9 48	−0 11·37	−5 32·4	3 44	7	*e*

Comparison Stars.

a	B.D. −5° No. 5663	R.A. 21 47 10·8	N.P.D. 95 57·0	(1855·0)
b	B.D. −4° No. 5568	21 46 35·9	94 57·2	(„)
c	B.D. −2° No. 5643	21 42 21·9	92 38·6	(„)
d	Anon. (Mag. 8)	21 43	91 17	(„)
e	7 Pegasi	21 35 45·16	84 54 39·2	(1870·0)

[Star *b* = Radcliffe 5900 = Piazzi XXI. 320.]

Notes.

June 24. The star B.D. −5° No. 5664, mag. 9·9, closely preceded the Comet, the star being a little to the north. Nucleus of Comet apparently about same mag. as this star. Nebulosity about 10′ in diameter surrounded nucleus. Central condensation, with radius about 1′, considerable. Comet a conspicuous object in the 3-inch finder.

June 25. Sky misty, but central condensation well seen. Nebulosity more extended, but nucleus less stellar than yesterday.

June 26. Comet brighter. Sky misty and cloudy. Dense central condensation seen, but stellar nucleus not visible. Estimated place of Comet at 8ʰ 45ᵐ G.M.T. = R.A. 21ʰ 45ᵐ·6, N.P.D. 93° 45′ (1855). Comet's Hour-angle East = about 4ʰ 43ᵐ. Extension of tail suspected in s.p. direction.

June 27.—Nebulosity similar to June 26. Stellar nucleus appears about = 10 mag. At 9ʰ 15ᵐ G.M.T. a faint star was involved in the dense part of the central portion of the Comet, and could not be distinguished from the nucleus.

June 28.—Nebulosity most extended in s.p. direction.

July 1. At 8ʰ 40ᵐ G.M.T. a very faint star preceded the Comet on the north. Nucleus seen, but not stellar. Tail (surrounding nucleus equally in all directions) very faint and diffused.

July 2. Nucleus not stellar. Faint extension of tail considerable. Sky misty.

Note "On the Positions of 166 Stars around Nova Geminorum" &c. By F. A. Bellamy.

In a letter received from Father Hagen he states that he satisfied himself that the two stars Nos. 57 and 58, referred *Monthly Notices*, lxiii. p. 520, are Oxford 158 and 17 respecti In the Georgetown College Observatory Circular for *Geminorum* the figures on page 4 for $\Delta\delta$ for Nos. 57 and 58 been partly interchanged ; they should be $+9''.4$ and $-1''.7$ ins of $+1''.4$ and $-9''.7$; the chart is correct. The Hagen nu for Oxford 37 should be 27 ; the figure 2 had dropped out passing the proof sheet. Further, he identifies Oxford 15 Hagen 72.

University Observatory, Oxford:
1903 *October* 21.

Measures of Southern Double Stars made at Shanghai, 1902–1903. By James L. Scott.

The following measures of southern doubles were made the same 5-inch refractor as those published in vol. lix. of *Monthly Notices*. Bright wire illumination was used through and nearly all the stars were measured within an hour of meridian.

Star's Name.	R.A.	S. Dec.	P.A.	Dist.	No. of Nights.	Mags.	D
	h m	° ′	° ′	″			
O.S. 51	0 3	14 51	108 4	10·1	2	9, 9	190
β 391 (κ^1 Sculptoris)	0 4	28 32	272 8	1·10	2	6·5, 6·5	
h 3377	0 28	26 38	58 1	19·1	2	6, 9·5	
h 3375	0 29	35 32	166 9	6·05	2	6·5, 9·4	190
h 3395	0 41	42 27	76 4	7·40	3	8·5, 9	190
C.G.C. 784 ...	0 47	23 9	268 5	2·15	2	7·5, 8	190
Washington I. ...	0 48	25 19	12 7	5·70	2	6·7, 8·5	190
C.G.C. 815 ...	0 48	25 31	27 0	13·2	1	7, 7·4	
LL. 1662... ...	0 53	16 13	215 2	6·31	2	8, 8·4	
Cord. Z.C. 1^h, 333	1 14	27 2	312 0	2·05	2	8, 8·5	
h 2036	1 15	16 20	15 1	1·50	3	7, 7·3	
h 3447 (τ Sculptoris)	1 32	30 25	97 0	1·80	3	6, 7	
h 3461	1 41	25 33	53 7	4·70	3	5, 9·3	
292 Ceti	1 54	23 24	305 0	8·44	1	7, 7·4	
Hastings I. ...	2 11	18 42	353 6	2·05	3	8, 8·5	190

Star's Name.	R.A. h m	S. Dec. ° ′	P.A. °	Dist. ″	No. of Nights.	Mags.	Date.
C.G.C. 2739 ...	2 30	21 59	97 8	14·7	1	7·3, 9	1902·912
Σ 3527	2 40	40 58	44 1	2·01	2	7, 7	·931
Σ 3532	2 45	37 50	147 1	5·40	2	6·8, 8·5	·939
Β 741	2 53	25 22	167 7	1·48	3	8, 8·4	·939
Σ 3565	3 14	18 55	115 2	6·24	2	5·8, 8·5	·950
Σ 3596	3 45	32 5	135 9	8·02	2	8, 8·3	·964
f Eridani ...	3 45	37 56	205 4	7·40	2	5, 5·5	·964
Β 1004	3 58	34 46	141 2	1·90	3	7·3, 7·8	·983
Β 184	4 23	21 43	259 9	1·25	2	7, 8	1903·137
Σ 3750	5 16	21 20	280 1	3·50	2	4·8, 9·5	·137
Σ 3752	5 18	24 53	101 4	3·05	2	5·5, 7·5	·137
Β 757	7 9	36 22	69 5	3·01	2	6·5, 9·4	·162
Jacob 4	7 15	36 35	209 6	2·90	2	9·8, 10	·162
Σ 3950	7 15	21 52	345 4	4·14	2	8·3, 8·3	·162
Σ 3949	7 15	30 37	77 8	2·98	2	8, 8·4	·162
Σ 1104	7 24	14 47	332 7	2·35	2	7, 9	·162
P. VII., 124 ...	7 25	31 38	52 0	8·60	1	6·5, 7·5	·164
e Arghs	7 26	43 6	74 2	22·3	1	3, 9	·164
Howe 8	7 49	34 27	280 1	2·58	2	5, 9	·162
Σ 1157	7 49	2 32	245 9	1·20	2	8, 8	·260
Β 583	8 4	6 25	69 8	1·75	2	9, 9·5	·167
Β 334	8 4	21 51	352 4	2·80	2	8, 9	·167
O.S. (O.A. S124)	8 5	26 50	261 2	3·30	2	8·5, 9·3	·162
Σ 4063	8 11	37 0	349 3	18·1	1	7·5, 9·3	·260
Β 454	8 12	30 37	15 6	2·50	2	6·8, 9	·293
Σ 4093	8 23	38 43	122 9	8·05	2	7, 7·5	·260
Σ 1295	8 51	7 35	353 2	4·30	2	7, 7·3	·290
Β 210	8 52	17 3	183 6	2·70	2	7, 7	·293
C.G.C. 12,339 ...	8 58	33 7	153 0	13·7	1	7·3, 7·8	·293
Σ 1316 (A–C) ...	9 2	6 44	169 4	7·50	2	8, 10	·290
Σ 4200	9 16	31 20	72 8	2·59	2	7·8, 8·5	·337
f Antliæ	9 26	31 27	212 5	8·00	1	5·8, 6·5	·290
Σ 4224	9 32	30 47	117 5	7·01	2	8, 9	·337
Σ 4249	9 44	34 33	126 5	4·16	2	8, 8	·337
C.G.C. 13,722 ...	9 58	17 30	273 6	21·6	1	6·5, 7·4	·337
Β 217	10 2	24 14	279 4	1·95	2	8, 8	·337
Β 25	10 17	9 16	174 3	1·80	2	8·3, 8·8	·348
e Velorum ...	10 28	44 31	217 4	13·3	1	6·5, 6·8	·337
Σ 1474	10 43	14 44	196 5	6·77	2	6·7, 7	1903·293

Star's Name.	R.A.	S. Dec.	P.A.	Dist.	No. of Nights.	Mags.	Date.
	h m	° ′	° ′	″			
Σ 1476	10 44	3 29	2 3	2·54	2	7, 8	1903·348
LL. 21,178 ...	10 57	15 9	16 2	2·90	2	8, 8·5	·364
Howe 15 ...	10 58	26 58	335 2	2·10	2	7·5, 9·3	·348
h 4423	11 12	45 20	275 2	2·20	3	7, 7·3	·348
Jacob 7	11 25	23 55	78 3	8·51	2	6, 8·5	·348
N. Hydræ ...	11 27	28 43	209 5	9·24	2	5·5, 5·5	·293
h 4455	11 32	33 10	243 7	3·83	2	6·3, 9·4	·293
C.G.C. 15,942 ...	11 34	37 25	95 7	16·8	1	7, 9	·337
Howe 16 ...	11 35	36 52	103 6	3·31	2	8, 8·5	·364
β Hydræ ...	11 48	33 21	352 6	1·76	3	5, 6·5	·348
h 4481	11 52	21 59	197 4	3·25	2	8, 8	·389
h 4495	12 0	32 23	318 1	6·58	1	6·5, 9	·389
C.G.C. 16,612 ...	12 3	34 0	202 7	3·86	2	6·5, 9	·348
Jacob 8	12 5	34 8	20 5	3·07	2	6·5, 8·8	·364
LL. 22,863 ...	12 6	16 14	285 4	6·07	2	6·5, 9	·348
D Centauri ...	12 9	45 10	241 5	3·10	2	5·5, 7	·397
δ Corvi	12 25	15 57	214 3	24·40	1	3, 8·5	·389
γ Virginis ...	12 37	0 54	330 1	5·88	3	3, 3	·397
h 4556	12 49	27 25	81 9	6·41	1	7, 8·5	·397
γ 4563	12 55	33 5	236 7	6·80	2	7·5, 9	·364
β 342	13 10	18 23	33 8	3·96	3	8, 8·5	·457
O.A. 12,867 ...	13 21	22 43	356 2	1·60	2	8·8, 9	·457
β 114	13 29	8 6	142 8	1·50	2	8, 8	·449
h 4608	13 36	33 28	178 6	4·52	1	7·5, 7·5	·397
h 4617	13 45	29 22	260 2	5·02	2	7·5, 9·5	·457
κ Centauri ...	13 46	32 30	109 5	7·91	1	4·8, 6	·457
β 343	13 46	31 7	120 7	1·02	3	6, 7	·457
Lac. 5751 ...	13 48	31 36	3 8	14·32	1	7, 8·5	·485
Σ 1788	13 49	7 34	79 5	2·70	2	6·8, 8·5	·444
h 4661	14 6	28 51	228 7	4·50	1	9, 9	·485
Howe 28 ...	14 12	27 3	119 7	3·20	2	9·3, 9·3	·449
h 4672	14 14	42 36	304 0	4·00	2	6·5, 9·5	·449
Σ 1837	14 19	11 13	300 5	1·42	2	7, 8·5	·449
β 117	14 26	15 11	95 9	2·30	2	8, 9	·485
h 4690	14 31	45 41	25 6	19·2	1	5·5, 8	·485
Howe 29	14 31	37 6	213 6	4·20	2	8, 8·5	·512
54 Hydræ ...	14 40	25 1	130 0	8·70	1	5·5, 7·5	·512
β 106 (A–B) ...	14 44	13 44	342 1	1·80	2	5·5, 6·5	·449
β 347 (A–C) ...	14 48	32 5 3	242 7	58·1	1	6, 9·8	1903·485

Star's Name.	R.A. h m	S. Dec. ° ′	P.A. ° ′	Dist. ″	No. of Nights.	Mags.	Date.
Σ 4718	14 51	34 58	63 3	2·20	2	7, 9	1903·457
P. XIV., 212 ...	14 52	20 57	294 9	16·76	2	6, 8	·449
Σ 4722	14 53	30 18	339 8	8·70	2	7·5, 9	·512
Σ 4727	14 57	27 26	36 8	7·62	2	8·3, 8·3	·485
Howe 31 ...	15 7	36 52	46 4	6·50	2	7·2, 7·5	·496
Anon.	15 10	36 45	197 7	20·7	1	7, 7·5	·496
Σ 227	15 13	23 54	175 1	2·10	2	7, 8·5	·496
Σ 4783	15 23	19 48	281 6	10·93	1	6·8, 8·5	·485
Σ 4776	15 23	41 34	225 6	5·40	2	7, 8·5	·457
Σ 4788	15 29	44 37	1 5	2·20	2	5, 8	·496
Cord. Z.C. 15ʰ 2046	15 31	31 11	44 7	2·05	2	8, 8·5	·575
Anon.	15 36	14 30	270 5	5·40	2	7·5, 8	·575
Howe 37 ...	15 38	41 30	350 6	3·80	2	6·5, 8·5	·594
Σ 35	15 38	15 41	100 6	2·60	2	7, 9	·594
ξ Lupi	15 50	33 40	48 6	10·80	1	5·5, 6	·485
ξ Serpii (AB–C)	15 59	11 6	63 2	7·10	2	4·5, 7·5	·600
Σ 120 (C–D) ...	16 6	19 12	48 7	2·05	2	7, 8	·594
Σ 4836	16 11	34 35	298 7	4·25	2	8, 8·5	·600
Σ 4848	16 17	32 58	154 5	6·09	1	7, 7·5	·594
Σ 4850	16 18	29 28	350 6	6·25	1	6, 6·5	·659
ρ Ophiuchi ...	16 20	23 13	353 4	3·35	2	5·5, 6	·659
σ Serpii... ...	16 23	26 13	274 7	3·30	2	1, 7·5	·662
P. XVI., 236 ...	16 51	19 23	230 5	4·85	2	6·8, 8	·643
Howe 45... ...	17 0	35 45	23 4	5·15	2	8, 8·5	·643
36 Ophiuchi ...	17 9	26 27	188 8	4·15	2	5·3, 5·5	·594
38 Ophiuchi ...	17 11	26 31	337 5	5·97	2	6·5, 9	·594
Σ 416	17 12	34 53	287 5	2·25	5	6, 8	·659
O.S. (Ophiuchi) ...	17 12	16 58	290 3	16·9	1	8, 9·3	·643
Σ 126	17 14	17 39	262 5	2·06	2	6·5, 7·5	·668
Howe 47... ...	17 23	33 38	324 5	4·60	2	7, 9·5	·659
Lac. 7465 ...	17 43	30 31	188 8	10·79	1	7, 8	·600
Σ 5003	17 53	30 15	104 5	5·37	2	5·3, 7	·600
π Ophiuchi ...	17 57	8 11	258 7	2·02	2	5·5, 6	·600
Σ 5014	17 59	43 25	241 6	1·62	3	6, 6	·643
Howe 50... ...	17 59	36 35	3 8	3·22	2	7·8, 8·5	·717
70 Ophiuchi ...	18 0	S 2 32	194 4	1·75	6	4, 6	·693
Σ 5023	18 4	40 27	277 5	8·61	1	8, 8·3	·659
Σ 245	18 4	30 45	353 4	4·20	2	6, 8	1903·643

Star's Name.	R.A. h m	S. Dec. ° ′	P.A. ° ′	Dist. ″	No. of Nights.	Mags.	Date.
β 132	18 5	19 52	218 6	elongated	2	7, 7·3	1903·643
β 759 (A–B)	18 5	39 22	120 7	1·70	3	8·5, 9	·717
(AB–C)			147 8	15·2	1	9, 9·3	·717
β 639 (AB–C) ...	18 13	18 40	52 0	17·4	1	7, 7·5	·668
Stone 10,006 ...	18 16	42 50	137 9	3·50	2	8·5, 8·8	·668
Jacob 10	18 19	20 36	288 0	1·85	3	5, 8·5	·668
Cor. Z.C. 18^{h}, 1122	18 19	27 28	358 6	5·52	2	8, 8	·643
β 133	18 22	26 42	260 3	1·70	2	7·5, 7·5	·643
Hh. 567	18 23	25 6	102 0	3·05	2	7·5, 7·8	·643
κ Cor. Australis ...	18 26	38 48	358 8	21·30	1	6, 7	·643
O.S. (Lac. 7763)...	18 28	34 54	137 1	1·75	3	7·8, 8·3	·698
λ Cor. Australis ...	18 37	38 25	214 3	29·6	1	5·5, 9·3	·717
Σ 2373	18 40	10 36	338 5	3·65	2	7, 8	·717
Lac. 7947 ...	18 54	37 12	282 5	12·30	1	7·5, 7·5	·698
Washburn 33 ...	18 56	28 47	58 1	2·70	2	8, 8·8	·668
γ Cor. Australis...	19 0	37 12	134 0	1·90	6	5, 5	·676
South 710 ...	19 0	16 25	0 2	6·28	1	5·8, 8·9	·659
h 5094	19 5	34 0	205 2	15·6	1	7, 7	·643
Cordoba 55 ...	19 8	27 29	329 5	2·25	3	8, 8·3	·698
O. Arg. S. 19,295	19 8	16 11	160 8	5·32	2	8·5, 8·5	·698
h 1381?	19 12	16 7	15 6	15·5	1	6, 7·5	·698
h 596	19 12	16 9	14 1	6·09	2	7·3, 7·8	·698
h 5113	19 19	29 30	164 6	16·5	1	6, 9·3	·698
β 142	19 22	12 21	337 6	1·60	3	8, 8	·698
H. 119	19 24	27 11	140 1	7·47	1	6, 8·8	·668
LL. 37,207 ...	19 33	10 22	323 2	3·65	2	7·3, 9	·728
O.S.	19 42	22 4	5 2	1·65	2	7·8, 8·3	·704
P. XIX., 365 ...	19 57	0 28	294 4	2·15	2	8, 8·5	·734
Cord. Z.C. 20^{h}, 20	20 3	39 1	261 5	3·40	2	9, 9·3	·704
h 5178	20 7	34 25	9 4	2·85	2	7, 8·5	·704
Dunlop 230 ...	20 11	40 30	116 3	9·65	1	7·3, 7·5	·668
β 762	20 11	32 55	303 9	2·30	2	8, 8·3	·704
Cord. Z.C. 20^{h}, 417	20 14	27 30	52 2	2·45	2	9, 9·5	·728
A.Oe$_{2}$ 20,449–50	20 16	18 45	108 6	2·40	2	8, 8·5	·704
Lac. 8390 ...	20 19	40 20	320 5	30·8	1	8, 8·5	·734
π Capricorni ...	20 21	18 36	145 9	3·30	2	5, 9	·717
Howe O.A. 20,494	20 21	26 57	52 4	2·90	2	8·5, 9	·734
ρ Capricorni ...	20 23	18 13	169 2	2·75	2	5, 7·5	·717
Jacob 11	20 27	40 54	223 8	4·30	2	7·5, 8·5	1903·717

Star's Name.	R.A.	S. Dec.	P.A.	Dist.	No. of Nights.	Magn.	Date.
	h m	° ′	°	″			
Â 1537	20 31	15 40	20 8	3·30	2	8·8, 9·5	1903·720
Â 5226	20 43	27 46	66 6	19·1	1	7·5, 8·5	·720
C.G.C. 29,052 ...	21 4	23 37	303 2	8·27	1	8, 9	·717
h 5252	21 7	15 25	319 7	3·53	2	8, 8·5	·704
Σ 767	21 21	42 59	140 6	2·30	2	5·8, 9	·720
LL. 41,705 ...	21 22	13 52	133 3	2·85	2	8, 9	·734
C.G.C. 29,568 ...	21 35	18 53	66 1	5·03	2	8, 9·4	·717
ν Piscis Australis	21 55	28 56	116 6	1·85	3	6, 6·5	·717
29 Aquarii ...	21 57	17 27	242 6	3·90	2	7, 8	·717
β 170	22 3	18 58	58 8	1·65	3	8·5, 8·5	·720
h 5319	22 6	38 48	122 7	2·00	2	8, 8	·720
South 508 ...	22 19	20 52	49 9	7·09	2	7, 8	·734
		x					
Σ 2920 (AB–C)	22 19	20 21	325 2	67·44	2	6, 9·3	1902·956
ζ Aquarii ...	22 24	0 32	319 7	3·29	2	4·5, 4·6	1903·720
h 5356	22 34	28 52	63 2	3·10	2	7·8, 8·5	·737
Σ 2928	22 34	13 8	313 3	4·18	2	8·8, 8·8	·720
Σ 2944 (A–B) ...	22 43	4 45	258 8	3·16	2	7, 8·3	1902·953
γ Piscis Australis	22 47	33 24	267 6	3·65	2	4·5, 8·5	1903·720
Σ 3008	23 19	9 2	237 4	3·89	2	7·5, 8	·720
h 5417	23 39	26 48	320 9	8·95	2	6, 9·5	·734
σ Aquarii ...	23 41	19 14	138 2	5·90	2	5·5, 7	·720
		x					
Σ 3041 (A–C) ...	23 43	16 31	351 8	66·11	2	7·3, 8	1902·964
" (A–B) ...			178 1	3·32	2	7·3, 8	·964
Dunlop 253 ...	23 49	27 36	269 6	6·96	2	6·8, 7·5	1903·734
		x					
Σ 3050	23 54	30 10	214 6	2·50	2	6, 6	1902·983

Shanghai: 1903 *September.*

Results of Double Star Measures with the 8-inch Equatorial at Windsor, New South Wales, in 1902.

By John Tebbutt.

Ref. No.	Star.	Observed Mags.	Approx. Place of Star. 1902. R.A. h m	Dec. S. ° ′	Fraction of Year.	Position-angle. °	Distance. ″	No. of Obs.	Mag. Power.	Eyes.	Hour-angles. h m	h m	Weight 1 to 5.
1	β Phœnicis	...	1 1·7	47 15	·090	20·1	1·28	10	300	P	2 54 W	3 19 W	4
2	,,	...	,,	,,	·093	19·4	2·16	10–6	300	P	2 49 W	3 10 W	4
3	,,	...	,,	,,	·101	20·6	...	10	230	P	3 4 W	3 32 W	2
4	,,	...	,,	,,	·101	18·6	...	10	300				3
5	*p* Eridani	...	1 36·1	56 42	·047	222·3	7·37	10–3	300	R	1 39 W	2 9 W	3
6	,,	...	,,	,,	·079	222·4	7·95	10–8	300	R	2 13 W	2 34 W	3
7	,,	...	,,	,,	·082	222·4	7·68	10	300	R	2 5 W	2 29 W	3
8	,,	...	,,	,,	·085	222·6	7·68	10	300	R	2 5 W	2 35 W	4
9	θ Eridani	4½, 5	2 54·5	40 42	·104	84·6	8·26	10–7	300	P	1 11 W	1 43 W	4
10	,,	...	,,	,,	·107	84·4	8·27	10–8	300	P	1 9 W	1 45 W	4
11	,,	...	,,	,,	·107	85·0	...	10	535				3
12	,,	...	,,	,,	·194	87·0	...	10	300	R	3 15 W	4 3 W	4
13	,,	...	,,	,,	·194	86·4	8·53	10	535 / 300	R	3 15 W	4 3 W	4
14	,,	4, 5	,,	,,	·203	87·3	8·38	10	300	R	3 12 W	3 41 W	4
15	12 Eridani	5, 9	3 7·9	29 23	·194	338·4	...	8	300	P	4 1 W	4 39 W	2
16	*f* Eridani	5⅓, 5½	3 45·0	37 55	·203	207·2	7·43	10	300	P	3 16 W	3 44 W	5
17	*h* 3752	6, 7½	5 17·7	24 52	·200	102·4	3·62	10	300	R	2 0 W		5

Ref. No.	Star.	Observed Magn.	Apparent Place of Star, 1900 R.A. h m	Dec. S. ° ′	Fraction of Year.	Position-angle. ″	Distance. ″	No. of Obs.	Mag. Power.	Rjns.	Hour-angles. h m	h m	Weight.
18	Lacaille 2145	6½, 7	6 2·2	48 27	·131	41·8	...	10	300	R	1 1 E	0 33 E	2
19	"	7, 7½	"	"	·298	45·3	2·13	10	300	R	2 56 W	3 31 W	4
20	"	...	"	"	·301	43·1	...	10	300	P			4
21	"	...	"	"	·301	45·6	...	10	300	R	3 33 W	4 18 W	4
22	"	...	"	"	·301	...	1·91	8	300	P			4
23	"	...	"	"	·315	41·7	1·87	10–8	300	P	3 29 W	3 53 W	4
24	"	...	"	"	·320	45·6	...	10	300	R	3 25 W	3 55 W	3
25	λ 3891	6, 8½	6 41·8	30 51	·146	224·0	...	10	300	R	1 37 E		3
26	γ Piscis Vol.	5, 7	7 9·6	70 20	·110	300·1	13·59	10–6	300	R	2 42 E	2 17 E	4
27	"	5, 6½	"	"	·112	300·8	13·63	10–8	300	R	2 50 E	2 26 E	3
28	λ 4087	7½, 7½	8 18·7	40 41	·140	295·5	...	10	300	R	2 47 E	2 21 E	3
29	δ Argûs	4, 8	8 42·0	54 21	·121	168·9	...	10	300	P	4 11 E	3 40 E	3
30	H Velorum	...	8 53·4	52 21	·131	343·0	...	10	300	P	2 17 E	1 52 E	2
31	"	5, 8	"	"	·137	344·1	2·97	10–8	300	P	3 14 E	2 30 E	2
32	λ 4220	5, 5½	9 30·2	48 34	·137	207·7	2·70	10–6	300	P	2 44 E	2 19 E	2
33	"	6, 6½	"	"	·145	208·5	2·34	10–8	300	P	3 26 E	3 3 E	4
34	"	5½, 6	"	"	·151	208·4	1·93	10–6	300	P	4 19 E	4 1 E	2
35	"	6, 6½	"	"	·153	208·6	2·27	10–7	300	P	4 22 E	3 58 E	3
36	ν Argûs	3, 7½	9 44·7	64 37	·153	130·2	...	10	230	R	3 18 E	2 43 E	3
37	"	"	"	"	·153	...	5·01	7	300	R			3

Ref. No.	Star.	Observed Mags.	Approx. Place of Star. 1902. R.A. h m	Dec. S. ° ′	Fraction of Year.	Position-angle. °	Distance. ″	No. of Obs.	Mag. Power.	Eyes.	Hour-angles. h m	h m	Weight 1 to 5.
38	ν Argûs	4, 7	9 44·7	64 37	·162	127·1	5·41	10–7	300	R	4 28 E	4 5 E	3
39	„	4, 7	„	„	·178	126·8	5·32	10–8	300	R	4 13 E	3 51 E	3
40	h 4252	8, 8¼	9 45·4	64 40	·153	124·3	12·49	10–7	140	P	2 43 E	2 12 E	3
41	Lacaille 4102	7, 8¼	9 53·1	68 43	·153	210·1	8·94	10–7	140	P	1 52 E	1 27 E	4
42	h 4310	8, 8¼	10 10·3	83 36	·162	272·2	4·16	10–6	230	R	4 7 E	3 36 E	4
43	h 4329	5, 8	10 27·6	53 13	·137	83·8	32·16	10–7	140	R	2 50 E	2 15 E	3
44	„	5, 8	„	„	·178	84·0	32·27	10–8	140	R	4 10 E	3 35 E	4
45	μ Argûs	4½, 8	10 42·5	48 54	·178	69·3	2·63	10–6	300	R	3 32 E	3 4 E	3
46	„	..	„	„	·189	67·9	...	10	300	R	3 37 E	3 12 E	2
47	„	4½, 8	„	„	·192	65·0	2·65	10–8	300	R	4 39 E	3 55 E	3
48	„	4½, 8	„	„	·192	65·9	...	10	300	R			
49	h 4373	9, 9	10 44·5	40 55	·153	343·5	8·27	8–1	140	P	1 50 E	1 31 E	3
50	„	8½, 9	„	„	·162	345·0	9·88	8–3	140	P	3 47 E	...	3
51	β Hydræ	5, 5½	11 48·0	33 22	·178	351·7	1·80	10	300	P	3 5 E	2 32 E	4
52	„	...	„	„	192	351·9	2·00	10–7	300	P	4 3 E	3 26 E	4
53	α Crucis	...	12 21·2	62 33	·370	117·3	4·90	10	535	R	4 14 E	3 46 E	4
54	„	...	„	„	·372	118·4	4·87	10	535	R	1 17 E	0 35 E	5
55	„	...	„	„	·452	119·1	5·17	10	300	R	3 19 E	2 36 E	4
56	„	...	„	„	·452	118·0	4·98	10	535	R	3 19 E	2 36 E	5
57	„	...	„	„	·460	119·3	4·98	10	300	R	2 39 E	2 16 E	3

Red No.	Star	Magnitudes	Apparent Place of Star for Jan. 1	Proportion of Year	Position angle	Distance	No. of Obs.	Mag. Power	Rys.	Hour angles	Weights
			h m s							h m	
58	γ Centauri		12 36 1 48 25	·320	355°9	1″77	10	300	P	2 26 E	1 59 E 4
59	,,	4·4	,,	·350	354·1	1·95	10	300	P	2 19 E	1 55 E 5
60	,,	4·4	,,	·356	354·4	1·87	10	300	P	2 9 E	1 41 E 5
61	,,	...	,,	·367	354·2	1·93	10	300	P	2 15 E	1 50 E 5
62	,,	...	,,	·386	354·3	1·96	10	535	P	3 29 E	3 8 E 5
63	,,	...	,,	·400	355·1	1·66	10	535	P	3 24 E	3 1 E 5
64	,,	...	,,	·403	355·3	1·84	10	535	P	3 45 E	3 31 E 5
65	,,	...	,,	·405	355·3	1·91	10	300	P	3 11 E	2 45 E 5
66	,,	...	,,	·435	355·1	1·95	10	300	P	2 26 E	2 3 E 3
67	,,	...	,,	·441	354·0	2·09	10	535	P	2 15 E	1 51 E 5
68	,,	...	,,	·449	355·3	1·84	10	300	P	1 43 E	1 17 E 4
69	γ Virginis	...	12 36·7 0 55	·370	147·7	6·04	10–6	535	P	3 47 E	3 26 E 4
70	,,	...	,,	·370	149·3	6·15	10	300	P	2 26 E	2 1 E 5
71	,,	...	,,	·372	150·2	...	10	300	R	0 34 E	0 5 E 5
72	,,	...	,,	·372	...	5·99	10	300	R		4
73	,,	...	,,	·375	148·7	6·17	10	300	P	1 35 E	1 8 E 5
74	β Muscæ	...	12 40·3 67 34	·320	343°0	1·55	10	300	R	1 48 E	1 16 E 4
75	,,	4·4	,,	·326	341·7	1·95	10	300	R	1 39 E	... 3
76	,,	5·5	,,	·350	342·4	1·71	10	300	R	1 50 E	1 24 E 4
77	,,	4·4	,,	·386	345·9	1·78	10	300	R	2 3 E	1 36 E 4

Ref. No.	Star.	Observed Mags.	Approx. Place of Star. 1902. R.A. h m	Dec. S. ° ′	Fraction of Year.	Position-angle. °	Distance. ″	No. of Obs.	Mag. Power.	Eyes.	Hour-angles. h m	h m	Weight 1 to 5.
78	β Muscæ	4, 4	12 40·3	67 34	·397	344·9	1·80	10	300	R	1 52 E	1 24 E	4
79	,,	...	,,	,,	·403	342·5	...	10	535	P	3 9 E	2 41 E	5
80	,,	...	,,	,,	·403	...	1·55	10	535	P			4
81	,,	...	,,	,,	·405	342·6	1·66	10	300	R	1 36 E	1 4 E	4
82	θ Muscæ	5, 7½	13 1·8	64 47	·370	187·8	5·48	10–6	300	P	2 6 E	...	4
83	,,	5, 7½	,,	,,	·372	187·2	5·86	10–7	300	P	2 35 E	2 8 E	4
84	k Centauri	5, 6	13 46·2	32 30	·361	109·3	8·35	10	300	P	2 29 E	2 0 E	3
85	,,	5½, 7	,,	,,	·386	110·0	8·49	10	300	P	2 23 E	1 35 E	4
86	y Centauri	6, 6	13 47·8	35 11	·403	90·5	1·60	10	300	R	2 35 E	2 3 E	4
87	Lacaille 5750	7½, 8	13 50·8	55 33	·350	26·9	14·96	10–6	140	P	2 13 E	1 46 E	3
88	Lacaille 5893	5, 7	14 15·6	58 1	·403	162·1	9·79	10	300	P	2 10 E	1 35 E	5
89	α Centauri	...	14 32·9	60 26	·044	212·0	...	10	140	R	3 14 W	3 33 W	2
90	,,	...	,,	,,	·192	211·4	21·65	10	300	P	4 39 W	5 28 W	4
91	,,	...	,,	,,	·192	211·3	21·96	10	535	P			
92	,,	...	,,	,,	·194	211·0	...	10	300	P	3 51 W	5 30 W	4
93	,,	...	,,	,,	·194	211·3	22·24	10–2	535	P			4
94	,,	...	,,	,,	·194	...	22·14	10	535	P			4
95	,,	...	,,	,,	·194	211·0	...	10	535	P			3
96	,,	...	,,	,,	·197	211·6	22·03	10	300	R	3 21 W	4 26 W	4
97	,,	...	,,	,,	·197	210·9	21·83	10	535	P			5

Ref. No.	Star	Nebular Rings	Epoch. Time of Mid Transit (G.M.T.)	Position of Vernier.	Position angle.	Distance.	No. of Obs.	Mag. Power.	Illum.	Hour angle.	Weight Obs.
98	α Centauri	..	14 32 0	200	211·2	21·56	10	300	R	h m 4 14 W	5
99	..	..	..	200	211·8	...	10	535	P	4 14 W	5
100	..	..	..	200	...	21·96	10	535	P	5 0 W	4
101	..	..	..	205	211·6	21·58	10	300	R	3 47 W	5
102	..	..	..	205	210·9	22·15	10	535	P	4 11 W	5
103	..	..	..	205	...	21·93	10	170	P	4 30 W	4
104	..	..	..	205	...	21·77	10	170	P	4 56 W	5
105	..	..	..	205	...	21·52	10	170	P	5 11 W	4
106	..	..	..	227	...	21·66	10	170	P	4 22 W	3
107	..	..	..	227	...	21·59	10	300	P	4 38 W	4
108	..	..	..	227	211·1	22·24	10	535	P	5 8 W	4
109	..	..	..	227	211·5	...	10	300	P	5 29 W	3
110	..	..	..	241	210·9	21·90	10	300	P	4 53 W	4
111	..	..	..	241	211·2	22·14	10	535	P	5 39 W	
112	..	1, 2	..	252	211·0	22·10	10	300	P	4 18 E	3
113	..	..	..	255	211·7	21·85	10–6	300	P	4 13 E	3
114	..	..	..	257	211·2	...	8	300	R	4 40 E	3
115	..	..	..	263	211·2	21·94	10	300	P	4 18 E	3
116	..	..	..	266	211·4	22·03	10	300	P	3 56 E	2
117	..	..	..	268	211·3	22·10	10	300	P	3 48 E	3

Ref. No.	Star.	Observed Mags.	Approx. Place of Star. 1902. R.A. h m	Dec. S. ° ′	Fraction of Year.	Position-angle. °	Distance. ″	No. of Obs.	Mag. Power.	Eyes.	Hour-angles. h m	h m	Weight 1 to 5.
118	α Centauri	...	14 32·9	60 26	·274	212·0	22·06	10	300	P	3 59 E	3 30 E	4
119	,,	..	,,	,,	·282	211·4	22·10	10	300	P	3 54 E	3 27 E	3
120	,,	...	,,	,,	·400	211·6	21·78	10	535	P	4 51 E	4 20 E	4
121	,,	...	,,	,,	·449	211·3	...	10	300	P	4 21 E	3 52 E	4
122	,,	...	,,	,,	·449	...	21·79	10	300	P			3
123	,,	...	,,	,,	·460	211·2	...	10	300	P	4 12 E	3 49 E	3
124	,,	...	,	,,	·460	...	22·01	6	300	P			2
125	,,	...	,,	,,	·476	211·1	...	10	300	P	3 30 E	2 59 E	4
126	,,	...	,,	,,	·476	211·7	21·88	10	535	P			
127	,,	...	,,	,,	·479	211·0	21·63	10	535	P	3 23 E	2 56 E	4
128	,,	...	,,	,,	·487	211·0	...	10	300	P	3 33 E	3 0 E	3
129	,,	...	,,	,,	·487	...	21·78	10	535	P			
130	,,	...	,,	,,	·674	...	21·68	10	170	R	0 15 W	0 35 W	3
131	,,	...	,,	,,	·676	...	21·57	10	170	R	1 28 W	1 41 W	3
132	,,	...	,,	,,	·685	...	21·88	10	170	P	3 39 E	3 19 E	2
133	,,	...	,,	,,	·685	...	21·47	10	170	R	2 10 W	2 21 W	3
134	,,	...	,,	,,	·690	...	21·31	10	170	R	3 51 W	4 10 W	3
135	,,	...	,,	,,	·701	...	21·24	10	170	R	1 29 W	1 45 W	3
136	,,	...	,,	,,	·704	...	21·49	10	170	R	1 56 W	2 21 W	3
[illegible]		...	,,	,,	·717	...	21·52	10	170	R	1 12 W	1 28 W	3

No.	Star	Observed Mags.	Approx. Place of Star 1900 R.A.	Dec.	Position of Year	Position angle	Distance.	No. of Obs.	Mag. Power	Eyes	Hour angles		Weight
			h m				"				h m	h m	
138	α Centauri		14 32·9	(sic) 36	·717	°	21·65	8	170	R	...	3 26 W	3
139	..		..	..	·720	...	21·40	10	170	R	1 46 W	2 3 W	3
140	..		..	..	·720	...	21·45	10	170	R	...	2 49 W	4
141	..	...	..	..	·720	...	21·50	10	170	R	3 0 W	3 12 W	3
142	..	..	..	..	·728	...	21·47	10	170	R	2 17 W	2 31 W	4
143	..	...	..	..	·728	...	21·56	10	170	R	2 38 W	2 52 W	4
144	..	...	..	..	·728	...	21·52	10	170	R	2 54 W	3 8 W	5
145	..	...	..	..	·728	...	21·54	10	170	R	3 8 W	3 24 W	3
146	..	...	..	..	·731	...	21·56	10	170	R	2 22 W	2 35 W	4
147	..	...	..	..	·731	...	21·56	10	170	R	2 35 W	2 49 W	4
148	α Circini	3. 8	14 34·6	64 33	·350	236·8	...	10	300	P	2 14 E	1 46 E	4
149	..	3. 8	..	..	·350	...	16·39	6	140	P			
150	..	4. 8	..	..	·367	237·7	...	10	300	P	3 37 E	3 4 E	4
151	..	4. 8	..	..	·367	...	15·81	6	140	P			
152	α Hydræ	5. 6½	14 40·3	25 2	·392	129·0	...	10	300	P	2 30 E	2 5 E	4
153	..	5. 6½	..	..	·392	...	9·14	10	300	P			5
154	..	...	..	..	·405	131·3	9·08	10–2	300	P	2 44 E	2 11 E	4
155	..	5. 8	..	..	·408	130·8	8·63	10	300	P	3 17 E	2 33 E	5
156	59 Hydræ	6½. 6½	14 52·9	27 16	·408	318·8	...	10	300	P	2 32 E	2 9 E	4
157	ν Lupi		14 58·5	46 40	·367	86·7	1·66	10	300	R	3 18 E	2 51 E	5

Ref. No.	Star.	Observed Mags.	Approx. Place of Star. 1902. R.A. h m	Dec. S. ° ′	Fraction of Year.	Position-angle. °	Distance. ″	No. of Obs.	Mag. Power.	Eyes.	Hour-angles. h m	h m	Weight 1 to 5.
158	π Lupi	5, 5	14 58·5	46 40	·375	87·4	...	10	300	R	2 30 E	2 9 E	5
159	,,	5, 5	,,	,,	·375	...	1·51	10	300	R			4
160	,,	5, 5	,,	,,	·408	85·0	...	10	300	R	2 4 E	1 33 E	5
161	,,	5, 5	,,	,,	·408	87·2	1·50	10–6	535	R			
162	κ Lupi	4½, 6	15 5·1	48 22	·392	144·4	27·40	10	300	P	2 15 E	1 38 E	5
163	μ Lupi	5, 5	15 11·7	47 31	·340	156·3	1·75	10	300	P	3 25 E	2 56 E	4
164	,,	5. 5	,,	,,	·386	155·1	1·89	10	300	P	4 8 E	3 41 E	5
165	,,	5, 5	,,	,,	·441	155·6	2·09	10	300	P	2 41 E	2 17 E	5
166	Dunlop 180	5. 7	,,	,,	·340	130·8	...	10	300	P	2 56 E	2 27 E	4
167	,,	5, 7	,,	,,	·340	...	23·18	5	140	P			
168	γ Circini	5, 5½	15 15·6	58 58	·367	80·1	1·64	10	300	R	2 49 E	2 23 E	4
169	,,	5, 5	,,	,,	·375	81·8	...	10	300	R	2 15 E	2 1 E	4
170	,,	5, 5½	,,	,,	·386	78·8	1·60	10	300	R	2 45 E	2 17 E	4
171	,,	5, 5½	,,	,,	·441	79·1	1·89	10	300	P	2 6 E	1 37 E	4
172	e Lupi	5, 8	15 16·0	44 20	·449	276·2	1·39	10	300	R	2 46 E	2 22 E	5
173	e Lupi & Dunlop 182	5, 7	15 16·0	44 20	·449	172·5	...	10	140	P	2 46 E	2 22 E	4
174	d Lupi	5, 7	15 29·0	44 38	·340	358·1	2·47	10	300	P	2 23 E	1 52 E	4
175	,,	5½, 7½	,,	,,	·356	359·0	2·58	10	300	P	3 52 E	3 26 E	5
176	,,	5, 8	,,	,,	·375	356·3	2·63	10–8	300	P	3 49 E	3 21 E	3
177	Lacaille 6477	6½, 6½	15 38·9	65 8	·397	153·7	2·52	10	300	P	3 56 E	3 22 E	5

Ref. No.	Star.	Observed Mags.	Approx. Place of Star. 1900. R.A. h m	Dec. S. ° ′	Fraction of Year.	Position-angle. ″	Distance. ″	No. of Obs.	Mag. Power.	Eyes.	Hour-angles. h m	h m	Weight 1 to 5.
178	A Scorpii	5, 8	15 47·7	25 2	·449	277·7	3·08	10	300	R	2 16 E	1 51 E	5
179	,,	5, 8	,,	,,	·476	276·5	3·17	10	300	R	2 32 E	1 59 E	4
180	,,	5, 8	,,	,,	·487	275·5	2·97	10	300	R	2 6 E	1 38 E	5
181	η Lupi	5, 8	15 53·6	38 7	·356	20·8	15·12	10–5	300	P	3 31 E	3 6 E	4
182	ε Scorpii	4, 6	16 6·3	19 12	·400	7·5	...	5	535	P	2 52 E	2 6 E	3
183	,,	4, 6	,,	,,	·400	10·9	...	10	300	P			
184	,,	5½, 7	,,	,,	·427	8·2	...	10	300	P	3 24 E	2 35 E	2
185	,,	5½, 7	,,	,,	·427	4·9	...	10	300	P			3
186	,,	5, 6½	,,	,,	·452	0·9	...	10	300	P	2 36 E	2 8 E	4
187	Jacob 9	7½, 7¾	16 6·3	19 12	·400	49·6	...	10	300	R	2 52 E	2 6 E	3
188	,,	7, 7½	,,	,,	·427	51·4	...	10	300	R	3 24 E	2 35 E	3
189	,,	6½, 7	,,	,,	·452	50·2	...	10	300	R	2 36 E	2 8 E	4
190	ρ Ophiuchi	5, 5½	16 19·7	23 13	·394	355·0	3·70	10	300	P	2 56 E	2 28 E	5
191	,,	...	,,	,,	·427	356·3	3·80	10	300	P	2 35 E	2 8 E	4
192	A Ophiuchi	5, 5	17 9·3	26 27	·394	192·4	4·49	10	300	P	3 3 E	2 25 E	5
193	,,	5½, 5½	,,	,,	·427	191·8	4·76	10	300	P	2 39 E	2 12 E	5
194	,,	6, 6	,,	,,	·452	190·8	4·98	10	300	P	2 52 E	2 26 E	4
195	Lacaille 7194	6, 8½	17 11·6	46 32	·441	83·6	...	10	300	R	3 15 E	2 52 E	3
196	,,	5½, 9	,,	,,	·449	84·4	2·83	10–7	300	R	2 48 E	2 26 E	5

Ref. No.	Star.	Observed Mags.	Approx. Place of Star. 1902. R.A. h m	Dec. S. ° ′	Fraction of Year.	Position-angle. °	Distance. ″	No. of Obs.	Mag. Power.	Eyes.	Hour-angles. h m	h m	Weight 1 to 5.
158	π Lupi	5, 5	14 58·5	46 40	·375	87·4	...	10	300	R	2 30 E	2 9 E	5
159	,,	5, 5	,,	,,	·375	...	1·51	10	300	R			4
160	,,	5, 5	,,	,,	·408	85·0	...	10	300	R	2 4 E	1 33 E	5
161	,,	5, 5	,,	,,	·408	87·2	1·50	10–6	535	R			
162	κ Lupi	4½, 6	15 5·1	48 22	·392	144·4	27·40	10	300	P	2 15 E	1 38 E	5
163	μ Lupi	5, 5	15 11·7	47 31	·340	156·3	1·75	10	300	P	3 25 E	2 56 E	4
164	,,	5. 5	,,	,,	·386	155·1	1·89	10	300	P	4 8 E	3 41 E	5
165	,,	5, 5	,,	,,	·441	155·6	2·09	10	300	P	2 41 E	2 17 E	5
166	Dunlop 180	5. 7	,,	,,	·340	130·8	...	10	300	P	2 56 E	2 27 E	4
167	,,	5, 7	,,	,,	·340	...	23·18	5	140	P			
168	γ Circini	5, 5½	15 15·6	58 58	·367	80·1	1·64	10	300	R	2 49 E	2 23 E	4
169	,,	5, 5	,,	,,	·375	81·8	...	10	300	R	2 15 E	2 1 E	4
170	,,	5, 5½	,,	,,	·386	78·8	1·60	10	300	R	2 45 E	2 17 E	4
171	,,	5, 5½	,,	,,	·441	79·1	1·89	10	300	P	2 6 E	1 37 E	4
172	e Lupi	5, 8	15 16·0	44 20	·449	276·2	1·39	10	300	R	2 46 E	2 22 E	5
173	e Lupi & Dunlop 182	5, 7	15 16·0	44 20	·449	172·5	...	10	140	P	2 46 E	2 22 E	4
174	d Lupi	5, 7	15 29·0	44 38	·340	358·1	2·47	10	300	P	2 23 E	1 52 E	4
175	,,	5½, 7½	,,	,,	·356	359·0	2·58	10	300	P	3 52 E	3 26 E	5
176	,,	5, 8	,,	,,	·375	356·3	2·63	10–8	300	P	3 49 E	3 21 E	3
177	Lacaille 6477	6½, 6½	15 38·9	65 8	·397	153·7	2·52	10	300	P	3 56 E	3 22 E	5

Ref. No.	Star.	Observed Magn.	Approx. Place of Star, 1900. R.A. h m	Dec. S. + -	Fraction of Year.	Position-angle.	Distance.	No. of Obs.	Mag. Power.	Eyes.	Hour-angles. h m	h m	Weight 1 to 5.
178	A Scorpii	5, 8	15 47·7	25 2	·449	277·7	3·08	10	300	R	2 16 E	1 51 E	5
179	,,	5, 8	,,	,,	·476	276·5	3·17	10	300	R	2 32 E	1 59 E	4
180	,,	5, 8	,,	,,	·487	275·5	2·97	10	300	R	2 6 E	1 38 E	5
181	η Lupi	5, 8	15 53·6	38 7	·356	20·8	15·12	10–5	300	P	3 31 E	3 6 E	4
182	ν Scorpii	4, 6	16 6·3	19 12	·400	7·5	...	5	535	P	2 52 E	2 6 E	3
183	,,	4, 6	,,	,,	·400	10·9	...	10	300	P			
184	,,	5½, 7	,,	,,	·427	8·2	...	10	300	P	3 24 E	2 35 E	2
185	,,	5½, 7	,,	,,	·427	4·9	...	10	300	P			3
186	,,	5, 6½	,,	,,	·452	0·9	...	10	300	P	2 36 E	2 8 E	4
187	Jacob 9	7½, 7½	16 6·3	19 12	·400	49·6	...	10	300	R	2 52 E	2 6 E	3
188	,,	7, 7½	,,	,,	·427	51·4	...	10	300	R	3 24 E	2 35 E	3
189	,,	6½, 7	,,	,,	·452	50·2	...	10	300	R	2 36 E	2 8 E	4
190	ρ Ophiuchi	5, 5½	16 19·7	23 13	·394	355·0	3·70	10	300	P	2 56 E	2 28 E	5
191	,,	...	,,	,,	·427	356·3	3·80	10	300	P	2 35 E	2 8 E	4
192	A Ophiuchi	5, 5	17 9·3	26 27	·394	192·4	4·49	10	300	P	3 3 E	2 25 E	5
193	,,	5½, 5½	,,	,,	·427	191·8	4·76	10	300	P	2 39 E	2 12 E	5
194	,,	6, 6	,,	,,	·452	190·8	4·98	10	300	P	2 52 E	2 26 E	4
195	Lacaille 7194	6, 8½	17 11·6	46 32	·441	83·6	...	10	300	R	3 15 E	2 52 E	3
196	,,	5½, 9	,,	,,	·449	84·4	2·83	10–7	300	R	2 48 E	2 26 E	5

Ref. No.	Star.	Observed Mags.	Approx. Place of Star. 1902. R.A. h m	Dec. S. ° ′	Fraction of Year.	Position-angle.	Distance.	No. of Obs.	Mag. Power.	Eyes.	Hour-angles. h m	h m	Weight 1 to 5.
197	Lacaille 7194	5½, 8½	17 11·6	46 32	·476	85°·0	″ ...	10	300	R	2 56 E	2 29 E	4
198	,,	5½, 8½	,,	,,	·476	...	2·24	7	300	R			3
199	,,	5½, 8½	,,	,,	·485	81·7	2·54	10	300	R	3 42 E	3 8 E	4
200	,,	5, 8½	,,	,,	·487	81·8	2·16	10	300	R	2 26 E	1 55 E	4
201	,,	5. 8½	,,	,,	·493	83·6	2·20	10	300	R	3 7 E	2 22 E	3
202	β 416	6½, 7	17 12·2	34 53	·479	288·7	2·56	10	300	R	2 29 E	1 59 E	4
203	,,	6, 6½	,,	,,	·485	289·7	...	10	300	R	2 22 E	1 55 E	4
204	,,	6, 7	,,	,,	·490	291·0	2·52	10	300	P	3 36 E	3 1 E	3
205	,,	6, 7	,,	,,	·493	291·1	2·56	10–5	300	R	2 7 E	1 40 E	3
206	,,	7, 8	,,	,,	·496	289·5	2·63	10	300	R	3 6 E	2 38 E	5
207	h 4949	6, 6½	17 19·6	45 45	·485	262·2	2·76	10	300	R	3 9 E	2 43 E	5
208	h 5014	6, 6	17 59·7	43 26	·479	242·6	2·07	10	300	R	1 44 E	1 13 E	5
209	γ Coronæ Aust.	5½, 5½	18 59·8	37 12	·394	136·1	1·87	10	300	P	3 48 E	3 21 E	5
210	,,	5, 5	,,	,,	·427	135·6	...	10	300	P	3 47 E	3 12 E	4
211	,,	5, 5	,,	,,	·427	...	2·04	10	300	P			3
212	,,	5½, 5½	,,	,,	·476	136·3	2·22	10	300	P	4 3 E	3 35 E	3
213	,,	5, 5	,,	,,	·479	136·4	2·24	10	300	P	3 25 E	3 0 E	5
214	,,	5½, 5½	,,	,,	·485	135·2	2·20	10	300	P	3 38 E	3 6 E	4
215	,,	5, 5	,,	,,	·487	133·3	2·45	10	300	P	3 27 E	3 3 E	5
216		5, 5	,,	,,	·496	136·3	2·13	10	300	P	4 2 E	3 33 E	5

Remarks.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 25, 26, 27, 34, 35, 38, 39, 62, 63, 65, 66, 67, 69, 79, 80, 120, 124, 125, 126, 127, 133, measures in twilight. 53, 55, 56, 57, 64, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 123, 130, 131, 132, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, measures in sunlight. 1, 2, 3, 4, 5, 6, 7, 8, 59, 60, 65, 66, 71, 72, 75, 77, 78, 86, 156, 157, 158, 159, 160, 161, 163, 164, 165, 177, 192, 193, 194, 208, 209, 210, 211, 212, 213, 214, 215, 216, components equal. 16, north component probably the brighter. 17, principal component yellow, companion blue. 18, hazy and ill-defined. 19, preceding and south component the brighter. 20, 21, 22, 23, nearly equal. 26, 27, 36, 37, 38, 39, principal component orange and companion blue. 29, 31, observations very difficult. 36, 37, 38, 39, erroneously called *h* 4252 in Innes' Reference Catalogue. 40, not in the Reference Catalogue. 43, 44, principal component white and companion reddish. 45, principal component yellow, companion pale green. 47, 48, principal component orange, companion greenish. 49, south component the brighter. 51, a neat and beautiful pair. 74, 190, following and south component the brighter. 79, 80, south component certainly slightly the brighter. 82, principal component white, companion blue. 103, 104, 105, 106, 107, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, measures with the Grubb distance filar micrometer. 121, 122, 128, 129, 141, measures partly in sunlight and partly in twilight. 156, components just separated, distance less than 1″. 166, 167, the position-angle is referred to the mean of μ Lupi. 173, Innes' Catalogue makes the companion = 10 mag. 175, a minute companion suspected midway between the components, but a little west of the line joining them. 182, 183, measures difficult, estimated distances = 1″. 184, 185, distance about 1″. 188, distance about 2″. 195, principal component reddish and rather blurred, distance about 3″. 196, 199, 200, companion hazy. 205, 206, a companion about 8½ or 9 mag. south, and following at a distance of 20″ or 30″.

On March 7 I attempted to observe γ *Carinæ* = Innes' No. 6, 11^h, but the definition was not good enough to enable me to distinguish the companion measured here in 1901, nor could I see the companion = 10 mag. recorded by Herschel and Pollock. Brisbane 3574 = Innes' No. 22, 11^h, was woolly and could not be seen double. On May 5 I examined θ *Centauri* = No. 2, 14^h of Innes' Catalogue, but could find no trace of a companion.

The arrangement of the preceding table is similar to that adopted in former communications. All the measures, except where otherwise stated, have been made with the Cooke position and distance filar micrometer.

Observatory, Peninsula, Windsor, N.S. Wales:
1903 *August.*

Additional Note to Paper on a New Method of Determining Time, Latitude, and Azimuth with a Theodolite. By W. Ernest Cooke, M.A.

In my article on a method of determining time, &c. (*Monthly Notices*, lxiii. p. 161), an error in the formula for latitude occurs.

The quantity L should be taken out for each clock star, as well as for those observed especially for latitude. Since the values of Z, L, &c., differ from one another so considerably in the latitude stars it will be better to compute $\Delta\phi$ separately for each pair, rather than take the mean of all the east and west stars respectively, as may be done in the case of clock stars.

Now let L_1 Z_1 be the mean of all the L*s* and Z*s* for clock stars, considered positive throughout, no matter whether east or west; and let L_2 Z_2 be the *sum* of the L*s* and Z*s* for the latitude pair, also considered positive throughout. Then instead of $c = L - L'$, we must put

$$c = L_2 - \frac{L_1}{Z_1} Z_2$$

A misprint on p. 160 may also cause confusion. In the middle section of that page the quantity Z should in each case be written *z* (small); and on p. 159 at the end of § 1 read

$$\tan \tfrac{1}{2} A = \sin \phi \tan t$$

If a number of observations are to be taken in the same place, it may be as well to compute t_o, from an approximate value of $\delta = \delta_o$ (say). Then the t corresponding to the apparent δ at date, from the *N.A.*, $= t_o + D(\delta_o - \delta)$ for the southern, and $= t_o + D(\delta - \delta_o)$ for the northern hemisphere. In this case N declination is + and south −.

Also if t_o has been computed for any assumed latitude ϕ_o, and if it be desired to correct it for a latitude ϕ, differing not very greatly from ϕ, we have

$$t = t_o + (\phi - \phi_o)L$$

Perth Observatory, Western Australia:
1903 *October* 13.

Ephemeris for Physical Observations of the Moon, 1904.

By A. C. D. Crommelin.

Greenwich Midnight.		Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.	C.
1904.		°	°	°	°	°	°	°
Jan.	1	77·60	+1·52	-4·79	+6·44	8·02	36·6	356·82
	2	89·73	1·52	−3·08	+6·47	7·17	25·5	3·43
	3	101·86	1·51	−1·17	+6·06	6·17	10·9	9·86
	4	113·98	1·50	+0·79	+5·21	5·27	351·4	15·55
	5	126·10	1·49	+2·60	+4·01	4·78	327·0	20·08
	6	138·23	1·48	+4·15	+2·56	4·88	301·7	23·17
	7	150·37	1·47	+5·34	+0·97	5·43	280·3	24·75
	8	162·51	1·46	+6·15	−0·63	6·18	264·2	24·84
	9	174·66	1·45	+6·60	−2·17	6·95	251·8	23·57
	10	186·82	1·44	+6·72	−3·55	7·60	242·2	21·08
	11	198·99	1·43	+6·56	−4·71	8·08	234·3	17·53
	12	211·16	1·42	+6·17	−5·61	8·34	227·7	13·14
	13	223·34	1·41	+5·58	−6·23	8·36	221·8	8·14
	14	235·52	1·40	+4·83	−6·54	8·13	216·4	2·75
	15	247·71	1·38	+3·92	−6·55	7·63	210·9	357·30
	16	259·89	1·37	+2·89	−6·26	6·90	204·8	352·05
	17	272·08	1·36	+1·73	−5·69	5·95	196·9	347·23
	18	284·27	1·35	+0·46	−4·88	4·90	185·4	343·05
	19	296·46	1·34	−0·90	−3·86	3·96	166·9	339·66
	20	308·65	1·33	−2·30	−2·68	3·53	139·4	337·15
	21	320·83	1·32	−3·70	−1·38	3·95	110·5	335·59
	22	333·01	1·31	−5·04	−0·01	5·04	90·1	335·02
	23	345·18	1·30	−6·23	+1·37	6·38	77·6	335·50
	24	357·35	1·28	−7·19	+2·72	7·69	69·3	337·04
	25	9·51	1·27	−7·83	+3·98	8·78	63·1	339·70
	26	21·67	1·26	−8·07	+5·07	9·53	57·9	343·46
	27	33·82	1·24	−7·83	+5·92	9·82	52·9	348·29
	28	45·96	1·23	−7·07	+6·47	9·59	47·5	354·03
	29	58·10	1·21	−5·81	+6·63	8·81	41·2	350·39
	30	70·23	1·19	−4·12	+6·35	7·57	33·0	6·91
	31	82·35	1·17	−2·12	+5·63	6·01	20·6	13·03
Feb.	1	94·48	1·15	+0·01	+4·50	4·50	359·9	18·21
	2	106·60	1·13	+2·08	+3·05	3·69	325·7	22·04
	3	118·73	+1·10	+3·93	+1·40	4·17	289·6	24·31

Greenwich Midnight.	Selenographical Colong.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.	C.
1904.	°	°	°	°	°	°	°
Feb. 4	130·86	+1·08	+5·44	−0·32	5·45	266·6	24·97
5	143·01	1·05	+6·52	−1·96	6·81	253·3	24·10
6	155·15	1·03	+7·18	−3·45	7·97	244·3	21·87
7	167·31	1·01	+7·42	−4·69	8·78	237·7	18·51
8	179·47	0·98	+7·29	−5·66	9·23	232·2	14·24
9	191·64	0·96	+6·85	−6·32	9·32	227·3	9·31
10	203·82	0·94	+6·14	−6·66	9·06	222·7	4·00
11	216·00	0·92	+5·22	−6·70	8·49	217·9	358·56
12	228·19	0·90	+4·14	−6·44	7·65	212·7	353·27
13	240·38	0·88	+2·93	−5·89	6·58	206·4	348·36
14	252·57	0·85	+1·64	−5·10	5·36	197·8	344·02
15	264·77	0·83	+0·29	−4·09	4·10	184·1	340·42
16	276·97	0·81	−1·09	−2·90	3·10	159·4	337·68
17	289·17	0·79	−2·47	−1·59	2·94	122·8	335·88
18	301·37	0·77	−3·80	−0·20	3·81	93·0	335·07
19	313·56	0·75	−5·04	+1·21	5·18	76·5	335·29
20	325·75	0·73	−6·13	+2·59	6·66	67·1	336·58
21	337·93	0·70	−7·01	+3·86	8·00	61·2	338·91
22	350·12	0·68	−7·62	+4·98	9·11	56·8	342·30
23	2·29	0·66	−7·88	+5·88	9·83	53·3	346·70
24	14·46	0·63	−7·74	+6·49	10·10	50·0	351·99
25	26·63	0·61	−7·16	+6·76	9·84	46·6	357·95
26	38·78	0·58	−6·14	+6·63	9·04	42·8	4·24
27	50·93	0·55	−4·71	+6·07	7·68	37·8	10·40
28	63·07	0·53	−2·95	+5·10	5·89	30·0	15·95
29	75·21	0·49	−0·98	+3·75	3·83	11·7	20·41
Mar. 1	87·35	0·46	+1·03	+2·14	2·37	334·3	23·45
2	99·49	0·43	+2·94	+0·37	2·96	277·2	24·87
3	111·63	0·40	+4·60	−1·39	4·80	253·2	24·64
4	123·77	0·37	+5·90	−3·02	6·63	242·9	22·89
5	135·92	0·34	+6·78	−4·42	8·09	236·9	19·82
6	148·08	0·31	+7·23	−5·52	9·10	232·6	15·70
7	160·25	0·28	+7·26	−6·29	9·61	229·1	10·81
8	172·42	0·25	+6·90	−6·72	9·62	225·8	5·47
9	184·61	0·23	+6·21	−6·82	9·22	222·3	359·96
10	196·79	0·20	+5·25	−6·61	8·45	218·5	354·58
11	208·99	0·17	+4·10	−6·12	7·37	213·8	349·54
12	221·18	+0·15	+2·81	−5·36	6·05	207·7	345·05

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.	C.
1903	°	°	°	°	°	°	°
Mar. 13	233·39	+0·12	+1·40	−4·38	4·60	197·7	341·26
14	245·59	0·10	+0·05	−3·22	3·22	180·9	338·29
15	257·81	0·07	−1·31	−1·91	2·32	145·5	336·25
16	270·02	0·05	−2·61	−0·51	2·66	101·1	335·18
17	282·23	0·02	−3·79	+0·92	3·90	76·4	335·14
18	294·45	0·00	−4·84	+2·33	5·37	64·3	336·17
19	306·65	−0·02	−5·70	+3·64	6·77	57·4	338·28
20	318·86	0·05	−6·35	+4·81	7·96	52·9	341·45
21	331·06	0·08	−6·75	+5·75	8·66	49·6	345·61
22	343·26	0·10	−6·88	+6·42	9·41	47·0	350·63
23	355·46	0·13	−6·71	+6·77	9·53	44·7	356·32
24	7·64	0·16	−6·21	+6·74	9·17	42·7	2·38
25	19·82	0·18	−5·39	+6·32	8·31	40·5	8·43
26	32·00	0·21	−4·26	+5·50	6·96	37·8	14·04
27	44·16	0·24	−2·88	+4·32	5·19	33·7	18·80
28	56·32	0·28	−1·30	+2·83	3·12	24·7	22·34
29	68·48	0·31	+0·38	+1·14	1·20	341·6	24·44
30	80·64	0·34	+2·04	−0·62	2·13	253·1	24·94
31	92·79	0·37	+3·57	−2·33	4·26	236·9	23·86
Apr. 1	104·95	0·40	+4·85	−3·86	6·20	231·5	21·31
2	117·11	0·43	+5·81	−5·12	7·75	228·6	17·51
3	129·28	0·46	+6·37	−6·04	8·78	226·5	12·76
4	141·45	0·49	+6·53	−6·60	9·29	224·7	7·40
5	153·63	0·51	+6·27	−6·81	9·25	222·6	1·78
6	165·81	0·54	+5·66	−6·68	8·76	220·3	356·21
7	178·01	0·56	+4·74	−6·25	7·85	217·2	350·97
8	190·20	0·59	+3·59	−5·56	6·61	212·8	346·26
9	202·41	0·61	+2·28	−4·63	5·16	206·2	342·25
10	214·62	0·63	+0·90	−3·51	3·62	194·4	339·04
11	226·83	0·66	−0·49	−2·24	2·29	167·7	336·73
12	239·05	0·68	−1·80	−0·87	2·00	115·8	335·37
13	251·28	0·70	−2·99	+0·55	3·04	79·6	335·05
14	263·50	0·72	−3·99	+1·97	4·45	63·7	335·79
15	275·74	0·74	−4·78	+3·31	5·81	55·3	337·63
16	287·97	0·76	−5·32	+4·52	6·99	49·6	340·57
17	300·20	0·78	−5·63	+5·51	7·88	45·6	344·54
18	312·42	0·80	−5·69	+6·24	8·45	42·4	349·43
19	324·64	−0·82	−5·51	+6·64	8·63	39·7	355·03

Greenwich Midnight.	Selenographical Colong.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat. of the Earth.	Combined Amount.	Direction.	C.
1904.	°	°	°	°	°	°	°
Apr. 20	336·86	−0·85	−5·11	+6·69	8·42	37·4	1·03
21	349·07	0·87	−4·51	+6·35	7·79	35·4	7·05
22	1·28	0·89	−3·73	+5·63	6·75	33·5	12·70
23	13·48	0·92	−2·80	+4·57	5·36	31·5	17·58
24	25·67	0·94	−1·72	+3·21	3·64	28·2	21·40
25	37·85	0·96	−0·54	+1·64	1·73	18·2	23·90
26	50·03	0·99	+0·70	−0·04	0·70	266·7	24·94
27	62·21	1·01	+1·94	−1·72	2·59	228·5	24·47
28	74·38	1·04	+3·11	−3·28	4·52	223·5	22·52
29	86·55	1·06	+4·13	−4·61	6·19	221·9	19·22
30	98·72	1·08	+4·92	−5·64	7·49	221·1	14·81
May 1	110·90	1·10	+5·41	−6·33	8·33	220·5	9·60
2	123·08	1·12	+5·54	−6·65	8·66	219·8	3·95
3	135·27	1·14	+5·30	−6·62	8·48	218·7	358·21
4	147·46	1·16	+4·71	−6·27	7·84	216·9	352·72
5	159·65	1·18	+3·82	−5·64	6·81	214·1	347·73
6	171·86	1·19	+2·68	−4·77	5·47	209·3	343·43
7	184·06	1·21	+1·37	−3·70	3·94	200·3	339·94
8	196·28	1·22	−0·01	−2·48	2·48	179·8	337·34
9	208·50	1·24	−1·38	−1·16	1·80	130·1	335·69
10	220·73	1·25	−2·65	+0·23	2·66	85·0	335·03
11	232·97	1·26	−3·73	+1·63	4·07	66·4	335·44
12	245·20	1·27	−4·57	+2·97	5·45	57·0	336·93
13	257·44	1·28	−5·12	+4·19	6·62	50·7	339·55
14	269·69	1·30	−5·35	+5·23	7·48	45·2	343·25
15	281·93	1·31	−5·26	+6·01	7·99	41·2	347·97
16	294·18	1·32	−4·89	+6·47	8·11	37·1	353·50
17	306·41	1·33	−4·29	+6·57	7·84	33·1	359·54
18	318·65	1·34	−3·52	+6·28	7·20	29·3	5·69
19	330·88	1·36	−2·65	+5·62	6·21	25·2	11·52
20	343·10	1·37	−1·72	+4·61	4·92	20·5	16·62
21	355·32	1·38	−0·78	+3·31	3·40	13·3	20·66
22	7·53	1·40	+0·16	+1·82	1·83	355·0	23·44
23	19·74	1·41	+1·07	+0·21	1·09	281·1	24·81
24	31·94	1·43	+1·95	−1·41	2·41	234·1	24·75
25	44·14	1·44	+2·79	−2·93	4·05	223·6	23·27
26	56·33	1·45	+3·54	−4·27	5·55	219·7	20·46
27	68·51	−1·46	+4·17	−5·34	6·78	218·0	16·48

Greenwich Midnight.	Selenographical Colong. of the Sun.	Selenographical Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Geocentric Libration Lat.	Combined Amount.	Direction.	C.
time.	°	°	°	°	°	°	°
May 28	80·70	−1·47	+4·62	−6·09	7·64	217·2	11·57
29	92·88	1·48	+4·84	−6·49	8·09	216·7	6·06
30	105·07	1·49	+4·78	−6·54	8·10	216·2	0·29
31	117·26	1·49	+4·42	−6·26	7·66	215·2	354·63
June 1	129·46	1·50	+3·74	−5·68	6·80	213·4	349·37
2	141·65	1·50	+2·81	−4·85	5·54	209·0	344·78
3	153·86	1·50	+1·64	−3·82	4·16	203·2	340·97
4	166·07	1·51	+0·32	−2·63	2·65	186·9	338·08
5	178·29	1·51	−1·07	−1·33	1·71	141·2	336·11
6	190·51	1·51	−2·44	+0·03	2·44	89·3	335·14
7	202·74	1·51	−3·68	+1·39	3·93	69·3	335·19
8	214·97	1·51	−4·71	+2·72	5·44	60·0	336·31
9	227·21	1·51	−5·45	+3·95	6·73	54·1	338·51
10	239·46	1·51	−5·83	+5·01	7·69	49·3	341·82
11	251·70	1·51	−5·82	+5·83	8·23	44·9	346·29
12	263·95	1·51	−5·41	+6·36	8·35	40·4	351·52
13	276·20	1·51	−4·66	+6·52	8·02	35·6	357·52
14	288·45	1·51	−3·64	+6·30	7·27	30·0	3·83
15	300·70	1·51	−2·44	+5·67	6·17	23·3	9·94
16	312·95	1·51	−1·18	+4·68	4·83	14·2	15·41
17	325·19	1·51	+0·06	+3·39	3·39	359·0	19·82
18	337·42	1·51	+1·18	+1·90	2·25	327·8	22·94
19	349·65	1·51	+2·18	+0·29	2·20	277·6	24·64
20	1·87	1·52	+3·03	−1·32	3·30	246·5	24·89
21	14·09	1·51	+3·73	−2·83	4·68	232·8	23·72
22	26·30	1·51	+4·28	−4·16	5·97	225·8	21·26
23	38·50	1·51	+4·68	−5·24	7·03	221·8	17·63
24	50·70	1·51	+4·93	−6·01	7·78	219·4	13·03
25	62·90	1·51	+5·00	−6·45	8·16	217·8	7·76
26	75·09	1·50	+4·87	−6·54	8·16	216·7	2·10
27	87·28	1·49	+4·51	−6·31	7·76	215·6	356·41
28	99·48	1·49	+3·91	−5·77	6·97	214·1	351·01
29	111·67	1·48	+3·07	−4·96	5·84	211·8	346·17
30	123·87	1·47	+2·01	−3·95	4·43	207·0	342·08
July 1	136·07	1·46	+0·77	−2·77	2·87	195·5	338·88
2	148·28	1·45	−0·60	−1·48	1·60	157·9	336·62
3	160·49	1·43	−2·01	−0·12	2·01	93·4	335·33
4	172·70	−1·42	−3·39	+1·24	3·61	69·9	335·06

Greenwich Midnight.	Selenographical Colong.	Lat. of the Sun.	Geocentric Libration Sel. Long.	Lat. of the Earth.	Combined Amount.	Direction.	C.
1904.	°	°	°	°	°	°	°
July 5	184·93	−1·41	−4·64	+2·56	5·30	61·1	335·82
6	197·16	1·40	−5·66	+3·79	6·81	56·2	337·62
7	209·39	1·39	−6·37	+4·87	8·02	52·6	340 50
8	221·63	1·37	−6·69	+5·74	8·81	49·4	344·43
9	233·87	1·36	−6·58	+6·33	9·13	46·1	349·36
10	246·12	1·35	−6·01	+6·58	8·91	42·4	355·10
11	258·37	1·34	−5·02	+6·45	8·18	37·9	1 37
12	270·63	1·33	−3·70	+5·91	6·97	32·0	7·69
13	282·88	1·31	−2·15	+4·96	5·41	23·4	13·57
14	295·14	1·30	−0·51	+3·68	3·71	7·9	18·51
15	307·38	1·29	+1·09	+2·15	2·41	333·1	22·16
16	319·63	1·28	+2·54	+0·49	2·59	280·9	24·32
17	331·86	1·27	+3·76	−1·19	3·94	252·4	24·95
18	344·09	1·26	+4·74	−2·76	5·48	239·8	24·09
19	356·31	1·25	+5·44	−4·13	6·83	232·8	21·88
20	8·53	1·23	+5·89	−5·25	7·89	228·3	18·50
21	20·74	1·22	+6·09	−6·05	8·58	225·2	14·15
22	32·95	1·20	+6·06	−6·52	8·90	222·9	9·04
23	45·15	1·19	+5·81	−6·65	8·83	221·1	3·53
24	57·34	1·17	+5·34	−6·45	8·37	219·6	357·89
25	69·54	1·15	+4·68	−5·94	7·56	218·2	352·43
26	81·73	1·13	+3·81	−5·16	6·41	216·4	347·47
27	93·92	1·11	+2·76	−4·16	4·99	213·6	343·13
28	106·11	1·09	+1·55	−2·98	3·36	207·5	339·69
29	118·31	1·06	+0·22	−1·68	1·69	187·5	337·16
30	130·51	1·04	−1·20	−0·32	1·24	104·9	335·60
31	142·71	1·02	−2·62	+1·06	2·83	68·0	335·04
Aug. 1	154·91	1·00	−4·00	+2·41	4·67	58·9	335·50
2	167·12	0·97	−5·24	+3·66	6·39	55·1	336·98
3	179·34	0·95	−6·26	+4·76	7·86	52·8	339·47
4	191·55	0·93	−6·99	+5·68	9·01	50·9	342·98
5	203·78	0·91	−7·33	+6·34	9·70	49·1	347·47
6	216·01	0·88	−7·25	+6·68	9·87	47·3	352·79
7	228·25	0·86	−6·69	+6·67	9·45	45·1	358·78
8	240·49	0·84	−5·67	+6·25	8·44	42·2	5·05
9	252·73	0·82	−4·26	+5·43	6·90	38·1	11·15
10	264·98	0·80	−2·55	+4·22	4·93	31·1	16·58
11	277·23	−0·78	−0·67	+2·71	2·79	13·9	20·87

Greenwich Midnight.	Selenographical Colong.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.	C.
h.	°	°	°	°	°	°	°
Aug. 12	289·48	−0·76	+1·22	+1·00	1·58	309·3	23·69
13	301·72	0·74	+2·99	−0·77	3·09	255·6	24·91
14	313·96	0·72	+4·52	−2·46	5·15	241·4	24·51
15	326·21	0·70	+5·74	−3·96	6·97	235·4	22·62
16	338·43	0·68	+6·60	−5·17	8·39	231·9	19·45
17	350·65	0·66	+7·11	−6·06	9·34	229·6	15·22
18	2·86	0·64	+7·26	−6·60	9·81	227·7	10·22
19	15·06	0·61	+7·10	−6·78	9·82	226·3	4·78
20	27·26	0·59	+6·64	−6·62	9·38	225·1	359·18
21	39·46	0·56	+5·94	−6·16	8·56	224·0	353·68
22	51·65	0·53	+5·02	−5·41	7·39	222·9	348·61
23	63·84	0·51	+3·93	−4·43	5·92	221·6	344·16
24	76·02	0·48	+2·69	−3·27	4·23	219·4	340·49
25	88·21	0·45	+1·36	−1·97	2·39	214·6	337·71
26	100·39	0·42	−0·04	−0·60	0·60	176·2	335·91
27	112·58	0·39	−1·46	+0·81	1·67	61·0	335·09
28	124·76	0·36	−2·85	+2·18	3·59	52·6	335·30
29	136·95	0·33	−4·17	+3·46	5·42	50·3	336·51
30	149·14	0·30	−5·35	+4·61	7·06	49·3	338·73
31	161·34	0·28	−6·32	+5·57	8·43	48·6	341·92
Sept. 1	173·54	0·25	−7·03	+6·28	9·43	48·2	346·04
2	185·74	0·22	−7·41	+6·71	10·00	47·8	351·00
3	197·95	0·20	−7·41	+6·80	10·05	47·5	356·62
4	210·17	0·17	−6·98	+6·53	9·56	46·9	2·64
5	222·39	0·14	−6·12	+5·86	8·48	46·2	8·70
6	234·62	0·12	−4·85	+4·81	6·83	45·2	14·34
7	246·85	0·09	−3·22	+3·41	4·69	43·4	19·12
8	259·09	0·06	−1·36	+1·75	2·22	37·9	22·62
9	271·32	0·04	+0·62	−0·04	0·62	266·3	24·57
10	283·56	0·02	+2·57	−1·83	3·16	234·5	24·86
11	295·80	0·00	+4·33	−3·47	5·55	231·3	23·51
12	308·02	+0·03	+5·81	−4·85	7·57	230·1	20·70
13	320·25	0·05	+6·90	−5·88	9·06	229·6	16·67
14	332·46	0·08	+7·57	−6·54	10·00	229·2	11·74
15	344·67	0·10	+7·81	−6·81	10·37	228·9	6·26
16	356·88	0·13	+7·64	−6·73	10·19	228·6	0·57
17	9·07	0·16	+7·11	−6·32	9·51	228·4	354·98
18	21·26	+0·19	+6·28	−5·63	8·44	228·1	349·78

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat. of the Earth.	Combined Amount.	Direction.	C.
1904.	°	°	°	°	°	°	°
Sept. 19	33·45	+0·22	+5·21	−4·69	7·01	228·0	345·16
20	45·63	0·25	+3·97	−3·56	5·33	228·1	341·30
21	57·81	0·28	+2·62	−2·29	3·48	228·8	338·31
22	69·98	0·31	+1·21	−0·92	1·52	232·8	336·26
23	82·15	0·34	−0·20	+0·48	0·52	22·6	335·21
24	94·32	0·37	−1·57	+1·87	2·44	40·0	335·17
25	106·49	0·40	−2·87	+3·18	4·28	42·1	336·13
26	118·66	0·43	−4·03	+4·36	5·93	42·7	338·12
27	130·83	0·46	−5·05	+5·36	7·37	43·3	341·10
28	143·01	0·48	−5·87	+6·13	8·49	43·8	344·98
29	155·18	0·51	−6·46	+6·62	9·25	44·3	349·70
30	167·36	0·54	−6·79	+6·79	9·60	45·0	355·08
Oct. 1	179·55	0·56	−6·82	+6·61	9·50	45·9	0·87
2	191·74	0·59	−6·51	+6·07	8·90	47·0	6·78
3	203·94	0·61	−5·86	+5·17	7·81	48·6	12·42
4	216·14	0·63	−4·86	+3·94	6·25	51·0	17·39
5	228·35	0·66	−3·52	+2·42	4·27	55·5	21·32
6	240·57	0·68	−1·91	+0·72	2·03	70·2	23·90
7	252·79	0·70	−0·11	−1·06	1·07	174·1	24·92
8	265·01	0·72	+1·75	−2·76	3·27	212·4	24·30
9	277·23	0·74	+3·55	−4·26	5·54	219·8	22·09
10	289·45	0·76	+5·12	−5·46	7·49	223·2	18·48
11	301·67	0·78	+6·36	−6·27	8·93	225·4	13·75
12	313·88	0·80	+7·18	−6·68	9·80	227·1	8·28
13	326·08	0·83	+7·55	−6·71	10·09	228·4	2·45
14	338·28	0·85	+7·45	−6·38	9·81	229·4	356·65
15	350·47	0·87	+6·95	−5·74	9·02	230·4	351·21
16	2·65	0·90	+6·09	−4·85	7·78	231·5	346·35
17	14·83	0·92	+4·97	−3·77	6·24	232·8	342·24
18	27·00	0·95	+3·66	−2·53	4·45	235·3	339·01
19	39·17	0·97	+2·25	−1·20	2·55	241·9	336·72
20	51·33	0·99	+0·83	+0·18	0·85	282·2	335·40
21	63·49	1·02	−0·55	+1·56	1·65	19·4	335·08
22	75·64	1·04	−1·83	+2·88	3·41	32·4	335·79
23	87·80	1·07	−2·95	+4·08	5·04	35·9	337·52
24	99·95	1·09	−3·90	+5·10	6·42	37·4	340·28
25	112·10	1·11	−4·65	+5·91	7·52	38·2	343·99
26	124·25	+1·13	−5·19	+6·44	8·27	38·9	348·57

Greenwich Midnight.	Selenographical Colong.	Selenographical Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Geocentric Libration Lat.	Combined Amount.	Direction.	C.
	°	°	°	°	″	°	°
Oct. 27	136·41	+1·15	−5·52	+6·65	8·64	39·7	353·83
28	148·56	1·17	−5·64	+6·53	8·63	40·8	359·54
29	160·72	1·19	−5·55	+6·06	8·22	42·5	5·40
30	172·89	1·20	−5·25	+5·25	7·42	45·0	11·04
31	185·06	1·22	−4·71	+4·13	6·27	48·7	16·09
Nov. 1	197·24	1·23	−3·94	+2·74	4·80	55·2	20·24
2	209·43	1·25	−2·92	+1·17	3·15	68·2	23·18
3	221·62	1·26	−1·68	−0·51	1·76	106·9	24·73
4	233·82	1·27	−0·26	−2·17	2·19	173·2	24·75
5	246·02	1·28	+1·28	−3·70	3·92	199·1	23·22
6	258·23	1·29	+2·83	−4·97	5·72	209·7	20·22
7	270·43	1·31	+4·25	−5·91	7·28	215·7	15·94
8	282·64	1·32	+5·42	−6·45	8·42	220·0	10·68
9	294·83	1·33	+6·24	−6·59	9·07	223·4	4·84
10	307·03	1·34	+6·64	−6·36	9·20	226·2	358·84
11	319·22	1·36	+6·60	−5·79	8·78	228·7	353·10
12	331·41	1·37	+6·13	−4·95	7·88	231·1	347·89
13	343·59	1·38	+5·30	−3·89	6·58	233·7	343·48
14	355·76	1·40	+4·18	−2·68	4·97	237·3	339·90
15	7·93	1·41	+2·87	−1·38	3·18	244·3	337·31
16	20·09	1·42	+1·46	−0·02	1·46	269·2	335·69
17	32·25	1·44	+0·04	+1·33	1·33	358·3	335·08
18	44·40	1·45	−1·29	+2·64	2·94	26·0	335·49
19	56·55	1·46	−2·47	+3·84	4·57	32·8	336·91
20	68·69	1·47	−3·43	+4·88	5·97	35·2	339·36
21	80·82	1·48	−4·13	+5·71	7·05	35·9	342·82
22	92·96	1·49	−4·58	+6·27	7·76	36·1	347·21
23	105·09	1·50	−4·76	+6·53	8·08	36·1	352·39
24	117·23	1·51	−4·72	+6·44	7·98	36·2	358·12
25	129·36	1·51	−4·48	+6·01	7·50	36·7	4·06
26	141·51	1·51	−4·09	+5·23	6·64	38·0	9·85
27	153·65	1·51	−3·57	+4·14	5·47	40·8	15·07
28	165·80	1·51	−2·94	+2·80	4·06	46·4	19·41
29	177·96	1·52	−2·21	+1·28	2·55	59·9	22·60
30	190·12	1·51	−1·37	−0·32	1·41	103·1	24·46
Dec. 1	202·29	1·51	−0·42	−1·92	1·97	167·7	24·89
2	214·47	1·51	+0·62	−3·41	3·47	190·3	23·88
3	226·65	+1·51	+1·74	−4·69	5·01	200·3	21·44

Greenwich Midnight.	Selenographical Colong.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.	C.
1904.	°	°	°	°	°	°	°
Dec. 4	238·84	+1·51	+2·85	−5·67	6·35	206·7	17·70
5	251·03	1·51	+3·89	−6·29	7·39	211·7	12·86
6	263·22	1·51	+4·75	−6·52	8·07	216·1	7·26
7	275·42	1·51	+5·33	−6·37	8·30	219·9	1·27
8	287·61	1·51	+5·59	−5·87	8·11	223·6	355·32
9	299·80	1·51	+5·46	−5·07	7·45	227·1	349·79
10	311·98	1·51	+4·96	−4·04	6·40	230·8	344·97
11	324·16	1·51	+4·13	−2·83	5·01	235·6	341·02
12	336·33	1·51	+3·02	−1·52	3·38	243·3	338·07
13	348·50	1·51	+1·71	−0·17	1·72	264·3	336·11
14	0·65	1·51	+0·30	+1·19	1·23	345·9	335·17
15	12·82	1·50	−1·10	+2·49	2·72	23·8	335·24
16	24·97	1·50	−2·41	+3·70	4·42	33·1	336·32
17	37·12	1·50	−3·53	+4·75	5·92	36·6	338·42
18	49·26	1·49	−4·39	+5·61	7·12	38·0	341·51
19	61·39	1·49	−4·93	+6·22	7·94	38·4	345·59
20	73·52	1·48	−5·14	+6·53	8·31	38·2	350·53
21	85·65	1·47	−5·01	+6·49	8·20	37·7	356·16
22	97·78	1·46	−4·59	+6·10	7·64	37·0	2·19
23	109·90	1·45	−3·93	+5·34	6·63	36·4	8·20
24	122·03	1·44	−3·10	+4·25	5·26	36·1	13·76
25	134·16	1·43	−2·19	+2·90	3·64	37·1	18·47
26	146·30	1·41	−1·23	+1·36	1·83	42·1	22·00
27	158·44	1·40	−0·29	−0·27	0·40	133·0	24·18
28	170·58	1·38	+0·63	−1·87	1·97	198·6	24·92
29	182·74	1·37	+1·51	−3·36	3·68	204·2	24·22
30	194·90	1·35	+2·35	−4·64	5·20	206·9	22·14
31	207·07	+1·34	+3·13	−5·64	6·45	209·0	18·80

The longitudes are reckoned in the plane of the Moon's equator, the axis of reference being the radius which passes through the mean centre of the visible disc. The axis therefore rotates with the Moon, and is not fixed in space.

The inclination of the Moon's equator to the ecliptic is taken as 1°·523, the value used in the *Nautical Almanac* from 1903.

The physical librations in longitude and latitude, as given by Professor Franz's formulæ, have been applied; their values are taken from the *Berliner Jahrbuch* for the days given there, and interpolated by a graphical method for the other days. But the

signs in the *Jahrbuch* require to be reversed in order to reduce to the system used here.

The colongitude of the Sun is 90° (or 450°) *minus* his selenographical longitude. It also is the selenographical longitude of the morning terminator reckoned eastward from the mean centre of the disc. Hence its value is approximately 270°, 0°, 90°, 180° at new Moon, first quarter, full Moon, last quarter respectively. The longitude of the evening terminator is of course 180° greater or less than that of the morning one.

When the geocentric libration in longitude is positive, the region brought into view is on the west limb; when negative, on the east.

When the geocentric libration in latitude is positive, the region brought into view is at the Moon's north pole; when negative, at the south.

The column "Combined Amount" gives the distance between the apparent and mean centres of the disc, and the column "Direction" gives the position-angle of the apparent centre from the mean centre, or, which is the same thing, the position-angle of the region which is most carried into view by libration. The angles are reckoned eastward from the northern extremity of the Moon's axis.

The question of discontinuing the columns "Combined Amount" and "Direction" is under consideration. I shall be glad to hear whether any observers make use of them. The information contained in them is of course implicitly the same as in the two preceding columns.

C denotes the geocentric position-angle of the northern extremity of the Moon's axis measured eastward from the northernmost point of the disc. It has been computed by the second formula given in the Preface to the *Nautical Almanac*.

The terms "East" and "West" are used throughout with reference to our sky, and not as they would appear to an observer on the Moon.

I give the method for finding the altitude of the Sun at a given point on the Moon whose position is defined: (1) by selenographical longitude and latitude; (2) by direction cosines.

In either case the Sun's selenographical colongitude and latitude (K, L supposed) must be found by interpolation from the ephemeris for the given time.

In the first case let the given point be in the position longitude M, latitude N. Longitudes are reckoned from the meridian passing through the main centre of the disc, and the positive direction is that towards Mare Crisium. North latitudes are considered positive.

Then

$$\sin \text{Sun's altitude} = \sin \mathrm{L} \sin \mathrm{N} + \cos \mathrm{L} \cos \mathrm{N} \sin (\mathrm{K} + \mathrm{M}).$$

In the second case let ξ, η, ζ be the direction cosines of the given point. The axes are (1) that diameter of the Moon's

equator which is 90° from the main centre of the disc ; (2) the Moon's polar axis ; (3) the diameter through the mean centre of the disc. The positive directions are as above. Mr. Saunder has issued some maps of portions of the Moon's surface from which the coordinates ξ, η, ζ can be taken at sight.

Then the Sun's direction cosines are :

$$\cos \mathrm{K} \cos \mathrm{L}, \sin \mathrm{L}, \sin \mathrm{K} \cos \mathrm{L},$$

and sine Sun's altitude

$$= \xi \cos \mathrm{K} \cos \mathrm{L} + \eta \sin \mathrm{L} + \zeta \sin \mathrm{K} \cos \mathrm{L}.$$

Neither formula is convenient when the Sun's altitude is very great, for an angle near 90° cannot be accurately determined from its sine. However, when the Sun is high the shadows are so inconspicuous that it is not necessary to compute his altitude with great accuracy.

Benvenue, 55 *Ulundi Road, Blackheath, S.E.*,
1903 *September* 19.

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

APPENDIX TO VOL. LXIV.

[*From Proceedings of the Royal Society, Vol. LXXII.*]

With indication of the original pagination.

No. 1.

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Professor J. H. Poynting, Radiation in the Solar System: its Effect on Temperature and its Pressure on Small Bodies . . . . . [1]

"Radiation in the Solar System; its Effect on Temperature and its Pressure on Small Bodies." By J. H. POYNTING, Sc.D., F.R.S., Professor of Physics in the University of Birmingham. Received June 16,—Read June 18, 1903.

(Abstract.)

PART I.—*Temperature.*

We can calculate an upper limit to the temperatures of fully absorbing or "black" surfaces receiving their heat from the sun, and on certain assumptions we can find the temperatures of planetary surfaces, if we accept the fourth power law of radiation, since we know approximately the solar constant, that is, the rate of reception of heat from the sun, and the radiation constant, that is, the energy radiated at 1° abs. by a fully radiating surface.*

The effective temperature of space calculated from the very uncertain data at our command is of the order 10° abs. Bodies interplanetary space and at a much higher temperature may, therefore, be regarded as being practically in a zero temperature enclosure except in so far as they receive heat from the sun.

The first case considered is that of an ideal earth, more or less resembling the real earth, and it is shown that the temperature of its surface is, on the average, 325°, 302°, or 290° abs. according as we take for the solar constant Ångström's value 4 cal./min., Langley's value 3 cal./min. or a value deduced from Rosetti's work 2·5 cal./min. The lowest value found, 290° abs., is very near the average temperature of the earth's surface, which may be taken as 289° abs. As the earth's effective temperature must, if anything, be below this, and cannot differ much from that of the ideal planet, Rosetti's value for the solar constant, 2·5 cal./min. or $0{\cdot}175 \times 10^7$ ergs./sec., is probably nearest to the true value and is therefore used in the following calculations.

The preceding calculations may be turned the other way. It is shown that, on certain assumptions, the effective temperature of the sun is 21·5 times that of the ideal earth. If we consider that the real earth with a temperature 289° abs. sufficiently resembles the ideal, we get a solar temperature $21{\cdot}5 \times 289 = 6200°$ abs.

The upper limit to the temperature of the surface of the moon is determined and is shown to be 412° abs. when no heat is conducted inwards. But Langley finds that the actual temperature is not much

* W. Wien ('Cong. Int. de Physique,' 1900, vol. 2, p. 30) has pointed out that Stefan's law enables us to calculate the temperature of celestial bodies which receive their light from the sun, by equating the energy which they radiate to the energy which they receive from the sun, and remarks that the temperature of *Neptune* should be below

above the freezing point on the average. This leads us to the conclusion that it is not higher than four-fifths the highest possible value, the reduction being due to inward conduction.

The temperature of a small body, dimensions of the order of 1 cm. or less, but still so large that it absorbs radiation, is shown to be nearly uniform, and at the distance of the earth from the sun about 300° abs.

Under otherwise similar conditions temperatures must vary inversely as the square root of the distance from the sun. Thus Mars, if an earth-like planet, has a temperature nowhere above 253° abs., and if a moon-like planet, the upper limit to the temperature of the hottest part is about 270°.

Part II.—*Radiation Pressure.*

The ratio of radiation pressure due to sunlight to solar gravitation increases, as is well known, as the receiving body diminishes in size. But if the radiating body also diminishes in size this ratio increases. It is shown that if two equal and fully radiating spheres of the temperature and density of the sun are radiating to each other in a zero enclosure, at a distance large compared with their radii, then the radiation push balances the gravitation pull when the radius of each is 535 metres. If the temperature of two equal bodies is 300° abs. and their density 1, the radius for a balance between the two forces is 19·62 cm. If the density is that of the earth, 5·5, the balance occurs with a radius 3·4 cm. If the temperatures of the two are different, the radiation pressures are different and it is possible to imagine two bodies, which will both tend to move in the same direction, one chasing the other, under the combined action of radiation and gravitation.

The effect of Döppler's principle will be to limit the velocity attained in such a chase. The Döppler effect on a moving radiator is then examined and an expression is found for the increase in pressure on the front, and the decrease in pressure on the back of a radiating sphere of uniform temperature moving through a medium at rest. It is proportional to the velocity at a given temperature. The equation to the orbit of such a body moving round the sun is found, and it is shown that meteoric dust within the orbit of the earth will be swept into the sun in a time comparable with historical times, while bodies of the order of 1 cm. radius will be drawn in in a time comparable with geological periods.

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXIV. DECEMBER 11, 1903. NO. 2

Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

Commander Philip Dumas, R.N., F.R.G.S., H.M.S. 'Duncan,' Mediterranean Squadron;
Henry Eichbaum, 3 Devonshire Terrace, Ventnor, Isle of Wight;
Frank Flowers, Map Office, Ginsberg Chambers, Johannesburg, Transvaal, South Africa; and
Louis George Macrory, M.D., M.B., B.C., B.A., Clifton House, Bridge Road, Battersea, S.W.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

J. D. Bharda, Joint Principal, New High School, Bombay, India (proposed by A. Fowler);
Tyson Crawford, Optician, Messrs. Dollond & Co., 35 Ludgate Hill, E.C. (proposed by W. H. Maw);
E. Vincent Heward, Hereward Cottage, Westgate-on-Sea (proposed by A. C. D. Crommelin);
Eber Jachin Sharpe, ex-Master, Board of Trade, Nautical Surveyor, Newport, Monmouthshire (proposed by M. J. O'Sullivan);
E. Spiegel, Banker, 120 Bishopsgate Street Within, E.C. (proposed by R. T. A. Innes); and
Philip Edward Vizard, Chief Clerk of the Summons and Order Department, Royal Courts of Justice, Belsize Lodge, Belsize Lane, Hampstead, N.W. (proposed by the Hon. Justice Bruce).

Fifty-six presents were announced as having been received since the last meeting, including amongst others :—

A. Auwers, Neue Reduction der Bradley'schen Beobachtungen, Band 1, presented by Professor Auwers ; Carte photographique du Ciel (23 charts), and Loewy and Puiseux, Atlas photographique de la Lune, fasc. 7, presented by the French Minister of Public Instruction ; Four original negatives of the Moon, taken with the Equatorial coudé of the Paris Observatory, presented by M. Puiseux ; Photograph of Langrenus's map of the Moon, presented by M. Loewy ; Portrait of Sir R. S. Ball (enlarged photograph), presented by Sir R. S. Ball.

Note on Photographs of Comet c 1903 (*Borrelly*), *taken with the 30-inch Reflector at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

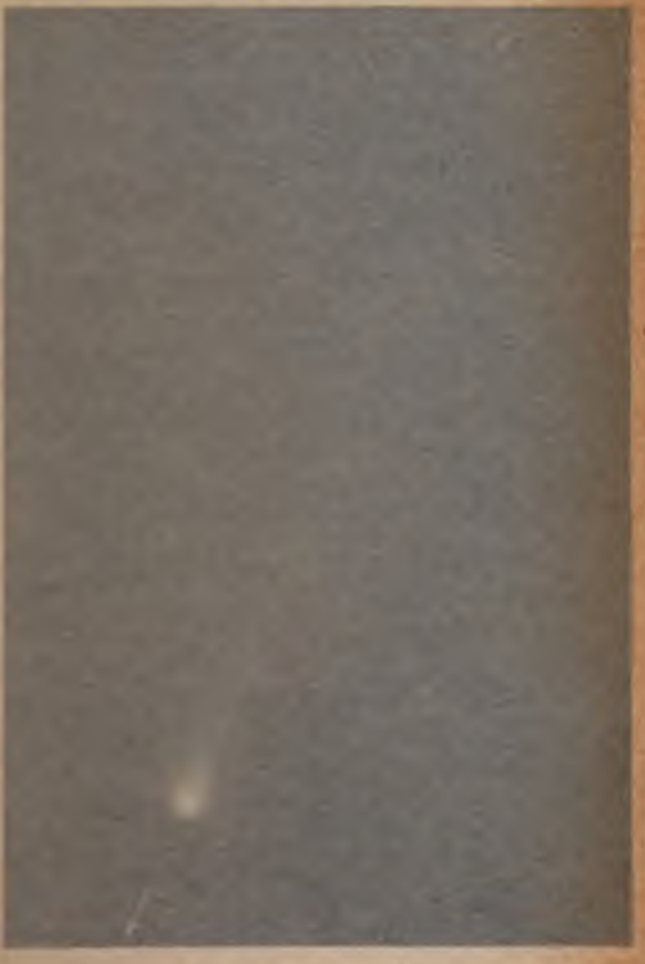
Photographs of this comet for determination of its position have been obtained on twenty-six nights from June 24 to August 14. The instrument was not guided, as the exposures given were short.

The following is a list of the dates on which photographs were obtained and the exposures given :—

Date. 1903.	Exposures.	Date. 1903.	Exposures.
June 24	4^m, 4^m	July 15	30^s, 30^s, 30^s, 30^s, 30^s, 30^s
26	3^m, 3^m, 2^m, 2^m	20	30^s, 30^s, 30^s, 30^s
27	2^m, 2^m, 2^m, 2^m	24	,,
29	2^m, 2^m	26	,,
30	2^m, 2^m, 2^m, 2^m	30	,,
July 1	2^m, 2^m, 2^m, 2^m	Aug. 1	,,
3	1^m, 1^m, 1^m, 1^m	4	,,
6	1^m, 1^m, 1^m, 1^m	5	,,
7	1^m, 1^m, 1^m, 1^m	6	,,
9	1^m, 1^m, 1^m	7	20^s, 20^s, 20^s, 20^s
10	30^s, 30^s, 30^s, 30^s	10	30^s, 30^s, 30^s, 30^s
12	30^s, 30^s, 30^s, 30^s	13	20^s, 20^s, 30^s, 30^s
13	30^s, 30^s, 30^s, 30^s	14	30^s, 30^s, 30^s, 30^s

Photographs with a long exposure, to show the form and structure of the tail, were taken with the 30-inch reflector as follows :—

Date.	Exposure.	Date.	Exposure.
July 20	60^m	July 30	12^m
24	80^m, 30^m	Aug. 1	45^m
26	58^m	4	30^m



COMET 1903 c BORRELLY 1903 AUGUST 1

Eighteen presents were announced as having been received since the last meeting, including amongst others :—

A. Auwers, Neue Reduction der Bradley'schen Beobachtungen, Band 1, presented by Professor Auwers; Carte photographique du Ciel (23 charts), and Loewy and Puiseux, Atlas photographique de la Lune, Isac. 7, presented by the French Minister of Public Instruction; Four original negatives of the Moon, taken with the Equatorial coudé of the Paris Observatory, presented by M. Puiseux; Photograph of Langrenus's map of the Moon, presented by M. Loewy; Portrait of Sir R. S. Ball (enlarged photograph), presented by Sir R. S. Ball.

Note on Photographs of Comet c 1903 (Borrelly), taken with the 30-inch Reflector at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

Photographs of this comet for determination of its position have been obtained on twenty-six nights from June 24 to August 14. The instrument was not guided, as the exposures given were short.

The following is a list of the dates on which photographs were obtained and the exposures given :—

Date, 1903.	Exposures.	Date, 1903.	Exposures.
June 24	2ᵐ, 2ᵐ	July 15	30ˢ, 30ˢ, 30ˢ, 30ˢ, 30ˢ, 30ˢ
26	3ᵐ, 3ᵐ, 3ᵐ, 3ᵐ	20	30ˢ, 30ˢ, 30ˢ, 30ˢ
27	4ᵐ, 4ᵐ, 4ᵐ, 4ᵐ	24	"
29	2ᵐ, 2ᵐ	28	"
30	2ᵐ, 2ᵐ, 2ᵐ, 2ᵐ	30	"
July 1	2ᵐ, 2ᵐ, 2ᵐ, 2ᵐ	Aug. 1	"
3	1ᵐ, 1ᵐ, 1ᵐ, 1ᵐ	4	"
6	1ᵐ, 1ᵐ, 1ᵐ, 1ᵐ	5	"
7	1ᵐ, 1ᵐ, 1ᵐ, 1ᵐ	6	"
9	1ᵐ, 1ᵐ, 1ᵐ	7	20ˢ, 20ˢ, 20ˢ, 20ˢ
10	30ˢ, 30ˢ, 30ˢ, 30ˢ	10	30ˢ, 30ˢ, 30ˢ, 30ˢ
12	30ˢ, 30ˢ, 30ˢ, 30ˢ	13	20ˢ, 20ˢ, 30ˢ, 30ˢ
13	30ˢ, 30ˢ, 30ˢ, 30ˢ	14	30ˢ, 30ˢ, 30ˢ, 30ˢ

Photographs with a long exposure, to show the form and structure of the tail, were taken with the 30-inch reflector as follows :—

Date.	Exposure.	Date.	Exposure.
July 20	80ᵐ	July 30	140ᵐ
24	80ᵐ, 30ᵐ	Aug. 1	45ᵐ
26	58ᵐ	4	58ᵐ

COMET 1903 *f* BORRELLY 1903 AUGUST 1.

Photographed with the 30-Inch Reflector at the Royal Observatory, Greenwich.

Exposure 45 Minutes.

A photograph on a smaller scale was also obtained with the 4-inch Dallmeyer rapid rectilinear lens of 34 inches focus on July 26, exposure 125^m. The most successful photographs with the reflector are the one taken with an exposure of 80^m on July 24 and the one taken on August 1, which is reproduced in Plate 1. The photograph taken on August 1 shows a sharply defined head with nine distinct tails, also sharply defined, the longest extending to the edge of the field, a distance of 125′.

On the Semi-diameter, Parallactic, Inequality, and Variation of the Moon from Greenwich Meridian Observations, 1847·0 to 1901·5. By P. H. Cowell.

In the following paper an account is given of an investigation of the solar parallax from observations of the Moon.

The material used consists of all Greenwich meridian observations with the transit circle from 1847·0 to 1901·5. This interval consists of forty-eight periods of 400 lunar days. Each period of 400 lunar days is treated separately, and 400 lunar days are approximately equal to fifteen anomalistic months and fourteen lunations. Associated, therefore, with any particular value of D, the age of the Moon, there will be found in one period fourteen values of g, the mean anomaly, uniformly distributed round 360°, so that certain classes of error should be to a great extent eliminated in each period.

The tabular places used are based upon Hansen's tables with Newcomb's corrections, and Hansen's error of sign corrected. From 1847–1861 the tabular places are taken from vol. l. of the *Monthly Notices*, and Newcomb's corrections have been applied. From 1862 onwards the tabular places have been taken from the Greenwich annual volumes, and from 1862–1882 Newcomb's corrections and a correction for Hansen's error of sign have been applied.

After this all tabular places received the following corrections:

$$
\begin{aligned}
&+1''\cdot362 \sin D\\
&-0''\cdot893 \sin (g+2\varpi-2J)\\
&-0''\cdot676 \sin (g+2\varpi+3V-5E)\\
&-0''\cdot548 \sin g\\
&+\{N+\Delta g+4''\cdot06 \sin \Omega\}\ 2e \cos g
\end{aligned}
$$

where in the last term N denotes Newcomb's corrections, and is therefore to be taken as zero from 1883 onwards; Δg is a correction to the mean anomaly investigated by myself; its value is $\Delta g = +4''\cdot63-13''\cdot99T+4''\cdot743T^2$ (see *Monthly Notices*,

1903 November) where the rest of the notation and the source of the corrections can be found. It is not necessary to repeat them here.

The present determination is therefore based upon 5647 observations, each observation being compared with a corrected theory, and therefore subject to a less accidental error; for a periodic error in the tabular places is of the nature of an accidental error when terms of a different period are under discussion (unless the argument of the error and the argument under discussion differ by a low multiple of D, the mean elongation). I believe, therefore, that the present discussion is based upon material of greater weight than any previously employed in deducing the solar parallax from lunar observations.

Twice the circumference of the circle, or 720°, has been divided into fifty-seven equal parts; and the observations grouped under the corresponding values of D. The fifty-seventh part of 720° is nearly equal to the movement of D in one lunar day. The difference between the true value of D and the value under which the observation is grouped is not necessarily less than half of 360°÷57; for in order to make the grouping mechanical the observations are not always grouped under the nearest value of D in the series of 57. The error may, in fact, amount to 7°, and its average value will be about 2°·5.

The periods of 400 lunar days are numbered from 86 to 133 because period No. 1 begins, with Airy's reductions, on 1750 September 13.

For the sake of greater clearness of explanations I give here the figures for period 86.

TABLE I.

Period 86.

Ref. No.	Value of D.	Sum of Errors in units of one-tenth of second of arc.	No. of observations.	Ref. No.	Value of D.	Sum of Errors in units of one-tenth of second of arc.	No. of observations.
	°				°		
0	0·0			12	151·6	−45	5
1	12·6			13	164·2	+ 4	4
2	25·3			14	176·8	− 5	3
3	37·9			15	189·5	+67	3
4	50·5	−12	1	16	202·1	+56	4
5	63·2			17	214·7	+42	1
6	75·8	+11	2	18	227·4	+ 7	2
7	88·4	+ 8	2	19	240·0	−10	1
8	101·0	−30	2	20	252·6	+61	3
9	113·7	+43	6	21	265·3	− 2	5
10	126·3	− 7	6	22	277·9	+35	2
11	139·0	+12	3	23	290·5	−14	4

Ref. No.	Value of D.	Sum of Errors in units of one-tenth of second of arc.	No. of observations.
	°		
24	303·2	−15	2
25	315·8		
26	328·4		
27	341·0		
28	353·7		
29	6·3		
30	19·0		
31	31·6		
32	44·2		
33	56·8		
34	69·5		
35	82·1	− 11	3
36	94·7	+ 14	3
37	107·4	+ 82	3
38	120·0	− 14	3
39	132·6	+ 8	5
40	145·3	+ 33	3
41	157·9	+ 124	5
42	170·5	− 10	4
43	183·2	+ 43	4
44	195·8	+ 46	2
45	208·4	+ 43	4
46	221·0	+ 29	2
47	233·7	+ 74	4
48	246·3	− 20	2
49	259·0	+ 32	3
50	271·6	+ 3	2
51	284·2	+ 57	4
52	296·8	+ 13	1
53	309·5		
54	322·1		
55	334·7		
56	347·4		

Since the mean error is +6·4 (in units of 0″·1) all errors have been decreased by 6, so as to render this period comparable with other periods. Line 9 therefore reads six errors with a sum +7.

It will be noted that the value of D is proportional to the reference number.

The periods are now arranged in six groups of eight periods each. The following table gives the results for each group and the sums for the whole forty-eight periods, or 54·5 years. The last column but one gives the mean error corresponding to the separate values of D, and the last column gives the same quantities corrected for a change of 0″·45 in semi-diameter, which appears to be indicated by the three preceding columns. Two lines nearest to full Moon are not filled in, as the observations do not always correspond to the same limb.

Table III. that follows gives the values of sin D, cos D, sin 2D, cos 2D, cos 3D, cos 4D for the different values of D: these quantities are required for a least-squares solution. Also in the same table are given the products of some of the above quantities by "*n*," the number of observations in the group, and Σ, the sum of the errors of the group.

The sums of lines 0–14, 15–28, 29–42, 43–56 are also taken for certain columns under the titles Σ_1, Σ_2, Σ_3 and Σ_4.

In the last two lines Σ denotes $\Sigma_1+\Sigma_2+\Sigma_3+\Sigma_4$, and Σ′ denotes $\Sigma_1-\Sigma_2+\Sigma_3-\Sigma_4$.

TABLE

Periods.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.
	88–93.		94–101.		102–109.		110–117.	
0	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...
3	− 9	1	+ 39	1	...	...	...	...
4	− 89	8	− 54	9	+ 50	3	− 45	4
5	−179	14	−130	8	+ 72	10	+ 50	12
6	− 70	21	−338	15	− 79	17	−184	20
7	− 43	26	−140	25	− 18	19	−142	24
8	− 92	24	−184	31	−203	32	−176	32
9	− 83	31	−110	29	+ 59	28	−243	32
10	−188	30	−170	34	−121	31	+ 28	32
11	−136	25	−213	33	−186	32	+ 36	30
12	−280	30	−305	33	− 74	32	+ 32	28
13	− 6	29	−249	33	− 60	27	+229	34
14	−198	26	−162	30	− 88	36	+ 48	27
15	−238	25	+219	32	− 84	29	+178	30
16	−292	30	+240	33	+119	36	+ 86	24
17	−371	23	+414	31	+ 10	22	+211	26
18	−176	19	+275	21	+180	22	− 51	18
19	+ 38	24	+250	20	+234	26	+ 30	19
20	+ 93	28	+213	17	+ 30	10	− 61	12
21	+155	23	+ 96	13	−135	15	+ 22	13
22	+158	21	+188	17	− 41	16	+ 8	14
23	+ 46	18	+205	10	+157	9	+146	15
24	+ 84	16	+ 47	8	+ 17	6	− 86	7
25	...	...	+ 9	1	− 11	2	...	...
26	...	...	...	...	...	...	...	...
27	...	...	...	...	...	...	...	...
28	...	...	...	...	...	...	...	...
29	...	...	...	...	...	...	...	...
30	...	...	...	...	...	...	...	...
31	...	...	...	...	...	...	...	...
32	− 55	8	+ 76	5	+ 80	7	...	...
33	−164	17	− 31	6	+ 55	8	− 48	10
34	−117	21	−154	19	− 80	13	− 86	12
35	−305	17	−214	24	− 43	18	−130	18

II.

Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Mean in units of $0''.01$.	Value of $\pm 0''.346 + 0''.120 \sin D - 0''.030 \sin 2D$ in units of $0''.01$.
118–125.		126–133.		86–133.			
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	+ 30	2	...	...
− 2	5	− 67	4	− 177	33	− 54	+41
− 11	8	−207	19	− 405	71	− 57	+43
− 59	23	− 73	18	− 803	114	− 70	+45
+100	22	−139	17	− 382	133	− 27	+46
− 68	28	− 27	29	− 650	176	− 37	+48
+ 60	23	−214	25	− 471	168	− 28	+48
+ 24	17	−105	26	− 532	170	− 31	+47
− 22	29	−199	27	− 700	176	− 40	+46
+ 36	27	−269	29	− 900	179	− 50	+43
− 11	18	−140	32	− 357	173	− 21	+39
− 66	31	−120	36	− 466	185	− 25	+36
+157	28	+303	31	+ 991	175	+ 57	−38
+158	25	+291	33	+1156	183	+ 63	−41
+ 91	20	+252	29	+1369	151	+ 91	−44
+130	22	+ 74	27	+ 704	129	+ 55	−47
+ 16	20	+158	15	+ 735	126	+ 58	−48
− 85	14	+186	15	+ 385	106	+ 36	−48
−150	17	+107	24	+ 13	117	+ 1	−47
+ 76	14	+ 40	13	+ 409	106	+ 39	−46
− 33	16	+ 96	12	+ 538	89	+ 60	−44
+ 39	4	+ 30	6	+ 91	41	+ 22	−42
− 18	1	+ 53	1	+ 33	4	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	+ 2	1	...	...
− 1	3	− 50	4	− 71	13	− 55	+40
−221	9	− 44	13	− 297	50	− 59	+42
− 58	18	−121	17	− 553	100	− 55	+44
−277	15	−144	19	− 934	118	− 79	+46

Periods.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.
	86—93.		94—101.		102—109.		110—117.	
16	−270	23	− 96	24	+ 45	22	−112	20
17	+ 30	22	−240	27	+112	35	− 76	25
18	−430	25	−242	25	−235	38	− 4	26
19	− 81	20	−273	28	−105	28	−121	25
20	−175	27	−328	34	− 60	36	− 11	33
21	73	36	−206	30	−156	31	0	21
22	102	33	263	37	− 1	35	+104	35
23	+250	23	− 62	31	− 20	27	− 91	33
24	+284	27	+323	32	− 34	27	+139	23
25	+243	26	−143	19	−111	24	−177	28
26	+230	26	−128	32	−236	30	−183	19
27	+106	22	+286	21	− 30	15	− 6	17
28	+178	29	+176	32	− 72	26	− 75	21
29	+ 81	26	+212	28	− 47	22	− 89	15
30	+ 22	20	+ 36	21	− 55	17	− 20	17
31	+ 55	28	+ 68	25	− 28	20	− 20	11
32	+ 11	7	+ 12	12	− 25	4	− 27	10
33	+ 28	5	+ 10	4	...	...	− 4	4
34	+ 28	1	+ 2	1	...	...	...	—
35					...	...	...	—
36					...	...	...	—
Total	+ 2	482	− 84	1002	− 72	652	−1175	876

Table

									− 180
									− 1
									− 6
									42
									100
									133
									224
									−31

Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Mean in units of o″·01.	Value of ±o″·346 +o″·120 sin D −o″·030 sin 2D in units of o″·01.
218–225.		126–133.		86–133			
+ 55	21	−182	25	− 566	135	− 42	+47
+ 101	27	−207	31	− 271	167	− 16	+48
−130	30	−215	32	−1256	176	− 71	+48
− 47	28	−189	31	− 818	169	− 48	+47
−251	28	−296	38	−1121	196	− 57	+44
+ 16	28	− 29	26	− 450	172	− 26	+41
+ 187	28	−125	40	− 200	200	− 10	+38
+ 25	30	− 47	31	+ 219	187	+ 12	−36
− 47	25	+278	37	+ 941	181	+ 52	−39
− 58	21	+235	27	+ 871	147	+ 59	−43
− 68	25	+214	32	+1024	155	+ 66	−46
+ 43	17	+232	29	+ 733	124	+ 59	−47
− 44	19	+139	19	+ 385	143	+ 27	−48
+ 9	12	− 11	18	+ 273	110	+ 25	−48
− 4	16	+158	23	+ 447	114	+ 39	−46
+ 42	12	+203	27	+ 581	104	+ 56	−45
− 14	5	+ 82	11	+ 257	52	+ 42	−43
+ 18	2	+ 29	4	+ 278	24	+116	−41
—	...	...	...	+ 15	2	...	...
—	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
−142	830	− 60	1002	+ 100	5647	...	...

III.

100 cos 3D.	n cos 3D.	100 cos 4D.	n cos 4D.	E.	E sin D.	E sin 2D.
+100		+100		...	...	...
79		+ 64		...	...	...
+ 25		− 19		...	...	...
− 40	− 1	88	− 2	+ 30	+ 18	+ 29
88	29	93	31	− 177	− 136	− 173
99	70	− 30	− 21	− 405	360	328
68	78	+ 55	+ 63	− 803	779	385
− 8	−11	99	132	− 382	382	− 23
+ 55	+97	+ 72	+127	− 650	637	+ 247
95	160	− 8	− 13	− 471	433	349
95	162	82	139	− 532	431	505

n,	Ref. No.	100 sin D.	n sin D.	100 cos D.	n cos D.	100 sin 2D.	n sin 2D.	100 cos 2D.	n cos 2D.
176	11	66	116	75	132	99	174	+ 14	+ 25
170	12	48	86	88	158	84	150	55	98
173	13	27	47	96	166	52	90	85	147
185	14	+ 6	+ 11	100	− 185	− 11	− 20	99	+ 183
175	15	− 16	− 28	99	− 173	+ 32	+ 56	95	+ 166
183	16	38	70	93	170	70	128	72	132
151	17	57	86	82	124	94	142	+ 35	+ 53
129	18	74	95	68	88	100	129	− 8	− 10
126	19	87	110	50	63	87	110	50	63
106	20	95	101	30	32	57	60	82	87
117	21	100	117	− 8	− 9	− 16	− 19	99	116
100	22	98	100	+ 14	+ 15	− 27	− 28	96	102
89	23	94	84	93	83	96	86	75	67
41	24	84	34	95	39	93	38	− 41	− 16
4	25	72	3	72	+ 3	100	− 4	+ 3	0
	26	52		85		89		45	
	27	32		95		21		72	
	28	11		98		− 11		96	
	29	− 11		98		− 32		96	
	30	− 31		95		− 51		77	
1	31	− 50	+ 1	87	+ 1	− 68	− 1	63	0
13	32	− 67	9	74	10	− 83	− 11	− 53	− 8
36	33	− 81	29	59	21	− 94	− 34	− 34	− 12
72	34	− 91	66	41	30	− 99	− 72	17	12
117	35	− 97	114	21	25	− 98	− 115	91	107
164	36	− 99	163	0	0	− 91	− 150	164	0
203	37	− 97	197	− 21	− 43	− 77	− 156	203	41
224	38	− 91	204	− 41	− 92	− 56	− 125	224	50
224	39	− 81	181	− 59	− 132	− 30	− 67	224	50
203	40	− 68	138	− 74	− 150	0	0	203	0
164	41	− 52	85	− 87	− 143	+ 30	+ 49	164	− 27
117	42	− 34	40	− 95	− 111	+ 56	+ 65	117	− 65
72	43	− 15	11	− 99	− 71	+ 77	+ 55	72	− 55
36	44	+ 4	+ 1	− 98	− 35	+ 91	+ 33	36	− 33
13	45	+ 23	3	− 95	− 12	+ 98	+ 13	13	− 13
1	46	+ 41	0	− 87	− 1	+ 99	+ 1	1	− 1
4	47	+ 58	+ 2	− 74	− 3	+ 94	+ 4	4	− 4
41	48	+ 73	30	− 59	− 24	+ 83	+ 34	41	− 33
89	49	+ 85	76	− 41	− 37	+ 68	+ 61	89	− 73
151	50	+ 93	140	− 21	− 32	+ 50	+ 75	151	− 75
224	51	+ 98	220	0	0	+ 30	+ 67	224	0
289	52	+ 99	287	+ 21	+ 61	+ 11	+ 32	289	+ 64
324	53	+ 95	308	+ 41	+ 133	− 11	− 36	324	+ 117
324	54	+ 87	282	+ 59	+ 191	− 30	− 97	324	+ 162
289	55	+ 74	214	+ 74	+ 214	− 50	− 144	289	+ 144
224	56	+ 59	132	+ 87	+ 195	− 68	− 152	224	+ 109
151	57	+ 41	62	+ 95	+ 144	− 83	− 126	151	+ 72
89	58	+ 23	20	+ 99	+ 88	− 94	− 84	89	+ 39
41	59	+ 4	+ 2	+ 98	+ 40	− 98	− 40	41	+ 16
13	60	− 15	2	+ 95	+ 12	− 99	− 13	13	+ 13
1	61	− 34	0	+ 87	+ 1	− 94	− 1	1	+ 1
4	62	− 52	− 2	+ 74	+ 3	− 87	− 4	4	+ 4
36	63	− 68	− 24	+ 59	+ 21	− 77	− 27	36	+ 27
72	64	− 81	− 58	+ 41	+ 30	− 59	− 43	72	+ 43
117	65	− 91	− 107	+ 21	+ 25	− 36	− 42	117	+ 42
164	66	− 97	− 159	0	0	− 11	− 18	164	0
203	67	− 99	− 201	− 21	− 43	+ 11	+ 23	203	− 23
224	68	− 97	− 218	− 41	− 92	+ 30	+ 67	224	− 50
224	69	− 91	− 204	− 59	− 132	+ 56	+ 125	224	− 125
203	70	− 81	− 164	− 74	− 150	+ 77	+ 156	203	− 156
164	71	− 68	− 112	− 87	− 143	+ 94	+ 154	164	− 154
117	72	− 52	− 61	− 95	− 111	+ 99	+ 116	117	− 116
72	73	− 34	− 24	− 99	− 71	+ 98	+ 71	72	− 71
36	74	− 15	− 5	− 98	− 35	+ 91	+ 33	36	− 33
13	75	+ 4	+ 1	− 95	− 12	+ 83	+ 11	13	− 11
1	76	+ 23	0	− 87	− 1	+ 68	+ 1	1	− 1
4	77	+ 41	+ 2	− 74	− 3	+ 50	+ 2	4	− 2
41	78	+ 58	24	− 59	− 24	+ 30	+ 12	41	− 12
89	79	+ 73	65	− 41	− 37	+ 11	+ 10	89	− 10
151	80	+ 85	128	− 21	− 32	− 11	− 17	151	+ 17
224	81	+ 93	208	0	0	− 30	− 67	224	0
289	82	+ 98	283	+ 21	+ 61	− 50	− 144	289	+ 144
324	83	+ 99	321	+ 41	+ 133	− 68	− 220	324	+ 220
324	84	+ 95	308	+ 59	+ 191	− 83	− 269	324	+ 269
289	85	+ 87	251	+ 74	+ 214	− 94	− 272	289	+ 272
224	86	+ 74	166	+ 87	+ 195	− 99	− 222	224	+ 222
151	87	+ 59	89	+ 95	+ 144	− 98	− 148	151	+ 148
89	88	+ 41	37	+ 99	+ 88	− 95	− 85	89	+ 85
41	89	+ 23	9	+ 98	+ 40	− 87	− 36	41	+ 36
13	90	+ 4	+ 1	+ 95	+ 12	− 74	− 10	13	+ 10

$\frac{\text{true}}{\cos 3D.}$	$\frac{n}{\cos 3D.}$	$\frac{\text{true}}{\cos 4D.}$	$\frac{n}{\sin 4D.}$	E.	$\frac{E}{\sin D.}$	$\frac{E}{\sin 2D.}$
+ 55	+ 97	96	169	− 700	462	693
− 8	− 14	− 40	− 72	− 900	432	756
68	118	+ 45	+ 78	− 357	96	186
99	−183	98	+ 181	− 466	− 28	+ 51
88	−154	79	+ 138	+ 991	− 159	+ 317
− 40	− 73	+ 3	+ 5	+ 1156	439	809
+ 25	+ 38	− 75	−113	+ 1369	780	1287
79	102	99	128	+ 704	521	704
100	126	− 50	− 63	+ 735	639	639
79	84	+ 35	+ 37	+ 385	366	219
+ 25	+ 29	95	111	+ 13	13	+ 2
− 40	− 42	85	90	+ 409	405	− 110
88	78	+ 14	+ 12	+ 538	506	355
99	41	− 68	− 28	+ 91	76	84
68	− 3	100	− 4	+ 33	− 23	− 33
− 8		− 59		...	...	...
+ 55		+ 25		...	...	...
95		90		...	...	...
95		90		...	...	...
+ 55		+ 25		...	...	...
− 8	0	− 59	− 1	+ 2	+ 1	+ 2
68	− 9	100	13	− 71	− 50	− 71
99	50	− 68	− 34	− 297	249	273
88	88	+ 14	+ 14	− 553	520	365
− 40	− 47	85	100	− 934	925	− 252
+ 25	+ 34	95	128	− 566	566	+ 91
79	132	+ 35	+ 58	− 271	257	154
100	176	− 50	− 88	− 1256	1093	1093
79	134	99	167	− 818	605	818
+ 25	+ 49	− 75	−147	− 1121	639	1054
− 40	− 69	+ 3	+ 5	− 450	171	315
88	−176	79	+ 158	− 200	− 32	+ 64
99	−185	98	+ 183	+ 219	− 13	+ 24
68	123	+ 45	+ 81	+ 941	254	489
− 8	− 12	− 40	− 59	+ 871	418	732
+ 55	+ 85	96	149	+ 1024	676	1014
95	118	82	102	+ 733	594	696
95	136	− 8	− 11	+ 385	354	285
+ 55	+ 60	+ 72	+ 79	+ 273	268	+ 104

	Ref. No.	100 sin D.	n sin D.	100 cos D.	n cos D.	100 sin 2D.	n sin 2D.	100 cos 2D.	n cos 2D.
114	50	100	114	+ 3	+ 3	− 6	− 7	100	114
104	51	97	101	25	26	48	50	88	92
52	52	89	46	45	23	81	42	59	31
24	53	77	18	64	15	98	24	− 19	− 5
2	54	61	− 1	79	+ 2	97	− 2	+ 25	0
	55	43		90		77		+ 64	
	56	− 22		+ 98		− 43		+ 90	
Σ_1 1580	...	+ 1058		− 753		− 632		− 156	
Σ_2 1227	...	− 833		− 587		+ 514		− 110	
Σ_3 1497	...	+ 1044		− 693		− 649		− 198	
Σ_4 1343	...	− 853		− 688		+ 532		− 36	
	...	...		...		...		...	
Σ + 5647	...	+ 426		− 2721		− 235		− 500	
Σ' + 507	...	+ 3788		...		− 2327		...	

The hypothesis is now made that E, the sum of the errors in any group, may be equated to

$$n\{\epsilon\Sigma_1\mu + \delta_1 \sin D + \delta_2 \sin 2D\}$$

where

ϵ represents the mean error;
μ, the error of semi-diameter, which is here supposed constant;
δ_1, a correction to the parallactic coefficient;
δ_2, a correction to the variation coefficient.

Let the observations now be divided into four groups 0–14, 15–28, 29–42, 43–56, and let Σ_1, Σ_2, Σ_3, Σ_4 denote summations over these four groups; then the normal equations are:

$$5647\epsilon + 507\mu + (\Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4)(\delta_1 \sin D + \delta_2 \sin 2D) = + 100$$
$$507\epsilon + 5647\mu + (\Sigma_1 - \Sigma_2 + \Sigma_3 - \Sigma_4)(\delta_1 \sin D + \delta_2 \sin 2D) = - 24796$$
$$\epsilon(\Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4)\sin D + \mu(\Sigma_1 - \Sigma_2 + \Sigma_3 - \Sigma_4)\sin D + \delta_1 \Sigma \sin^2 D + \delta_2 \Sigma \sin D \sin 2D = - 17231$$

and

$$\epsilon(\Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4)\sin 2D + \mu(\Sigma_1 - \Sigma_2 + \Sigma_3 - \Sigma_4)\sin 2D + \delta_1 \Sigma \sin D \sin 2D + \delta_2 \Sigma \sin^2 2D = + 10475$$

The calculations of Table III., remembering that

$$\sin^2 D = \tfrac{1}{2}(1 - \cos 2D)$$
$$\sin^2 2D = \tfrac{1}{2}(1 - \cos 4D)$$
$$\sin D \sin 2D = \tfrac{1}{2}(\cos D - \cos 3D)$$

100 / cos 3D.	s / cos 3D.	100 / cos 4D.	s cos 4D.	E.	E sin D.	E sin 2D.
− 8	− 9	99	113	+ 447	447	− 27
68	71	+ 55	+ 57	+ 581	564	279
99	51	− 30	− 16	+ 257	229	208
88	21	93	22	+ 278	214	272
− 40	− 1	88	− 2	+ 15	− 9	− 15
+ 25		− 19		...	...	...
+ 79		+ 64		...	...	...
+ 12		+ 134		− 5813	− 4158	+ 1907
− 12		+ 57		+ 6424	− 3927	+ 3395
+ 86		+ 13		− 6535	− 5106	+ 2630
− 74		+ 152		+ 6024	− 4040	+ 2543
...		...		...	...	...
+ 12		+ 356		+ 100	− 17231	+ 10475
—		...		− 24796	...	...

enable us to write the normal equations in the numerical form

$$\begin{aligned} 5647\epsilon + 507\mu + 416\delta_2 - 235\delta_3 &= +100 \\ 507\epsilon + 5647\mu + 3788\delta_2 - 2327\delta_3 &= -24796 \\ 416\epsilon + 3788\mu + 3074\delta_1 - 1366\delta_2 &= -17231 \\ -235\epsilon - 2327\mu - 1366\delta_1 + 2645\delta_3 &= +10475 \end{aligned}$$

We do not want the value of ϵ, and it is clear that we may drop the first equation and the terms containing ϵ in the others.

Solving the three equations that remain, I obtain

$$\begin{aligned} -\mu &= +0''{\cdot}346 \\ +\delta_1 &= -0''{\cdot}120 \\ +\delta_2 &= +0''{\cdot}030 \end{aligned}$$

or the tabular semi-diameter requires a correction

$$-0''{\cdot}346$$

The tabular parallactic coefficient requires a correction

$$+1''{\cdot}562 + 0''{\cdot}120$$

(of which the first term had been applied to the individual observations before grouping).

The tabular variation coefficient requires a correction

$$-0''{\cdot}030$$

About this method of treatment the following points are to be remarked:

(1) The observations of line 14 are not all first-limb observations, nor those of line 43 all second-limb. It has not been thought worth while to undertake the labour of applying the necessary correction. Probably not more than fifty observations, or less than 1 per cent., have been assigned to the wrong limb.

(2) The hypothesis that the error of semi-diameter is a constant is the best that I can make. It will be noticed that the observations grouped in line 7, for example, contain some daylight and some night observations, according to the season of the year. I am not concerned with any difference in the correction applicable to the individual observations of this group, but merely to the mean correction. The mean correction for different ages of the Moon is no doubt a function of the age of the Moon and not a constant. If there were any means of obtaining its numerical values they could be analysed into the form $\beta(D) + \alpha \sin D + \beta \sin 2D$, when α, β would be corrections applicable to the results of this paper. As far as I can see the difficulty is inherent in this method of finding the solar parallax, which is rendered uncertain by an amount $\frac{1}{2}\alpha$. I do not think that α can possibly be as large as 2″·30, and it is possibly a good deal smaller, so that the parallax deduced in this paper should not be as much as 0″·02 in error.

The weights of the values found for p_1, k_1 and k_2 are 781, 3·13,* and 2·14 times the weight of a single observation.

The pure error of observation has been found from the transit circle and altazimuth to have a probable amount 0″·75. But we have to deal here with the probable discordance from the tables as corrected for the terms set out at the beginning of this paper. Perhaps 1″·3 is a fair estimate. If so, the probable error of this determination of the parallactic coefficient is

$$\frac{1''\cdot 3}{\sqrt{781}} = 0''\cdot 047$$

and of the resulting solar parallax 0″·085.

Having now completed the investigation of the correction required by Hansen's parallactic terms, it remains to deduce the value of the solar parallax.

Newcomb (*Astronomical Constants*, p. 102) gives the following equation:

$$\text{Parallactic Coefficient} = 0''\cdot 1245 \frac{\text{Solar Parallax}}{8''\cdot 790}$$

This equation assumes the mass of the Moon to be $\frac{1}{81\cdot 45}$

[illegible] the mass is in the ratio of [illegible] the error of [illegible] observations are [illegible] to unity, if the difficulty did not [illegible]

times that of the Earth ; and as Newcomb considers the denominator of this fraction uncertain by only $\pm 0''.15$ (p. 189), this introduces an inappreciable uncertainty into the value of the solar parallax.

In deducing this equation, however, Newcomb assumes that Hansen's theory is correct, whereas he has himself thrown doubt upon this very point. I have transformed Brown's results (*Memoirs R.A.S.* vol. liii. pp. 89 and 96), and I find that they confirm Delaunay's theory with Newcomb's estimate of the correction on account of terms of high order not calculated by Delaunay, and Hansen is therefore $0''.55$ in error. I therefore obtain the above equation in the revised form :

$$\text{Parallactic Coefficient} = -125''.2 \frac{\text{Solar Parallax}}{8''.790}$$

Hansen's tables and Hansen's theory differ by $0''.033$ in the coefficient of the parallactic term for the same value of the solar parallax ; and Hansen's theory leads to a coefficient of $-122''.032$ (Newcomb, *Astron. Papers Amer. Eph.* vol. i.), and Hansen's tables consequently to

$$-122''.065 \times 1.03573 = -126''.43$$

The coefficient with which the observations have been compared is therefore

$$-126''.43 + 1''.56 = -124''.87$$

and the further correction deduced in this paper alters this to

$$-124''.87 + 0''.120 = -124''.75$$

The solar parallax is therefore

$$8''.76$$

The correction to the mean semi-diameter used since 1847 is $-0''.346$. The semi-diameter corresponding to a parallax of $57'\ 2''.3$ used since 1856 is $15'\ 34''.10$. This quantity should therefore be about $15'\ 33''.75$.

Lastly Hansen's theory, Delaunay and Hill agree as to the coefficient of the variation, but in his tables Hansen has added $0''.34$ to this coefficient to correct for an assumed difference between the Moon's centre of figure and centre of gravity. This paper confirms the coefficient of Hansen's tables. This suggests that the evection should be analysed to see whether it supports Hansen's assumption, and also it appears advisable to transform Hansen's tables in the same way that Newcomb has already transformed his theory.

It will be also noticed that if the semi-diameter and variation are determined from other sources, and values differing by $\Delta\mu$ and $\Delta\delta_1$ from the results of this paper are found for them, then

if $\Delta\delta$, and $\Delta\pi$ be the corrections to the parallactic coefficient and solar parallax

$$\Delta\pi = +\frac{1}{14\cdot2}\Delta\delta_1 = \frac{1}{14\cdot2}\left\{-\frac{3788}{3074}\Delta\mu + \frac{1366}{3074}\Delta\delta_2\right\}$$
$$= -\frac{1}{11\cdot5}\Delta\mu + \frac{1}{32\cdot0}\Delta\delta_2$$

It should be mentioned that Newcomb in 1876 attributed the apparent increase of the variation to systematic errors of observation varying with the age of the Moon.

On Oscillating Satellites. (Second Paper.)

By H. C. Plummer, M.A.

1. In the earlier paper (*Monthly Notices*, vol. lxiii. p. 436), to which the present is a continuation, periodic orbits which are possible in the neighbourhood of centres of libration have been investigated with a degree of approximation corresponding to the second order of the relative coordinates. The results appear to suggest that a still closer approximation may be made with some advantage, although the discussion relates only to a small part of the restricted problem of three bodies, and has apparently no direct practical bearing. The notation of the former paper will be followed generally, but some changes will be admitted. The three collinear centres of libration alone will be considered here.

2. The general equations of motion of the satellite are taken in the form

$$\ddot{x} - 2\dot{y} = \Omega_x, \qquad \ddot{y} + 2\dot{x} = \Omega_y,$$

where

$$2\Omega = \mu\left(r_1^2 + \frac{2}{r_1}\right) + \nu\left(r_2^2 + \frac{2}{r_2}\right), \quad \Omega_x = \frac{\partial^{x+y}\Omega}{\partial x^x \partial y^y}.$$

Now at the collinear centres of libration $y = 0$ and $\Omega_{10} = \Omega_{01} = 0$; and $\Omega_{x0} = 0$ when x is odd. Hence in relative coordinates the equations of motion are

$$\left.\begin{aligned} \ddot{\xi} - 2\dot{\eta} &= \Omega_{20}\xi + \tfrac{1}{2}(\Omega_{30}\xi^2 + \Omega_{12}\eta^2) + \tfrac{1}{6}(\Omega_{40}\xi^3 + 3\Omega_{22}\xi\eta^2) + \dots \\ \ddot{\eta} + 2\dot{\xi} &= \Omega_{02}\eta + \Omega_{11}\xi\eta \qquad + \tfrac{1}{2}(3\Omega_{31}\xi^2\eta + \Omega_{03}\eta^2) + \dots \end{aligned}\right\} \quad \dots (1)$$

so long as the satellite remains within the region over which the expansions are possible. It is known that periodic solutions of these equations exist, and these can be appropriately expressed by Fourier series of the form

$$\left.\begin{aligned} \xi &= a_0 + a_1\cos nt + a_2\cos 2nt + a_3\cos 3nt + \dots \\ \eta &= \quad b_1\sin nt + b_2\sin 2nt + b_3\sin 3nt + \dots \end{aligned}\right\} \quad \dots (2)$$

the orbits being symmetrical with respect to the ξ-axis.

If these expressions for ξ and η be substituted in (1) and powers and products of the circular functions be expressed in terms of circular functions of multiples of mt, there will result an identity between two cosine series and an identity between two sine series. Now it is known that a_1, b_1 are of the first order, and a_0, a_2, b_2 of the second order. The order of the other coefficients then corresponds to their suffixes. For if the series (2) be supposed to terminate at the argument imt, the series obtained by substitution in (1) must terminate at the same argument; since it is easy to see that the argument $i'mt$ $(i'>i)$ can only arise in connexion with coefficients of order higher than i. Hence the two identities will give $(2i+1)$ equations involving the $(2i+1)$ coefficients and m; and these equations, together with the Jacobian integral, will suffice to furnish a solution to the order of approximation considered.

3. The immediate object is to find a solution which shall be valid to the third order. The expansions of (1) and (2) may therefore be supposed to end with the terms actually written down. Then when (2) are substituted in (1) the following seven equations are obtained by neglecting terms of higher order than the third, and equating the coefficients of the sines and cosines of the different multiples of mt:

$$0 = \Omega_{00}a_0 + \tfrac{1}{2}\Omega_{20}a_1^2 + \tfrac{1}{2}\Omega_{12}b_1^2 \quad \dots \quad \dots \quad \dots \quad (3)$$
$$0 = (m^2+\Omega_{20})a_1 + 2mb_1 + \mathrm{L} \quad \dots \quad \dots \quad \dots \quad (4)$$
$$0 = (m^2+\Omega_{02})b_1 + 2ma_1 + \mathrm{M} \quad \dots \quad \dots \quad \dots \quad (5)$$
$$0 = (4m^2+\Omega_{00})a_2 + 4mb_2 + \tfrac{1}{2}\Omega_{20}a_1^2 - \tfrac{1}{2}\Omega_{12}b_1^2 \quad \dots \quad (6)$$
$$0 = (4m^2+\Omega_{02})b_2 + 4ma_2 + \tfrac{1}{2}\Omega_{11}a_1b_1 \quad \dots \quad \dots \quad (7)$$
$$0 = (9m^2+\Omega_{20})a_3 + 6mb_3 + \mathrm{A}_3 \quad \dots \quad \dots \quad \dots \quad (8)$$
$$0 = (9m^2+\Omega_{02})b_3 + 6ma_3 + \mathrm{B}_3 \quad \dots \quad \dots \quad \dots \quad (9)$$

The quantities L, M, A_3, B_3 are of the third order and represent the expressions

$$\mathrm{L} = \tfrac{1}{2}\Omega_{30}a_1(2a_0+a_2) + \tfrac{1}{2}\Omega_{12}b_1b_2 + \tfrac{1}{2}\Omega_{40}a_1^3 + \tfrac{1}{2}\Omega_{22}a_1b_1^2 \quad \dots \quad (4')$$
$$\mathrm{M} = \tfrac{1}{2}\Omega_{12}(2a_0b_1 - a_2b_1 + a_1b_2) + \tfrac{1}{2}\Omega_{22}a_1^2b_1 + \tfrac{1}{8}\Omega_{04}b_1^3 \quad \dots \quad (5')$$
$$\mathrm{A}_3 = \tfrac{1}{2}\Omega_{30}a_1a_2 - \tfrac{1}{2}\Omega_{12}b_1b_2 + \tfrac{1}{24}\Omega_{40}a_1^3 - \tfrac{1}{8}\Omega_{22}a_1b_1^2 \quad \dots \quad (8')$$
$$\mathrm{B}_3 = \tfrac{1}{2}\Omega_{12}(a_1b_2 + a_2b_1) + \tfrac{1}{8}\Omega_{22}a_1^2b_1 - \tfrac{1}{24}\Omega_{04}b_1^3 \quad \dots \quad (9')$$

4. An eighth equation is required, and this can be obtained from the Jacobian integral. Now

$$\xi = -ma_1 \sin mt - 2ma_2 \sin 2mt - 3ma_3 \sin 3mt$$
$$\eta = \quad mb_1 \cos mt + 2mb_2 \cos 2mt + 3mb_3 \cos 3mt$$

and

$$2\Omega = 2\Omega_0 + \Omega_{20}\xi^2 + \Omega_{02}\eta^2 + \tfrac{1}{3}\Omega_{30}\xi^3 + \Omega_{12}\xi\eta^2 + \tfrac{1}{12}\Omega_{40}\xi^4 + \tfrac{1}{2}\Omega_{22}\xi^2\eta^2 + \tfrac{1}{12}\Omega_{04}\eta^4$$

351535

In substituting in the equation

$$V^2 = C_0 - C + 2(\Omega - \Omega_0)$$

terms of the fourth order are to be retained, and there results an equation involving cosines of the first four multiples of mt. Their coefficients must clearly vanish, since the integral must be satisfied throughout the motion, and the four relations thus obtained must evidently be involved implicitly in the equations (3)–(9). There will thus remain an equation involving the energy constant which takes this form

$$0 = C_0 - C + \tfrac{1}{2}a_1^2(\Omega_{20} - m^2) + \tfrac{1}{2}b_1^2(\Omega_{02} - m^2) + N \quad \ldots \quad (10)$$

where N stands for this expression of the fourth order

$$\begin{aligned} N = {} & \tfrac{1}{8}\Omega_{20}(2a_0^2 + a_2^2) + \tfrac{1}{8}\Omega_{02}b_2^2 \\ & + \tfrac{1}{4}\Omega_{30}a_1^2(2a_0 + a_2) + \tfrac{1}{4}\Omega_{12}b_1(2a_0b_1 - a_2b_1 + 2a_1b_2) \\ & + \tfrac{1}{32}\Omega_{40}a_1^4 + \tfrac{1}{16}\Omega_{22}a_1^2b_1^2 + \tfrac{1}{32}\Omega_{04}b_1^4 - 2m^2(a_2^2 + b_2^2) \\ = {} & \tfrac{1}{4}(a_1L + b_1M) - 4m^2(a_2^2 + b_2^2) - 4ma_2b_2 \quad \ldots \quad (10') \end{aligned}$$

The eight equations (3), (4), . . . (10) are sufficient to determine m and the seven coefficients a_0, a_2, . . .

5. Since (3), (6), and (7) give a_0, a_2, and b_2 in terms of a_1, b_1, and m, the latter quantities are determined by (4), (5), and (10). An approximate solution is obtained by neglecting L, M, and N. This will give

$$\frac{a_1}{b_1} = \frac{-2m}{m^2 + \Omega_{20}} = \frac{m^2 + \Omega_{02}}{-2m} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

$$0 = (m^2 + \Omega_{20})(m^2 + \Omega_{02}) - 4m^2 \quad \ldots \quad \ldots \quad (12)$$

$$a_1^2 = (m^2 + \Omega_{02})a^2 \; ; \; b_1^2 = (m^2 + \Omega_{20})a^2 \quad \ldots \quad (13)$$

$$C_0 - C = (m^4 - \Omega_{20}\Omega_{02})a^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

Then, if L, M, and N be reinstated, it will be necessary to replace a_1 by $a_1 + \delta a_1$, b_1 by $b_1 + \delta b_1$, and m by $m + \delta m$, the three increments being clearly of the second order. Hence

$$\begin{aligned} 0 &= (m^2 + \Omega_{20})\delta a_1 + 2m\delta b_1 + 2(ma_1 + b_1)\delta m + L \\ 0 &= 2m\delta a_1 + (m^2 + \Omega_{02})\delta b_1 + 2(mb_1 + a_1)\delta m + M \\ 0 &= (\Omega_{20} - m^2)a_1\delta a_1 + (\Omega_{02} - m^2)b_1\delta b_1 - m(a_1^2 + b_1^2)\delta m + N \end{aligned}$$

the quantities L, M, N and the coefficients in these equations being calculated with the approximate values of a_1, b_1, and m. Forms more convenient for calculation can be deduced by the help of (11), (12), and (13). Thus if

$$\frac{\delta a_1}{a_1} - \frac{\delta b_1}{b_1} = \delta_1, \quad \frac{\delta a_1}{a_1} + \frac{\delta b_1}{b_1} = \delta_2$$

the following form an equivalent set of equations :

$$(m^2-\Omega_{20}\Omega_{02})\delta m = m^2(b_1^{-1}\,\mathrm{L}+a_1^{-1}\,\mathrm{M}) \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

$$4m\delta_2 = (\Omega_{20}-\Omega_{02})\delta m + b_1^{-1}\,\mathrm{L} - a_1^{-1}\,\mathrm{M} \ldots \quad \ldots \quad (16)$$

$$(m^2-\Omega_{20}\Omega_{02})\delta_2 = m^2(\Omega_{20}-\Omega_{02})\delta_2 - m(2m^2+\Omega_{20}+\Omega_{02})\delta m + a^{-2}\,\mathrm{N} \quad \ldots \quad (17)$$

The first of these determines approximately the variation of the period from its limiting value; the two latter give successively δ_2 and δ_0, whence δa_2 and δb_2 are obtained immediately. The coefficients a_3 and b_3 are found from (8) and (9), and thus the solution to the third order is complete.

6. To the derivatives of the force function given in the former paper must be added those of the fourth order. Their values are given by

$$3\Omega_{40} = -6\Omega_{22} = 8\Omega_{04} = 72(\mu r_1^{-5} + \nu r_2^{-5})$$

A table of numerical values for the case in which $11\mu = 10$, $11\nu = 1$, will now be given, many of the numbers being taken from the earlier paper :

	Family *a*.	Family *b*.	Family *c*.
r_1	0·71751	1·34700	0·94693
r_2	0·28249	0·34700	1·94693
C_0	3·65292	3·53418	3·17322
Ω_{20}	+ 13·9878	+ 6·0955	+ 3·1660
Ω_{02}	− 5·4939	− 1·5478	− 0·0830
$\Omega_{22} = -2\Omega_{13}$	+ 65·076	− 39·279	+ 6·822
$3\Omega_{40} = -6\Omega_{22} = 8\Omega_{04}$	+ 3982·86	+ 1315·79	+ 86·202
m	2·6082	1·6763	1·0706
$m^4 - \Omega_{20}\Omega_{02}$	123·13	17·330	1·5766
$a^{-1}\,.\,a_1$	− 1·1439	− 1·1234	− 1·0312
$a^{-1}\,.\,b_1$	+ 4·5597	+ 2·9841	+ 2·0766
$a^{-2}\,.\,a_0$	+ 10·5685	− 5·1400	+ 0·5887
$a^{-2}\,.\,a_2$	− 4·1352	+ 2·6261	− 0·5487
$a^{-2}\,.\,b_2$	− 1·9209	+ 1·5800	− 0·2895
$a^{-3}\,.\,\mathrm{L}$	+ 1234·6	+ 73·93	+ 2·8618
$a^{-3}\,.\,\mathrm{M}$	+ 3493·8	+ 47·45	+ 1·4737
$a^{-4}\,.\,\mathrm{N}$	+ 2981·3	− 118·74	− 2·4169
$a^{-3}\,.\,\mathrm{A}_3$	− 2044·8	− 288·49	− 8·3943
$a^{-3}\,.\,\mathrm{B}_3$	− 2190·5	− 225·81	− 6·5525
$a^{-3}\,.\,\delta m$	− 153·79	− 2·832	− 0·03708
$a^{-3}\,.\,\delta a_2$	− 92·47	− 6·227	− 0·3670
$a^{-3}\,.\,\delta b_2$	+ 224·79	− 3·652	− 0·5638
$a^{-3}\,.\,a_3$	+ 20·19	+ 7·109	+ 0·4530
$a^{-3}\,.\,b_3$	+ 33·64	+ 6·500	+ 0·3559

7. In the former paper the results for orbits of family *a* were compared with four orbits given by Professor Darwin. The comparison can now be repeated with improved figures. The values of the energy constant C and the parameter *a* are given by

$$11\,C = 40{\cdot}0,\quad 39{\cdot}0,\quad 38{\cdot}5,\quad 38{\cdot}0$$
$$\log a = 8{\cdot}06428,\ 8{\cdot}47046,\ 8{\cdot}54705,\ 8{\cdot}60356$$

The general equations for the family are

$$\xi = +10{\cdot}5685a^2-(1{\cdot}1439a+92{\cdot}47a^2)\cos mt-4{\cdot}1352a^2\cos 2mt+20{\cdot}19a^3\cos 3mt$$
$$\eta = (4{\cdot}5597a+224{\cdot}79a^2)\sin mt-1{\cdot}9209a^2\sin 2mt+33{\cdot}64a^3\sin 3mt$$
$$m = 2{\cdot}6082-153{\cdot}79a^2$$

which become in the four cases considered

(*a*) $\xi = +{\cdot}0014-{\cdot}0134\cos mt-{\cdot}0006\cos 2mt+{\cdot}0000\cos 3mt$
$\eta = +{\cdot}0533\sin mt-{\cdot}0003\sin 2mt+{\cdot}0001\sin 3mt$
$m = 2{\cdot}588$

(*β*) $\xi = +{\cdot}0092-{\cdot}0362\cos mt-{\cdot}0036\cos 2mt+{\cdot}0005\cos 3mt$
$\eta = +{\cdot}1405\sin mt-{\cdot}0017\sin 2mt+{\cdot}0009\sin 3mt$
$m = 2{\cdot}474$

(*γ*) $\xi = +{\cdot}0131-{\cdot}0443\cos mt-{\cdot}0052\cos 2mt+{\cdot}0009\cos 3mt$
$\eta = +{\cdot}1705\sin mt-{\cdot}0024\sin 2mt+{\cdot}0015\sin 3mt$
$m = 2{\cdot}418$

(*δ*) $\xi = +{\cdot}0170-{\cdot}0579\cos mt-{\cdot}0067\cos 2mt+{\cdot}0013\cos 3mt$
$\eta = +{\cdot}1975\sin mt-{\cdot}0031\sin 2mt+{\cdot}0022\sin 3mt$
$m = 2{\cdot}361$

The following table gives the points where the orbits cross the axis of x (ξ_0 and ξ_1), the points (ξ', η') for which $mt = \frac{1}{2}\pi$ and the angle ($2\pi m^{-1}$) through which the system turns in one period of the satellite. In the comparison numbers η' is the ordinate corresponding to the calculated ξ'.

	ξ_0	ξ_1	ξ'	η'	$2\pi m^{-1}$
(*a*)	−·0126	+·0142	+·0090	+·0532	139·1
(*b*)	−·0125	+·0147	„	+·0530	138·3
(*β*)	−·0301	+·0413	+·0128	+·1396	145·5
(*b*)	−·0304	+·0413	„	+·1395	145·7

	ξ_0	η_0	$\frac{1}{2}\eta'$	η'	$2\pi m^{-1}$
(γ)	−·0354	+·0514	+·0182	+·1690	148°·9
D.	−·0361	+·0523	,,	+·1691	149·6
(δ)	−·0403	+·0609	+·0237	+·1953	152·5
D.	−·041	+·060	,,	+·197	(not determined)

The limiting value of $2\pi m^{-1}$ for an infinitesimal orbit is 138°·0. The agreement shown in the above table is much better than in the corresponding table of the previous paper (vol. lxiii. p. 442), and may be considered satisfactory.

8. For the two orbits of family *b* given by Professor Darwin the values of C and *a* are given by

$$11\,C = 38{\cdot}5, \qquad 38{\cdot}0$$
$$\log a = 8{\cdot}64750, \quad 8{\cdot}83116$$

The general equations for the family are:

$$\xi = -5{\cdot}1400\,a^2 - (1{\cdot}1234a + 6{\cdot}227a^3)\cos mt + 2{\cdot}6261a^2\cos 2mt + 7{\cdot}109a^3\cos 3mt$$
$$\eta = (2{\cdot}9841a - 3{\cdot}652a^3)\sin mt + 1{\cdot}5800a^2\sin 2mt + 6{\cdot}500a^3\sin 3mt$$
$$m = 1{\cdot}6763 - 2{\cdot}832a^2$$

which become in the two cases considered

(α) $\xi = -{\cdot}0101 - {\cdot}0504\cos mt + {\cdot}0052\cos 2mt + {\cdot}0006\cos 3mt$
$\eta = +{\cdot}1322\sin mt + {\cdot}0031\sin 2mt + {\cdot}0006\sin 3mt$
$m = 1{\cdot}6707$

(β) $\xi = -{\cdot}0236 - {\cdot}0781\cos mt + {\cdot}0121\cos 2mt + {\cdot}0022\cos 3mt$
$\eta = +{\cdot}2012\sin mt + {\cdot}0073\sin 2mt + {\cdot}0020\sin 3mt$
$m = 1{\cdot}6633$

The comparison is made as in the preceding paragraph

	ξ_0	η_0	$\frac{1}{2}\eta'$	η'	$2\pi m^{-1}$
(α)	−·0547	+·0449	−·0153	+·1316	215°·5
D.	−·0551	+·0445	,,	+·1312	[213·9]
(β)	−·0874	+·0644	−·0357	+·1991	216·4
D.	−·0875	+·0679		+·1987	208

The limiting value of $2\pi m^{-1}$ for an infinitesimal orbit is 214°·8. There is therefore a discrepancy between the two methods in

regard to the periods of the orbits, Professor Darwin's results indicating a decrease of the period as the orbits increase in size. This is not supported by my figures. The value [213°·9] really belongs to a neighbouring orbit which is not quite periodic, so that this number is not strictly comparable. On the other hand the orbit (β) contains in the value of ξ_1 a second discrepancy, which is greater than the evidence based on other comparisons would lead us to expect. On the whole it seems quite possible that the determination of this orbit by the method of quadratures has not been so accurate as in other cases.

9. In Professor Darwin's results the range of the energy constant has the limits given by $11C = 40{\cdot}0$ and $11C = 38{\cdot}0$. Now orbits of family e only begin to appear after $11C = 35{\cdot}0$, and therefore orbits of this family for comparison are wanting. But, although the results cannot be submitted to this kind of check, it seems worth while to apply them to drawing a few new orbits of this family. The general equations are

$$\xi = +{\cdot}5887\, a^2 - (1{\cdot}0312\, a + {\cdot}3670\, a^2)\cos mt - {\cdot}5487\, a^2 \cos 2mt + {\cdot}4530\, a^2 \cos 3mt$$

$$\eta = (2{\cdot}0788\, a - {\cdot}3638\, a^2)\sin mt - {\cdot}2895\, a^2 \sin 2mt + {\cdot}3559\, a^2 \sin 3mt$$

$$n = 1{\cdot}0708 - {\cdot}3708\, a^2.$$

The selected values of C and a are given by

$11C =$	34°5	34°0	33°5	33°0
$\log a =$	9·78439	9·88886	9·96434	9·02043

The equations of the four orbits then become

(γ) $\xi = +{\cdot}5887 - {\cdot}9590\cos mt - {\cdot}0138\cos 2mt + {\cdot}0216\cos 3mt$

$\eta = +{\cdot}3705\sin mt - {\cdot}0088\sin 2mt + {\cdot}0213\sin 3mt$

$n = 1{\cdot}0661\,;\quad 2\pi n^{-1} = 5{\cdot}8955$

(δ) $\xi = +{\cdot}5887 - {\cdot}9450\cos mt - {\cdot}0087\cos 2mt + {\cdot}0254\cos 3mt$

$\eta = +{\cdot}3705\sin mt - {\cdot}0055\sin 2mt + {\cdot}0249\sin 3mt$

$n = 1{\cdot}0661\,;\quad 2\pi n^{-1} = 5{\cdot}8955$

(ε) $\xi = +{\cdot}5887 - {\cdot}9313\cos mt - {\cdot}0054\cos 2mt + {\cdot}0295\cos 3mt$

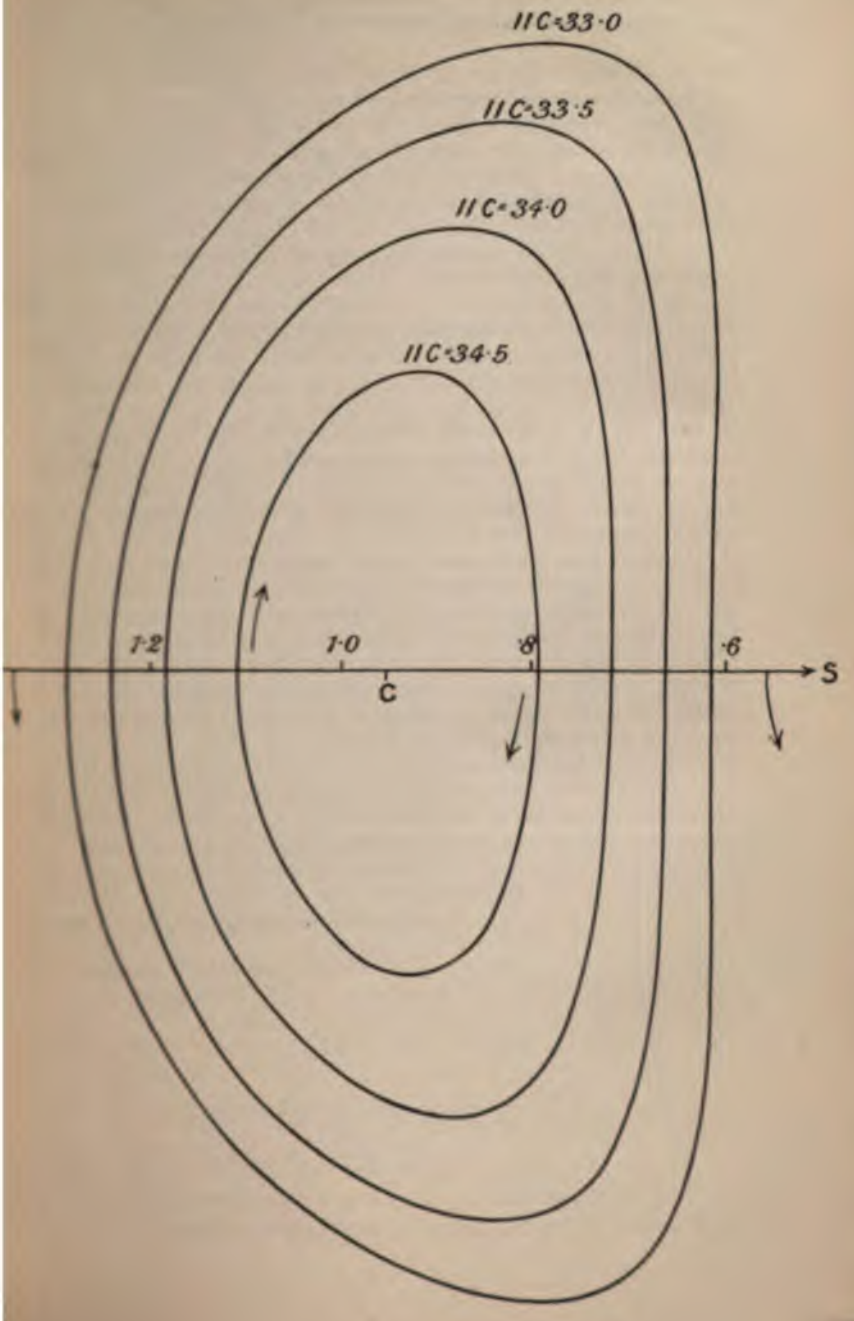
$\eta = +{\cdot}3705\sin mt - {\cdot}0034\sin 2mt + {\cdot}0289\sin 3mt$

$n = 1{\cdot}0661\,;\quad 2\pi n^{-1} = 5{\cdot}8955$

(ζ) $\xi = +{\cdot}5887 - {\cdot}9178\cos mt - {\cdot}0034\cos 2mt + {\cdot}0335\cos 3mt$

$\eta = +{\cdot}3705\sin mt - {\cdot}0021\sin 2mt + {\cdot}0328\sin 3mt$

$n = 1{\cdot}0661\,;\quad 2\pi n^{-1} = 5{\cdot}8955$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ASTOR, LENOX AND
TILDEN FOUNDATIONS.

The curves corresponding to these equations are represented in the accompanying plate (Plate 2), in which the unit of length is 5 in., this being the scale on which most of Professor Darwin's orbits are reproduced.

10. A comparison of §§ 7 and 8 with §§ 10 and 11 of the previous paper will show the degree of improvement obtained for families *a* and *b* by increasing the order of approximation, and give some idea of the convergence (in a practical and not technical sense) of the series. It has now been found possible to calculate the variation of the period, an element which, with the lower approximation, is supposed constant for each family. In every case the period is found to increase with the scale of the orbits, the variation being most marked in the case of family *a* and least in the case of family *c*. As regards the shape and position of the orbits, the general features which characterise the departure from the small elliptic paths surrounding the centres of libration are similar in all cases. The mean position of the satellite tends always in the direction of the nearer body of finite mass, while at the intersection of the orbit with the line of syzygies in the same direction, the curvature first decreases and finally changes sign. These features are shown, though not in a very marked degree, by the orbits of family *c* drawn in this paper. Where comparison was possible with the results of Professor Darwin's researches it has been made, and in one case among the six orbits available a discrepancy is found which is pointed out in § 8. The method of the present paper is extremely simple and elementary; yet it might be extended without serious difficulty to higher orders of approximation if this were desirable. The next order in particular might easily be added. But it has not seemed likely that anything of value would be gained thereby. A series of orbits of oscillating satellites has been found by M. Burrau (*A.N.* 3251) to be limited by a "trajectory of ejection." The real obstacle in the way of following up to this limit the continuation of the class of solutions considered in this paper is the limited range over which the force function can be expanded in ascending powers of the relative coordinates. Whether this difficulty can be surmounted is a question which must be left for further consideration.

University Observatory, Oxford:
1903 December 7.

Preliminary Note on the Effect of the Direction of Gravity on Lunar Observations. By E. B. H. Wade, M.A.

(*Communicated by Professor H. H. Turner.*)

That the Moon's apparent place is affected by the spheroidal figure of the Earth is well known. If $d\phi$ is the difference between geocentric and geographic latitude, the Moon's declination on the meridian requires a supplementary correction of $p \times \cos\zeta \, d\phi$, which in extreme cases amounts to $1''$ for every $1'$ of $d\phi$.

Unless this correction is made, latitudes found by altitudes of the Moon will differ to that extent from those found by altitudes of stars. Again, if there be admitted a distinction between geocentric and geographic longitude ($d\lambda$), then the Moon's R.A. at transit requires a correction for parallax $p \cos t \dfrac{\cos\phi}{\cos\delta} d\lambda$, instead of being free from parallax as usually assumed. For every $1'$ of $d\lambda$ this means, in equatorial latitudes, $1''$ error about in R.A. Now since longitudes found by Moon culmination are about thirty times as erroneous as the R.A. from which they are deduced, we find that if $d\lambda$ attains $1'$, the longitude will differ 2^s from that given by star-signals. If this has not been emphasised before, it is no doubt due to the practical certainty that $d\lambda$ cannot attain to $1'$, but we shall nevertheless pursue the idea somewhat further.

The 2^s in longitude, though a large quantity, means only $1''$ in R.A., not a large quantity perhaps to detect by observation with the transit instrument. In the case of occultations near the meridian, the disappearance would be accelerated or delayed by 2^s, which could not escape observation, as times of disappearance are reliable to $0^s{\cdot}2$. We conclude that a difference of only $6''$ between geocentric and geographical longitude has a measurable effect on an occultation.

Application.

It is immaterial to the above argument whether the $d\phi$ or $d\lambda$ arise from (*a*) systematic ellipticity, (*b*) accidental causes (so-called local attraction), (*c*) a regional drift of gravity prevailing over large areas, or any combination of these.

(*a*) We have shown that if this effect attain to $6''$ in longitude it will be noticeable, but it would only be detected in geodetic operations if correct to 1 in 45,000, both in the triangulation and in the astronomical work.

(*b*) Is sufficiently detected by ordinary geodetic operations.

(*c*) Might suitably be dealt with by lunar observations.

Major Burrard, R.E., in a paper read to the British Association (*B.A. Report*, 1902, p. 541), expresses a fear that a large

regional drift may remain uneliminated, so that it is not possible to apply a map of India to a map of the whole world with certainty. As this doubt has been authoritatively expressed, it seems worth while to point out that lunar observation would detect such drift, whether in latitude or longitude. In the case of latitude, that found from the Moon would differ by 1″ from that found from stars, and in the case of longitude that found from occultations would differ 2^s (30″) from that found from star signals, if $d\phi$ or $d\lambda$ attain to 1′.

Or, better, on the same night, longitudes found from occultations at large hour-angles would differ nearly 2^s (30″) from those observed near the meridian.

Occultations appear to be very suitable, but as long as they are corrected for ephemeris errors obtained with transit instruments, they are no better than transit observations. In the case when two occultations are observed on one night, or one in two localities, this objection is removed.

It may be added that a marked discrepancy was found in the fundamental Indian longitude based on a long series of Moon culminations, and the same later determined telegraphically, and this note may help to explain it.

Abbasia Observatory, Egypt: 1903 *October* 26.

Remarks on a Paper by Mr. Cooke on a New Method of Determining Time, Latitude, and Azimuth. By E. B. H. Wade, M.A.

(*Communicated by Professor H. H. Turner.*)

At the beginning of the present year Mr. Cooke described a new method of determining time, latitude, and azimuth (*Monthly Notices, R.A.S.*, vol. lxiii. p. 156). As I have been employing precisely his method since May 1902, I thought it would be permissible to make a few additional remarks on the subject.

Having been instructed to determine the latitude of Tamia in the Fayum Province of Egypt, I proceeded thither with an 8-inch theodolite, intending to rely on circle readings. However, a very strong wind was blowing, and this caused such inconvenience in illuminating graduations, that Talcott's method seemed preferable. For this a vertical micrometer is necessary, and the one furnished with my instrument could not be turned into the vertical without structural alterations. I therefore decided to make my chronometer a substitute for the micrometer. For example, if the second of a pair of stars pass the meridian at a greater altitude than the first, then by placing the instrument out of the meridian for the second star we may still secure it at the same altitude as the first. It at once became evident that

pairs of stars could thus be utilised which would not come within the limited range of Talcott's method. Moreover, it was now necessary to know the error of the chronometer, and for this purpose I included stars in the neighbourhood of the prime vertical. Thus, by degrees, I had come to employ as a substitute for Talcott's method, one differing in no respect from that described by Mr. Cooke.

As the results were satisfactory, I contemplated publishing them, but a study of the literature of the subject led me to think that there was little if anything original in the method. I therefore contented myself with writing a departmental minute on the subject, and inserting an appendix on it in the annual report of the Observatory for 1902. In the first place the method differed only from Chandler's* in the use of a spirit level ; again, Chauvenet had described a method for finding latitude and time by the observation of three or more stars at equal altitudes.† At the time when he wrote, Chandler's device had not been invented, and the instrument which he had in mind must have been a sextant, a zenith telescope, or a theodolite. Even if he had not a theodolite in mind, the use of a theodolite as zenith telescope did not appear to me new. For instance, Sir David Gill has used theodolites in place of zenith telescopes on the survey of South Africa.‡ However, the method, even if not strictly new, has probably escaped the notice of surveyors, and it is therefore fortunate that Mr. Cooke has drawn attention to the subject.

Applications.

In my own case I have used the method for latitudes and time only. I deliberately rejected it as a means of obtaining azimuths.

Mr. Cooke refers to the difficulties arising from circle graduations and collimation. To these we may add the reading of a striding level and the pivot errors. All these must be reckoned with to obtain the azimuth of a terrestrial mark. I have preferred a method specially designed to mitigate these evils. An account of it will appear in the report above referred to.

Definitions.

In this note I propose to use the following definitions :—

ϕ = observer's latitude.

θ = sidereal time of observation.

h = star's altitude, A = star's azimuth } at time θ.

δ = star's declination.

* *Annals of Harvard College Observatory*, vol. xvii.
† *Spherical and Practical Astronomy*, vol. i. p. 280.
‡ *Report on the Geodetic Survey of South Africa*, 1894.

Practice.

My practice appears to be the same as Mr. Cooke's in all details except the following :—

1. I usually point the instrument to *Polaris* at the beginning of the set, instead of the altitude ϕ + refraction. But the method of calculation (given below) leaves me quite free in this matter, and the set may be taken at any convenient altitude. If the method is to apply to equatorial latitudes, the constant altitude must evidently differ much from that of the pole.

2. The azimuth circle is never read for the reasons stated above. I would add that if it is to be read, and the observer's object is to find latitude and azimuth only and in one operation, the timekeeper may be dispensed with, which might be a great convenience in the field. Consider a special case. A star of known declination north of the zenith cuts the circle of equal altitude twice, and the horizontal readings are A_1A_2. Another to the south does so at readings A_3A_4. Either $\frac{A_1+A_2}{2}$ or $\frac{A_3+A_4}{2}$ is the meridional reading of the theodolite, and the relation between A_1-A_2 and A_3-A_4 leads to the latitude. A general theory for any number of stars is easily worked out.

Computation.

From the fact that we have been working independently, my method of computation is totally different from Mr. Cooke's. I begin by assuming a probable value for the latitude of the place and the error of my chronometer. I then calculate on these assumptions the zenith distance of every star in the set from the formula,

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t.$$

I make no attempt, as does Mr. Cooke, to group my stars into pairs. All stars are calculated in exactly the same way. Now, supposing that the assumption as to time and latitude is correct, and further that the "bubble" readings are throughout the same, and that the error of observation is negligible, then all these computed zenith distances will work out exactly equal; but, if not, each star will give us an equation of the form,

$$h + \Phi d\phi + T dt - h_o = 0.$$

In these equations the quantities Φ and T are the differential coefficients expressing the result of errors of chronometer or latitude upon the calculated zenith distance. These coefficients may be obtained by the ordinary rules of spherical trigonometry, and the unknown quantities, $d\phi$, dt, h_o, may then be found by the method of least squares. In my earliest observations I

proceeded thus, but in later ones I have employed the modification about to be described.

In the table below the first column is the ordinary calculation of an altitude. The second shows the changes (in units of the seventh place) produced by increasing ϕ by 10″; these are found at once from the proportional parts 44 and −10 for sin ϕ and cos ϕ respectively. Similarly, the third column shows the effect of an increase in t by 10″; it is found from the proportional parts for cos t; namely, 104.

	I.	II.	III.
log sin ϕ	1. 6377 167	44	...
log sin δ	1. 8075 742	...	...
sum	1. 4452 909	44	...
nat (1)	0. 2757 991	29	...
log cos ϕ	1. 9546 301	−10	...
log cos δ	1. 8846 006	...	...
log cos t	1. 2958 659	...	−104
sum	0. 1350 966	−10	−104
nat (2)	0. 1364 887	− 3	− 33
(1)+(2)	0. 4152 878	26	− 33
log (1+2)	0. 6153 492	27	− 35
h=	24° 32′ 14″.8	6″.0	−7″.5

The equation for the star is therefore

$$0{\cdot}60\, d\phi - 0{\cdot}75\, dt + 14''{\cdot}8 - h_0 = 0$$

Results.

I proceed to give some results with the method. I would first mention that they are, strictly speaking, field results. Without wishing to be unduly critical, I question whether Mr. Cooke's results were obtained at such a distance from his observatory that they can really be compared with field results obtained by other observers. Working at hundreds of miles from the nearest centre of civilisation, we are beset with petty difficulties which tend to diminish accuracy. The further, however, that we go into the field, the more we are struck by the suitability of the present method for field purposes.

1. *Station Tamia, June* 21, 1902.

Latitude found 29° 28′ 0″−4″.65. $\Delta\theta = +46''.2$.

Residuals given by stars

	″
β Cassiopeiæ	2·1
α Cassiopeiæ	2·4

δ Capricorni	3″·6	
γ Capricorni	1·0	East of meridian.
β Capricorni	−1·7	
γ Capricorni	−4·3	West of meridian.
δ Cassiopeiæ	−2·2	
ϵ Herculis	−2·2	

This example is given chiefly on account of its early date.

2. *Station near Luxor.*

	Latitude found.	$\Delta\theta$.
	° ′ ″	″
1903 Feb. 26	25 44 12, 0	+11, 0
27	10, 4	+0, 0
28	11, 0	+12, 0

In this set as the result of increased experience the residuals are very much smaller than at Tamia. On the last two nights no single residual exceeded 1″·5.

The observations near Luxor are more suitable than those at Tamia for a discussion of accuracy. In the first place, there are three nights of observation; and secondly, the result can be compared with an independent method. Owing to the clearness of the sky in Egypt it was found possible to observe the altitude of the pole star in broad daylight, reading the circles of the instrument with micrometer microscopes under the most favourable conditions as to illumination. The arc of the instrument employed in this set was subsequently examined at the Observatory, and the following latitudes were obtained by the ordinary methods. For convenience the results obtained by the two methods are placed side by side:

	By Polaris.	By Method.		By Polaris.	By Method.
	° ′ ″	″		° ′ ″	″
Feb. 25	25 44 10·7		Feb. 26	25 44 11·4	
	12·2			10·8	
	11·0	...			
	11·0		27	12·0	
				11·3	10·4
26	9·9			12·5	
	10·8				
	8·4	12·0	28	—	11·0
	9·8				
	Mean by Polaris.			Mean by Method.	
	25 44 10·9			25 44 11·1	

Apparently the accuracy which I have reached is rather less than Mr. Cooke's, but my results, I think, confirm his opinion as to the convenience of the method. I may add that the times which I have obtained by this method have only been needed for approximate purposes, but I believe them to be quite accurate. In May of the present year I received instructions to determine the latitude and longitude of Tema, and made my arrangements to obtain both latitude and accurate local time by the method; but the telegraphic arrangements fell through, so that the method was only applied to the determination of latitude. I am convinced, however, that more accurate local times can be obtained in this way than by any other field method.

Relative Accuracy of Mr. Cooke's and other Methods.

It is interesting to inquire to what circumstance this method owes its supposed superiority over other field methods, including Talcott's. Given a faultless micrometer, Talcott's is a special case of Mr. Cooke's, and the one in which the error in zenith distance has the minimum effect on the latitude. If the superiority is confirmed, it would seem to imply that the accuracy of Talcott's method has hitherto been limited by micrometer errors.

My best thanks are due to Professor Turner for encouragement in this investigation, to the Survey Department of Egypt for providing facilities for the observations, and to the Under Secretary of State for Egyptian Public Works for permission to publish this note.

Abbasia Observatory, Egypt:
1903 *October.*

On the use of the Stereo-comparator for Plates on which a Réseau has been Impressed. By Dr. Max Wolf.

[*Introductory Note by Professor H. H. Turner.*—The accounts given of the ease and rapidity with which two plates of the same region can be compared with the stereo-comparator naturally create a desire to possess such an instrument. A generous offer has been made to present one to the Oxford University Observatory, *provided that it will clearly be of use in our work.* The very proper condition specified at once raises the question whether plates which have a réseau impressed upon them, and which have several images of the same star, can be scanned as readily by the stereoscopic method as those containing only a single image of each star and no réseau; for all our plates accumulated hitherto at Oxford are of the former class. For some time I hoped to be able to obtain a stereo-comparator on loan to make

the experiment; the makers (Messrs. Zeiss) courteously offered to put at my disposal for this purpose the first one available. But the demand for the instruments has been, I am glad to learn, so great, that hitherto they have been unable to do more than keep pace with it; and ultimately I ventured to ask Dr. Max Wolf to make the experiments for me, which he very kindly consented to do. Two plates were sent him, both impressed with a réseau, and containing, the first three, and the second two, images of each star. They had been exposed for the planet *Eros*, and thus afforded material for tests of various kinds. It will be seen from the following letter that, even in this extreme case, the stereo-comparator can be used with advantage under certain conditions. The letter suggested that if *one* of the plates were free from a réseau, many of the difficulties would disappear, and this was found to be the case. A second pair of plates was sent to Heidelberg, one with a réseau, the other without, and a mere glance was sufficient to show Dr. Wolf that "there was now no trouble in observing two such plates with the stereo-comparator. The best position is to bring the chains of three stars in the line joining the eyes, and then it gives a splendid effect, so that the examination of the plates is easily made." In proof of which, Dr. Wolf remarked that there was a planet on one of the plates, and indicated its approximate position. The place was carefully measured when the plates were returned to Oxford, and the results—which seem to show that the planet was *Ausonia* (63)—have been communicated to the *Astronomische Nachrichten*. Hence, although it will be seen below that Dr. Max Wolf expresses himself rather doubtfully as to the value of the stereo-comparator for cases when both plates have a réseau, these doubts need not seriously concern us. For the instrument can be brought into effective use by taking a new plate without a réseau.

But Dr. Max Wolf's account of his experiments when both plates have a réseau will probably interest others; and I therefore give it at full length.

I should add that Dr. Pulfrich of Jena also offered to make similar experiments, and another pair of plates was accordingly sent to him. His full account of the examination substantially agrees with Dr. Wolf's results. The réseau *disturbs* the examination, but does not *prevent* it—this is the verdict of both.

As a result of these experiments I felt justified in assuring Mr. C. L. Brook, F.R.A.S., who had made the generous offer above referred to, that the instrument would be most useful to us; and he promptly purchased it for the Oxford University Observatory. It was on view at the R.A.S. rooms on November 27 and following days.]

Dear Professor Turner,—It is indeed a very difficult problem which you have set to me, and I can hardly give you advice whether to procure a stereo-comparator or not. Indeed,

a réseau makes the use of a stereo-comparator much more difficult than it is for our plates without réseau.

You have forwarded me two plates No. 1619 and 1620. No. 1619 contains of each star three images, No. 1620 two images :

FIG. 1.

FIG. 2.

Now, if I put the two plates as seen here in the stereo-comparator and adjust the plates so that the middle images of 1619 coincide with the upper images of 1620 (viz. 2 with 4), then the star 1 comes out of the "infinity" plane and is close to the observer,* but the point 3 (combined with 5) approaches also to the observer, but not so much as the point 1, because the distance 2–3 is not equal to the distance 4–5.

So it happens that you see (of each star) always three images freely suspended in space : No. 1 very near to the observer, No. 2 at a very great distance far behind, and No. 3 at a mean distance between the two.

This appearance is preserved all over the plate if you move the two slides of the stereo-comparator ; you always see this beautiful roomy chain ; and at any place, where there is some trouble, you see it immediately by the change of this order.

On the two plates is also the planet *Eros*. Like all the stars he has made on 1619 three images, on 1620 two. Since some time has elapsed between the two exposures, the pictures do not coincide, but are situated at wholly different places. On each plate, therefore, the planet is seen immediately by the stereo-effect. On the plate with the three images *all three* points of *Eros* come far out of the ∞ plane and seem to be as near to the observer as points No. 1 of the stars. The same is with the left plate (I did not know the positions before).

In the same manner every wrong star is easily detected by the stereo-comparator. The stereo-effect is, of course, not necessary to detect faint stars, because all stars are threefold,

* There is, of course, no direct stereoscopic effect for an object which appears on one plate only ; but the appearance suggested is that of an object close to the observer.

and every spot only once upon the plate. But, notwithstanding, it is very comfortable to go over the plate using the slides of the stereo-comparator, and to feel the errors without thinking.

I am sorry that I could not detect a variable star on the plates, but there seems to be no variable.

Going over the plate I found the following wrong stars, which are all caused by holes in the réseau plate:

$x = 15{\cdot}5$	:	$y = 2{\cdot}4$
25·8		10·8
0·8		10·5
20·4		11·2
21·6		13·7 = (artificial cluster)
20·2		17·0

&c.

These spots are on the two plates at the same places of the réseau, so that it is certain that they are holes in this.

So far as this, the stereo-comparator is of great value to find out changes in the stars and errors of the plates.

But now we come to the difficulty!

If the stereo-comparator is adjusted to the stars as above, the lines of the two réseaux fall tolerably separate; and after practice for a day or so, one is accustomed to neglect the lines and to see the stars free in space on a distant indefinite network. If you look firmly upon the net then, you see the following view:

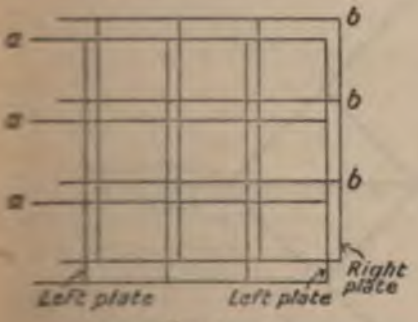


Fig. 3.

Fig. 4.

Fig. 3 gives the relative situation roughly of the two nets of the two plates, if the stars are adjusted as described above; Fig. 4 gives the view seen with the two eyes by the stereo-comparator. We see the vertical lines join together, whilst the horizontal remain separated. This results from the situation of the two eyes, which are in a horizontal. The horizontal lines of

the réseau of the left plate seem much thicker and somewhat nearer to the observer.

If starpoints 2 and 4 coincide over the whole plate, and if then the réseaux are adjusted absolutely parallel in the two

Fig. 5.

pictures, then we see in the stereo-comparator the horizontal lines not parallel to the eyes. It looks as if the lines *b* were inclined to the lines *a* (fig. 3). This is a very striking appearance. (Fig. 5 somewhat exaggerates the appearance.) The reason was found very soon. The connecting lines 1, 2, 3 and 4, 5 (figs. 1 and 2) are not parallel, and not parallel to the réseau. You see how the stereo-comparator can give the *errors of the orientation of your apparatus.*

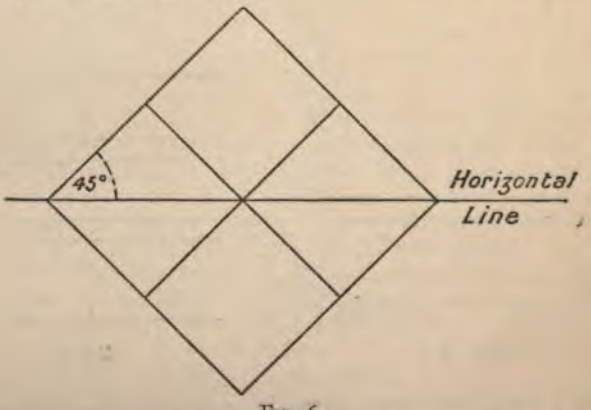


Fig. 6.

If we proceed to another adjustment of your plates we have still further difficulties. We will now put the two plates in the stereo-comparator, so that the lines of the réseau are inclined 45° to the connecting-line of your eyes (horizontal), fig. 6. If we

now adjust the plates so that the starpoints 2 and 4 coincide all over the plate, then all the lines join together in our mind, and we see only *one* net, *one* réseau ; but we cannot any longer see the stars coincident at the same time. By some effort it is possible to look firmly upon the stars, and to receive again the stereo-view of the stars. This is done most easily by moving the slides of the stereo-comparator and regarding the stars.

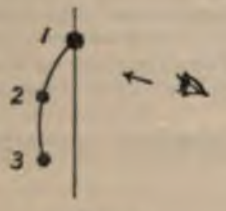


Fig. 7.

Point 1 is again very near to the observer.
2 very far distant.
3 lies nearer than at the first adjustment.

Eros seems very near to the observer. Pictures 3 and 5 of *Eros* seem very much nearer than the others.

The third position of the stereo-comparator was the following,

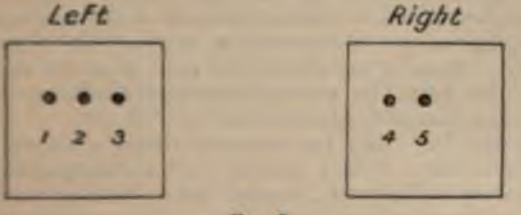


Fig. 8.

adjusting 2 upon 4 all over the plate. In this position picture 1 was still nearer to the observer, picture 2 still further, the effect a maximum, as natural. But now it was nearly impossible to seek over the plate and to fix the eyes upon the stars. At every moment the lines of the réseau will coincide in the eyes, and it is impossible for the eyes to hold upon the stars and to join the images of the stars in the mind.

Now you see the difficulty. If the two plates are so exposed that the lines fall *near* together when the stars are in coincidence, then the eyes are unable to hold the lines from together and to join the stars. If, on the other hand, the plates are so exposed that the lines are *far* from together, regarding the same starpoints, then the stereo-comparator is *very* useful and easily applied for the research of the plates.

You see also from the above that for these two plates, 1619 and 1620, there was one position which was useful; and it is not impossible that on a great part of your plates you will be able to find one position for stereo-comparing work.

Now you can imagine how difficult it is for me to bear the responsibility to give you advice whether to procure a stereo-comparator or not. There is no doubt you will not have the full benefit of the instrument, because in many cases the réseau makes the use of the stereo-comparator impossible; on the other hand, in many cases you will have some great *profit* of its use.

Excuse me that I can give you no deciding answer, but the reason lies not in my hands, it reposes on the difficulty of the matter.

With kind regards, yours very truly,
MAX WOLF.

Heidelberg: 1903 *July* 5.

An Examination of the Relative Star-density in Different Parts of the Plates forming the Harvard Photographic Sky-map. By J. C. W. Herschel, B.A.

In *Monthly Notices*, lxii. 1902 April, p. 444, Professor Turner gave diagrams for five different observatories, which use essentially similar objectives, showing that the relative number of star images photographed in different parts of a plate varies by about 50 per cent., and can be expressed in terms of the distance from the centre; there is no continuous area of useful uniformity; on the other hand, he showed that the *doublet* used for the C.P.D. gives a sensibly uniform field over 6°×6°.

Professor Pickering has recently issued a set of fifty-five photographs taken with a doublet as "a photographic map of the entire sky" (*Harvard Circular*, No. 72), being picked plates chosen out of those with which a continuous watch has been kept upon the sky at Harvard College and Arequipa for many years.

It seems useful, therefore, to ascertain whether any considerable area of *these* plates has reasonable uniformity of star-density, and if so to what extent.

I am indebted to Professor Turner for suggesting this question, and for the use of the set of plates at the Oxford University Observatory.

The photographs were taken with a portrait lens of one inch aperture and thirteen inches focal length, and they cover about 30° square. Twelve plates (Nos. 1, 2, 6, 19, 16, 22, 28, 34, 40, 46, 50, and 54) were chosen, to cover an entire belt of the sky passing through the poles and cutting the Milky Way nearly at right angles; in fact, the plate centres lie along the equinoctial colure, at declinations +60°, +30°, 0°, −30°, −60°, on both the 2ʰ and xiiʰ

circles, and one at $0^h+90°$ and $9^h-85°$. The plates not having réseaux themselves, a separate plate with a 5 mm. réseau of thirty-two squares to a side was used, and this covered most of the photograph. Some difficulty was found at first owing to grit in the film of the réseau (due perhaps to its having unfortunately been washed in the hard Oxford water instead of distilled water). One naturally puts réseau and photograph film to film, so that both may be in focus together; but when, as in this case, there is only a single exposure, spots in focus are easily mistaken for stars. For counting purposes, where one is not *measuring* from the réseau lines, this difficulty is got over by putting réseau and plate glass to glass, films outwards instead of together, and the spots are then so much out of focus as to be invisible, or readily distinguishable from stars, while the réseau is still in good enough focus for the purpose of defining a small area.

As a rapid way of estimating the number and distribution of star images, I counted only those in the diagonal line of réseau squares. I therefore mounted the plates diagonally in the counting microscope, so that the eye followed along a diagonal when the frame was moved horizontally.

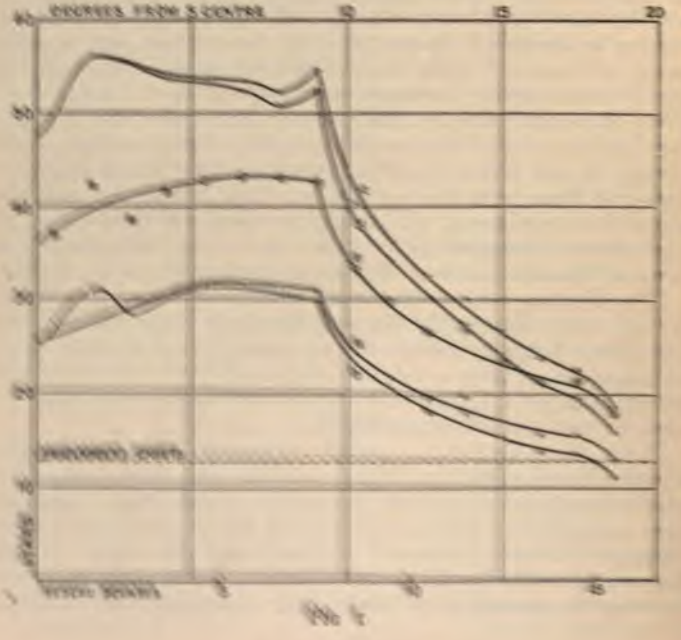
I need not give the figures for each plate. The star-density curves, when reduced to percentages of the maximum density, are much the same for all of them. One plate, No. 50, was so much in the thick of the Milky Way (near the Southern Cross) that the number of stars—as many as 250 in one réseau square—quite swamped the other counts, and it would give a false impression to include it in ascertaining the *average* star-density, though it does not make much difference in the *form* of the density curve itself. The table below shows this. (The northern Milky Way plate happens to come on a poor region of the Milky Way, and the number of stars photographed, though well above the average, is not disturbingly abnormal.) The réseau lines are numbered from 0 at the centre to 16 at the corner, and the figures for equal areas on the sky are deduced from those for equal areas on the plate by multiplying them by the cube of the secant of the distance from the centre, given in the last column.

The diagram shows the result better. We see that up to about 9° from the centre the star-density is fairly uniform, the undulations being doubtless due to incomplete compensation of the lenses, remarkably so at the centre of view. A sharp fall follows, and at 15° from the centre the density has fallen to about 50 per cent. of the maximum.

Here, then, we have a central area approaching to the uniformity of the doublet type of lens used for the C.P.D., apparently combined with the falling off beyond it of the single objective type; but with this difference, that whereas the curve required by Professor Turner's formula* is anticlinal, the curve of the Harvard plates is synclinal, and also the outer part is not a continuation by reversal of the inner part. (Compare Figs. 1 and 2.)

* P. 441 in *loc. cit.*

Réseau square.	Number of stars photographed at different distances from the plate centre on equal areas				Star-density (Number per sq. deg.)	Distance from centre.
	on the plate		on the sky			
	Incl. pl. 50.	Excl. pl. 50.	Incl. pl. 50.	Excl. pl. 50.	Excl. pl. 50.	° ′
0 to 1	49·8	27·0	49·8	27·0	36·9	0 37
1 „ 2	56·0	30·9	56·0	30·9	42·2	1 50
2 „ 3	55·1	28·2	55·1	28·2	38·5	3 4
3 „ 4	53·5	30·0	54·0	30·3	41·4	4 17
4 „ 5	53·1	31·2	53·6	31·5	43·0	5 30
5 „ 6	52·3	30·9	53·3	31·5	43·0	6 43
6 „ 7	50·7	30·4	52·2	31·3	42·8	7 56
7 „ 8	52·4	29·8	54·5	31·0	42·4	9 8
8 „ 9	38·5	23·2	40·4	24·4	33·3	10 20
9 „ 10	34·2	20·6	36·3	21·8	29·8	11 30
10 „ 11	30·3	18·0	32·7	19·4	26·5	12 41
11 „ 12	27·5	18·0	30·0	19·6	26·8	13 50
12 „ 13	24·2	15·3	26·9	17·0	23·2	15 0
13 „ 14	21·1	13·7	23·8	15·5	21·2	16 8
14 „ 15	19·5	13·5	22·4	15·5	21·2	17 15
15 „ 16	15·7	11·0	18·4	12·9	17·6	18 23
Curve in diagram.	*a*	*b*	*c*	*d*	*e*	



The change of star-density, once it begins, is more rapid with the doublet than with the single objective. Central uniformity is gained at the expense of the outlying parts, and it is possible that the Cape doublet would show a falling off of the same character were the field extended a few degrees further. The advantage of uniformity over the *whole* plate is obvious: it is the ideal of the lensmaker and the astronomer. The attractiveness of the C.P.D. plates in this respect is attained by confining the field of view within the limit indicated by the curves. The Harvard plates exceed this limit, but nevertheless it is satisfactory to find that the Harvard plates over their large area of $30° \times 30°$ have the same range of star-density (from 50 to 100 per cent.) as the astrographic plates over their area of $2° \times 2°$, with the additional merit of having the considerable area of 18° in diameter of practically uniform star-density.

Fig. 2. (From Professor Turner's paper.)

It may be of interest to compare this star-density with Argelander's charts. To find this approximately, I drew circles on the charts a degree across at the plate centres and réseau corners, and laid down a pair of rulers touching them in pairs, centre to corner, and counted the stars between them. In this way I was counting approximately the same parts of the sky as I had counted on the photographs. The projection of Argelander's charts is not such as to make it exactly so, but it is near enough for the present purpose. I could, of course, only do this for the regions included in Argelander and Schonfield's charts, namely plates 1, 2, 6, 10, 16, 22, and 28. The count gave an average star-density of 12·9—less than the corner réseau square (which was not quite the corner of the plate). The mean Harvard star-density within the 18° circle is 41·3, so that the Harvard map, *over its area of reasonably uniform star-density*, shows nearly three times as many stars as Argelander, and is therefore comparable with the astrographic catalogue plates as well in respect to the magnitude of stars shown as to the relative star-density over the plate.

1903 *December.*

The Rotation Period of the Planet Saturn. By G. W. Hough.

The white spot first observed by Professor E. E. Barnard on June 15 was a conspicuous object during the month of July.

I secured an observation on June 27, but supposed for some time that it was another spot, since I imagined the rotation period would not differ very materially from that found by Professor Asaph Hall in 1877. Owing to the low altitude of the planet, combined with much cloudy weather and poor seeing, but few observations of spots were secured.

At my request Professor S. W. Burnham made micrometer measures of spots on *Saturn* with the 40-inch telescope of the Yerkes Observatory, which he has kindly communicated to me.

I found the Barnard spot had been observed on three nights by myself and on two nights by Burnham, covering a period of fifty-three days, or 119 rotations. I have also reduced a micrometer observation by Professor Barnard made on June 23 (*Astronomical Journal*, pp. 542, 543). The time given by the micrometer differs 6·8 minutes from that adopted by the observer as the most probable value.

It is to be regretted that astronomers generally have not used the micrometer to ascertain the time of passage of the spot over the central meridian of the disc in place of a single estimation. For about twenty minutes of time, when the spot is near the central meridian, the motion may be regarded as uniform, and the time of passage over the central meridian may be ascertained by micrometer measures without taking into account the size of the disc, the latitude of the object, or the rotation period.

At mean distance one minute of rotation time is equal to 0″·087 of arc. Hence one should expect a large accidental error from an eye estimate.

With the micrometer, on the contrary, one can make as many measures as is desirable, any one of which is of far greater value than an eye estimate.

Besides the principal spot there were others which were not so well defined.

The following notation is used :—

t = mean time of observation (six hours slow of Greenwich).

m = distance from the central meridian, reduced to mean distance.

Δt = reduction to the central meridian.

T = time of passage of the spot over the central meridian of the disc.

The constants for reduction were computed by using Barnard's value of the size of the disc, viz. $a = 8''{\cdot}90$, $b = 8''{\cdot}12$ (*A.N.* No. 3760).

In determining rotation period the observations have been corrected for motion in longitude, annual parallax, and aberration time.

Micrometer Measures, Barnard Spot.

Date 1903.	t h m	m "	Δt m	T h m	App. Disc. "	Obs.
June 23	15 03·0	+2·68	+32·2	15 35·2	+2·63	Bar.
27	14 45·3	+2·75	+33·0	15 18·3	...	...
	56·9	+1·86	+22·2	19·1	+2·74	...
Mean	...	...	...	15 18·7	...	Ho.
July 13	14 17·0	−0·13	− 1·6	14 15·4	...	...
	20·7	−0·47	− 5·4	15·3	+2·69	...
Mean	...	...	...	14 15·3	...	Ho.
18	12 11·0	−2·61	−31·3	[11 39·7]	+2·98	Ho.
29	13 27·0	−0·51	− 6·1	13 20·9	+2·90	β
Aug. 19	9 58·0	−2·24	−26·8	9 31·2	[+1·55]	β

Minor Spots.

July 6	13 38·9	−4·02	−49·5	12 49·4	+3·54	Ho.
22	12 42·2	−2·99	−36·2	12 06·0	+1·31	β
23	11 26·1	+0·13	+ 1·5	11 27·6	...	...
	34·2	−0·50	− 5·8	28·4	+0·68	β
Mean	...	...	...	11 28·0	...	...
23	foll. spot	+2·42	+29·0	11 57·0	...	β
30	11 20·0	+0·54	+ 6·5	11 26·5	...	β
Aug. 9	10 59·3	−0·11	− 8·3	10 51·0	...	Ho.
	foll. spot	+3·41	+41·6	11 32·6	...	Ho.
20	9 24·2	−2·44	−29·2	8 55·0	...	Ho.
20	9 31·0	+0·12	+ 1·3	9 32·3	...	Ho.

July 18.—Seeing very bad; approximate.
Aug. 19.—Spot faint and difficult.

By combining the observation of June 27 with the following ones successively we get for the rotation period—

d.	h m s
0 to 16	R = 10 38 19·0
0 „ 32	26·1
0 „ 53	29·0

These values indicate that the rotation period was not constant.

I have shown that the rotation periods for spots in the planet *Jupiter* are a function of the time, and are sensibly constant only for short intervals.

The following table shows the comparison of the observations with an ephemeris :—

First, for a uniform rotation period

$$R_1 = 10^h\ 38^m\ 27^s$$

and, secondly, for an increasing period

$$R_2 = 10^h\ 38^m\ 18^s + n \times 0^s{\cdot}1856$$

n = the number of rotations since June 27.

$$R_1 = 10\ 38\ 27$$
$$R_2 = 10\ 38\ 18 + n \times 0{\cdot}1856$$

Date 1903.	Obs.	Days.	T.	O−E$_1$.	O−E$_2$.
				m	m
June 23	Bar.	−4	15 35·2	+ 1·8	+0·4
27	Ho.	0	15 18·7	+ 0·0	−0·3
July 13	Ho.	16	14 15·3	− 4·8	−1·7
18	Ho.	21	[11 39·7]	[+15·5]	...
29	β	32	13 20·9	− 1·1	+1·4
Aug. 19	β	53	9 31·2	+ 4·1	−0·2

The constant period satisfies the observations with a mean residual (O−E$_1$) $\neq 2^m{\cdot}3$, or a probable error on rotation period $\neq 1^s{\cdot}1$.

For the variable period the mean residual O−E$_2$ is $\neq 0^m{\cdot}8$, which is better than one ought to expect from the difficulty of the observations.

The kronocentric latitude of the spot was deduced from the measures made on June 27 and July 13, viz. + 36° 33′ and +36° 22′ respectively.

The mean of the two values was adopted in the reduction of the observations.

The spots observed on July 6, 22, and 30 are identical. The rotation period for this spot is $10^h\ 38^m\ 30^s{\cdot}5 \neq 3^s{\cdot}2$.

Dearborn Observatory of North Western University,
Evanston, Illinois : 1903 *November* 11.

Observations of the Leonid Meteors of 1903 *made at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

Until 4^h A.M. on November 16 meteors were appearing at a regular rate (about twenty per hour), and were easily kept under observation by Messrs. Showell and Parkinson, the two observers on duty. At 4^h, as there appeared to be no prospect of a shower, observations were suspended. Shortly afterwards an increase in the rate of appearance took place, and between 4^h 30^m and 5^h A.M. it was estimated by Mr. Furner that meteors were falling at the rate of about eighty per hour; a result closely confirmed by the regular observers on duty, who resumed observing from 5^h 54^m till 6^h 16^m A.M. During the night's watch (about five hours) the number of observations amounted to 107, nearly 90 per cent. of which were *Leonids*.

It should be added that Mr. Crommelin, who was on duty with the altazimuth on November 15 from 4^h 30^m to 6^h 0^m (civil time), and was looking at the sky fairly continuously during the time, saw no meteors.

The morning of November 17 was cloudy.

Royal Observatory, Greenwich:
1903 December 8.

The Shower of Leonids in 1903. By W. F. Denning.

A strikingly abundant display of these meteors occurred on the morning of Monday, November 16. Watching the clear north-eastern sky between 12^h and 14^h (November 15) I found the horary number of *Leonids* about eighteen for one observer, while from 14^h to 16^h the rate increased to fifty. Between 16^h and 18^h there was a further rise to about 130 per hour. The maximum was from 17^h 30^m to 17^h 45^m, when forty-two were counted. Between 17^h 30^m and 18^h 5^m the rate of apparition must have exceeded 200 per hour for one observer watching the sky from a good open position. Here the view is somewhat restricted by trees and buildings, so that the meteors enumerated were certainly less than the numbers actually visible under the best circumstances. The shower was about five times as rich as a normal display of *Perseids*.

I counted the meteors at alternate intervals of 15^m, so that I might not only be able to trace the strength of the shower, but also to record some of the brighter objects and note special features. The following is a summary of the chief results.

The meteors were bright generally, and nearly all 1st and 2nd mags. Few, however, were of sufficient brilliancy to outshine *Venus*. Perhaps about one in twenty-five appeared as bright as *Jupiter*, while one in 100 was equal to *Venus*. In the showers of August *Perseids* and November *Andromedids* a considerable number of very small meteors are intermingled with the more brilliant members, but in this respect they offer a marked distinction to the *Leonids*.

The radiant was diffused over an area of about 6°, though a large proportion of the flights were accurately directed from a central point at 151°+22°. Many short and well-observed paths in and near the sickle showed the dispersion very distinctly from R.A. 147° to 155° and Dec. +20° to 26°.

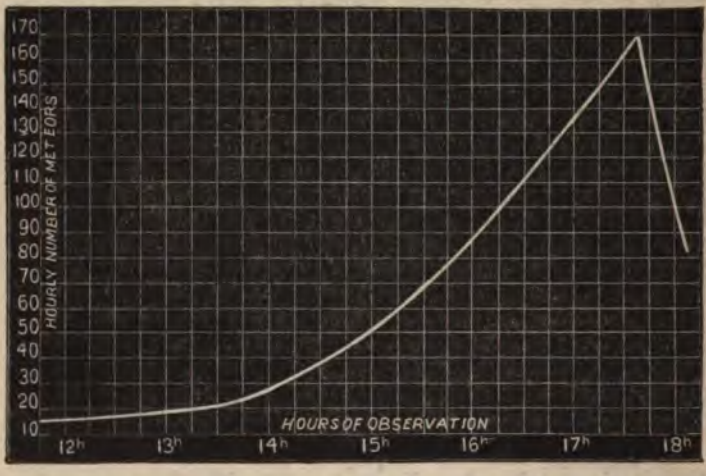


FIG. 1.—Hourly number of meteors seen by one observer. 1903 November 15, 12^h to 18^h.

In the majority of instances the apparent motions were "very swift" or "swift," but the velocities varied in a manner which could hardly be explained by differences in the individual distances or in the angles under which the lines of flight were presented. Several *Leonids* seen here when the radiant was low were described as "slow" or "rather slow," and if at ordinary heights, they could scarcely have travelled with velocities exceeding 30 or 35 miles per second, owing probably to the effects of atmospheric resistance.

The really active portion of the shower was confined to the morning of November 16. On the previous morning I watched a beautifully clear sky for three quarters of an hour (November 14 17^h to $17^h\ 45^m$); but though "the Sickle" was high in the south not a single meteor shot from it, though five were observed from

other directions. On the morning of November 17 the firmament was not very favourable, but during occasional observation

FIG. 2.—The Radiant of the *Leonid* shower, 1903 November 15, 12h to 18h

meteors were found to be rare, while the *Leonid* shower afforded only scanty evidence of its presence.

Taking the maximum as having occurred on November 15, $17^h\ 40^m$, the Earth's longitude was 52° 41′, while the bright shower observed in America in 1901 reached a maximum on November 14 at about $22\frac{1}{2}^h$ G.M.T., when the Earth's longitude was 52° 23′. The latter shower appears to have been decidedly stronger than the former, for at the richest time in 1901 various observers found the rate of appearance six or seven per minute. In 1903, during the half-hour when meteors were falling in most plentiful numbers, the rate, as determined under the best surrounding conditions, was about four per minute.

As usual in such displays the meteors came in bursts : two or three were frequently noticed at the same moment pursuing parallel flights in nearly the same region. During the most active period there were successive volleys of four, five, or six. At $16^h\ 45^m$ two bright flashing *Leonids* (one = *Jupiter*, the other > *Venus*) were seen almost at the same instant : one crossed the head of *Auriga* at δ, the other left a streak for more than a minute on the fore paws of *Leo* between *o Leonis* and *ζ Hydræ*.

I observed twenty-seven meteors during the night belonging to the numerous minor showers of the epoch. An exceedingly slow red meteor, of mag. 4, was recorded, at $15^h\ 41^m$, struggling along a short arc from 219°+63° to 209°+61¼° in about three seconds. It very probably belonged to a long-enduring radiant at 262°+62° near *ζ Draconis*. At $15^h\ 59^m$ an unusually slow streak-leaving meteor of mag. 2 traversed a long flight of 45° from 178°+19° to 223°+41° (approximately from *β Leonis* to *β Boötis*). Its visible course may have been even longer than this, for it sailed into my view from behind the corner of a house. The meteor was also seen by Mr. W. Parkinson at the Royal Observatory, Greenwich ; by the Rev. S. J. Johnson at Bridport, Dorset; and by Mr. F. H. Wright, Northampton; and its radiant appears to have been low in the south-eastern sky at 147°−11°. A preliminary determination of the real path indicates the flight, as an extended one of 128 miles, from over Kent to Lincoln, at a height gradually descending from 91 to 45 miles. Velocity about 29 miles per second. At the end of its flight the meteor slowed up considerably, and became almost stationary, as though, its material being well nigh spent, the increasing pressure of the atmosphere had effectually retarded its further progress. Additional observations of this interesting object would be valuable. Its individuality of direction and slow, sailing flight over an extensive arc of the heavens must have caused it to arrest the special notice of many persons who were watching the *Leonid* shower.

Duple or multiple observations of several bright *Leonids* were obtained during the recent display, and the real paths of these will be computed. A 2nd mag. *Leonid* was seen on November 15 $13^h\ 18^m$ by Mr. C. L. Brook at Meltham, near Huddersfield, and also by the writer at Bristol. It fell from a height of 73 to 58 miles, along a path of 36 miles, at a velocity

of about 48 miles per second. The radiant was at 151°+24°. Mr. Brook has also calculated the real path, and makes the height 77 to 61 miles, length of course 34 miles, and velocity 45 miles per second.

For comparison with this and other similar results it may be useful to give the average heights, &c., of seventeen *Leonids* doubly observed in England during the last seven years.

Height at first appearance	...	...	84·1 miles
„ disappearance ...	...	...	55·9 „
Length of visible course ...	...	...	45·1 „
Velocity per second	...	...	49·8 „
Radiant point	...	...	150·65+23·1

Subjoined are the apparent paths of a few bright *Leonids* and of several other shooting stars recorded here during the recent display, and it would be interesting to have duplicate observations of any of these objects.

Date. 1903	G.M.T.	Mag.	Path From α	Path From δ	Path To α	Path To δ	Length.	Duration.	Remarks.	Probable Radiant.
	h m		°	°	°	°	°	sec.		
Nov. 15	12 36	1	122	+24½	103	+23½	17½	1·6	slow, stk.	L.
	12 53	1½	187½	+44	211	+48	17	1·4	slow, stk.	L.
	13 9	3	265	+70	245	+63	10½	1·0	slow	332+71 / 13+39
	13 13	1	119½	+48	97	+53½	15	0·8	v. swift, stk.	L.
	13 35	5	129	+26½	138	+28	8	0·9	slow	Orion
	13 40	3	130	+60	147	+59	8½	1·0	slow	α Aurigæ
	14 32	1½	141½	+35	138½	+39	5	0·5	swift, stk.	L.
	14 36	>1	117½	+26½	98	+26½	18	0·9	swift, stk.	L.
	14 41	2	32	+71	2	+81	12	0·7	swift	48+43
	14 43	1	162	+38½	168	+44	7	0·6	swift	L.
	14 43	½	73	+72½	17	+63	22	1·2	swift, stk.	L.
	15 9	5	227	+60½	234	+57½	5	0·9	slow	194+67
	15 13	1½	173½	+34½	187	+38½	11½	0·7	slow, stk.	L.
	15 38	4	134	+58½	147	+49	12	1·2	slow	13+39
	15 41	4	219	+63	209	+61½	5	3·0	v. v. slow	262+62
	15 59	2	178	+19	223	+41	45	5·0	v. v. slow	147−11
	16 4	1	168½	+37	186	+45	15	1·0	swift, stk.	L.
	16 12	½	70	+70	19	+59	23	1·3	swift, stk.	L.
	16 35	2	202	+62	217	+46½	17½	1·3	swift	α Aurigæ
	16 37	½	138	+35	133	+39	6	0·5	swift, stk.	L.
	16 45	½	92	+53	68	+52½	15	0·9	swift, stk.	L.

Date. 1903.	G.M.T.	Mag.	Path From α	Path From δ	Path To α	Path To δ	Length.	Duration.	Remarks.	Probable Radiant.
	h m		°	°	°	°	°	sec.		
Nov. 15	16 45	♀	138	+12	134	+7½	6	0·5	swift, stk.	L.
	17 7	4	165½	+41	161½	+36	6	0·5	swift, stk.	177+49
	17 11	1	213	+29	223	+27	9	0·7	swift, stk.	L.
	17 14	♃	173	+40½	190	+47	13½	0·9	swift, stk.	L.
	18 5	>♃	182	+51	195	+55½	9	0·7	swift, stk.	L.
	16 5 59	>1	343	+38½	308	+34	29	3·5	v. slow, yellow train	42+20 58+9

Fig. 1 exhibits the increase in the hourly number observed. This was due to two causes—viz. the much greater density of that part of the stream through which the Earth passed towards 18^h and the improving position (higher altitude) of the radiant as the night progressed.

In fig. 2 the diffuse character of the radiant is shown by the intersecting lines of flight of a number of well-observed *Leonids*. In some cases the meteors appeared outside the limits of the diagram, but their paths have been carried backwards through the radiant.

Bishopston, Bristol: 1903 *November* 25.

Cape Double Star Results, 1903. By R. T. A. Innes.

(*Communicated by Sir David Gill, K.C.B., F.R.S., H.M. Astronomer.*)

The observations in 1903 were confined to a few nights in February and March.

All the measures have been made with the Repsold micrometer on the 18-inch refractor. The assumed value of one revolution of the micrometer screw has been taken as 17″·600.

An explanation of the columns headed "Diff. of Mags." and "Colours" is given on p. 471 of the *Monthly Notices* for 1902 April, vol. lxii.

My measures of double stars with the above instrument extend altogether from 1899 November to 1903 March, and are now brought to a close by my transfer to the Transvaal. My thanks are due to Sir David Gill for his constant encouragement and help, and for the clerical assistance placed at my disposal. The work was greatly facilitated by the mechanical conveniences of the McClean Observatory, the rising floor in particular being of the utmost value for double-star work.

Star.	Mag.	1900. R.A.	1900. S. Dec.	1903+	Pos. Angle.	Dist.	Diff. of Mags.	Colours A	Colours B	No. of Nights.	Remarks.
		h m	° ′		°	″					
Innes 387	8·0	3 1	60 23	16	142·0	0·51	0·0	2, 2		1	
„ 388	8·3	3 14	63 39	16	314·5	0·62	0·2	...		1	
Cape 38	9·0	3 42½	40 29	16	211·7	3·96	0·5	...		1	
Å 3642	6·7	4 15	34 9	16	159·8	6·05	2·0	2, *b*		1	
Innes 413	7·8	4 23	24 41	16	349·8	0·80	0·8	...		1	
Å 3683	6·6	4 39	59 8	16	80·6	1·54	0·1	3, 3		1	1
Cordoba [9]	7·4	4 39	48 1	16	234·8	3·73	2·0	2, *b*		1	2
„ [10]	7·5	4 46	50 3	16	258·4	4·82	2·5	2, *b*		1	
Innes 342	7·5	4 47	54 4	16	175·5	0·91	0·5	3, 3		1	
„ 343	7·8	4 48	54 37	16	45·6	2·56	3·5	...		1	
λ 42 ..	6·9	4 49	51 54	16	Single	...	...	...		1	
Å 3707	9·2	4 51	59 55	16	270·6	8·50	3·0	...		1	
Innes 274	8·0	5 0	50 56	16	259·4	3·34	1·7	2, *b*		1	
Cordoba [12]	8·4	5 2	40 42	17	241·8	2·14	0·4	1, 1		2	
Å 3743	8·5	5 12	60 6	17	131·4	5·11	1·7	*b*, 2		2	
Cordoba [13]	8·8	5 14	27 36	16	274·0	3·21	0·4	...		1	
Innes 61	7·8	5 15	41 9	22	100·5	0·91	0·5	1, 1		1	
„ 414	6·4	5 21	56 15	19	Single	...	...	...		2	3
8 Pictoris	6·3	5 23	52 24	19	194·3	0·48	0·3	3, 3		2	
Innes 346	5·9	5 24	41 2	19	180±	15±	8·1	...		1	4
Å 3770	7·6	5 29	24 24	22	16·4	4·17	3·2	3, *b*		2	
Innes 278	8·5	5 43	68 45	19	174·2	1·20	2·5	...		2	
„ 279	8·5	5 43	43 18	20	2·0	0·75	1·8	3, 3		3	
„ 63 AB	7·1	5 46	48 57	22	12·5	1·19	1·8	...		1	
„ „ AC				22	209·6	32·3	6·5	...		1	
Å 3807	8·3	5 49	41 42	16	270·8	5·15	2·7	2, *b*		1	
Cape 39	9·4	6 19	73 26	17	259·9	5·49	1·1	...		2	
G Puppis	6·0	6 23	48 7	16	132·5	1·30	3·0	2, *b*		1	
Innes 350	7·5	6 26	45 43	22	10·6	15·9	5·0	...		1	
Lac. 2333 AB	5·7	6 27	50 10	19	261·1	0·64	0·1	3½, 3½		2	
„ CD	6·7			19	117·8	0·40	0·0	6½, 6½		2	
℞ 754	7·0	6 31	33 56	19	36·3	0·87	0·4	3, 3		2	
Innes 178	8·3	6 31	32 14	19	203·1	0·80	2·0	...		2	
CZ. 1518	7·6	6 33	48 12	19	201·2	10·7	1·8	2½, *b*		2	
Dunlop 31	5·0	6 36	48 8	16	319·2	13·5	2·0	4, *b*		1	
Innes 180 AB	8·5	6 42	52 19	16	336·5	0·81	1·0	...		1	
„ 157	7·0	6 45	54 35	16	343·6	1·92	3·0	2, *b*		1	
„ 158	7·2	6 43	48 27	22	184·0	1·69	3·5	3, *b*		1	

Star.	Mag.	1900. R.A.	1900. S. Dec.	1902+	Pos. Angle.	Dist.	Diff. of Mags.	Colours	No. of Nights.	Remarks.
		h m	° ′		°	″		A B		
Innes 181	... 8·0	6 47	44 56	22	255·0	0·68	1·0	0, 0	1	
„ 182	... 8·0	6 47	28 36	22	136·3	0·79	2·0	0, 0	1	
„ 431	... 8·5	6 47	28 37	22	321·5	0·37	0·0	...	1	5
„ 432	... 7·8	6 53	28 36	22	213·2	1·65	0·0	...	1	
„ 65	... 6·3	6 54	35 23	18	216·3	0·30	0·0	$3\frac{1}{2}$, $3\frac{1}{2}$	3	6
Innes 184	... 8·0	7 8	60 25	18	179·3	0·71	0·4	...	3	
Harvard	... 6·0	7 11	63 1	19	153·2	0·53	0·4	2, 2	2	
Innes 7	... 7·3	7 15	46 49	22	212·0	1·10	...	4, *b*	1	
„ 391 AB	8·7	7 16	21 42	16	275·2	4·67	0·2	...	1	
„ AC				16	30 ±	5 ±	4·3	...	1	
Innes 312	... 7·7	7 19	75 41	22	166·9	0·88	0·0	3, 3	1	
„ 352	... 8·5	7 22	37 57	19	356·6	0·67	0·6	0, 0	2	
h 3997	... 6·1	7 37	74 3	16	293·2	1·65	0·1	2, 2	1	
h 4003	... 9·2	7 44	23 56	18	117·0	31·2	1·7	$\frac{1}{2}$, 6	2	7
Innes 161	... 5·3	7 44	38 16	18	84·5	10·6	4·0	1, *b*	1	
Cape 20	... 8·5	7 45	44 18	18	88·1	3·49	1·0	...	1	
ξ Argûs	... 3·4	7 45	24 37	18	241 ±	...	...	...	1	8
Howe 8	... 5·0	7 49	34 27	18	289·2	2·94	3·0	$2\frac{1}{2}$, *b*	2	
β 333	... 7·7	7 57	22 4	18	45·1	1·83	2·5	...	2	
Howe	... 8·5	7 58	26 59	18	331·1	2·49	2·5	...	1	
h 4041	... 7·8	8 1	22 9	17	Sing'r	...	...	...	1	
Lac. 3146 AB	6·5	8 2	33 17	20	87·4	22·3	2·0	4, *b*	2	
„ AC				20	58·3	27·9	4·0	...	2	
Innes 164	... 7·5	8 3	39 49	22	83·9	0·62	1·0	...	1	
„ 193	... 8·1	8 10	34 26	17	103·8	2·48	1·5	...	2	
Howe 9	... 8·7	8 13	26 58	17	118·1	3·26	0·1	0, 0	2	
Cordoba [77]	8·2	8 29	50 38	18	99·3	3·53	1·3	3, *b*	2	
Innes 68	... 8·6	8 33	30 29	17	67·2	7·49	2·0	...	2	
Cordoba [19]	8·0	8 37	31 27	18	328·8	5·49	2·7	3, *b*	2	
Innes 315	... 8·5	8 38	42 14	18	166·8	0·99	0·2	...	1	
„ 70	... 7·0	8 44	38 34	18	111·9	1·58	2·0	...	1	
„ 317	... 8·5	8 52	43 5	18	304·3	2·19	1·5	1, *b*	1	
h 4164	... 7·5	8 56	65 49	18	145·9	10·7	2·5	4, B	1	
b_2 Carinæ	... 5·2	8 57	58 42	18	20·8	28·2	...	...	2	
Innes 357	... 8·4	8 58	23 21	24	177·1	0·67	0·1	...	1	
h 4165	... 5·4	8 59	51 48	21	103·6	1·46	2·0	2, —	1	
Innes 319	... 9·1	9 10	82 37	18	86·2	4·47	1·0	...	1	
„ 11	... 6·5	9 12	45 8	22	271·7	0·91	1·5	...	1	

Star.	Mag.	1900. R.A.	1900. S. Dec.	1900+	Pos. Angle	D'st.	Diff. of Mags.	Colours	No. of Nights.	Remarks.
		h m	° ′		°	″		A B		
Innes 358	... 5·4	9 16	68 16	18	133·0	18·1	7·1	3, —	1	
Cape 33	... 8·4	9 16	69 58	20	339·0	4·76	0·6	2½, 0	2	
Innes 198	... 8·0	9 16	28 48	22	178·9	0·33	...	...	1	
„ 170	... 7·5	9 17	34 56	22	32·4	1·31	3·0	...	3	
„ 12	... 5·4	9 18	74 28	22	263·3	0·31	0·3	2½, 2½	2	
„ 72	BC 9·6	9 23	52 57	21	32·9	3·14	1·0	...	1	
„ 199	... 7·8	9 23	69 59	18	136·1	2·69	2·5	0, *b*	1	
φ Argus	... 3·5	9 27	40 2	20	24·1	0·29	...	3, 3	6–5	
Innes 203	... 8·2	9 36	62 6	22	313·8	0·45	0·3	...	1	
Hargrave 28	... 7·8	9 42	59 1	22	284·6	2·02	2·5	3, B	1	
Innes 204	... 7·2	9 44	69 39	16	127·7	1·87	2·5	...	1	
„ 174	... 8·4	10 28	61 4	16	46·0	1·06	0·5	...	1	
Jacob [8]	.. 6·4	12 5	34 9	23	21·2	3·54	...	2, *b*	1	
Harvard AB	... 6·2	12 23	55 51	23	275·4	27·2	6·8	...	1	
AC	...			23	290·2	48·9	5·8	...	1	
γ Centauri	... 2·4	12 36	48 25	22	353·6	1·42	0·0	3, 3	3	
Cordoba [27]	8·3	12 39	61 40	23	98·1	4·80	...	6, *b*	1	
Harvard	... 6·7	12 47	53 17	23	210·6	6·05	5·0	4½, —	2	
Innes 399	... 7·5	13 8	62 7	23	137·6	8·32	4·0	3, —	1	
„ 400	... 8·8	13 22	42 41	23	151·5	4·75	3·0	3, *b*	1	
h 4398	... 7·0	13 33	74 37	23	46·5	13·1	4·0	3, —	1	
h 4616	... 9·5	13 45	70 41	23	349·5	4·55	1·0	...	1	
Harvard	... 5·2	14 8	56 37	23	169·3	33·6	...	...	1	
„	... 5·9	14 9	66 7	23	307·0	24·1	7·1	...	1	
Dunlop 159	AB 5·0	14 15	58 0	18	160·2	9·51	2·0	4, B	1	
AC				18	255·0	20·7	8·5	...	1	
AD				18	4·0	45·6	5·5	...	1	
Innes 426	... 5·6	14 24	44 52	18	313·6	10·5	5·0	2, —	1	
Innes 367	... 8·0	14 32	64 17	18	357·6	4·15	4·0	3, —	1	
α Centauri	... 0·2	14 33	60 25	21	211·2	21·66	1·2	2·3, 4	10	
ε Circini	... 3·4	14 34	64 32	20	236·0	15·8	4·0	2, 7½	2	
Innes 334	... 9·0	15 54	71 37	23	141·0	2·53	1·5	2, *b*	1	

Remarks.

1. Fixed in angle; some decrease in distance.
2. P.M. of 0″·094 towards 238°·1.
3. To be rejected as single.
4. Too faint to measure.
5. New. The chief star of the old pair *h* 3896.

6. The angle increases 10° a year.
7. Mag. from Cor. D.M. Estimated 8·0. Is 8·7 in C.P.D. and marked: Angle decreasing, distance decreasing.
There is a 13th mag. star 200° ±, 35″ ±.
8. Change. Two careful estimations with the chief star out of the field.

Short Method for the Calculation of the Orbits of Celestial Bodies.

By D. A. Pio.

(*Communicated by the Secretaries.*)

1. *Gain of Time is the Object.*—The determination of the elements of the orbits described by a celestial body has been carried by Gauss to the highest pitch of perfection the present state of mathematical science allows us to reach. The author has not the pretension to propose a new method for solving the same problem; he only intends to substitute short, simple, and easy calculations for those which the method of Gauss requires, and limits himself to the most simple case: how to determine the first approximate values of the elements of a newly discovered celestial body with the least loss of time, so as to render the calculation of a new orbit an easy performance.

On the authority of the illustrious French mathematician M. H. Poincaré, the writer begs to state that the calculation of the first approximate orbit, by Gauss's method, requires from fifteen to seventeen hours. A reduction to about six to seven hours is what the author is working for. A gain of about eleven hours with a sufficiently approximate determination of the elements would be a service to practical astronomers.

The author proposes, for attaining this short calculation of orbits, a modification of Gauss's method containing innovations of great practical interest for the efficient simplification of the mathematical work.

The writer begs to remark that the problem of orbits has been really solved long ago; that the question is only how to render tedious calculations more easy, but not to present new theories by which the work of La Place, Olbers, and Gauss may be swept away. This is the point of view from which the author begs his work to be considered. Practical improvement, not radical subvertment, is the only amelioration Gauss has left posterity to accomplish.

2. *Preliminary Remarks.*—It is supposed that the data furnished by the observations and those taken from the solar tables are corrected for precession, nutation, and aberration of the fixed stars. Parallax and aberration in time are not taken into consideration. The calculations are made with logarithms to not more than five decimal places.

3. *Simplifications of Gauss's Method.* – (1) Gauss established the equation

$$\mathfrak{M} \cdot \sin^4 z = \sin(z+\omega)$$

which leads to the value of r_2. This part of the solution cannot be improved. However, modern authors have brought confusion into the signs (+ or −) of the many auxiliary quantities occurring in the calculation: especially the case where r_2 is *less* than unity offers difficulties. The writer has reverted to the *Theoria Motus*, and has removed all ambiguities (see articles 4 and 5).

(2) The solution of the equation

$$\mathfrak{M} \cdot \sin^4 z = \sin(z+\omega)$$

has been rendered shorter by using logarithms with three decimals for the first approximations. The method of logarithmic differences has been reduced to its simplest form (see articles 7 and 8)

(3) Gauss deduces the elements of the orbit from the known values of r_1 and r_3, and from the angle $2f = v_3 - v_1$ contained between them. The calculation of these quantities requires the previous calculation of the distances from the Earth, ρ_1 and ρ_3, which as done in the usual manner is the most lengthy and the most tedious part of the whole work. The usual formulæ, as given in Tisserand, *Leçons sur la Détermination des Orbites*, pp. 76 and 77, are twenty-four in number, and some are very troublesome. The writer has reduced them to six by using the "fundamental equations" for the calculation of $r_1 = \rho_1 \cos\beta_1$ and $r_3 = \rho_3 \cos\beta_3$. The time thus gained amounts to some hours (see article 9).

(4) The parameter, the eccentricity, and the longitude of the perihelium of an orbit are the most difficult points in the determination of the elements. Gauss invented a most ingenious method for the calculation of the parameter by successive approximations through auxiliary tables. The author has much shortened the calculation of the parameter by reverting to the formula of Euler given in *Theoria Motus*, § 86 (VII.). It is the shortest existing.

(5) The formulæ usually employed for the calculation of ϕ, the angle of eccentricity, and the true anomalies v_1 and v_3 are very tedious. They occasion a considerable loss of time (see Tisserand, p. 80). The arc v_1 has been determined by a simple formula taken from *Theoria Motus*. The value of the eccentricity may be calculated very easily when the parameter, one of the radii vectores, and the corresponding true anomaly are known. The author, profiting by this circumstance, calculates *two* separate values for e and takes their *geometrical mean* as the most probable value.

Each of the above modifications is certainly a detail of no theoretical importance by itself, and has no other merit than

either to shorten the calculation or to increase the accuracy. The author begs to submit that all these modifications, taken as a whole, give quite a new appearance to the method of Gauss.

The question is, then, whether the above modifications satisfy or not the requirements of practical astronomers.

4. *Rules of Signs.*—The calculations preliminary to the solution of

$$\mathfrak{M} \cdot \sin^4 z = \sin(z+\omega)$$

are relative to the determination of the quantities ψ, δ, τ, T, Σ, σ, S, d, u, ω, Ω, and h. The angles ψ, δ, τ, and u are to be taken always as positive arcs *less* than 180°. The arc σ is always less than 90° : it may be positive or negative, but it *must* be the *least* of all values that result from the absolute number found for $tg\,(\sigma+\delta)$; and the sign of $tg\,(\Sigma = \delta+\sigma)$ must be determined in conformity. The arc ω must always be less than 90°, but it may be negative or positive. The signs of the linear quantities T, S, d, Ω, and h are + or − according as the calculation determines them. Every other rule, even given by great authorities, leads to impossible results.

NOTE.—Oppolzer's rule to make T always positive is false when r_2 is less than 1. In this last case ω also is positive and not negative.

5. *A Difficulty in the Solution of* $\mathfrak{M} \cdot \sin^4 z = \sin(z+\omega)$.—The equation

$$\mathfrak{M} \cdot \sin^4 z = \sin(z+\omega)$$

is usually applied only to the planetoids where r_2 is comprised between 1·5 and 4, and then ω is usually negative and nearly equal to z. In such cases the unknown value of z is found by successive approximations, the value of $-\omega$ being the first, and the method of logarithmic differences leads very rapidly to a value sufficiently approximate for the first determination of the orbit. However, when r_2 is less than R_2, then δ is greater than 90°, and the angle ω is not only positive but very different from z, so that it becomes difficult to find a first approximation for z. As usually ω is small, by assuming $\sin\omega = 0$ the equation

$$\mathfrak{M} \sin^4 \omega = \sin(z+\omega)$$

becomes

$$\mathfrak{M} \cdot \sin^3 z = \cos\omega$$

and then

$$\sin z = \sqrt[3]{\frac{\cos\omega}{\mathfrak{M}}}$$

which may serve as first approximation.

6. *Logarithms to three Decimal Places.*—In the usual manner of calculating the orbits much time is wasted by using logarithms to many decimal places, while the results found out with so

much labour are scarcely trustworthy to the third figure. This happens with the first attempts to solve the equation

$$\mathfrak{M} \cdot \sin^4 z = \sin(z+\omega)$$

Also the correction of the approximate value of the parameter (Euler's formula) needs only logarithms to three decimal places.

7. *Logarithmic Differences.*—The method of logarithmic differences given by Gauss in *Theoria Motus*, § 11, for the solution of Kepler's Problem is excellent for solving the equation

$$\mathfrak{M} \cdot \sin^4 z = \sin(z+\omega)$$

Its shortest form is that the author proposes.

EXAMPLE.—$N[1{\cdot}0822321] \cdot \sin^4 z = \sin(z-12^\circ)$, where $\mathfrak{M} = N[1{\cdot}0822321]$, $\omega = -12^\circ$, λ = logarithmic difference of $\sin z$, for one minute (respectively one second), μ = log dif. for 1′ (or 1″) of $\sin(z_0+\omega)$, $\Delta = \log\sin^4 z_2 - \log\sin^4 z_1$, $z_1 = -\omega$.

$\log\sin z_1$	9·318	$\lambda = 0{\cdot}5$	$\log\sin^4 z_2$	9·362
	4			4
$\log\sin^4 z_1$	7·272	$\mu = 5{\cdot}6$	$\log\sin^4 z_3$	7·448
$\log\mathfrak{M}$	1·082	$\mu - 4\lambda = 3{\cdot}6$	Δ	
			$\overline{\mu - 4\lambda}$ =	49′
$\log\sin(z_0+\omega)$	8·354	$\Delta = 176$		
$z_0+\omega = 1^\circ\ 17'{\cdot}5$		$\omega = -12^\circ$	$z_2 =$	13° 17′·5
			$z_3 =$	14° 6′·5

Six lines with logarithms are sufficient.

8. *Calculation of* $\rho_1 \cdot \cos\beta_1$ *and* $\rho_3 \cdot \cos\beta_3$.—The condition that the centre of the Sun must lie in the plane determined by the three observed positions of the celestial body furnishes three "fundamental equations," (5), (6), (7), in Tisserand, p. 47, containing the unknown quantities $r_1 = \rho_1 \cdot \cos\beta_1$, $r_2 = \rho_2 \cdot \cos\beta_2$, $r_3 = \rho_3 \cdot \cos\beta_3$. One of them, r_2, being known, the determination of r_1 and r_3 becomes very easy. Equation (7) furnishes directly r_1, and from (6), where r_1 and r_2 are known, r_3 is easily found.

9. *Concluding Remarks.*—(1) The abridgments introduced by the author will furnish the values of the elements with sufficient accuracy, when the intervals of time between two successive observations are not more than ten days for planetoids.

(2) The purpose of a first determination of an orbit being first the calculation of a provisory ephemeris, and next the setting-up of numerical values for the elements approximate enough to allow the easy correction of these numerical values by comparison with a series of observations, any luxury of calculations, as those indicated in the text-books, is a mere waste of time. Whatever be the care bestowed upon the first determination of an orbit, the result arrived at by so much work will be discarded. Therefore brevity ensuring sufficient accuracy is the ideal of the first determination of an orbit.

(3) The calculation of an orbit is considered as the trial-piece of an incipient astronomer. It is, indeed, a proof of proficiency, especially if carried out by the method of Gauss, as explained in Oppolzer's text-book. But even the dreaded method of Gauss may be popularised, and that is what the author has attempted. In about six to seven hours, when the observations have been already reduced and corrected, to determine with sufficient accuracy the numerical values of the elements of an orbit without employing special auxiliary tables, only using the common logarithmic tables of numbers and trigonometrical functions to no more than five places of decimals, this is certainly no achievement in theory, but only a sort of mechanical craft. However, is it useful or not? Has the writer succeeded in building up an engine of calculation proper for the use of practical astronomers?

10. *A Test for Asteroids.*—The writer has chosen as an example for the application of the simplified method of Gauss the same observations from which Gauss himself deduced the orbit of *Juno* in *Theoria Motus*, § 150 to § 155. His calculations are a model of accuracy, and a comparison with those made by the writer shows the degree of precision the first values for the elements may reach when the modifications proposed by the author are used. The following table contains in its first column the results of the "last hypothesis" of Gauss:

	Gauss.	Pio.	Differences.
r_2	2·1183	2·1187	+0·0004
$2f'$	7° 34′ 54″	7° 35′ 7″	+13″
a	2·6450	2·6499	+0·0049
e	0·24532	0·24744	+0·00212
i	13° 6′ 44″	13° 7′ 30″	+46″
Ω	171° 7′ 49″	171° 8′ 29″	+40″
Π	52° 18′ 9″	52° 30′ 23″	+12′ 14″
m''	824″·80	822″·58	−2″·3
L_0	41° 52′ 22″	41° 58′ 22″	+6′ 0″

Formulæ.

1. *Symbols.*—For the three observations of the planet at the epochs E_1, E_2, and E_3, we have:

$E_2 - E_1 = t_1$.
$E_3 - E_2 = t_2$.
R_1, R_2, R_3, radii vectors of Earth.
L_1, L_2, L_3, geocentric longitudes of Sun.
λ_1, λ_2, λ_3, geocentric longitudes of planet.
β_1, β_2, β_3, geocentric latitudes of planet.
l_1, l_2, l_3, heliocentric longitudes of planet.
b_1, b_2, b_3, heliocentric latitudes of planet.

r_1, r_2, r_3, radii vectores of planet.
v_1, v_2, v_3, true anomalies of planet.
ρ_1, ρ_2, ρ_3, distances of planet from Earth.
e, eccentricity of planet.
p, half parameter of planet.
a, half major axis of planet.
T^d, planet's revolution in mean solar days.
m'', its mean diurnal motion in seconds of arc.
i, inclination of planet's orbit.
Ω, longitude of planet's ascending node.
Π, longitude of planet's perihelium.
L_0, mean longitude of planet at epoch E_0.
E_0, epoch of departure.

2. *Preliminary Calculations.*—

$$A = R_3[tg\beta_3 . \sin(\lambda_1 - L_1) - tg\beta_1 . \sin(\lambda_3 - L_1)] \quad \dots \quad (1)$$

$$B = R_3[tg\beta_3 . \sin(\lambda_2 - L_2) - tg\beta_2 . \sin(\lambda_3 - L_2)] \quad \dots \quad (2)$$

$$C = R_2[tg\beta_3 . \sin(\lambda_1 - L_2) - tg\beta_1 . \sin(\lambda_3 - L_2)] \quad \dots \quad (3)$$

$$D = -R_2[tg\beta_3 . \cos(\lambda_2 - L_2) - tg\beta_2 . \cos(\lambda_3 - L_2)] \quad \dots \quad (4)$$

$$tg . \psi = \frac{tg\beta_2}{\sin(\lambda_2 - L_2)} \quad \psi \text{ positive and } < 180° \quad \dots \quad \dots \quad (5)$$

$$tg\hat{c} = \pm \frac{tg(\lambda_2 - L_2)}{\cos . \psi} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6)$$

NOTE.—$\hat{c} > 90°$ when $(\lambda_2 - L_2) \begin{matrix} < 90° \\ > 270° \end{matrix}$; $\hat{c} < 90°$ when $(\lambda_2 - L_2)$ $\begin{matrix} > 90° \\ < 270° \end{matrix}$

$\hat{c}$ positive and $< 180°$.

$$tgr = \frac{D}{R_3 . \sin(\lambda_3 - \lambda_1)} \quad r \text{ positive and } < 180° \quad \dots \quad \dots \quad (7)$$

$$T = \frac{D}{\sin r} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (8)$$

$$tg\Sigma = \pm \frac{C}{T . \sin(r + \psi)} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

$$\sigma = \Sigma - \hat{c} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (10)$$

NOTE.—σ always $< 90°$, positive or negative. Take Σ positive or negative, so that σ has always its least value :

$$S = \frac{C . \sin \hat{c}}{\sin \Sigma} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (11)$$

$$P = \frac{t_1}{t_2} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (12)$$

$$d = \frac{P+1}{A+PB} S \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (13)$$

$$tg u = \frac{1}{d \,.\, \cos \,.\, \sigma}\ u \text{ positive and } < 180^\circ \ldots \quad \ldots \quad \ldots \quad (14)$$

$$tg \omega = \frac{tg \sigma \,.\, \cos u}{\sqrt{2} \,.\, \sin (45^\circ - u)} \quad \text{comp. log } \sqrt{2} = 9{\cdot}84949 - 10 \quad (15)$$

NOTE.—ω is always $< 90^\circ$, positive or negative.

$$\Omega = \frac{d \,.\, \sin \sigma}{\sin \omega} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

$$h = \Omega(A + PB) R_2^3 \sin^3 \delta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

$$j_1 = \frac{k^2}{6} t_1 (t_1 + 2t_2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

$$j_2 = \frac{k^2}{6} t_2 (2t_1 + t_2) \quad \log \frac{k^2}{6} = 5{\cdot}69301 - 10 \quad \ldots \quad \ldots \quad (19)$$

$$\mathfrak{M} = \frac{A j_1 + PB j_2}{h} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

3. *Solution of* $\mathfrak{M} \,.\, \sin^4 z = \sin (z + \omega)$.

4. *Calculation of* r_1 *and* r_3.—

$$r_2 = \frac{R_2 \sin \delta}{\sin z} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

$$r_2 = \frac{R_2 \,.\, \cos \beta_2 \,.\, \sin (\delta - z)}{\sin z} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

$$Y_1 = \frac{j_1}{r_2^3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

$$Y_2 = \frac{j_2}{r_2^3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

$$n_1 = \frac{t_2}{t_1 + t_2} (1 + Y_1) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

$$n_2 = \frac{t_1}{t_1 + t_2} (1 + Y_2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

$$r_1 = r_2 \,.\, \sin (\lambda_2 - \lambda_3) + n_1 R_1 \sin (L_1 - \lambda_3) + n_2 R_3 \sin (L_3 - \lambda_3) - R_2 \sin (L_2 - \lambda_3) \Big\} : n_1 \sin (\lambda_1 - \lambda_3) \quad (27)$$

$$r_3 = \frac{r_2 tg \beta_2 - n_1 r_1 tg \beta_1}{n_2 tg \beta_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

$$ctg \tilde{c}_1 = \frac{r_1}{R_1 \sin (\lambda_1 - L_1)} - \cot (\lambda_1 - L_1) \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

$$l_1 = \lambda_1 + \tilde{c}_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

$$ctg\,\gamma_3 = \frac{r_3}{R_3 \sin(\lambda_3 - L_3)} - \cot(\lambda_3 - L_3) \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

$$l_3 = \lambda_3 + \gamma_3 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

$$tg\,b_1 = \frac{r_1\, tg\beta_1 \sin(l_1 - \lambda_1)}{R_1 \sin(\lambda_1 - L_1)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

$$tg\,b_3 = \frac{r_3\, tg\beta_3 \sin(l_3 - \lambda_3)}{R_3 \sin(\lambda_3 - L_3)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

$$r_1 = \frac{r_1\, tg\beta_1}{\sin b_1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

$$r_3 = \frac{r_3\, tg\beta_3}{\sin b_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

5. *Calculation of Inclination and Ascending Node.—*

$$tg\,D = \frac{tg b_1 \,.\, \sin(l_3 - l_1)}{tg b_3 - tg b_1 \,.\, \cos(l_3 - l_1)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (37)$$

$$\Omega = l_1 - D \quad D < 180° \text{ or } > 180° \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

Note whether the motion is direct or retrograde.

$$tg\,i = \frac{tg b_1}{\sin D} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

For direct motion $i < 90°$; for retrograde $i > 90°$.

6. *Calculation of Eccentricity and Major Axis.—*

$$\sin(v_3 - v_1) = \frac{\sin(l_3 - l_1) \,.\, \cos b_1 \,.\, \cos b_3}{\cos i} \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

$$\sqrt{p'} = \frac{r_1 r_3 \sin(v_3 - v_1)}{k(t_1 + t_3)} \quad \log k = 8{\cdot}23558 \quad \ldots \quad \ldots \quad (41)$$

$$\sqrt{p} = \left[1 + \frac{\sin^2(v_3 - v_1) \,.\, \sqrt{r_1 r_3}}{6p'}\right] \sqrt{p'} \quad \ldots \quad \ldots \quad (42)$$

$$tg\,v_1 = \cot(v_3 - v_1) - \frac{r_1(p - r_3)}{r_3(p - r_1)\sin(v_3 - v_1)} \quad \ldots \quad \ldots \quad (43)$$

$$e^2 = \frac{(p - r_1)(p - r_3)}{r_1 r_3 \cos v_1 \,.\, \cos v_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (44)$$

$$a = \frac{p}{1 - e^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (45)$$

$$T^d = a^{\frac{3}{2}} \times 365{\cdot}256^d \qquad \log 365{\cdot}256 = 2{\cdot}56260 \quad (46)$$

$$n'' = \frac{1296\,000''}{T^d} \qquad \log 1296\,000 = 6{\cdot}11260 \quad (47)$$

7. *Calculation of the Longitude of Perihelium and of Mean Longitude.*—

$$tgu_1 = \frac{tgD}{\cos i} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (48)$$

When $D < 180°$ $u_1 < 180°$; when $D > 180°$ $u_1 > 180°$.

$$\Pi = u_1 + \Omega - v_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (49)$$

$$tg\,\frac{w}{2} = tg\,\frac{v_1}{2}\sqrt{\frac{1-e}{1+e}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (50)$$

$$M = w - e \cdot \sin w \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (51)$$

M and w to be taken in parts of the radius.

$$t^d = E_1 - E_0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

$$L'' = t^d m'' + M \times 206\,265''$$

$$\log 206\,265 = 5{\cdot}31442 \quad \ldots \quad \ldots \quad (53)$$

$$L_0 = L + \Pi \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (54)$$

Calculation of Juno's Orbit.

1. *The Data.*—

$$t_1 = 11^d{\cdot}96324 \qquad t_2 = 9^d{\cdot}97119$$

$\log R_1 = 9{\cdot}99968$ $\log R_2 = 9{\cdot}99810$ $\log R_3 = 9{\cdot}99697$

$L_1 = 192° 28' 28''$ $L_2 = 204° 19' 49''$ $L_3 = 214° 16' 10''$

$\lambda_1 = 354° 44' 32''$ $\lambda_2 = 352° 34' 22''$ $\lambda_3 = 351° 34' 38''$

$\beta_1 = -4° 59' 31''$ $\beta_2 = -6° 21' 55''$ $\beta_3 = -7° 17' 51''$

2. *Preparatory Calculations.*—

$\log P = 0{\cdot}07910$ $t_1 + t_2 = 21^d{\cdot}93443$

$\log t_1 = 1{\cdot}07785$ $\log t_2 = 0{\cdot}99875$ $\log (t_1 + t_2) = 1{\cdot}34113$

$\log (t_1 + 2t_2) = 1{\cdot}50387$ $\log (2t_1 + t_2) = 1{\cdot}53017$

$\lambda_1 - L_1 = 162° 16' 4''$ $\lambda_2 - L_2 = 150° 24' 43''$

$\lambda_1 - L_3 = 140° 28' 22''$

$\lambda_3 - L_1 = 159° 6' 2''$ $\lambda_3 - L_2 = 147° 14' 41''$

$\lambda_3 - L_3 = 137° 18' 20''$

$\lambda_3 - \lambda_1 = -3° 10' 2''$ $\lambda_3 - \lambda_2 = -59' 52''$

$\lambda_2 - \lambda_1 = -2° 10' 10''$ $\lambda_2 - L_2 = 148° 14' 33''$

$L_3 - L_2 = 9° 56' 21''$ $L_3 - L_1 = 21° 47' 42''$

$L_2 - L_1 = 11° 51' 21''$

3. *Preliminary Calculations for* A, B, C, *and* D.—

log $tg\beta_3$	9·10741 (*n*)	log $tg\beta_3$	9·10741 (*n*)	log $tg\beta_3$	9·10741 (*n*)
log sin (λ_1-L_1)	9·48368	log sin (λ_1-L_3)	9·80376	log sin (λ_1-L_2)	9·69351
log (s_1)	8·59109 (*n*)	log (s_3)	8·91117 (*n*)	log (s_5)	8·80092 (*n*)
$(s_1)=$	−0·039003	$(s_3)=$	−0·81502	$(s_5)=$	−0·063230
log $tg\beta_3$	8·94124 (*n*)	log $tg\beta_1$	8·94124 (*n*)	log $tg\beta_1$	8·94124 (*n*)
log sin (λ_3-L_1)	9·55234	log sin (λ_3-L_3)	9·83129	log sin (λ_3-L_2)	9·73324
log (s_2)	8·49358 (*n*)	log (s_4)	8·77253 (*n*)	log (s_6)	8·67448 (*n*)
$(s_2)=$	−0·031159	$(s_4)=$	−0·059229	$(s_6)=$	−0·047259
$(s_1)-(s_2)=$	−0·007844	$(s_3)-(s_4)=$	−0·022273	$(s_5)-(s_6)=$	−0·015971
log $tg\beta_1$	8·94124 (*n*)	log $tg\beta_3$	9·10741 (*n*)	$(s_7)=$	0·073457
log cos (λ_3-L_3)	9·92479 (*n*)	log cos (λ_1-L_3)	9·93932 (*n*)	$(s_8)=$	0·111360
log (s_7)	8·86603	log (s_8)	9·04673	$(s_4)-(s_8)=$	−0·037903

4. *Calculation of* A, B, C, *and* D. (1) *to* (4).—

log R_1	9·99968	log R_3	9·99697	log R_2	9·99810	log R_2	9·99810
log $[(s_1)-(s_2)]$	7·89454 (*n*)	log $[(s_3)-(s_4)]$	8·34778 (*n*)	log $[(s_5)-(s_6)]$	8·20333 (*n*)	log $[(s_7)-(s_8)]$	8·57867 (*n*)
log A	7·89422 (*n*)	log B	8·34475 (*n*)	log C	8·20143 (*n*)	log D	8·57677 (*n*)
		log P	0·07910	A =	−0·0078382		
		log PB	8·42385 (*n*)	PB =	−0·0265370		
		log (A+PB) =	8·53624 (*n*)	A+PB =	−0·0343752		

5. *Calculations preceding the Equation of Gauss.* (5) *to* (20).—

log $tg\beta_2$	9·04748 (*n*)	log $tg(\lambda_2-L_2)$	9·79169 (*n*)	og D	8·57677 (*n*)
log sin (λ_2-L_2)	9·72125	log cos ψ	9·99046 (*n*)	c. log R_2	0·00190
log $tg\psi$	9·32623 (*n*)	log $tg\delta$	9·80123	c. log sin $(\lambda_3-\lambda_1)$	1·25766 (*n*)
$\psi = 168°\ 2'$	$\delta = 32°\ 19'\ 24''$	$r = 34°\ 27'\ 2''$		log tgr	9·83633

log D	8·57677 (*n*)	log C	8·20143 (*n*)	$\Sigma =$	$31°\ 56'\ 10''$
log sin r	9·75259	c. log T	1·17582 (*n*)	$\delta =$	$32°\ 19'\ 24''$
log T	8·82418 (*n*)	c. log sin $(r+\psi)$	0·41746 (*n*)	$\sigma =$	$-\ 23'\ 14''$
$r+\psi = 202°\ 29'\ 2''$		log $(tg\Sigma)$	9·79471 (*n*)	P+1 =	2·1998
log C	8·20143 (*n*)	log S	8·20611 (*n*)	log *d*	0·01225
log sin δ	9·72811	log (P+1)	0·34238	log cos σ	9·99999
c. log sin Σ	0·27657	c. log (A+PB)	1·46376 (*n*)	log cot *u*	0·01224
log S	8·20611 (*n*)	log *d*	0·01225	$u =$	$44°\ 11'\ 34''$
log $tg\sigma$	7·82984 (*n*)	log *d*	0·01225	$45°-u =$	$48'\ 26''$
log cos *u*	9·85552	log sin σ	7·82983 (*n*)	log Ω	8·46858
c. log $\sqrt{2}$	9·84949	c. log sin ω	0·62650 (*n*)	log (A+PB)	8·53624 (*n*)
c. log sin $(45°-u)$	1·85114	log Ω	8·46858	log R_2^3	9·99430
log $tg\omega$	9·38599 (*n*)	$\omega = -13°\ 40'\ 11''$		log $\sin^3\delta$	9·18433
				log *h*	6·18345 (*n*)

	$\log \frac{k_1}{6}$	5·69301−10		$\log \frac{k^2}{6}$	5·69301−10
	$\log t_1$	1·07785		$\log t_2$	0·99875
	$\log (t_1+2t_2)$	1·50387		$\log (2t_1+t_2)$	1·53017
	$\log j_1$	8·27473−10		$\log j_2$	8·22193−10
$\log A$	7·89422 (n)	$\log PB$	8·42385 (n)	(s_9)	= −0·00014755
$\log j_1$	8·27473	$\log j_2$	8·22193	(s_{10})	= −0·00044236
$\log (s_9)$	6·16895 (n)	$\log (s_{10})$	6·64578 (n)	$(s_9)+(s_{10})$	= −0·00058991
		$\log [(s_9)+(s_{10})]$	6·77079 (n)		
		$\log h$	6·18345 (n)		
		$\log \mathfrak{M}$	0·58734		

6. *Solution of the Equation of Gauss.*—

$\log \sin \omega$	9·374	$\lambda =$	0·5	$\log \sin z_2$	9·394
	4				4
$\log \sin^4 \omega$	7·496	$\mu =$	10·5	$\log \sin^4 z_2$	7·576
$\log \mathfrak{M}$	0·587	$\mu-4\lambda =$	8·5	Δ	
				$\frac{\Delta}{\mu-4\lambda} =$	9′ 24″
$\log \sin (z_2+\omega)$	8·083	$\Delta =$	80		
$z_2+\omega =$	41′ 42″	$\omega =$	−13° 40′ 11″	$z_2 =$	14° 21′ 53″
				$z_2 =$	14° 31′ 17″

$\log \sin z_4$	9.39993
	4
$\log \sin^4 z$	7.59972
	Δ
	$= 26''$
	$\mu = 48$
	$z_4 = 14° 32' 44''$
	$z_5 = 14° 33' 10''$
$\log \sin z_0$	9.40014
	4
$\log \sin^4 z_0$	7.60056
	Δ
	$= 0''$
	$\mu = 48$
	$z_0 = 14° 33' 10''$
$\log R_0$	9.99810
$\log \cos \beta_0$	9.99731
$a. \log \sin z$	0.59986
$\log \sin (z_0 - z)$	9.48459
$\log \tau_0$	0.07986

$\log j_1$	8·27473	$\log t_2$	0·99875
$\log r_2^3$	0·97821	c. $\log (t_1+t_2)$	8·65887
$\log Y_1$	7·29652	$\log (1+Y_1)$	0·00086
$Y_1 = 0·001979$		$\log n_1$	9·65848

$\log j_2$	8·22193	$\log t_1$	1·07785
$\log r_2^3$	0·97821	c. $\log (t_1+t^0)$	8·65887
$\log Y_2$	7·24372	$\log (1+Y_2)$	0·00076
$Y_2 = 0·001753$		$\log n_2$	9·73748

$\log r_2$	0·07986
$\log \sin (\lambda_2-\lambda_3)$	8·24090
$\log (s_{11})$	8·32076
$\log n_1$	9·65848
$\log R_1$	9·99968
$\log \sin (L_1-\lambda_3)$	9·55234 (*n*)
$\log (s_{12})$	9·21050 (*n*)
$(s_{11}) =$	0·020929
$(s_{12}) =$	−0·162366
$(s_{13}) =$	−0·367908
$(s_{14}) =$	0·538688
$(S) =$	0·029343

$\log R_2$	9·99810
$\log \sin (L_2-\lambda_3)$	9·73324 (*n*)
$\log (s_{14})$	9·73134 (*n*)
$\log (n_2)$	9·73748
$\log R_3$	9·99697
$\log \sin (L_3-\lambda_2)$	9·83129 (*n*)
$\log (s_{13})$	9·56574 (*n*)
$\log (S)$	8·46750
c. $\log n_1$	0·34152
c. $\log \sin (\lambda_1-\lambda_3)$	1·25766
$\log r_1$	0·06668

$\log r_2$	0·07986	$\log n_1$	9·65848	$(s_{15}) =$ −0·134074
$\log tg\beta_2$	9·04748 (*n*)	$\log r_1$	0·06668	$-(s_{16}) =$ 0·046387
$\log (s_{13})$	9·12734 (*n*)	$\log tg B_1$	8·94124 (*n*)	$(S_1) =$ −0·087687
$\log (S_1)$	8·94294 (*n*)	$\log (s_{16})$	8·66640 (*n*)	

c. log n_2	0·26252	l_1	$= 362° 55' 13''$	l_3	$= 370° 20' 12''$
c. log $tg\beta_2$	0·89259 (*n*)	λ_1	$= 354° 44' 32''$	λ_3	$= 351° 34' 30''$
log r_2	0·09805	$l_1-\lambda_1 =$	$8° 10' 41''$	$l_3-\lambda_3 =$	$18° 45' 42''$

8. *Calculation of Heliocentric Coordinates.* (29) *to* (36).—

	log r_1	0·06668		log r_3	0·09805		
	c. log R_1	0·00032		c. log R_3	0·00303		
	c. log sin (λ_1-L_1)	0·51632		c. log sin $(\lambda_3$ $L_3)$	0·16871		
	log (z_{17})	0·58332		log (z_{18})	0·26979		
(z_{17})	3·83109	$\lambda_1 = 354° 44' 32''$	(z_{18})	1·86117	$\lambda_3 = 351° 34' 30''$		
$-\cot(\lambda_1-L_1)$	3·12734	$\delta_1 = 8° 10' 41''$	$-\cot(\lambda_3-L_3)$	1·08390	$\delta_3 = 18° 45' 42''$		
$\cot\bar{z}_1$	6·95843	$l_1 = 2° 55' 13''$	$\cot\delta_3$	2·94507	$l_3 = 10° 20' 12''$		
log r_1	0·06668	log r_1 0·06668	log r_3	0·09805	log r_3	0·09805	
log $tg\beta_1$	8·94124 (*n*)	log $tg\beta_1$ 8·94124 (*n*)	log $tg\beta_3$	9·10741 (*n*)	log $tg\beta_3$	9·10741	
log sin $(l_1-\lambda_1)$	9·15304	c. log sin b_1 1·32292 (*n*)	log sin $(l_3-\lambda_3)$	9·50736	c. log sin b_3	1·11665(*n*)	
c. log R_1	0·00032	log r_1 0·33084	c. log R_3	0·00303	log r_3	0·32211	
c. log sin (λ_1-L_1)	0·51632		c. log sin (λ_3-L_3)	0·16871			
	8·67760			8·88456			
log tgb_1		(*n*) $b_1 = -2° 43' 30''$	log tgb_3		(*n*) $b_3 = -4° 23' 3''$		

Motion direct. $i < 90°$.

9. *Calculation of Inclination and Ascending Node.* (37) *to* (39).—

$l tg b_1$	8·67760(*n*)	$tg b_3 = -0{\cdot}076665$	$l_3 =$	10° 20′ 12″
$l \cos(l_3 - l_1)$	9·99635	$-(s_{18}) =$ 0·047201	$l_1 =$	2 55 13
$l(s_{18})$	8·67395(*n*)	$(s_{19}) = -0{\cdot}029464$	$l_3 - l_1 =$	7 24 59

$l tg b_1$	8·67760(*n*)	$l tg b_1$	8·67760(*n*)	$l_1 =$	2° 55′ 13″ + 360°
$l \sin(l_3 - l_1)$	9·11085	$l \sin$ D	9·30993(*n*)	D =	11 46 44 + 180
c. $l(s_{19})$	1·53071(*n*)	$l tg i$	9·36767	Ω =	171 8 29
$l tg$ D	9·31916	i =	13° 7′ 30″		

10. *Calculation of Eccentricity and Major Axis.* (40) *to* (47).—

log sin $(l_3 \sim l_1)$	9·11085	log sin $(v_3 \sim v_1)$	9·12058
log cos b_1	9·99951	log r_1	0·33084
log cos b_3	9·99873	log r_3	0·32211
c. log cos i	0·01149	c. log k	1·76442
log sin $(v_3 \sim v_1)$	9·12058	c. log $(t_1 + t_2)$	8·65887
log p'	0·39364	log $\sqrt{p'}$	0·19682
log 6	0·77815	$v_3 \sim v_1 =$	7° 35′ 7″
log 6p'	1·17169		

log sin² $(v_3 \sim v_1)$	8·24116	log $(1 + (s_{20}))$	0·00108		
log $\sqrt{r_1}$	0·16542	log $\sqrt{p'}$	0·19682		
log $\sqrt{r_2}$	0·16105	log $\sqrt{p}$	0·19790		
c. log 6p'	8·82831	$p =$ 2·48768	log p	0·39580	
log (s_{20})	7·39594	$(s_{20}) =$ 0·0024885	$1 + (s_{20}) =$	1·00249	

$p =$	2·48768	$p =$	2·48768
$r_1 =$	2·14210	$r_3 =$	2·09948
$p - r_1 =$	0·34558	$p - r_3 =$	0·38820
log $(p - r_3)$	9·58906		
log r_1	0·33084	$-(s_{21}) =$	−8·68280
c. log r_3	9·67789	cot $(v_3 - v_1) =$	7·52104
c. log $(p - r_1)$	0·46145	$tg\ v_1 =$	−1·16176

c. log sin (u_3-u_1)	0·87942	$v_1 = -49^\circ\ 16'\ 46''$	
log (s_{21})	0·93866	$\frac{v_1}{2} = -24\ 38\ 23$	

log $(p-r_1)$	9·53855	$v_1 = -49^\circ\ 16'\ 46''$	
log $(p-r_3)$	9·58906	$v_3-v_1 = 7\ 35\ 7$	
c. log r_1	9·66916	$u_3 = -41\ 41\ 39$	
c. log r_3	9·67789	log p	0·39580
c. log cos v_1	0·18542	l. cos² ϕ	9·97256
c. log cos v_3	0·12686	log a	0·42324
log e^2	8·78694	$e = 0{\cdot}24744$	
log $e = l$ sin $\phi = 9{\cdot}39347$		$\phi = 14^\circ\ 19'\ 33''$	

log $a^{\frac{3}{2}}$	0·63486	log 1·296000	6·11260
log $365^d{\cdot}256$	2·56260	log T^s	3·19746
log T^s	3·19746	log m'' 2·91514	
$T^s = 1575^d{\cdot}65$		$m'' = 822''{\cdot}5$	

11. *Calculation of Longitude of Perihelium and Mean Longitude.* (48) to (54).—

log tg D	9·31916	$u_1 = 192^\circ\ 5'\ 8''$	
log cos i	9·98851	$\Omega = 171\ 8\ 29$	
log tg w_1	9·33065	$363\ 13\ 37$	
$1+e = 1{\cdot}24744$		$-v_1 = 49\ 16\ 46$	
$1-e = 0{\cdot}75256$		$\Pi = 52\ 30\ 23$	

log tg $\frac{v}{2}$	9·66151(n)	log e 9·39347	$w = -0{\cdot}684489$	
log $\sqrt{1-e}$	9·93827	log sin w 9·80091(n)	$-(s_{22}) =$	$0{\cdot}156452$
c. log $\sqrt{1+e}$	9·95199	log (s_{22}) 9·19438(n)	$M = -0{\cdot}528037$	
log tg $\frac{w}{2}$	9·55177(n)	$\frac{w}{2} = -19^\circ\ 36'\ 33''$	$w = -39^\circ\ 13'\ 6''$	

E_1 = 1804, Oct. 5·458644 log M. 9·72263(*n*)

E_0 = 1805, Jan. 0·000000 log 206265″ 5·31442

τ^d = 86^d·541356 log(s_{23}) 5·03705(*n*)

log τ^d 1·93722 (s_{23}) = −108″905 L = −10° 28′ 45″

log m'' 2·91514 (s_{24}) = 71180 II = 52 30 23

log (s_{24}) 4·85236 L″$_0$ = − 37725 L_0 = 41 58 22

Syra, Greece: 1903 *August* 20.

Ephemeris for Physical Observations of Saturn, 1903–4.
By A. C. D. Crommelin.

The discovery by Professor Barnard of a conspicuous white spot on *Saturn* last July has reawakened interest in the question of the planet's rotation, and has illustrated the remarkable variation of rate that prevails in different latitudes. At Mr. Denning's suggestion I have computed an ephemeris giving the longitudes of central meridian and the times of transit of the zero meridian on two assumptions as to the rotation period, viz.:

System I. 10^h 14^m·4 for equatorial spots (Hall).

System II. 10^h 37^m·92 for the spot discovered by Barnard last July (period provisionally deduced by Denning).

The quantities corresponding to P, B, *B* in the *Jupiter* ephemeris may be taken from the *Nautical Almanac* ephemeris of the ring, or more conveniently from the *Connaissance des Temps*, as this is fuller and contains the longitude of the Earth in the plane of the ring. In order to utilise the data of the *Connaissance* without interpolation, I give the longitude of the central meridian for Paris midnight, corresponding to 11^h 50^m·65 G.M.T.; but the transits of zero meridian are given for G.M.T. The quantities are to be interpolated for the time for which they are required, the equation of light having been already applied. The approximate increase of the longitude of central meridian in five days is 4219° in System I., 4063° in System II. I have commenced a new system of zero meridians, as Mr. Marth used a different period of rotation (10^h 13^m·6) in his ephemerides, of which the last appeared in vol. lv. p. 164. The small figures between the transits of zero meridians denote the number of intervening rotations, and the table at the end facilitates the determination of the times of intervening transits.

Paris Midnight. 1903.		Light Time.	Longitude of Central Meridian. I. 843°·750.	II. 812°·641.	G.M.T. of Preceding Transit of Zero Meridian. System I.		System II.	
		m	°	°	h m		h m	
June	8	77·49	0·00	0·00	1 36·3	12	1 12·8	12
	13		259·23	103·69	4 28·3	12	8 46·9	11
	18	76·45	158·47	207·36	7 20·2	12	5 43·2	11
	23		57·70	311·01	10 12·2	11	2 39·6	12
	28	75·63	316·94	54·66	2 49·8	12	10 13·8	11
July	3		216·17	158·33	5 41·8	12	7 10·1	11
	8	75·01	115·39	262·01	8 33·7	12	4 6·4	12
	13		14·60	5·68	11 25·7	11	11 40·7	11
	18	74·59	273·81	109·33	4 3·4	12	8 37·0	11
	23		172·98	212·94	6 55·5	12	5 33·4	11
	28	74·44	72·11	316·53	9 47·6	11	2 29·8	12
Aug.	2		331·21	60·09	2 25·6	12	10 4·2	11
	7	74·52	230·28	163·61	5 17·7	12	7 0·8	11
	12		129·31	267·10	8 10·0	12	3 57·4	12
	17	74·85	28·28	10·53	11 2·4	11	11 32·0	11
	22		287·20	113·92	3 40·5	12	8 28·8	11
	27	75·43	186·07	217·24	6 33·1	12	5 25·7	11
Sept.	1		84·89	320·53	9 25·8	11	2 22·7	12
	6	76·21	343·64	63·74	2 4·1	12	9 57·7	11
	11		242·34	166·91	4 57·0	12	6 54·9	11
	16	77·18	140·98	270·01	7 50·0	12	3 52·2	12
	21		39·57	13·07	10 43·1	11	11 27·5	11
	26	78·29	298·10	116·06	3 22·0	12	8 25·0	11
Oct.	1		196·56	218·98	6 15·2	12	5 22·6	11
	6	79·54	94·96	321·85	9 8·5	11	2 20·3	12
	11		353·31	64·67	1 47·6	12	9 56·1	11
	16	80·87	251·62	167·45	4 41·1	12	6 53·9	11
	21		149·88	270·18	7 34·7	12	3 51·8	12
	26	82·24	48·10	12·87	10 28·5	11	11 27·8	11
	31		306·27	115·52	3 7·9	12	8 26·0	11
Nov.	5	83·62	204·41	218·12	6 1·8	12	5 24·2	11
	10		102·51	320·69	8 55·7	12	2 22·4	12
	15	84·96	0·56	63·23	11 49·7	11	9 58·7	11
	20		258·61	165·75	4 29·2	12	6 56·9	11
	25	86·22	156·64	268·25	7 23·2	12	3 55·3	12
	30		54·66	10·74	10 17·3	11	11 31·6	11
Dec.	5	87·38	312·66	113·21	2 57·0	12	8 30·1	11
	10		210·66	215·67	5 51·1	12	5 28·5	11

Paris Midnight.	Light Time.	Longitude of Central Meridian. I. 843°·750.	II. 810°·641.	G.M.T. of Preceding Transit of Zero Meridian. System I.		System II.	
1903	m	°	°	h m		h m	
Dec. 15	88·41	108·65	318·13	8 45·2	12	2 26·9	12
20		6·65	60·60	11 39·3	11	10 3·3	11
25	89·27	264·65	163·06	4 18·9		7 1·7	
1904.							
May 13	81·68	301·20	164·29	3 16·5	12	6 59·5	11
18		200·30	267·77	6 8·8	12	3 56·2	12
23	80·30	99·36	11·27	9 1·1	11	11 30·7	11
28		358·45	114·80	1 38·9	12	8 27·2	11
June 2	78·98	257·57	218·36	4 31·1	12	5 23·8	11
7		156·72	321·96	7 23·2	12	2 20·2	12
12	77·72	55·91	65·58	10 15·2	11	9 54·5	11
17		315·10	168·22	2 52·9	12	6 50·8	11
22	76·60	214·30	272·86	5 44·9	12	3 47·2	12
27		113·52	16·52	8 36·9	12	11 21·4	11
July 2	75·62	12·74	120·19	11 28·9	11	8 17·7	11
7		271·97	223·87	4 6·5	12	5 14·0	11
12	74·84	171·20	327·54	6 58·5	12	2 10·3	12
17		70·41	71·20	9 50·5	11	9 44·5	11
22	74·27	329·62	174·86	2 28·1	12	6 40·8	11
27		228·82	278·51	5 20·2	12	3 37·2	12
Aug. 1	73·91	127·99	22·13	8 12·2	12	11 11·4	11
6		27·13	125·72	11 4·3	11	8 7·9	11
11	73·82	286·23	229·28	3 42·2	12	5 4·4	11
16		185·30	332·81	6 34·4	12	2 1·0	12
21	73·97	84·33	76·30	9 26·7	11	9 35·5	11
26		343·32	179·75	2 4·7	12	6 32·2	11
31	74·36	242·16	283·15	4 57·2	12	3 28·9	12
Sept. 5		141·15	26·50	7 49·8	12	11 3·7	11
10	74·99	39·98	129·79	10 42·4	11	8 0·7	11
15		298·75	233·03	3 20·8	12	4 57·7	11
20	75·82	197·47	336·21	6 13·6	12	1 54·9	12
25		96·12	79·33	9 6·6	11	9 30·1	11
30	76·84	354·71	182·39	1 45·3	12	6 27·5	11
Oct. 5		253·25	285·40	4 38·4	12	3 24·9	12
10	78·00	151·74	28·36	7 31·7	12	11 0·4	11
15		50·16	131·26	10 25·0	11	7 58·0	11
20	79·29	308·54	234·10	3 4·0	12	4 55·8	11
25		206·87	336·90	5 57·6	12	1 53·6	12
30	80·62	105·15	79·65	8 51·2	12	9 29·5	11

Paris Midnight. 1904.	Light Time.	Longitude of Central Meridian. I. 843°·750.	II. 812°·641.	G.M.T. of Preceding Transit of Zero Meridian. System I.		System II.	
	m	°	°	h m		h m	
Nov. 4		3·39	182·36	11 44·8	11	6 27·5	11
9	82·00	261·58	285·02	4 24·2	12	3 25·5	12
14		159·74	27·65	7 18·0	12	11 1·6	11
19	83·37	57·86	130·24	10 11·9	11	7 59·8	11
24		315·96	232·81	2 51·3	12	4 58·0	11
29	84·68	214·05	335·36	5 45·3	12	1 56·3	12
Dec. 4		112·10	77·88	8 39·3	12	9 32·6	11
9	85·92	10·13	180·38	11 33·4	11	6 30·9	11
14		268·15	282·87	4 12·9	12	3 29·3	12
19	87·04	166·16	25·35	7 7·0	12	11 5·7	11
24		64·17	127·83	10 1·1	11	8 4·1	11
29	88·01	322·18	230·31	2 40·7		5 2·5	

The following interpolation table gives the quantities to apply to any transit of the zero meridian in order to obtain the neighbouring transits :

Increase of Central Meridian in 5 days.		6 Rotations.	5 Rotations.	4 Rotations.	3 Rotations.	2 Rotations.	1 Rotation.
	°	h m	h m	h m	h m	h m	h m
System I.	4219·2	61 26·0	51 11·7	40 57·3	30 43·0	20 28·7	10 14·3
	4218·8	26·4	12·0	57·6	43·2	28·8	14·4
	4218·4	26·7	12·3	57·8	43·4	28·9	14·5
	4218·0	27·1	12·5	58·0	43·5	29·0	14·5
System II.	4063·7	63 47·1	53 9·2	42 31·4	31 53·5	21 15·7	10 37·8
	4063·3	47·4	9·5	31·6	53·7	15·8	37·9
	4062·9	47·8	9·8	31·9	53·9	15·9	38·0
	4062·5	48·2	10·2	32·1	54·1	16·1	38·0

The argument is, of course, found by taking the difference of consecutive values of "longitude of Central Meridian." Only the unit and first decimal figures of the difference need be considered.

55 *Ulundi Road, Blackheath, S.E.:*
1903 *December* 11.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor H. H. TURNER, M.A., Sc.D., F.R.S., PRESIDENT, in the Chair.

Henry John Wolverton Brennand, B.A., M.B., Ch.M., F.C.S., Government Statistician's Department, and 203 Macquarie Street, Sydney, New South Wales ;
Albert Edward Garrett, F.R.G.S., 127 Lothair Road, Finsbury Park, N. ;
Joseph Massey Harvey, 77 Cloudesdale Road, Balham, S.W. ;
John George Hatchard, C.S.A. Railway Works, Pretoria, Transvaal, South Africa ;
Rev. Gustavo Heredia, S.J., Stonyhurst College, Blackburn, Lancashire ;
Herbert Kitchin, Marshall Square, Johannesburg, Transvaal, South Africa ;
George Aimer Russell, M.A., B.Sc., 29 Glebe Road, Kilmarnock, Scotland ; and
Frank Herbert Shaw, Ferndale, Gledholt, Huddersfield ;

were balloted for and duly elected Fellows of the Society.

The following Candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended :—

Percy Morris, Holmwood, Camborne Road, Sutton, Surrey (proposed by H. F. Newall).

Sixty-five presents were announced as having been received since the last meeting, including, among others :—

Annals of the Royal Observatory, Cape of Good Hope, vol. 11 (S. S. Hough, Heliometer triangulation of the southern circumpolar area) ; Harvard College Observatory Annals, vol. 51 (W. H. Pickering, Photographic Atlas of the Moon) ; Helsingfors Observatory, Catalogue photographique du Ciel, coordonnées rectilignes, &c., tome 4; Rome, Specola Vaticana, Catalogo fotografico stellare coordinati rettilinei &c., vol. 1 (presented by the Observatories).

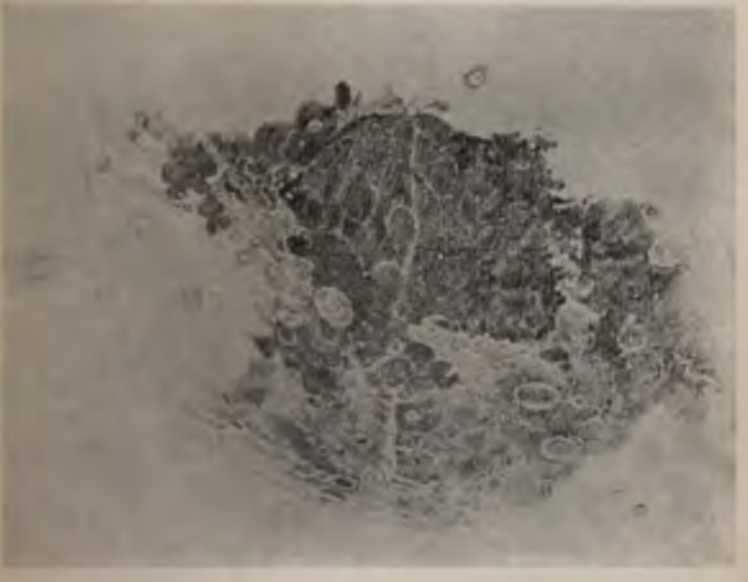
Two Drawings of the Mare Serenitatis by John Russell, R.A., affording some hitherto unpublished evidence as to the appearance of Linnè in the year 1788. By Arthur A. Rambaut, F.R.S., Radcliffe Observer.

In the *Monthly Notices* of the Royal Astronomical Society, vol. lvi. p. 3, appears a paper, by the late Mr. Stone, entitled "Note on a Crayon Drawing of the Moon, by John Russell, R.A., at the Radcliffe Observatory, Oxford." In this paper a passing reference is made to a large number of detailed pencil drawings of different portions of the Moon's surface, which are also in the possession of the Radcliffe trustees, and a photographic reproduction of one of them is given in the paper ; but no particulars are afforded with respect to them, Stone's attention being confined to the large picture representing the gibbous Moon, which still hangs in the library of the Radcliffe Observatory, and of which a photographic reproduction is also given in the paper.

Russell's drawings, which were for the most part made on separate sheets of paper, are all contained in a large folio volume bound in russia leather, and were presented to the Radcliffe Observatory by his grandson, Rev. S. H. Russell, B.D., on August 4, 1873. This volume contains 187 separate telescopic studies of limited regions of the lunar surface. They are all carefully mounted and protected from injury by sheets of tissue paper ; and although now more than 100 years old, having with one exception been made between 1785 and 1797, they are still in a very good state of preservation. Many of the drawings have memoranda in Byrom's system of shorthand written upon them, giving in most cases the date on which the drawing was made, but unfortunately this particular is sometimes omitted, and in such cases the only clue to the date of the drawing is afforded by its position in the volume with regard to the other dated drawings and the approximate phase of the Moon indicated by the drawing itself. I have, however, so far been unable to ascertain by whom and on what system the studies were arranged ; and as in some cases the year given is manifestly incorrect, this clue cannot always be relied upon. Such errors can, however, generally be

Fig. 1.

Fig. 2.



AWINGS OF THE MARE SERENITATIS, BY JOHN RUSSELL, R.A. 1788.

corrected without difficulty. As, for instance, when he writes, "1/2 after 7 in the evening 1790 January 19. Full Moon to-morrow at 6 in the morning." The full Moon of January 1790 occurred at 7 P.M. on the 30th, whereas in the following year it occurred at 6 A.M. on the 20th. Whence it is clear that the mistake is of a sort that most of us have made from time to time, especially in the month of January.

From a memorandum left by Professor Rigaud, we find that "Mr. R. had a 6-feet reflector of Dr. Herschell's with which he observed, and also a telescope provided with a micrometer: with this he took all his own measures."

We have no information as to the aperture or focal length of the latter instrument, but are told that it was an achromatic telescope, and that with it most of Russell's observations were made; "for, although Dr. Herschel had provided him with a reflector, Mr. Russell made very little use of it." On the same authority we learn: "Mr. R. always preferred a low power in examining the Moon."

My attention has recently been directed to these drawings by Mr. S. A. Saunder, F.R.A.S., in connexion with the question as to possibility of change in Linné. At his request I have had these drawings examined with a view to ascertaining what evidence they may afford with regard to the condition of this marking in Russell's time. Out of the 187 drawings it is found that only two represent this object. The first of these, which is represented in Plate 3, fig. 1, is particularly interesting.* In the original Linné is somewhat whiter and more conspicuous than it appears in fig. 1, Plate 3. Owing to the age of the paper on which the drawing was made, some effect of contrast has been lost in the process of reproduction. As indicated by the shorthand note inscribed upon it, this drawing represents the appearance of the Mare Serenitatis from 9 to 11 on 1788 April 12. It was thus executed in the same year as Schröter's drawing† of 1788 November 5, and under very nearly the same conditions as to the phase of the Moon. The shorthand note on this drawing runs as follows: "From 9 to 11 on the 12th April, 1788. First quarter to-morrow noon. This was drawn to show the small circular pit situated upon the white line, that it is deeper than the general surface, yet empty, the shadow of the rim bearing no proportion with the hollow centre." The rest of the legend is rather obscure, some of the characters being indistinct; but as the small circular pit referred to is Bessel, not Linné, this is not of so much consequence. Opposite Manilius, and just beyond the terminator, are the words "too large" in shorthand.

On this drawing Linné is represented as a small white spot

* The dark marking which appears on the surface of the Mare about halfway between Linné and Sulpicius Gallus is a stain on the paper, and not part of the original drawing.

† *Selenotopographische Fragmente*, vol. i., plate ix.

without any indication of a crater-like depression. On the southern side a slightly darker shading is faintly indicated, but whether this is intentional or due to some inequality in the paper or other accident it is impossible to say. From its position it cannot be the shadow of a crater ring. As represented in the drawing before us, this object resembles nothing so much as the appearance it offers at the present day with a small telescope and under fairly good conditions of seeing, or on a photograph of this region.

The other drawing, Plate 3, fig. 2, represents on a smaller scale the Mare Serenitatis and the region to the north of it. Unfortunately this is one of the undated pictures, but there is little doubt that it belongs to the same year (1788) as the other, and the state of the shadows shows that it was taken very near the time of full Moon. The only particulars given by Russell with this drawing are the names of three craters near the limb, written in longhand, "Zoroaster, Thales, Endimion" (sic). In this drawing Linné is represented as a conspicuous white spot, with perhaps a slight shading to the south. The absence of any indications of a crater within the white spot in this drawing is not under any circumstances to be wondered at, the illumination being almost vertical at the time. Even Menelaus, Bessel, and Sulpicius Gallus, which in the former drawing appear under an oblique illumination as deep well-marked cavities, are here represented as uniformly white patches. A remarkable feature of this drawing is the relatively large size of Linné. It is here represented as more conspicuous than Bessel, which exceeds it in size in the previous drawing, and about equal to Sulpicius Gallus A. In view, however, of the fact that the first of Russell's drawings was intended chiefly to exhibit the appearance of Bessel, and not Linné, it would be unsafe to attach much weight to the apparent rise in conspicuousness of the latter marking as the phase increases. Such a deduction would be opposed to the result of Professor W. H. Pickering's observations, viz. that it rapidly diminishes in size under the direct solar rays, attaining its minimum diameter about twenty-four hours after lunar midday.

It is not my intention at present to enter upon the general question as to the possibility of a change having occurred in Linné. The principal facts bearing on this point have been recently discussed by Mr. Saunder in his presidential address to the British Astronomical Association, and by Professor W. H. Pickering in the *Annals of Harvard College Observatory*, vol. xxxii., part ii., chap. vii. It is not to be supposed that the observations of such experienced selenographers as Lohrmann, Mädler, and Schmidt (who frequently studied this formation and described it as having been previous to 1843 a large and deep crater, which disappeared in the year 1866)* can be lightly set aside. At the same time the evidence afforded by these drawings of Russell, now published for the first time, corroborates

* See *Report of British Association*, 1867.

in a remarkable manner Schröter's isolated observation, and renders it probable that in the year 1788 this object presented much the same appearance as at the present day when viewed with a low power eyepiece attached to a small telescope.

I would merely venture to point out that the evidence of Grimaldi's two maps, as published by Riccioli in his *Almagestum Novum*, which is sometimes appealed to as proof of the existence of a large crater at this spot about the middle of the seventeenth century, appears to me to be of very little value indeed. In one of these two maps Linné is represented as a white spot, while Bessel and Sulpicius Gallus appear as craters with shaded interiors. In the other map, which seems to be the more carefully finished production, Linné is also provided with a shaded crater-like interior, in every way comparable with that of Sulpicius Gallus. The force of this evidence in favour of the crater-like form of Linné in 1651 is, however, very much weakened, if not entirely dissipated, by the fact that the peaks of the Apennine range, which in the former map are represented as white spots similar to but smaller than Linné, appear in the latter as a row of small carefully shaded craterlets.

Studies of Mersenius and Copernicus—both of which have been suspected of change—are also included amongst Russell's drawings.

Transformation of Hansen's Tables. By P. H. Cowell, M.A.

Newcomb (*Astron. Papers, Amer. Eph.* vol. i.) has transformed Hansen's theory into longitude and latitude in order to compare it with Delaunay. In this paper I give the transformation of Hansen's tables in order to compare them with the observations. Newcomb's paper has, of course, been of great assistance. It will be seen that my method is to vary Newcomb's results, and thus avoid performing the long calculations that would have been necessary in the absence of his paper.

Hansen's longitude is found from the formula

$$L = f + \Pi + R + R'$$

where f, R, R' are functions of e, I, $n\delta z$, s, and Hansen's latitude β is found from the formula

$$\sin\beta = \sin I \sin(f+\omega) + s$$

e and I are arbitrary constants to which Hansen assigns one value in the theory and a different value in the tables. $n\delta z$ and s are the symbols under which Hansen calculates the perturbations. As regards these quantities Hansen gives slightly different coefficients in his tables to those that he subsequently

gives in his theory. Moreover in his tables he omits certain terms as not worth tabulating, although in the opening pages of his tables he gives the coefficients that he has calculated, and these coefficients in some cases amount to upwards of a tenth of a second. He has also introduced other modifications such as multiplying all terms in $n\delta z$ by one factor and all parallactic terms by another factor. Hence it must be clearly understood that throughout this paper "Hansen's tables" refers to what Hansen has tabulated, and not to the preliminary expressions.

The variation between Hansen's tables and Hansen's theory will be denoted by the operator Δ.

Hence
$$\Delta e = +0{\cdot}00000728$$
$$\Delta \mathrm{I} = -8''{\cdot}04$$

Also $\Delta n\delta z$, Δs denote respectively the variations of $n\delta z$ and s. Newcomb has considered the parts of $n\delta z$ and s that arise from the action of the Sun only. Hence $\Delta n\delta z$ and Δs consist of two distinct parts, one part arising from the varied coefficients of arguments due to the action of the Sun, and the other part comprising the planetary and figure of Earth and figure of Moon terms in so far as these terms depend on fresh arguments.

These variations will here be considered separately.

I. *The Variation of the Solar Terms.*

Following Newcomb's notation

$$f = (e, g)_0 + (e\,g)_1\, n\delta z + (e\,g)_2\,(n\delta z)^2 + \ldots$$
$$\mathrm{R} = -\tfrac{1}{2}\tan^2\tfrac{1}{2}\mathrm{I}\sin 2(f+\omega) + \tfrac{1}{2}\tan^4\tfrac{1}{2}\mathrm{I}\sin 4(f+\omega) - \ldots$$
$$= \mathrm{R}_1 + \mathrm{R}_2 \ldots$$
$$\mathrm{R}_1 = \mathrm{R}_{10} + \mathrm{R}_{11}\, n\delta z + \mathrm{R}_{12}\,(n\delta z)^2 + \ldots$$
$$\mathrm{R}_2 = \mathrm{R}_{2\cdot0} + \mathrm{R}_{2\cdot1}\, n\delta z + \mathrm{R}_{2\cdot2}\,(n\delta z)^2 + \ldots$$

$\mathrm{R}' = s\mathrm{S}$ + three periodic terms that require no transformation + nutation that is not considered in this paper; and

$$s = \mathrm{S}_0 + \mathrm{S}_1 n\delta z + \mathrm{S}_2\,(n\delta z)^2$$

Hence
$$\Delta \mathrm{L} = \Delta(e, g)_0 + \Delta \mathrm{R}_{1\cdot0} + \mathrm{A}\Delta(n\delta z) + \mathrm{S}_0\Delta s$$

where $\mathrm{A} = (e, g)_1 + \mathrm{R}_{11} + s\mathrm{S}_1 + 2(e\,g)_2\, n\delta z + \ldots$

A may be reduced to its first term

$$(e, g)_1 = 1 + 0{\cdot}1098\cos g + 0{\cdot}0075\cos 2g$$

I also copy from Newcomb's paper

$$S_0 = +0''.005 \cos\omega - 0''.090 \cos(g+\omega) - 0''.005 \cos(2g+\omega)$$

Then

$$\Delta(e, g)_0 = +3''.000 \sin g + 0''.206 \sin 2g + 0''.015 \sin 3g$$
$$\Delta R_{r,0} = -0''.034 \sin(g+2\omega) + 0''.359 \sin(2g+2\omega) + 0''.033 \sin(3g+2\omega)$$

$\Delta(ne)$ and Δs are given in the third columns of the two tables in this paper. The terms in these columns enclosed in brackets are those that do not occur in Hansen's tables; the coefficients are therefore those of the theory with the signs changed. Hence, to form the solar part of ΔL, it is only necessary to perform the multiplications and additions indicated by the formula. Adding to L, as given in the last column of Newcomb's second table, $L+\Delta L$, the solar part of the longitude of Hansen's tables is obtained. The result is set down in the second column of the first table.

The following intermediate step of the calculations is given here.

$$S_0 \cdot \Delta s = -0''.288 \sin(2g+2\omega) + 0''.024 \sin(g+2\omega) - 0''.022 \sin(3g+2\omega) + 0''.018 \sin g$$

Lastly it must be noted that certain coefficients vary with the time. Hence the value of $L+\Delta L$ given in Table I. must be supplemented by the terms

$$-0''.182\,T \sin(-g-g') - 1''.637\,T \sin(-g') - 0''.278\,T \sin(g-g')$$
$$+0''.069\,T \sin(g-g'+2\omega-2\omega') + 0''.057 \sin(2g-g'+2\omega-2\omega')$$
$$-0''.493\,T \sin(g-3g'+2\omega-2\omega')$$
$$-0''.388\,T \sin(2g-3g'+2\omega-2\omega')$$

where T is reckoned in centuries from 1800·0.

For the latitude

$$\Delta\beta = (1+\tfrac{1}{2}\sin^2\beta)\,\Delta \sin\beta$$
$$= (1+\tfrac{1}{2}\sin^2\beta)\{\sin I \sin(f+\omega)\frac{\Delta \sin I}{\sin I} + \sin I \cos(f+\omega)\Delta f + \Delta s\}$$

Of the expression within brackets the first term is obtained from the second column of Newcomb's Table IV. by multiplying by the factor $\frac{\Delta \sin I}{\sin I}$, or $-0''.0004328$; its value is set down in

the fourth column of the second table in this paper; in the second term

$$\cos(f+\omega) = \cos(g+\omega) - 0''\cdot1098 \sin(g+\omega)\sin g$$
$$\Delta f = \Delta(e,g)_0 + (1 + 0\cdot1098\cos g)\Delta n\delta z + n\check{z}\cos g \,.\, 2\Delta e$$
$$\sin \mathrm{I}\,\Delta(e\,g)_0 = +0''\cdot269 \sin g + 0''\cdot018 \sin 2g$$
$$\sin \mathrm{I}\,\Delta n\check{z} = +0''\cdot014 \sin(-g') + 0''\cdot043 \sin(-g-2g')$$
$$+0''\cdot064 \sin(g-2g'+2\omega-2\omega') + 0''\cdot031 \sin(2g-2g'+2\omega-2\omega')$$
$$-0''\cdot013 \sin(-4g'+2\omega-2\omega') + 0''\cdot014 \sin 2\omega + 0''\cdot027 \sin(g+2\omega)$$
$$+0''\cdot025 \sin(2g'+2\omega') + 0''\cdot024 \sin(\omega-\omega') + 0''\cdot049 \sin(g+\omega-\omega')$$
$$-0''\cdot064 \sin(-g'+\omega-\omega') - 0''\cdot395 \sin(g-g'+\omega-\omega')$$

and $\sin \mathrm{I}\, n\delta z \cos g \,.\, 2\Delta e$ is less than $0''\cdot01$.

The second term of $\cos(f+\omega)$ is only sensible when taken in connection with the terms

$$+0''\cdot269 \sin g - 0''\cdot395 \sin(g-g'+\omega-\omega')$$

and this part of the product is

$$+0''\cdot007 \sin(-g+\omega) - 0''\cdot015 \sin(g+\omega) + 0''\cdot007 \sin(3g+\omega)$$
$$+0''\cdot011 \sin(g-g'+2\omega-\omega') + 0''\cdot011 \sin(g-g'-\omega')$$
$$-0''\cdot011 \sin(-g-g'-\omega') - 0''\cdot011 \sin(3g-g'+2\omega-\omega')$$

The complete value of the term is given in the fifth column of the second table, the excess of what is there set down over the expression just written representing $\cos(g+\omega) \,.\, \sin \mathrm{I}\,\Delta f$.

Δs is given in the third column

$$\tfrac{1}{2}\sin^2\beta \,.\, \Delta \sin\beta = -0''\cdot005 \sin(g+\omega) + 0''\cdot002 \sin(3g+3\omega)$$

and $\beta+\Delta\beta$ is given in the second column of the table, β being taken from Newcomb's fourth table. In both tables only those terms are given which have a coefficient as large as $0''\cdot1$, and the results set down must only be considered as accurate to $0''\cdot02$.

II. *The Planetary and other Terms.*

The planetary terms of $\Delta \mathrm{L} = \mathrm{AP} + \mathrm{S}_0\mathrm{Q}$, where P, Q are the planetary terms of $\Delta\,(n\check{z})$ and Δs respectively.

$$\mathrm{Q} = +8''\cdot764 \sin(f + \Pi + 169^\circ\,51')$$

The value of P can be taken from Hansen's tables.

$$\mathrm{A} = 1 + 0\cdot1098 \cos g + 0\cdot0075 \cos 2g$$
$$\mathrm{S}_0 = +0\cdot005 \cos\omega - 0\cdot090 \cos(g+\omega) - 0\cdot005 \cos(2g+\omega)$$

as before.

To this must be added

$$0''.022 \cos (g-2g'+2\omega-2\omega') + 0''.021 \cos (2g-2g'+2\omega-2\omega')$$

multiplied by certain terms of P; for Hansen directs that the sum of the terms in question is to be added to the argument of the evection and to the mean elongation.

The multiplications, so far as P is concerned, need not be performed, for P only contains the following short-period terms:

$$0''.256 \sin \Theta \cos g$$

for which A may be taken as unity.

For the long-period terms the product by the term $0''.1098 \cos g$ in A is not required for this reason. The observations will be divided into several periods of analysis, and in each period it will be assumed that an error $a \cos g$ runs through the observations. a will be found for each period and compared with $0''.1098$ P.

$$S_2 Q = -0''.394 \{\sin (2f+\Pi+\omega+169^\circ\ 51') + \sin (-\Theta+169^\circ\ 51')\}$$

or with sufficient accuracy

$$-0''.394 \{\sin (2g+\Pi+\omega+169^\circ\ 51') + \sin (-\Theta+169^\circ\ 51')\}$$
$$-0''.088 \cos (2g+\Pi+169^\circ\ 51') \sin g$$

The planetary terms of $\Delta\beta$ are

$$(1+\tfrac{1}{2}\sin^2\beta)\{\sin I \cos (f+\omega)\, P+Q-1''.00\}$$

the last term being a term that Hansen adds to all latitudes on account of the figure of the Moon.

Here also, and for a similar reason, it is not necessary to perform the multiplications.

TABLE I.

Terms in Hansen's Tabular Longitude transformed and in Δ $(n\delta s)$.

Arg. g	Arg. g'		$L+\Delta L$.	$\Delta(n\delta s)$.	Arg. g	Arg. g'		$L+\Delta L$.	$\Delta(n\delta s)$.
1	0	+	22640·168	...	2	−1	+	9·719	...
2	0	+	769·064	...	3	−1	+	0·670	...
3	0	+	36·127	...	−1	−2	+	+0·705	−0·475
4	0	+	1·932	...	0	−2	+	7·493	−0·014
5	0	+	0·113	...	1	−2	+	2·568	−0·025
−3	−1	+	0·551	...	2	−2	+	0·192	...
−2	−1	+	7·666	...	$2\omega-2\omega'$				
−1	−1	+	109·953	+0·037	0	0	−	0·266	−0·036
0	−1	+	670·006	+0·154	1	0	−	2·488	+0·047
1	−1	+	148·032	+0·024	2	0	−	0·194	...

Arg. g	Arg. g'		$L+\Delta L$.	$\Delta(n\delta t)$.
−1	−1	+	0·124	−0·053
0	−1	+	2·556	+0·035
1	−1	−	28·595	−0·036
2	−1	−	24·452	...
3	−1	−	2·926	...
4	−1	−	0·292	...
−2	−2	+	0·949	...
−1	−2	+	13·200	+0·011
0	−2	+	211·704	+0·010
1	−2	+	4586·690	+0·717
2	−2	+	2370·137	+0·352
3	−2	+	191·940	...
4	−2	+	14·374	...
5	−2	+	1·060	...
−1	−3	+	0·453	−0·022
0	−3	+	8·693	+0·033
1	−3	+	206·492	+0·060
2	−3	+	165·556	+0·039
3	−3	+	14·597	...
4	−3	+	1·182	...
0	−4	+	0·134	−0·146
1	−4	+	7·419	−0·013
$2\omega-2\omega'$				
2	−4	+	8·125	...
3	−4	+	0·758	...
1	−5	+	0·257	...
2	−5	+	0·344	...
2ω				
2	1	+	0·416	...
3	1	+	0·265	...
0	0	+	1·253	+0·153
1	0	−	39·281	+0·304
2	0	−	411·598	−0·012
3	0	−	45·080	...
4	0	−	3·997	...
5	0	−	0·329	...
3	−1	−	0·304	..
$2\omega'$				
−1	3	+	0·339	−0·063

Arg. g	Arg. g'		$L+\Delta L$.	$\Delta(n\delta t)$.
0	3	−	2·153	...
−2	2	+	0·425	...
−1	2	+	6·435	+0·057
0	2	−	54·980	+0·282
1	2	−	0·139	+0·021
2	2	+	0·561	...
0	1	+	1·499	−0·055
$\omega-\omega'$				
−1	0	+	0·384	...
0	0	+	1·582	+0·266
1	0	+	18·167	+0·551
2	0	+	1·276	+0·011
−2	−1	−	0·118	...
−1	−1	−	1·804	−0·020
0	−1	−	19·150	−0·711
1	−1	−	126·459	−4·388
2	−1	−	8·543	−0·053
3	−1	−	0·588	...
0	−2	−	0·153	+0·014
1	−2	−	0·594	−0·010
2	−2	−	0·127	...
$3\omega-3\omega'$				
2	−2	+	0·270	...
3	−2	+	0·150	...
1	−3	−	1·247	−0·063
2	−3	−	3·224	−0·081
3	−3	+	0·422	+0·027
2	−4	−	0·226	...
$\omega+\omega'$				
0	1	+	0·112	+0·037
1	1	+	0·571	+0·030
$3\omega-\omega'$				
3	−1	+	0·235	(−0·010)
$\omega-3\omega'$				
1	−3	−	0·300	+0·018
$4\omega-4\omega'$				
2	−3	−	0·356	...
3	−3	−	0·640	...
4	−3	−	0·293	...

Arg. g	Arg. g'		L+ΔL.	Δ(nds).	Arg. g	Arg. g'		L+ΔL.	Δ(nds).
1	−4	+	1·177	...	4	−2	−	5·753	(−0·010)
2	−4	+	30·778	...	5	−2	−	0·991	...
3	−4	+	38·426	...	6	−2	−	0·124	...
4	−4	+	13·900	...	3	−3	−	0·389	(+0·041)
5	−4	+	1·581	...	4	−3	−	0·384	...
6	−4	+	0·173	(−0·048)	2ω − 4ω'				
2	−5	+	2·746	...	1	−4	+	0·202	−0·021
3	−5	+	4·402	...	6ω − 6ω'				
4	−5	+	1·886	...	3	−6	+	0·292	...
5	−5	+	0·286	...	4	−6	+	0·572	...
2	−6	+	0·156	...	5	−6	+	0·395	...
3	−6	+	0·313	...	6ω − 4ω'				
4	−6	+	0·153	...	4	−4	−	0·150	(+0·016)
4ω − 2ω'					5	−4	−	0·203	...
2	−2	−	0·534	...	4ω				
3	−2	−	9·370	...	4	0	+	0·422	...

Table II.

Terms in Hansen's Tabular Latitude transformed and in Δs,
$\sin(f-\omega)\,\Delta \sin I$, $\sin I \cos(f+\omega)\,\Delta f$.

Arg. g − ω	Arg. g'	β+Δβ.	Δs.	$\frac{\sin(f+\omega)}{\Delta \sin I}$	$\frac{\sin I}{\cos(f+\omega)\,\Delta f}$
		"	"	"	"
−1	1	0·322	−0·017	...	...
0	1	5·665	...	+0·002	...
1	1	6·484	...	+ ·013	...
2	1	− 5·333	...	+ ·003	...
3	1	0·640	..	...	...
−2	0	1·585	...	...	...
−1	0	− 31·759	...	+ ·003	+0·007
0	0	− 999·533	− ·169	+ ·442	− ·120
1	0	− 18461·652	+6·405	−7·981	− ·015
2	0	+ 1010·006	− ·136	− ·437	+ ·134
3	0	− 61·891	+ ·010	− ·027	+ ·007
4	0	+ 3·977	...	− ·002	...
5	0	− 0·263	...	...	...
−1	−1	+ 0·312	...	...	...
0	−1	− 5·102	− ·022	− ·001	...
1	−1	− 4·867	...	− ·013	..
2	−1	− 6·759	...	− ·004	...

Arg. g g'		$\beta+\Delta\beta$.	Δs.	$\sin(f+\omega)$ $\Delta \sin L$.	$\frac{\sin I}{\cos(f+\omega)}$ Δf.
		"	"	"	"
3 −1	+	0·798	...	...	...
2 −2	+	0·142	(+ ·026)	...	...
$\omega-2\omega'$					
0 −1	−	0·797	...	...	...
1 −1	−	12·140	...	...	...
2 −1	−	0·832	...	...	...
−3 −2	+	0·134	...	...	...
−2 −2	+	1·518	...	− ·001	...
−1 −2	+	15·570	+ ·019	− ·009	...
0 −2	+	166·581	+ ·037	− ·091	+ ·032
1 −2	+	623·708	+ ·045	− ·043	+ ·004
2 −2	+	33·368	...	− ·001	...
3 −2	+	2·146	...	...	...
4 −2	+	0·145	...	...	...
−1 −3	+	0·655	...	...	...
0 −3	+	7·471	...	− ·004	...
1 −3	+	29·733	...	− ·003	...
2 −3	+	1·776	...	...	...
3 −3	+	0·121	...	...	...
0 −4	+	0·276	...	...	...
1 −4	+	1·095	...	...	...
$\omega+2\omega'$					
0 2	+	0·286	...	...	...
1 2	−	2·151	+ ·029	+ ·002	+ ·012
2 2	−	0·325	...	...	...
3ω					
1 0	+	0·108	...	...	...
2 0	−	2·790	...	+ ·002	+ ·014
3 0	−	6·301	...	...	...
4 0	−	1·021	...	...	...
5 0	−	0·119	...	...	...
$3\omega-2\omega'$					
2 0	−	0·116	...	...	...
2 −1	−	1·320	...	+ ·001	...
3 −1	−	1·276	...	+ ·001	...
4 −1	−	0·257	(− ·017)	...	...
0 −2	+	0·269	...	...	...

Arg.		$\beta+\Delta\beta$.	Δs.	$\sin(f+\omega)\,\Delta \sin I$.	$\sin I \cos(f+\omega)\,\Delta f$.
g g'		"	"	"	"
1 − 2	−	1·622	...	+ ·001	...
2 − 2	+	199·455	+ ·027	− ·087	+ ·032
3 − 2	+	117·243	+ ·020	− ·051	+ ·016
4 − 2	+	15·124	(+ ·015)	− ·006	...
5 − 2	+	1·518	...	− ·001	...
6 − 2	+	0·140	...	...	...
2 − 3	+	8·908	...	− ·004	...
3 − 3	+	7·993	...	− ·004	...
4 − 3	+	1·162	(+ ·022)	...	...
5 − 3	+	0·127	...	...	...
2 − 4	+	0·320	...	...	...
3 − 4	+	0·390	...	...	...
$3g-4g'$					
2 − 3	−	0·153	...	...	...
3 − 3	−	0·103	...	...	...
1 − 4	+	0·642	+ ·017	...	...
2 − 4	+	6·576	...	...	...
3 − 4	+	3·679	...	...	...
4 − 4	+	0·466	...	...	...
2 − 5	+	0·517	...	...	...
3 − 5	+	0·410	− ·012	...	...
$-g'$					
0 0	+	0·802	(− ·015)	...	+ ·024
−2 1	−	0·110	...	...	...
1 −1	−	0·467	+ ·010	...	− ·054
0 −1	−	4·906	− ·020	...	− ·198
1 −1	−	0·606	− ·022	...	...
$2g-g'$					
2 0	+	0·812	...	...	+ ·024
3 0	+	0·101	...	...	...
1 −1	+	0·122	+ ·029	...	− ·032
2 −1	−	5·412	+ ·037	...	− ·198
3 −1	−	0·672	...	...	− ·022
$2g-3g'$					
1 − 3	−	0·295	...	...	...
2 3	−	0·351	...	...	...
$5g-4g'$					

Arg.		$\beta+\Delta\beta$.	Δs.	$\frac{\sin(f+\omega)}{\Delta \sin L}$	$\frac{\sin I}{\cos(f+\omega)\Delta f}$.
g g'		′	″	″	″
3 −4	+	2·419	...	...	...
4 4	+	3·004	...	...	...
5 −4	+	1·193	...	...	...
6 −4	+	0·214	...	...	...
3 −5	+	0·218	...	...	...
4 −5	+	0·346	...	...	...
5 −5	+	0·161	...	...	...
$4\omega-3\omega'$					
3 −3	−	0·208	...	...	...
$5\omega-2\omega'$					
4 −2	−	0·245	...	...	...
5 −2	−	0·145	...	...	...

A Spectrographic Study of β Lyræ. By Rev. Walter Sidgreaves, S.J., Stonyhurst College Observatory.

Introduction.—The more the spectrum of *β Lyræ* is studied, the more evident it becomes that its photographs cannot be too greatly multiplied; and one of the objects of this paper is to show that a suitable telespectrograph would be well employed if devoted solely to this purpose until a series of plates had been collected containing many for each day of the light-period.

The attempt made here during the last two years has been greatly frustrated by the extraordinarily cloudy state of the nights. And the result, so far, is disappointing, inasmuch as many more photographs are wanted, in order to support or undermine the constructive explanations suggested during a rather lengthy study of the plates already collected.

All the photographs have been taken with the small prismatic camera of 4 inches aperture described in the December number of the *Monthly Notices*, 1901, in the introductory part of the paper on *Nova Persei*. They cover the spectrum from HS to the head of the hydrogen series, and on one of the stronger photographs the lines can be counted up to $H\pi$. But HS and the lines beyond $H\eta$ are too weak for any trustworthy comments, and the enlargements have been cut down accordingly.

Professor Vogel, discussing the Potsdam photographs of

1892–3,* has noted the importance of correct exposure in all scientific applications of photography. Defect of exposure has been a source of loss to the Stonyhurst series of spectrograms of 1902–3. The slitless spectrograph employed here has its own merits, but for successful exposures the slit has the advantage in this, that the time of exposure can be prolonged at will, to suit the state of the sky and the magnitude of the star. With a slitless spectrograph exposure can be quoted only as a velocity; and this is limited in slowness by a small oscillation of the star's image, caused by inequalities of the clock's driving-screw. The rate of trail must not be less than the amplitude of this oscillation in the time of the oscillation; otherwise the stellar image backs on the plate periodically and blurs the spectral lines.

Another source of loss to the series of spectrograms has been the difficulty often experienced in setting the spectrum on the plate in its true position of focus, owing to the considerable slope of the plate required by the spectral image as given by a visual objective. This presents no difficulty when the sky is really clear and the spectral colours can be distinguished; but real clearness has been exceptional in the last two years.

The criterion adopted for the selection of the plates has been a readable enlargement, and this has thrown out of service about 40 per cent. of the total number of plates.

These enlargements have been found of great service in studying the changes of the several lines. They are not of the same worth as the original negatives, but after an examination of each of the negatives in conjunction with its enlargement, the details of the negative can be recognised on the positive, and then comparisons are easily made on the card of mounted enlargements. But it should be understood that all remarks upon the lines in the course of this discussion of the spectrum are made on the original negatives, and these remarks refer only to the selected negatives; but the rejected plates are not all worthless, and a list of these is appended at the end of the paper, with remarks concerning their positions on the spectrographic chart or card of enlargements.

The order of the spectrograms on the chart is that of their time-intervals in days and hours from the previous principal minimum of the light period. These intervals are entered in the margin, and will be referred to as *periodic dates*, indicated generally by the letter D written after the date; and the civil date can be taken from the opposite margin, in which the date of the previous principal minimum is entered.

Mounted in this order on a long strip of cardboard the enlargements possess a double value—first, for the study of changes in connexion with the light period; and second, as a reference chart for subsequent photographs of the spectrum.

* *Über das Spectrum von β Lyræ*, Königl. Preuss. Akad. der Wissenschaften, vi. 1894.

A very cursory examination of the chart is enough to discover a regularity in the sequence of spectral changes so independent of absolute time-intervals that a word of caution seems necessary to protect the reader against erroneous conclusions which may be suggested by this arrangement of dates. It was probably this risk which prompted Professor Vogel's remark at the end of the paper already referred to : "Dass nach den hier gemachten Beobachtungen es nicht zulässig sein dürfte, Beobachtungen, welche Monate lang aus einander liegen, zu verbinden, um die Veränderungen der Linien innerhalb der Lichtperiode darzustellen." An illustration taken from the first plates of the series will serve this purpose. On the periodic dates 0–0 and 0–5 D the line Hζ consists of three parts, a dark centre and two bright sides. On 0–6 D the same line has only one bright side. There is no proof here of so sudden a change as that indicated by one hour difference of periodic time, for the absolute interval is nearly one year. But there is some probability of suddenness represented by the present state of the chart, which shows three components of the line on the two plates of 0–0 and 0–5 D, and only two components on 0–6, 0–16, and 0–17 D ; and if we had the confirmation of this change by many photographs of different absolute dates belonging to the same two periodic intervals, the probability of the sudden change would become very great. This confirmation is wanted for many of the inferences suggested in the following comments on this series of spectrograms, which is far from complete. Some of the inferences may be already contradicted by the photographs obtained with other instruments ; but discordances of different instruments should not be too easily set up as contradictions. A comparison series of this sort, in which the parts were filled in by instruments of unequal dispersion and light-power, would be for many conclusions entirely misleading.

Before discussing the changes exhibited on the photographs it is necessary to challenge a proposition in Miss Clerke's *Problems in Astrophysics*, p. 344, "that a dark line in the spectrum of one star cannot cut a slice out of a brilliant band proceeding from another." The proposition is an answer to a question of primary importance to the interpretation of the changes in the spectrum of β *Lyræ* ; for, if it cannot be gainsaid, the simple explanation of those changes which are periodic by relative motion of two stars must be abandoned, and nothing remains, at least to the writer, to take its place. Changes of pressure seem wholly inadequate, and their periodicity inexplicable. This must be the apology for the following, perhaps lengthy, discussion of the question.

Every dark line in the solar spectrum is dark only by contrast with the adjoining continuous spectrum, and it matters nothing whether the contrast is the result of subtraction or of addition. In the Sun or single star it is by subtraction (absorp-

tion) of a fraction of the light-intensity. In the case of two stars in the same sight-line it is by addition.

Let i and i' be the intensities of the continuous spectrum in the neighbourhood of λ by each of the two stars S and S′; and Δi, $\Delta i'$ the differences producing line λ. Then

the intensity of bright λ by S is $i+\Delta i$

and

the intensity of dark λ by S′ is $i'-\Delta i'$.

In superposition the intensity of the continuous spectrum is

$$i+i'=c$$

and that of λ is

$$i+\Delta i+(i'-\Delta i')=l$$

and λ is bright or dark according as $l \gtrless c$, *i.e.* as $\Delta i \gtrless \Delta i'$.

It seems clear therefore that bright and dark λ can be neutralised; for Δi can $= \Delta i'$. Therefore if the bright line is broad, and the dark line narrow, the contrast can be between neutralisation of the centre and integrated lateral brightness, and the contrast becomes greater when $\Delta i' > \Delta i$. This is the case with Hζ in the spectrum of *β Lyræ*. Professor Vogel has the dark component "absolut dunkel"* on the photographic plates, which is expressed by $\Delta i' = i'$. The intensity, then, of λ at the centre of the broad bright line becomes

$i+\Delta i$ between sides of intensity $i+\Delta i+i'$.

It must remain a matter of opinion whether there is here contrast enough for the appearance of a strong dark line crossing the broad bright line. It cannot be black physically, for its intensity is $i+\Delta i$; but can it *look* black by contrast? The writer thinks it can: for the red hydrogen line of the Sun is very brilliant, as seen *off* the solar limb, and this intensity is the least possible light-intensity of the line, which looks very dark *on* the solar surface. But, finally, can a physically bright line, which looks dark by contrast, fail to make its impression on the photographic film, and leave, as on the Potsdam plates, a line of clear glass? This is a matter of exposure. Time of exposure is the photographic analogue of ocular contrast. Vogel insists emphatically on correct exposure, presumably for the general picture; and this may be the exposure which leaves dark Hζ, on the negative clear glass. The slit spectrograph could decide the case of Hζ in *β Lyræ* by timing the exposure needed for a perceptible deposit of silver on the line. If time should fail, the consequence is clear: that the dark line can only be the result of absorption in the one star; and the importance of this determination needs no words.

* *Loc.* p. 120.

Hζ.—The line at the position of the hydrogen line Hζ is well described by Vogel as "bei weitem die auffallendste Linie in dem durch Photographie fixierten Spectrum."* But our photographs suggest the addition that its prominence in the spectrum is chiefly, if not wholly, due to its dark component. The line λ 447 $\mu\mu$ appears for the greater part of the light-period as the brighter line, and it passes through greater changes. But the dark component of Hζ has no competitor: it is unique in the spectrum. On some plates it appears, as described by Vogel, a clear-glass line. It is sharply cut, and it is narrower than any other of the hydrogen lines, except when Hγ and Hδ are nearly obliterated at the chief minimum, and when they are reduced to fine sharp lines soon after the secondary minimum of light.

During the first part of the light-period, from 1–11 to 6–20 D, bright Hζ differs in no appreciable manner from bright Hγ; and during the rest of the period the apparent difference between the two bright lines is attributable to the real difference between their dark lines. In the first part both bright lines are seen only on the red sides of their dark companions. On the five dates 7–4 to 7–19 D the bright component of Hγ is extinct, and that of Hζ is very weak and divided. The narrow dark Hζ cuts out a more or less central part of the broad bright line, while the more diffuse Hγ absorption extinguishes either the whole bright line when concentric, or one of its sides when relatively shifted.

The change from the appearance of a single to that of a double bright line begins on 5–9 D. On this date there is, on the more refrangible side of the dark line, a clear bright fringe, which is also seen on 6–20 D, but less clearly. On 7–4, 7–6, and 7–15 D, Hζ is a very feeble broad bright line, divided, perhaps centrally, by the dark component; but it is too weak for measurement. It is stronger on the plates of 7–17 and 7–19 D, but divided unequally, the red side part being the stronger and broader. And it retains the appearance of a double bright line throughout the rest of the period, including the principal minimum, up to 0–6 D, on which date is found the first return to the condition of a single bright line on the red side. The two bright parts are of nearly equal strength at the chief minimum; the red side part is the brighter on 0–5 D, and on 0–6 D the shorter wave part has disappeared.

This disappearance might be the effect of a relative shift of the dark line over the single broad bright line, or it might be the simple extinction of the more refracted of a pair of bright lines. The width of the remaining bright line, on the red side, measured on the negative, seemed to favour the latter possibility, for it was not wider than before the disappearance of its more refracted companion. And the position of the vanished line was found to be at λ 388·1 $\mu\mu$, the position of the strongest of Cyanogen lines;

* *l.c.* p. 120.

suggesting a periodic extinction and reappearance of this line, as was observed for a while in the spectrum of Nova *Persei*.

On this account very careful measures were made of the widths of the components of Hζ on all the photographic plates of the series, in order to compare them in the two conditions of the whole composite line—viz. first, when the bright line is seen on both sides of the dark line, and secondly, when it is seen on one side only. Measures of this sort cannot be altogether free from errors, attributable to varying conditions of the atmosphere and to relative excellence of the photographs. But these errors may be expected to balance one another in a large number of photographs taken on different nights. The results are presented in abridged form in the following table, taken from the details contained in the complete table at the end of the paper. In both tables the widths are entered in micrometer divisions (centimillimetres), and the periodic intervals in days and hours. In this abridged table the series is divided into four parts, according to the apparent uniformity of the widths.

Table of mean widths of the whole composite line Hζ

	d h		Periodic intervals. d h	d h	No. of dates.	Mean width.	Greatest width.	Least width.
Chief min.	12 22	...	From 12 12	to 0 5	5	20·8	22	18
First max.	3 2	...	0 6	2 13	11	15·5	17	14
Second min.	6 11	...	2 20	9 14	23	22·0	24	18
Second max.	9 18	...	10 0	12 12	11	16·5	18	15

From this analysis it appears that if the two bright lines, as seen one on each side of the dark line, belong to the same origin, the resulting broad band changes between twenty-one and sixteen units of width. We have therefore to choose between a temporary loss of width in bright Hζ and a temporary extinction of another line, possibly CN. In favour of the latter hypothesis a strong case would have been made out if the greater breadth of the line had been maintained through the last quarter of the period; but it must be observed that during this quarter all the bright lines are failing in strength, and bright Hζ may, on this account, have lost width on the more refractive side, which is weaker than the red side and shades down to the continuous spectrum. Accepting, provisionally, this explanation, we should have for the normal width of the whole line about twenty-one divisions, and this cut down to sixteen only for about two days shortly after the principal minimum. The extinction of a line during these two days could then be accounted for by dissociation at higher temperature, as in the writer's explanation of the same extinction in Nova *Persei*, but occurring, unlike the Nova, near the minimum, instead of at the maximum, of the light curve. There is, however, no contradiction in this difference, the light-changes being by eclipses in *β Lyræ* and by temperature in the

Nova. In both stars the extinction would be by dissociation effected by greater heat; and in *β Lyræ* greater heat is quite possible in conjunction with less light, more light being lost by eclipse than is gained by heat.

Tidal disturbance of a near approach at the principal minimum is perhaps the only assignable cause of the postulated rise of temperature, for its periodic recurrence can only be explained by the regularity of orbital motion; and although this supposition seems to be excluded by the form of the light-curve, which requires uniform velocity of revolution, it will not be out of place to point out in what ways the near approach of the two stars would better agree with the periodic changes of the spectrum. For at present there are, in every hypothesis, forbidding difficulties, which drew Professor Vogel's remark: "Ich halte den Zeitpunkt, eine einigermassen erschopfende Erklärung der sehr complicirten Erscheinung zu geben, noch nicht für gekommen."* And it is only by collating suppositions, however antagonistic, which better account for separate facts, that we may hope to hasten the time of arriving at the truth.

In the first place, the separation of the bright and dark lines so soon after the principal minimum is more in accord with the sharper turn at the periastron of an elliptical orbit than with the rounded transit of a circular path. In the latter motion the component velocity in the sight-line would change too slowly to give the observed displacements so early in the period, for the separation of the lines is well advanced within thirty-six hours after their conjunction at the minimum.

2. The second conjunction of the lines occurs after the secondary minimum of light, between 6–20 and 7–20 D; and this retardation on the conjunction of the two stars could be explained by an elliptic orbit having its major axis inclined a little to the sight-line in the plane of the orbit.

3. There would be an outburst of incandescent hydrogen and helium, &c., shortly after the nearest approach of the stars, giving the strong bright lines, while the higher temperature would kill the CN and its bright line. In the cooling process throughout the rest of the period the CN band would reappear and the other lines would fade. In all respects the changes would be according to the general appearances of the photographic chart.

4. This real increase of intensity is a more satisfactory explanation of the exceeding brightness of the hydrogen and other lines, shortly after the chief minimum, than contrast with a weaker continuous spectrum; for this contrast ought to be the same before and after the minimum, and the lines should be growing into prominence as the minimum is approached, whereas, according to the photographic chart, they are steadily fading all through the last quarter of the period.

* *l.c.* p. 130.

Hγ.—The hydrogen line Hγ has been already described along with Hζ, but some details have yet to be filled in. For some hours before and after the principal minimum the line is single and dark, but very weak. Both components, bright and dark, come into prominence as strong thick lines rather later than those of Hζ. The presence of the bright line is doubtful on 0–17 D of 1903–7–24 on a plate which shows Hζ and other lines very well; but on 0–16 D of 1902–9–17 both components are easily seen on a thinner but clearer photograph. Both components are in fullest strength, for the first time in the series, on 1–11 D. The intensity of the bright line falls off considerably, but there is no certainty of any change, either in the strength of the dark line or in the relative position of the two lines, until after the second minimum. The bright line remains fully on the red side through twenty-five successive periodic dates, including 6–20 D, nine hours after the second minimum. On 7–4 D the transition has begun, and Hγ is a single sharp fine dark line on 7–4, 7–6, 7–15, 7–17, and 7–19 D. The bright line reappears on 8–3 D wholly on the more refractive side of the dark line, where it remains without any apparent change of position until it vanishes some hours before the principal minimum. It is seen on nineteen successive periodic dates up to 12–3 D, when it has only a doubtful existence, and after this date it is quite extinct on all the seven plates covering the principal minimum.

There is therefore, in the behaviour of bright Hγ, fair evidence, first, of a retardation of the spectroscopic upon the eclipsing conjunction of the stars, of about twenty-four hours, the concentric superposition of the bright and dark lines occurring between 6–20 and 8–3 D, and the secondary minimum on 6–11 D; and secondly, of the general decline of bright-line intensity during the course of the light-period.

Hδ and Hε.—Both these lines follow Hγ in all the changes; but their bright components are much weaker, and probably on this account the dark lines recover strength after the secondary minimum better and sooner than that of Hγ.

The thinner dark companion line on the more refractive side of Hε gives to the hydrogen line the appearance of a double absorption line before and after the principal minimum, contracting to a single line, in the middle of the light-period. But the companion line is always well separated from Hε. It is a strong line only between the principal minimum and first maximum. For the rest of the period it remains a very fine line, not visible on all the plates, but becomes clearer towards the end of the period. The apparent separation of the two lines, however, differs very definitely in the two halves of the light-period; and the difference confirms the evidence of Hγ that the spectroscopic conjunction occurs later than the second stellar conjunction. In the first half of the light-period, from 0–0 to 6–20 D the mean separation is 13 divisions of the micrometer head, between extreme readings 12 and 14, from 14 plates; in the second half,

from 7-4 to 12-15 D, the mean separation is 19 divisions between extremes 15 and 22, from 12 plates; and the mean of these two is the measure on the plate of 7-4 D. But this mean may reasonably be taken for the normal separation of the lines when unaffected by displacements. And thus 7-4 D appears as the periodic date of no displacement. Unfortunately, on account of the weakness of the companion line at mid-period, we have only the one measure, on 7-4 D, between dates 5-9 and 9-1 D; but on this plate both lines are thin and sharp.

It must be noted, in addition, that the changes of separation of these two lines cannot be attributed to a Doppler effect of motion. In the first half of the period, when the separation is less, the bright component lies on the red side of the dark line, cutting down its width and thus shifting its apparent centre nearer to the companion line, an effect which is reversed in the second half of the period, when the separation is greater.

Elliptic orbit.—The spectroscopic evidence, therefore, in favour of an elliptical orbit is not small. The superposition of the bright and dark hydrogen lines does not coincide with the secondary light-minimum, but occurs some hours later, between 7-4 and 7-19 D; while at the principal minimum the coincidence is perfect; and these are the conditions of elliptic orbital motion, with the major axis inclined to the line of sight in the plane of the orbit: the angle being formed by rotating the axis in the direction of the orbital motion. Then, according to the excentricity of the ellipse, and the inclination of the axis, we may have the spectral conjunction considerably later than the stellar or eclipse conjunction near apastron, and no measurable difference between them at periastron: as may be seen in the diagram at the end of the table.

Radial velocity.—It is now generally recognised that the relative radial velocity of the components of a binary star cannot be determined by the displacement of lines which are too broad to escape overlapping. The positions of their centres cannot be fixed, and there is the additional uncertainty of the form of the widening, whether equally or unequally on either side of the true position in the spectrum. But we may derive an approximate measure of the relative velocity of the pair in *β Lyræ*, independently of the spectral centres of the lines, through the opposite displacements of bright H_γ before and after the secondary minimum of light. The character of H_γ makes this possible. The dark line has no diffusive edges, and the red side margin of the bright line is never concealed. The centre, therefore, of the dark line may be taken as a fiducial point for the measurement of the relative displacement of the red side edge of the bright line, caused by the opposite velocities of the same star, with reference to the other, at the two epochs of the light curve before and after the secondary minimum. These measures are taken from the table of widths at the end of the paper by adding half the width of the dark line in Column II. to that of the bright

line in Column III., and are entered in column r. An inspection of these measures shows that the greatest elongation of the red side edge of the bright line from the centre of the dark line occurs between 2-20 and 6-20 D, and the least between 10 and 11 D. The mean of the greater measures (r_1) from 18 plates is 16·3 divisions, at epoch 1, and the mean of the smallest measures (r_2) from 4 plates is 7·5 divisions. The difference is the displacement effected by the arithmetical sum of the two opposite velocities of the bright-line star, with reference to the other, at the two epochs; and half this difference is the displacement due to the relative velocity of the pair of stars at either *one* of the two epochs. All the measures were made by the writer, so that personal equation disappears in the differences, and the only systematic error that might vitiate the result is a possible shift of the apparent centre of the dark line by the position and strength of the bright line. Between the first maximum and second minimum, when the measures are greatest, the bright line lies for the most part so completely on the red side of the dark line that it might cut down its width on this side only; and thus the apparent centre of the dark line would be shifted away from the red side edge of the bright line. This would give the readings of r_1 too great, while the smaller readings (r_2) would be unaffected, the bright line being seen well on both sides, and the consequent thinning of the dark line presumably symmetrical. A correction therefore is wanted for the greater measures (r_1), and this may be taken from the measured widths of the dark line entered in the table of widths. The mean width at epoch (1) is 8·0 divisions, and at epoch (2) 6·0. Half this difference, or 1·0, is to be deducted from the mean of the greater measures, and we have for r_1 corrected 15·3 divisions, and for r_2 7·5. The difference 7·8 is the total displacement between the two epochs. The half, or 3·9, is the displacement at either epoch; and the resulting relative velocity of the two stars is 120 kilometre-seconds, which, on the supposition of equal velocities, gives 60 km. secs. for the velocity of either star.

No high degree of probability can be claimed for this measure of the velocity: it needs the confirmation of more photographs; but the method seems worthy of consideration, and might command confidence if the details were provided by an instrument of greater light-power. The result, however, as it stands at present is a remarkable, although perhaps to some extent fortuitous, confirmation of Belopolsky's measure of the apparent orbital velocity of the bright-line star. The Pulkowa measures were made between the bright lines, Hβ, as given by the star and by a Geissler hydrogen-tube; and the resulting orbital velocity in circular motion is 12·01 lieues géographiques,* or 66·7 km.-secs, to compare with the 60 kms. by the Stonyhurst measures.

Group 447-8.—This group of bright and dark lines presents

* *Le spectre de l'étoile variable β Lyræ: Mélanges mathématiques et astronomiques*, t. vii. liv. 3, p. 433.

a very perplexing study. It comes into prominence synchronously with $H\gamma$, early in the period, and dies away with it at the end, so that its cycle of changes is complete in the same light-period. But the changes in the grouping of the components are not easily accounted for on the single supposition of a simple binary star.

There are three principal phases of the grouping : First, *early in the period*, a strong bright line between two dark lines of unequal strength, the weaker one being on the red side of the group ; secondly, *an intermediate phase*, when only a single, narrow, and sharply cut bright line is seen ; and, thirdly, *after the second minimum*, a broad bright band divided by a fine dark line, and terminated on the red side by a stronger absorption.

The intermediate phase appears to correspond with that of the hydrogen lines, differing only by showing the single bright line instead of the single dark line ; but it occurs many hours earlier, halfway between the first maximum and second minimum, as seen on 4–16 to 4–20 D. On 6–20 D, nine hours after the second minimum, the bright line is already broad, divided, and terminated on the red side by a dark line as in phase 3. The phase is fully developed on 7–15 D ; and the group retains this form repeated on twenty-five successive periodic dates to the end of the light-period, except that the bright parts fade away towards the end, and are quite extinct on the five last plates, from 12–3 to 12–15 D ; when also the dark lines appear in equal strength, but much weaker than at the beginning of the period.

The two dark lines occupy the same positions with reference to dark $H\delta$ probably all through the period ; but they are not visible on the plates of the intermediate phase, and are too ill-defined for the micrometer on six of the plates after the second minimum. The measures on nine plates before the second minimum, and on seventeen plates after it, assign their positions in the spectrum at those of helium and magnesium, 447·1 and 448·2, the readings varying irregularly between 446·9–447·3 for the former, and between 448·1–448·5 for the latter.

The appearance of the group in the third phase, after the second minimum, is quite that of a pair of bright lines (helium and magnesium), each with its dark companion on the red side, in keeping with the hydrogen displacements. But these displacements should be reversed after the principal minimum. There should be a bright companion on the red side of the dark magnesium line ; and there is no sign of this, although the two bright lines in phase 3 are of equal strength. Also the intermediate phase has no explanation on this supposition. And, lastly, the relative displacements of both pairs of lines would be in opposition to those of hydrogen on the same plate of 6–20 D.

A better agreement between the phases of the group might be found on the supposition of a single broad bright line, which after the principal minimum, in phase 1, covers the region between the red side limits of the two dark lines, paling the

magnesium and leaving the helium in fullest strength. The width of the whole bright line, as seen in phase 3, is rather more than enough for this ; and the changes in the relative intensities of the two dark lines favours the supposition ; for as the relative shift proceeds, the bright line uncovers the magnesium line and covers the helium line, until on 5–9 D and following periodic dates the magnesium is the stronger dark line. Also the intermediate phase of a single bright line would appear when the broad bright line covered both dark lines nearly equally ; and the periodic time of this occurrence, being considerably earlier than that of the mean position of the oscillating line, would be in accord with the date of this phase in the period. And, finally, the mean position of the line would show a darker magnesium line and a broader bright line divided by the helium absorption, as seen on the plates of 7–4 and 7–6 D : a result of considerable weight in favour of this supposition, seeing that it brings the periodic time of the mean position of these lines into agreement with that of the hydrogen lines, and accounts for the apparent contradiction already noticed on plate 6–20 D of the hydrogen bright lines and those of this group in seemingly opposite displacements.

Conclusion.—There are other manifestations on some of the photographs which indicate, on the principle of Doppler displacements, temporary disturbances similar to the solar eruptions which distort the hydrogen lines. A line is bent or curved while the neighbouring lines are quite straight. The effect on a slit spectrograph would be a wider line, but the trailed line shows the progressive change of the line. It may, however, be delusive, a mere photographic freak ; and the only demonstration possible would be the same effect by two instruments running at the same time—a coincidence not to be looked for. But the possibility of a real stellar cause of the distortion is a possibility too probable in itself to be neglected. And for this reason, returning to the opening paragraph of this paper, it may be with some confidence asserted that the suitable telespectroscope for *β Lyræ* must be the slitless prismatic camera of as short focal length as may be consistent with sufficient light-gathering aperture. The 4-inch prism employed here is only just enough, and fails at the important epochs of least magnitudes, except in exceptionally clear atmospheric conditions ; and the disappointments are too frequent to hope for continued perseverance in the work. The ideal instrument would be a 5-inch prism and objective corrected for the photographic spectrum ; the prism mounted for first surface refraction only, and the second surface figured as part of the corrected objective ; the refractive angle of the prism and the focal length of the objective being adjusted to give length of spectrum from Hβ to Hη, about 2 centimetres. With an instrument of this description the bright lines would not suffer in contrast by over-magnification; disappointments would be exceptional, and the work could be carried on with the regularity

of Sun-spot photographs, supplying the material for a fruitful study of the variations of this star, which, as Vogel has it, "zält in spectralanalytischer Beziehung zu den interessantesten Objecten des nördlichen Himmels," and without which we cannot hope to unravel "the endless complications of the problem star." *

Notes on the following table of Hζ *components.*

D = Periodic date = interval from previous principal minimum, in days and hours.
M = Civil date (year, month, day) of previous principal minimum.
I., II., III stand for the components of Hζ in order of increasing wave-length; II. being the dark line.
W = Width in micrometer divisions.
S = Sum of the widths, or total width of composite line.
r = Length from apparent centre of dark line II to red side edge of bright line III.

a. Another photograph on the same night gave all the measures greater. The photograph was very weak, by a trail much too quick.
b. These two plates have been re-examined many times, on account of the seeming greater total width of Hζ on the positive enlargements. The measures entered always appeared to be the most probable readings.
c. The plate of 4–6 D is omitted from the table on account of a scratch on the film which passes through the important line III., and makes impossible a true pointing of the margin. But it is most probable that the width of III. is exceptionally small.
d. Probable fine bright edge.
e. Probably broad; not measurable.
f. Doubtful existence.

Hζ Components.

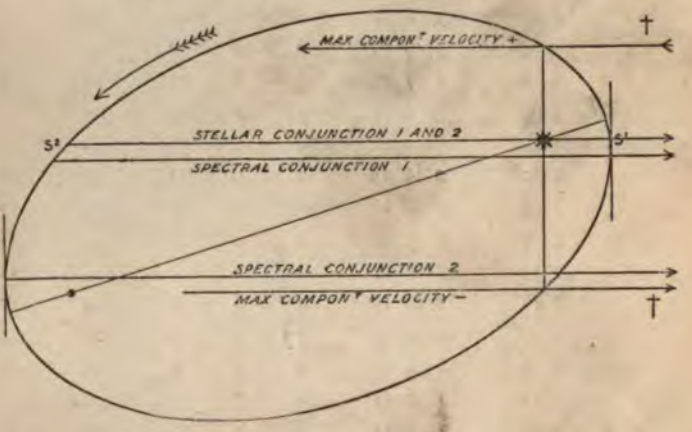
	D. d	h	M. y	m	d	I. w.	II. w.	III. w.	S w.	r
	12	12	1902	8	22	8	5	8	21	10.5
	12	15	2	9	4	5	7	6	18	9·5
Chief min.	12	22	2	5	23	8	6	7	21	10·0
	0	0	3	9	13	7	7	8	22	11·5
	0	5	1	8	25	9	6	7	22	10·0
	0	6	2	7	1	...	8	6	14	10
	0	16	2	9	17	...	8	7	15	11
	0	17	3	7	24	...	8	8	16	12
a	1	3	3	10	22	...	8	7	15	11
b	1	11	2	8	9	...	7	9	16	12·5
b	1	14	2	9	4	...	9	8	17	12·5
	2	0	3	9	13	...	8	7	15	11
	2	12	1	10	29	...	8	8	16	12
	2	12	2	8	22	...	8	7	15	11
		13	2	8	22	...	8	8	16	12
	2	20	1	6	22	...	8	12	20	16
	2	22	3	8	19	...	9	11	20	15·5

* A. M. Clerke, *Problems in Astrophysics*, *β Lyræ*.

D. d h	M. y m d	I. w.	II. w.	III. w.	s w.	r
Max. 1 (3 2)		...	...	...	...	...
3 12	1 10 29	...	8	13	21	17
3 12	2 8 22	...	8	10	18	14
3 14	2 9 4	*d*	8	11	19	15
3 18	1 6 9	3	7	15	25	18·5
3 19	2 9 30	*d*	7	13	20	16·5
3 21	3 8 19	*d*	9	12	21	16·5
e 4 12	1 10 29	*d*	8	12	20	16
4 15	1 11 24	*d*	7	15	22	18·5
4 16	1 5 27	*d*	8	14	22	18
4 20	1 6 22	*d*	10	12	22	17
4 21	2 10 25	3	8	11	22	15
4 21	3 8 31	*d*	9	11	20	15·5
4 23	3 9 13	*d*	9	12	21	16·5
5 9	2 7 27	3	6	11	20	14
5 23	3 8 31	3	8	13	24	17
Min. 2 (6 11)		...	...	...	...	...
6 20	2 10 25	2	9	12	23	16·5
7 4	1 8 12	10	5	9	24	11·5
7 6	1 9 7	*e*	4	9	...	11
7 15	2 9 17	*e*	6	8	...	11
7 17	2 10 13	8	8	8	24	12
7 19	2 10 13	7	5	9	21	12·5
8 3	1 8 12	6	6	*e*	...	...
9 1	1 7 17	6	7	*f*	...	...
9 14	1 5 14	6	5	11	22	13·5
9 14	3 6 8	5	6	8	19	12
Max. 2 (9 18)		...	...	...	...	..
10 0	2 5 23	5	7	4	16	7·5
10 3	1 8 12	5	7	4	16	7·5
10 21	3 8 31	5	5	5	15	7·5
10 23	3 8 31	5	5	5	15	7·5
11 10	2 8 9	5	6	5	16	8·0
11 12	2 8 9	5	6	6	17	9·0
11 13	1 11 11	5	7	5	17	8·5
11 16	1 5 27	4	5	...	...	...
11 16	3 7 11	5	5	8	18	10·5
12 3	1 8 12	5	5	6	16	8·5
12 4	2 6 18	4	7	5	16	8·5
12 12	1 11 11	5	7	5	17	8·5

Inferior plates: not full failures.

3-20 D. like	3-18 D.	on photographic chart
5-10	5-9	
9-3	9-1	at λ 447. Too weak at Hζ.
8-10	8-3	" "
1-16	1-11	
11-17	11-16	at Hζ; but 447 single absorption.
1-18	1-11	
2-18	2-20	
3-20	3 19	
11-19	11-16	
8-15	8-3	at λ 447. Hζ doubtful.

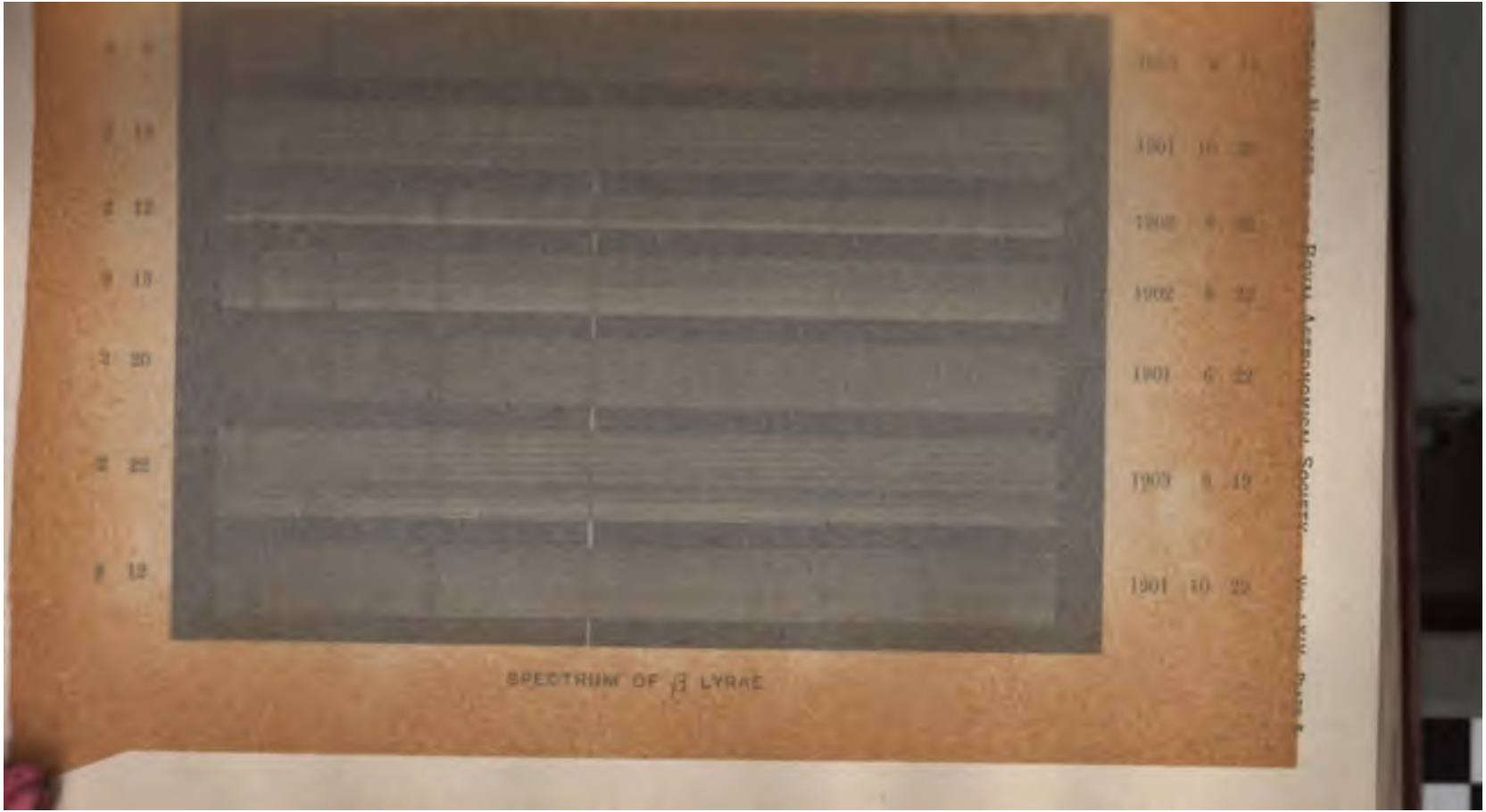


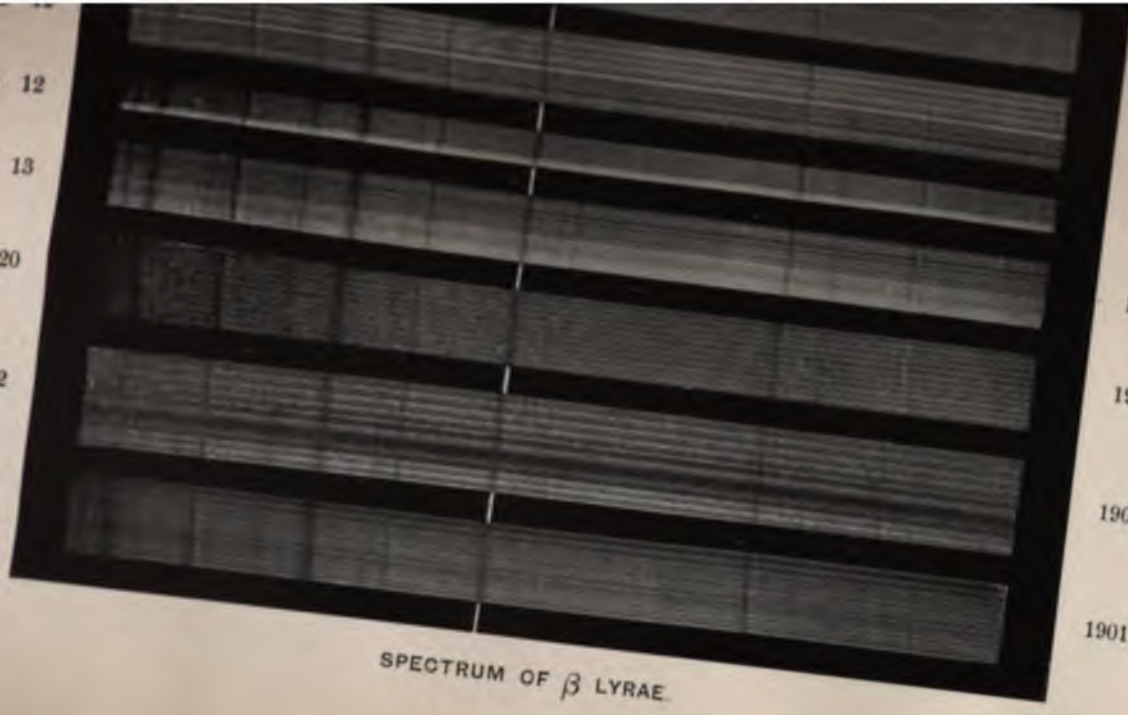
On the Chromatic Correction of Object-glasses. By A. E. Conrady.

The disproportionality of dispersion in crown- and flint-glass renders a perfectly achromatic object-glass an impossibility; a secondary (or in the case of a few special kinds of glass recently introduced by the Jena glassworks, a tertiary) spectrum being unavoidable. With all ordinary combinations of crown- and flint-glass the effect is that light of a particular wave-length is focussed at a minimum distance, while all light of sensibly different wave-length is focussed at a greater distance from the objective. The only control which the optician has over the colour correction, once the glass has been decided upon, consists in choosing the most favourable wave-length for which the

† Rambaut, *Monthly Notices*, vol. l., p. 305.

D	H	M Y	M	D
12	15	1902	9	4
0	0	1902	5	23
0	5	1901	8	25
0	6	1902	7	1
0	17	1903	7	24
1	11	1902	8	9
1	14	1902	9	4


SPECTRUM OF β LYRAE



SPECTRUM OF β LYRAE.

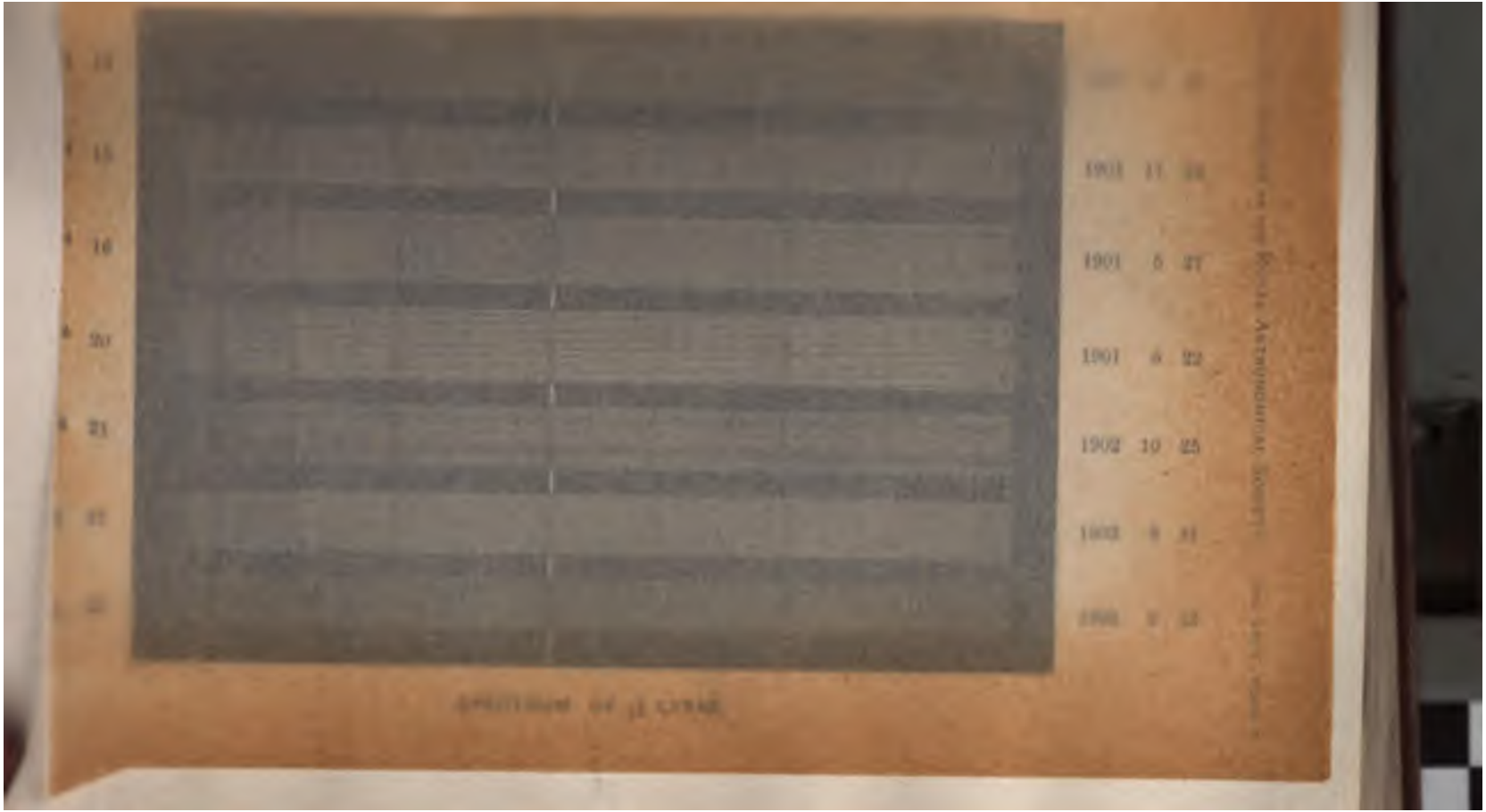
H. K. H_2 HELM H_3

H_1 HELM $\frac{\text{OFF}}{\text{ATT}}$

D	H		
8	12	1901	11
8	12	1902	8
8	14	1902	9
8	16	1901	6
8	19	1902	9
8	21	1903	6
8	4	1903	7

SPECTRUM OF β LYRAE.

D	H	M Y	M	D
3	12	1901	10	29
3	12	1902	8	22
3	14	1902	9	4
3	18	1901	6	9
3	19	1902	9	30
3	21	1903	8	19
4	6	1902	7	1



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D
H$_\zeta$ K H$_\varepsilon$ HELM H$_\delta$ H$_\gamma$ HELM 447 448
M
D H
Y M D
4 23
2
1903 9 13
5 9
1902 7 27
5 23
1903 8 31
6 20
1902 10 25
7 4
1901 8 12
7 6
1901 9 7
7 15
1902 9 17

SPECTRUM OF β LYRAE.

7	17	1902	10	13
7	19	1902	10	13
8	3	1901	8	12
9	1	1901	7	17
9	14	1901	5	14
9	14	1903	6	28
10	0	1902	5	23

SPECTRUM OF β LYRAE.

D
H
K
H
H
H

7	17	1902	10	13
7	19	1902	10	13
8	3	1901	8	12
9	1	1901	7	17
9	14	1901	5	14
9	14	1903	6	28
10	0	1902	5	23

SPECTRUM OF β LYRAE.

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D
H_ζ K H_ε HELM H_δ H_γ HELM 447 448
M
D H
Y M D
10 0
2
1902 5 23
10 3
1901 8 12
10 21
1903 8 31
10 23
1903 8 31
11 10
1902 8 9
11 12
1902 8 9
11 13
1901 11 11

SPECTRUM OF β LYRAE

M			D	
h m s			h m	
1901 11 11			11 13	
1902 8 9			11 12	
1902 8 28			11 10	
1903 8 31			10 22	
1905 8 25			11	
1907 8 27			11	
1908 8 —			11	

11	16		1901	5	27
11	16		1903	7	11
12	3		1901	8	12
12	4		1902	6	18
12	12		1901	11	11
12	12		1902	8	22
12	15		1902	9	4

SPECTRUM OF β LYRAE.

objective shall have the minimum focus; and this choice is so important that it is not unusual at the present time to see the selected wave-length stipulated by the astronomer when ordering an object-glass for a particular purpose.

Such a stipulation calls for new methods of computation based on the principles of the differential calculus, as the coloured "rays" to be united must have an indefinitely small difference of wave-length, while the usual trigonometrical computation can be successfully carried out only for widely different wave-lengths.

But it further calls for a mathematical formula connecting the refractive index, n, of the glass used with the wave-length, λ, of light, as otherwise it would be difficult to determine the rate of dispersion for any stipulated wave-length.

Several such formulæ have been proposed which are based on theoretical grounds, but they are too complicated for practical use. Among the empirical formulæ one consisting of the first three terms of Cauchy's used to be a favourite:

$$n_\lambda = n_0 + r_1\lambda^{-2} + r_2\lambda^{-4},$$

but when this is applied to glass by determining the three constants from the observed indices for three different wave-lengths, the discrepancies between observed and computed values for other wave-lengths are altogether too serious to be tolerated.

A number of years ago a Dr. Willibald Schmidt, by a most laborious series of calculations according to the method of least squares, tried all likely combinations of two *whole-number* powers of λ and came to the conclusion that the formula

$$n_\lambda = n_0 + r_1\lambda^{-1} + r_2\lambda^{-4}$$

brought about the most satisfactory agreement between observed and computed indices.

But on applying Schmidt's formula to the numerous varieties of glass in the Jena Catalogue, I found decidedly unpleasant discrepancies, smaller than those resulting from Cauchy's, but yet too large, especially for flint-glass. A closer discussion showed that the differences with Schmidt's formula were always of the opposite sign and generally of about one-third the magnitude of those found with Cauchy's formula. And so, after some trials, I adopted an intermediate formula, viz.

$$n_\lambda = n_0 + r_1\lambda^{-\frac{3}{2}} + r_2\lambda^{-4}.$$

This formula will render the refractive indices of all ordinary kinds of glass correctly throughout the visible spectrum within a few units of the fifth decimal, and is therefore sufficiently accurate for all ordinary purposes. It may confidently be used to interpolate the refractive index for any stipulated wave-length, and, by differentiation, to determine the rate of dispersion in the vicinity of any wave-length.

I now proceed to formulate a chromatic condition for use in computing objectives having minimum focus for a prescribed wave-length. From the point of view of the undulatory theory a combination of lenses transforms the spherical waves emanating from a luminous point forming the object into waves converging towards the conjugate point. Obviously a perfectly achromatic telescope would have to transform waves of any wave-length into refracted waves of exactly similar form and consequently of exactly the same focus. With the ordinary kinds of glass and the ordinary types of object-glasses we can attempt this ideal correction for a very short part of the spectrum only, as explained above. But, moreover, we cannot get waves of exactly similar form even within such a narrow range, because such object-glasses do not allow of establishing freedom from spherical aberration for more than one wave-length. Under these circumstances we shall get a good compromise by stipulating that the refracted waves of a wave-length slightly differing from that selected as the principal one shall, when tangent to the principal

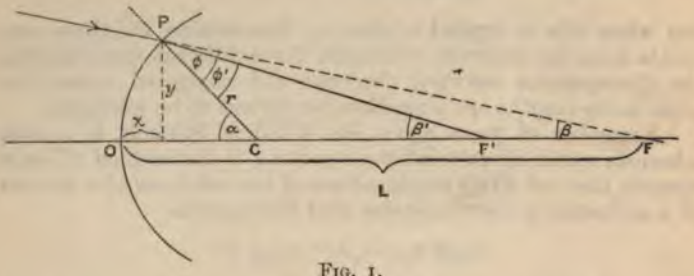


FIG. 1.

wave in the optical axis, also intersect that wave in the margin of the segment which the object-glass allows to pass. It is important to note that the chromatic condition thus formulated does not depend on the spherical correction for the principal wave-length; the formulæ to be given can therefore be applied with equal facility whether a system of lenses is spherically corrected or not.

These formulæ are based on and used in connection with the trigonometrical ones suitable for computing the course of rays through a system of lenses, and as these are not found in ordinary text-books I will give them first. In these calculations a ray, P F in fig. 1, is defined by the distance O F=L from the vertex of a refracting surface at which it cuts the optical axis—positive on that side of the surface towards which the light travels—and the angle β under which it cuts the optical axis—positive when the ray, in crossing, passes from above to below the axis. The sign of the radius of curvature r is determined by the same rule as that of L.

Calling the angle of incidence CPF ϕ, the triangle CPF gives

I. $$\text{Sin } \phi = \sin \beta \frac{\text{CF}}{\text{CP}} = \sin \beta \frac{\text{L} - r}{r}.$$

Next we obviously have the angle α at the centre of curvature.

II. $$\alpha = \beta + \phi.$$

Distinguishing the angles and coordinates after refraction by a dash, we have, from the law of refraction, when n is the index of the medium to the left, n' that of the medium to the right of our refracting surface.

III. $$n \sin \phi = n' \sin \phi' \quad \text{or} \quad \sin \phi' = \frac{n}{n'} \sin \phi, \text{ whence } \phi'.$$

The new triangle CPF' next gives

IV. $$\beta' = \alpha - \phi',$$

and further, in analogy with I.,

V. $$\text{L}' - r = r \frac{\sin \phi'}{\sin \beta'}, \text{ whence L}'.$$

We thus have the coordinates of the refracted ray and can at once proceed to any following refracting surface. We can also find the rectangular coordinates of point P; they are

$$x = \text{R}(1 - \cos \alpha) = 2\text{R} \sin^2 \frac{\alpha}{2} \text{ and } y = \text{R} \sin \alpha.$$

Returning to our problem, we must bear in mind that light travels the slower the denser the medium, taking the same time to traverse a distance d in a medium of refractive index n as a distance $n \cdot d$ in empty space. $n \cdot d$ is called the equivalent "optical path" of the distance d in medium of index n; any path may be reduced to its equivalent optical path by multiplying each portion by the index of the medium in which it lies.

Now, in any system of lenses the optical paths of all rays of any one wave-length connecting the waves sent out by a luminous point with the same waves after passing through the system are necessarily equal; in other words, light of any one wave-length will, owing to the retardation in glass, travel in the same time over the axial distance between two such waves as over the geometrically longer marginal distance. My chromatic condition stipulates that the axial and marginal paths shall also be optically equal between the same points for light of a slightly different wave-length.

It is obvious that a coloured ray of slightly different wave-length will follow a slightly different path from that taken by

the principal ray; we shall have to ascertain whether this difference is sensible. In fig. 2 let the thick line $r\,r'$ represent the principal ray, the thin line $\rho\,\rho'$ a coloured ray of slightly different wave-length. The path of the latter will form small angles with and be separated by small distances from that taken by the principal ray, and, the law of refraction being of a linear character, these angles and distances are of the same order of magnitude as the difference in refractive index. The angle between the two paths will make that of the coloured ray longer in proportion to its secant; as this differs from unity only by a term of the *second* order we may neglect this factor. We next notice that the path of $\rho\,\rho'$ is alternately longer and shorter than that of $r\,r'$; but here again there is compensation at each

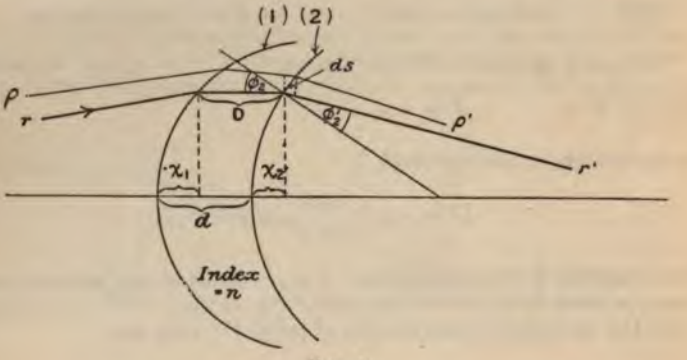


FIG. 2.

refracting surface; for, taking as an example the surface (2), we have obviously

The path of the principal ray within surface (2) shorter by $ds\,.\,\sin\phi_2$ or *optically* by $ds\,.\,n\,.\,\sin\phi_2$,
but the path of the principal ray beyond surface (2) longer by $ds\,.\,\sin\phi'_2$ or *optically* by $ds\,.\,n'\,.\,\sin\phi'_2$;

and as, by the law of refraction, we have $n\,.\,\sin\phi_2 = n'\sin\phi'_2$ there is complete compensation as far as first order terms are concerned.

Hence we may assume without error that the coloured rays (of an indefinitely small difference in wave-length) follow the same course as the principal ray, which greatly simplifies the problem. For if we call the geometrical paths of the principal axial and marginal rays d and D respectively, the refractive index for the principal ray n, that for the coloured ray $(n + dn)$, we shall have, in accordance with the explanations given above,

$$\Sigma n(d-\mathrm{D}) = 0$$

when this sum is taken over the entire path from a wave sent out by the object to the same wave after refraction.

Similarly, the total difference of the optical paths of the coloured rays will be

$$\Sigma(n + dn)(d - \mathrm{D}),$$

or, deducting the preceding equation,

$$\Sigma\, dn\,(d - \mathrm{D}),$$

and the system of lenses will be achromatic for the selected wave-length when this sum is made equal to zero; hence the mathematical formula expressing my new chromatic condition is

$$\text{VI.} \qquad \Sigma\, dn\,(d - \mathrm{D}) = 0.$$

Here the dn are to be found as explained in the early part of this paper. The d are given. The D are easily determined by projection upon the optical axis, which gives

$$\mathrm{D} = (d + x_2 - x_1)\sec\beta'_1$$

$$\text{or } \mathrm{D} = \left\{ d + 2r_2 \sin^2\frac{\alpha_2}{2} - 2r_1 \sin^2\frac{\alpha_1}{2} \right\} \sec\beta'_1.$$

As, in air, we have practically $n = 1$ and $dn = 0$, the D have to be computed only for those portions of the optical system which consist of glass or some other dense medium.

This formula is so simple and convenient in practical use that I always use it, even in cases when the older method of computing trigonometrically for two widely different wave-lengths would be equally correct. One further important advantage afforded by it is that it greatly reduces the number of trials necessary to obtain a properly corrected lens. For it easily yields a rigorous formula by which the last radius of a system of lenses may be at once determined so as to get perfect achromatism. With the older method even this has to be attained by successive approximations.

In ordinary cases it is not always necessary to obtain the dn by differentiation. Thus for visually corrected objectives it is generally sufficient to take the dispersion between the C and F lines as the value of dn, and similarly for ordinary photographic objectives the dispersion between the D and G lines may be used in a like manner.

I must add that the formulæ given are also applicable to paraxial rays by proper modification.

Taking the trigonometrical formulæ first, we note that for rays very close to the optical axis all the angles become very small, hence equal to their sines. Distinguishing these small paraxial angles by a left-hand index o and the corresponding

abscissæ of the paraxial rays by small letters l, we can write the formulæ down at once :

$$_{0}\text{I.} \quad -_{0}\phi = {}_{0}\beta \frac{l-r}{r}$$

$$_{1}\text{II.} \quad {}_{0}\alpha = {}_{0}\beta + {}_{0}\phi$$

$$_{0}\text{III.} \quad n \cdot {}_{0}\phi = n' \cdot {}_{0}\phi' \text{ or } {}_{0}\phi' = \frac{n}{n'} \cdot {}_{0}\phi$$

$$_{0}\text{IV.} \quad {}_{0}\beta' = {}_{0}\alpha - {}_{0}\phi'$$

$$_{0}\text{V.} \quad l'-r = r \frac{{}_{0}\phi'}{{}_{0}\beta'}$$

To determine the chromatic condition for paraxial rays, we first transform the term $(d-\text{D})$ in VI. by introducing the value of D given by VII., and obtain

$$d-\text{D} = d - \frac{d + 2r_2 \sin^2 \frac{a_2}{2} - 2r_1 \sin^2 \frac{a_1}{2}}{\cos \beta_1'}$$

$$= \frac{2r_1 \sin^2 \frac{a_1}{2} - 2r_2 \sin^2 \frac{a_2}{2} - d\,(1-\cos \beta'_1)}{\cos \beta'_1}$$

$$= \frac{2r_1 \sin^2 \frac{a_1}{2} - 2r_2 \sin^2 \frac{a_2}{2} - 2d \sin^2 \frac{\beta'_1}{2}}{\cos \beta'_1}$$

If now the angles become very small, the cos in the denominator will become equal to unity, and we shall have

$$d-{}_{0}\text{D} = \tfrac{1}{2}\{r_1 \cdot {}_{0}a_1^2 - r_2 \cdot {}_{0}a_2^2 - d \cdot {}_{0}\beta'^2_1\}$$

and the condition of achromatism for paraxial rays will be

$$_{0}\text{VI.} \quad \Sigma\, dn\,(d-{}_{0}\text{D}) = 0.$$

It will be clear that the fulfilling of this condition will give refracted coloured waves of the *same radius of curvature* as that of the principal waves ; in fact, this condition gives the identical result as the old one of equal focal point for principal and coloured paraxial rays.

When the conditions stated by VI. and $_{0}$VI. are fulfilled simultaneously, we attain a very high state of chromatic correction (semi-apochromatic correction) practically but not absolutely the same as that resulting from the well-known Gauss-condition.

Note on the use of Long-focus Mirrors for Eclipse work.
By H. H. Turner, F.R.S., Savilian Professor.

1. At recent eclipses the inner corona has been successfully photographed on a large scale with long-focus lenses: lengths of 40 feet, 60 feet, and even 100 feet do not seem to be unmanageable. Such lenses are naturally costly, and it is of interest to inquire how far they can be replaced by concave mirrors, which are cheaper.

2. It may be taken for granted that any telescope of such a focal length must be used in conjunction with a coelostat, or some form of heliostat which allows the telescope to remain fixed in position. We may therefore disregard at present any disadvantages which are introduced by the coelostat itself, and exist whether we use a refracting or reflecting telescope—such, for

FIG. 1.

instance, as the want of perfect flatness in the plane mirror. But there is an important difference between the two cases in respect of the available field, which it is the object of the present Note to briefly elucidate. Practically there is no difficulty about the field with a lens, unless it be placed unnecessarily far away from the coelostat (see *Monthly Notices*, lvi. p. 413).

3. As regards the mirror. Let A B (fig. 1) be the concave mirror, centre M and axis M O; and P any point in the image formed in the focal plane. If we trace the rays backwards from P to the mirror, and then towards the origin of light, they will form a sensibly circular cylinder which cuts the focal plane in a circle K L sensibly equal to the mirror, and having its centre at R, where O R = O P. For ideal definition at the point P this cylinder of rays must all fall on the coelostat mirror, otherwise only a portion of the concave will be used in forming the image at P. But in practice we shall be able to put up with something

short of this ; how much short of it remains to be determined, probably by actual experiment.

4. As we move P about to different portions of the image, R moves about over a precisely similar path, carrying the circular patch K L with it. And for ideal definition at all points of the image, the total area thus indicated must fall on the cœlostat mirror. Thus we can find the size and shape of the smallest cœlostat mirror which must be used in the focal plane, when we know the contour of the image. *Or conversely*, if we know the size and shape of the mirror we can find the contour of the image within which there will be ideal definition.

5. It is more convenient to consider the problem from this latter point of view, for the effective shape of the mirror (*i.e.* its projection on the focal plane) will usually be an ellipse. For a circular mirror of radius a placed near the focus of the concave, and inclined at an angle θ to the focal plane, the axes of this ellipse are $2a$ and $2a \cos \theta$; or, say, $2a$ and $2b$. In fig. 2 let X Y be this ellipse, and let a circle with centre R and radius c equal to

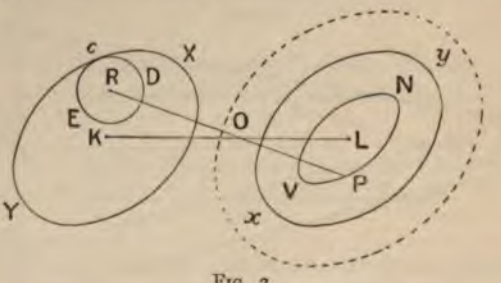


FIG. 2.

the radius of the concave mirror touch X Y internally. As R moves about, keeping this circle in contact with the ellipse, its image P will describe the contour of the image formed by the whole concave, and it is easily seen that this contour is an oval curve of axes $2(a-c)$ and $2(b-c)$. Thus unless the cœlostat mirror is so large that

$$a \cos \theta > c$$

then no part of the image will get light from all parts of the concave. But we can of course always cut down the concave by a diaphragm to secure this condition.

6. If we are satisfied with the rays from about *half* the concave mirror, then we can always get an available image equal in size and shape to the ellipse X Y. For if R lie on the edge of X Y, half the circle surrounding it (a little more indeed) will fall outside X Y, but the remaining portion will lie within ; and the image formed at P will still be as good as (say) a heliometer image, which uses half the object glass. As R moves round the edge of X Y, P will describe a precisely similar ellipse

xy ; and within this ellipse, which is equal in size and shape to the projection X Y of the cœlostat mirror, we accordingly get a fair image. A larger oval found by making R move so that the circle CDE touches X Y externally gives the limit of any image at all.

7. To fix the ideas, suppose that the cœlostat mirror is 16 inches in diameter and is inclined at an angle of 45° to the incident beam ; so that $2a = 16$ inches, $2b = 11{\cdot}3$ inches. Then with a concave of 12 inches diameter we shall get no field at all filled by the whole mirror; but if we cut the aperture of the concave down to 8 inches we get an oval field 8 inches long and 3·3 inches wide filled by the whole mirror. If we do not cut the 12-inch mirror down we get *more light* at every point of the image, but this light comes from a non-circular mirror. It is a question which can be worked out theoretically, but will probably be more easily settled experimentally, how far the extra light got from a large concave is a gain ?

8. One point seems clear. If we place the image of the eclipsed Sun centrally we shall lose the best part of the image in the central part obscured by the Moon. To make clear what is meant by central let us recur to fig 2. The important axis for us is the line joining K, the centre of the cœlostat, to the centre of the concave mirror, which cannot be represented in this figure—call it M, as in fig. 1. It is the line K M which forms the central ray, reflected along M L to the centre of the image. The normal M O to the concave mirror is only of secondary importance. Keeping M fixed, we may tilt the concave mirror and thus move O about to any point in the plane represented by fig. 2, carrying the point L and the patches surrounding it to positions represented by the law of images, i.e. K O = O L always. And if the dark centre of the Sun is viewed from M in the cœlostat in the direction of MK, the image of the dark centre will be formed at L, the best part of the field. Hence we must clearly arrange to see in the direction MK a part of the corona, which can be then photographed at L.

9. Before leaving the concave, remark that we cannot pass from one part of the corona to another by tilting it. Whatever part can be seen from M in the cœlostat (in the direction M K) is reflected to L, which should be the centre of the plate ; and by tilting the concave we merely select a different position for putting the plate. At the same time we alter the angle of incidence on the concave of the field we are photographing ; and since it is important to have this as small as possible, the proper tilt of the concave is immediately indicated—viz. it should be such that the line KOL is in the direction of the minor axis of X Y : and O should be as near to K as is possible without cutting off any portion of the image which it is wished to keep. When the concave is fixed in accordance with these conditions, the proper position for the plate is also fixed.

10. But we may, during the eclipse, alter the position of the cœlostat slightly in R A, and thus bring different parts of the corona into the field. We will consider this point presently.

11. Let us first get some sort of an idea how much of the corona can be photographed at once. The diameter of the Moon with different focal lengths is as follows :—

Diameter in inches	8	12	16
Focal length in feet	69	104	138

Thus even with a concave of 140 feet focus we could, with a 16-inch cœlostat, photograph about half the corona at once, if we are content with light from about half the concave mirror over some portions of the image. The definition will vary in quality in different parts of the image, the amount of variation depending on the size of the concave mirror. With shorter focal lengths we get more of the corona at a single exposure; with longer focal lengths less. But as the focal length increases the angle of incidence diminishes, and the portion which is photographed will therefore be better photographed.

12. Thus with a concave of 140 feet focus and a 16-inch cœlostat we could bring one limb of the Sun central during the early part of totality and take photographs of it; then, using the slow motion of the cœlostat in R A, bring the other limb central and photograph that. Note that the cœlostat is close to the observer and he could easily manage to do this, or to superintend it, and the position for plates is not altered. The two limbs would not necessarily be either the R A or N P D limbs, being determined entirely by the direction of the minor axis of the ellipse which represents the cœlostat mirror. Portions of the limb 90° away from these would suffer most from defective definition and loss of light.

13. It seems to me possible, though I have not at the moment time to examine the point, that these portions of the corona might be brought central by tilting the cœlostat mirror in N P D—*i.e.* inclining it slightly to the polar axis. Such a tilt is of course an error in adjustment, and produces a slow change in the image, but the change might not be serious enough to matter under the circumstances of an eclipse.

14. One small point may be noticed in conclusion. If such mirrors are used, the photographs taken will not be suitable for determining the relative brightness of different parts of the corona without a troublesome calculation of the amount of light received by different parts of the image.

On Graphical Methods of determining the Local or Greenwich Time of Sunset at different places within a given region. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. A few years ago my attention was directed in two very different ways to the problem of determining by some simple method—preferably a graphical one—the time of sunrise or sunset at various places within a limited region. First, Mr. Robert Sewell, in discussing Indian eclipses, wanted a correction for reducing the time of sunset or sunrise at any place in India to the time of sunset at Lanka. Secondly, it was decided that cyclists were to be guided in lighting their lamps by the time of local sunset, and not by that of sunset at Greenwich. It was easy to devise *some* graphical method, but it was not so easy to arrive at one which seemed to be final, for new, though slight, simplifications were apt to suggest themselves. In 1902 August I had arrived at the form described below, and no modification of value has since occurred to me; but in 1903 September, during the visit of the British Association to Belfast, I found that Mr. D. E. Benson and Mr. G. Napier Clark, who had also been working at the problem for some years, had arrived by a different route at a somewhat similar solution, of which a brief description is appended.

2. The ordinary processes involve two corrections, one for longitude and one for latitude. The former is a mere addition or subtraction; the latter is made usually by spherical trigonometry, put into the form of tables perhaps. To combine the two, remark that the places where the Sun is setting at any instant lie on a great circle of the Earth, if we neglect refraction; or in a small circle parallel to it at a distance of about 33′ if we take in refraction. As the Earth rotates, this circle travels over its surface, maintaining constant its position relatively to successive meridians.

3. Thus if we take any map on which the meridians are represented by parallel lines at equal distances for equal differences of longitude, and draw the sunset-line for any moment, its position for any other moment is obtained by simply sliding the sunset-line uniformly across the map in a direction perpendicular to the meridians. Thus, in fig. 1, if AB be the sunset-line on any date at Greenwich sunset, then by sliding it a horizontal distance aA, so as to pass through Hull, we obtain the sunset-line at Hull sunset, and the distance Aa measures the elapsed time.

4. But the sunset-line will on most maps be a curved line, excepting at an equinox, when it coincides with a meridian, if refraction be neglected. (If refraction be included there will generally be some date near an equinox for which the sunset-line is straight.) We can choose the form of the map so that the

sunset-line will be straight at some one particular date, for we can set our lines of latitude on any arbitrary scale, but at other dates the lines will still be curved. As this point is important we will examine it in detail before proceeding further.

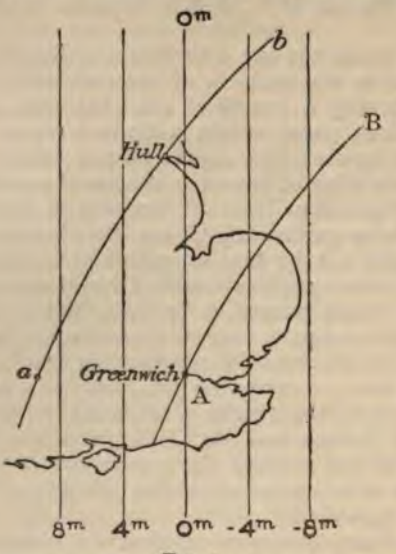


FIG. 1.

5. Neglecting refraction, the hour-angle of the Sun at setting differs from 90° by an angle

$$\theta = \sin^{-1}(\tan\phi \tan\delta)$$

where δ is the Sun's declination, and ϕ the latitude of the place. The hour-angle represents local time. If l be the longitude of the place, the G.M.T. of sunset will be

$$l+\theta$$

where l and θ are both expressed in time.

Since l increases uniformly in one coordinate, θ must increase uniformly in the other if we are to have a straight line; and thus we must lay off our latitudes so that

$$\sin^{-1}(\tan\phi \tan\delta)$$

increases uniformly. Now it is easily seen that if this is true for one value of δ (not zero) it will not be true for another.

6. But when we are dealing with a limited area—such as Great Britain, say—we can make all the lines *sensibly* straight, since δ is never greater than $23\frac{1}{2}°$. In illustration of this point let us make the lines straight when $\delta = 26° 36'$ and see what the de-

viation is when $\delta' = 14^\circ\ 5'$, which should give about the maximum error, remembering that $\delta = 0^\circ$ is always straight. These values of δ are chosen because $\tan 26^\circ\ 36' = \cdot 5$ and $\tan 14^\circ\ 5' = \cdot 25$, which makes the multiplications $\tan\phi \tan\delta$ easy. In Table I. column A gives the values of θ for $\delta = \tan^{-1} 0{\cdot}5$, and column B the values of θ for $\delta = \tan^{-1} 0{\cdot}25$. By comparing the two we find that, *approximately*,

$$B = 4^\circ{\cdot}4 + {\cdot}359\ A$$

and the values calculated from this formula are given in column C. The difference B—C never exceeds $0^\circ{\cdot}2$ or $0^m{\cdot}8$, which may perhaps be neglected for our present purpose.

Table I.

ϕ	A.	B.	C.	B–C.
°	′	°	°	°
50	36·6	17·3	17·5	−0·2
52	39·8	18·7	18·7	0·0
54	43·5	20·1	20·0	+0·1
56	47·8	21·8	21·6	+0·2
58	53·2	23·6	23·5	+0·1
60	60·0	25·7	25·9	−0·2

Thus between parallels 50° and 60° we can use straight sunset-lines without any great error if we lay off our parallels of latitude so as to make the extreme case of $\delta = 27^\circ$ straight. The error will be much less if we make a map of England alone, which extends from 50° to 56° only.

7. We have taken limits 50° to 60° of latitude in the above example. In Table II. we see that the proposition is very nearly true from *the equator* up to 60° for values which δ can actually assume. Taking $\tan\delta_1 = \frac{1}{3}$ (or $\delta = 18\frac{1}{2}^\circ$ about) as the case for actual straightness, the greatest deviations from straightness will occur when $\tan\delta_2 = \frac{1}{5}$, say, and $\tan\delta_3 = \cdot 44$ ($\delta_3 = 23\frac{1}{2}^\circ$). Column D gives the value of θ for δ_1; column E for δ_2, which is compared with ·49 D; and column F for δ_3, which is compared with 1·35 D.

Table II.

ϕ	D.	E.	E−·49 D.	F.	F−1·35 D.
°	°	°	°	°	°
0	0·0	0·0	0·0	0·0	0·0
10	3·4	1·7	0·0	4·4	−0·2
20	7·0	3·5	+0·1	9·2	−0·2
30	11·1	5·5	+0·1	14·7	−0·3
40	16·3	8·1	+0·1	21·6	−0·4
50	23·4	11·5	0·0	31·5	0·0
60	35·3	16·8	−0·5	49·6	+1·9

The extreme error in the last case is 1°·9, or nearly 8^m, which can scarcely be neglected; but it is evident that it is only near 60° that the error becomes large.

8. But now returning to a special problem, such as a sunset diagram for England, we have established this proposition.

When meridians are represented on a map by equidistant parallel lines it is possible to choose a variable scale of latitude so that the sunset-lines at all dates are straight within limits which we may neglect.

The proposition has been examined, for simplicity's sake, when refraction is neglected; but since refraction only slightly modifies the lines, it is readily seen to be also true when refraction is taken into account.

9. To make the best latitude scale for the purpose we may proceed as follows:—

Select the date for which $\tan\delta = \frac{1}{3}$ ($\delta = 18\frac{1}{2}°$) as that for which to make the line accurately straight, or some date near this. We shall get the maximum deviations from straightness at about $\delta = 9°$ (since $\delta = 0°$ is also straight) and $\delta = 23\frac{1}{2}°$, as in Table II.

Calculate the accurate times of sunset, including refraction, for each degree of latitude on this date, and let them be as follows:—

Latitude	50°	51°	52°	53° . . .
Local time of sunset	t_0	t_1	t_2	t_3 . . .

Then the space between 50° and 51° is to be proportional to $t_1 - t_0$; that between 51° and 52° to $t_2 - t_1$; and so on. This enables us to set out our latitude-lines.

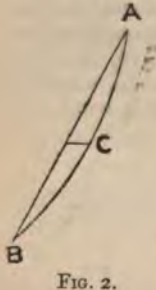


FIG. 2.

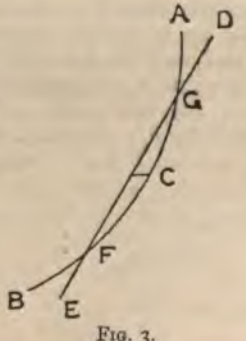


FIG. 3.

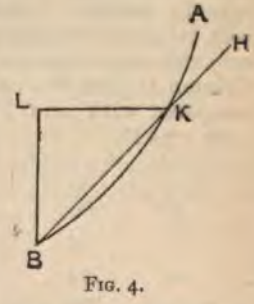
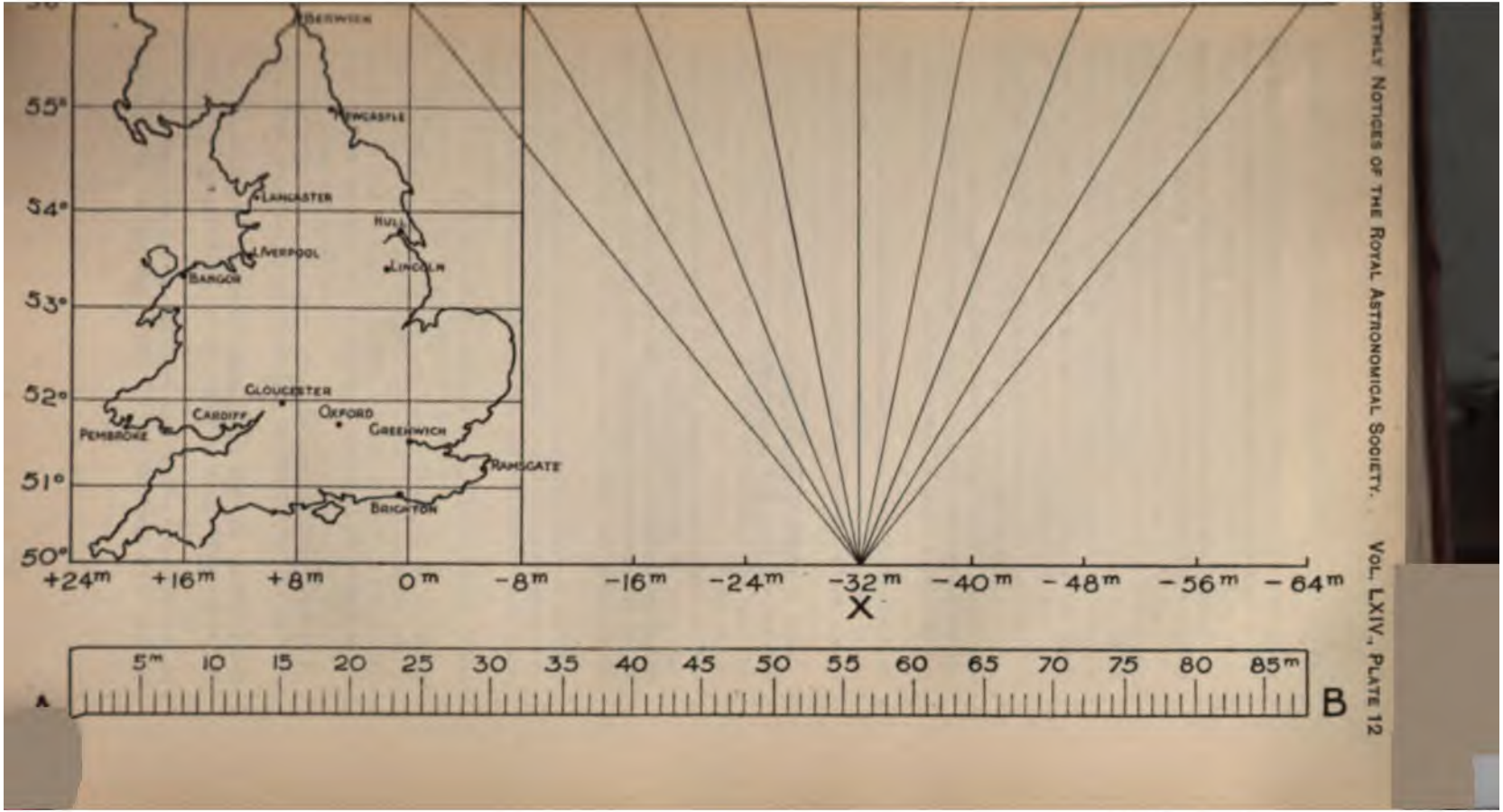


FIG. 4.

10. To find the sunset-line for any date.—We are really representing a curved line by a straight one, and can make the compromise in a variety of ways, none of them deviating much from the truth, for the straight line and the curve will cut at two points which may be varied in position. If we take these


BERWICK
NEWCASTLE
LANCASTER
HULL
LIVERPOOL
LINCOLN
BANGOR
GLOUCESTER
OXFORD
CARDIFF
GREENWICH
PEMBROKE
RAMSGATE
BRIGHTON
55°
54°
53°
52°
51°
50°
+24m +16m +8m 0m -8m -16m -24m -32m -40m -48m -56m -64m
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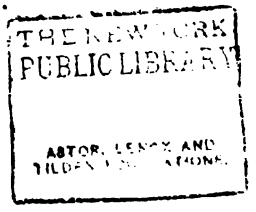
A
B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points of intersection at the top and bottom of the map, as in fig. 2, the error will be greatest in the middle, and always of one sign. It will be better to put the points of intersection, as in fig. 3, so that $DG = EF = \frac{1}{2}DE$, and then we shall distribute the error. But if we are drawing a series of lines from a fixed point B the best compromise may be BKH, as in fig. 4. The general principle of getting a good compensation will be obvious from these illustrations. In practice the rule comes to this: Select two definite latitudes, either those of F and G or those of Band K, as the case may be, and find the difference in time of sunset for these two latitudes at any date. If BL be a meridian, set off LK equal to this difference on the latitude-line through L; then BK will be the sunset-line for the date.

11. We have now considered everything but the *size* of the map, which is an important practical detail, and determines the scale of time. We may represent the G.M.T. of sunset in two ways:

(*a*) Either the map shall show the actual G.M.T. of sunset, or

(*b*) It shall show the *difference* which is to be added to (or subtracted from) the time of sunset at Greenwich, which can be given in a separate table.

12. If we adopt (*a*), then, since the time of sunset at Greenwich itself varies more than four hours, and that at Berwick more than five hours, with a map 6 inches long we can only allow 1 inch to an hour. If we adopt (*b*), the whole *difference* to be represented is only about 80 minutes, so that we can have nearly 4 inches to the hour. It is true that method (*b*) involves adding two quantities together instead of taking a single quantity from the map; but it seemed to me that the advantages of an open scale were too great to be neglected.

13. Having settled the scale of longitude, that of latitude follows generally from the consideration that the extreme sunset-lines should be inclined about 45° to the meridians—not more, and preferably less.

14. I will now give an indication of the form which I had arrived at for the map last year (see Plate 12).

Method (*b*) being adopted, it was determined to represent the difference of sunset-time *not* from that at Greenwich, which involves + and − signs, but from a point X, 32^m east of Greenwich, and in latitude 50°, so that the correction to sunset at X should always be additive.

The sunset-lines are represented by straight lines through X, the times of sunset being accurately calculated for the parallel 54°.

The scale of latitude expands as we go north, being accurately proportional to the differences of sunset-time for $\tan \delta = \frac{1}{4}$.

A certain number of the lines are drawn corresponding to dates shown at the top; those for intermediate dates can be

inferred. The other half of the year is shown on another scale of dates, which can be folded down over that represented in the figure.

Supplementing the map there must be given a table of times of sunset for the point X.

To find the time of sunset at any point P for any date, take the paper scale AB (which is a scale of minutes and should be detached from the map), lay the corner A on P, keeping the scale horizontal, and read where it cuts the date-line. This reading gives the number of minutes to be added to the time of sunset at X, which is given in the table.

15. The scheme of Mr. Benson and Mr. Clark may be indicated as follows :—

(*a*) They take Mercator's projection for the map. The latitude scale is a gradually expanding one, and happens therefore to be nearly the same as that we were led to theoretically in what precedes. Mr. Benson and Mr. Clark determined its suitability for straight sunset-lines by experiment.

(*b*) They place the reference point X in the centre of the map, which involves the use of + and — signs, but reduces the size of the map.

(*c*) The sunset-lines are not actually drawn. The dates are indicated at the top and bottom of the map, and a string fastened at X can be stretched to the date so as to form any required sunset-line.

(*d*) No separate scale AB is given. The distance of a place to right or left of the line is to be inferred from the distances between meridians. This is indeed not difficult when the point X is placed in the middle of the map, and the distances are all small.

(*e*) The table giving sunset at X is practically ranged alongside the dates given at the top and bottom of the map, in the form of an accompanying time-scale. This is in many ways convenient, and the only disadvantage is (as indicated in § 11) that the scale is rather small, as four hours are to be represented within the dimensions of the map.

Tables to facilitate the working of Combined Altitudes by Saint-Hilaire's Method. By Lieut. E. B. Simpson-Baikie, R.N.R.

Communicated by the Secretaries.

The most valuable of all methods for finding a ship's position at sea is no doubt the combined altitude method, which consists of either a combination of simultaneous observations of different heavenly bodies, or a combination of successive observations of the same one.

There are several systems of utilising such observations, according to the positions of the bodies. When a star is near the prime vertical, the hour angle and thence the longitude may be calculated from the altitude, declination, and assumed latitude; when near the meridian, the latitude may be found by the "ex-meridian" method.

In Marcq Saint-Hilaire's method the position of the observer is assumed, the altitude of the body calculated for the instant of observation from this assumed position, and compared with the true altitude.

In all methods the combination of the two or more observations is effected by drawing "lines of position" at right angles to the lines of azimuth of the observed bodies through the approximate positions resulting from each observation on a Mercator chart. These "lines of position," being short, may be considered arcs of circles of altitude of the respective bodies, and therefore geometrical "loci" for the "true position," which lies at their intersection.

To ensure accuracy the altitudes should not exceed 80°, and the angular difference (difference in azimuth) between two bodies should not be less than 25° or greater than 155°.

In Saint-Hilaire's method the "lines of position" are displaced as many miles towards (or from) the body as the number of minutes of arc that the true altitudes are greater (or less) than the calculated altitudes.

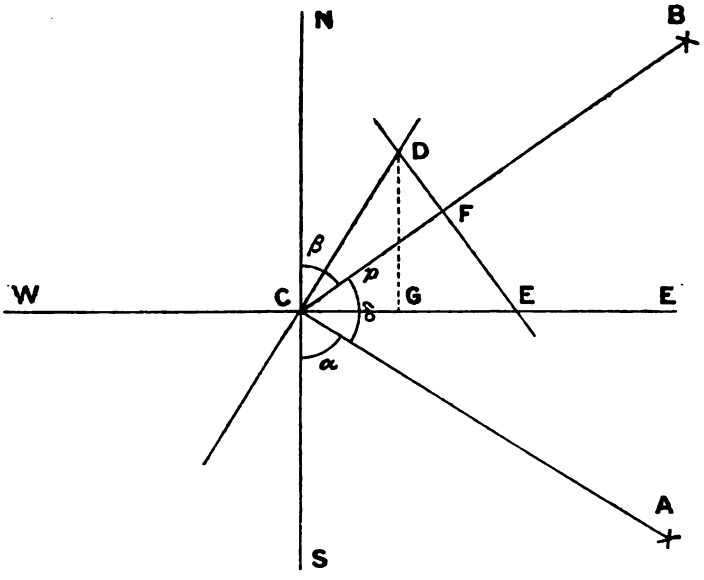
The great advantage of this method over all others is that it may be used with observations of heavenly bodies in any position in azimuth, with only the two restrictions mentioned above, which are common to all methods of combined altitudes. This advantage has been recognised in the Imperial German Navy, where Saint-Hilaire's method (*Höhenmethode*) is now taught almost to the exclusion of all others. In the Royal Navy the junior ranks are instructed in it, and it is a compulsory subject in the course for first-class navigators.

One disadvantage still exists. This is the necessity of obtaining the final result graphically from a chart or sheet of paper—a proceeding which is both clumsy and tiresome. The alternative of calculating the corrections to the assumed position is slightly confusing and somewhat lengthy.

To remove this disadvantage tables have now been computed by which the difference of latitude and difference of longitude between the intersection of the "lines of position" and the assumed position may be found and a final result obtained with few additional figures. These tables are founded on the following considerations, which are explained in "Brent, Walter, and Williams' Ex. Mer. Alt. Tables."

In fig. 1 let NS represent the meridian and WE the parallel of latitude passing through C, the "assumed position." A is one star, whose azimuth is a and whose true and calculated altitudes agree. CD is the line of position due to the observation

of A. B is another star, whose azimuth is β. Let the difference in azimuth between A and B be δ. The difference between the true and calculated altitudes of B is CF $= p$. The



line of position FE, determined by the second observation, cuts the first line of position at D, the "true position."

Now, $$DC = \frac{DG}{\sin DCG}$$

I. $\therefore DG = DC \cdot \sin DCG,$

also $$DC = FC \,.\, \sec DCF.$$

II. $$DG = FC \,.\, \sec DCF \sin DCG,$$

but, $$DCG = ACS = \alpha,$$

and $$DCF = (\alpha + \beta) - 90°$$

$\therefore \sec DCF = \operatorname{cosec} 180° - (\alpha + \beta) = \operatorname{cosec} \hat{c}.$

Therefore for II. we put: correction lat. $= \frac{p \,.\, \sin \alpha}{\sin \delta}$.

Similarly: $$CG = DC \,.\, \cos DCG$$
$$CG = FC \,.\, \sec DCF \,.\, \cos DCG$$

$\therefore$ correction departure $= \frac{p \,.\, \cos \alpha}{\sin \delta}$

And, correction longitude $= \frac{p \,.\, \cos \alpha}{\cos \text{lat} \,.\, \sin \delta}$.

The expressions $\frac{p \cdot \sin a}{\sin \delta}$ and $\frac{p \cdot \cos a}{\sin \delta}$ are so related as to make it possible, p being taken as unity, to construct a table with values of a as arguments at the head of the columns and the values of the corresponding complements of a at the foot, each line corresponding to successive values of δ. The table so constructed gives values of a from 0° to 90° for every whole degree, and values of δ from 20° to 70° for every whole degree, and from 70° to 90° for every alternate degree. When δ exceeds 90° its supplement must be used for entering the table.

To prevent error it must be remembered that the arguments for finding the corrections in latitude and longitude due to the difference between the true and calculated altitudes of a star B (see diagram) are the difference in azimuth (δ) between the two stars, and the azimuth (a) of the *other* star, A.

An additional table gives the values of difference of longitude corresponding to departure within suitable limits.

Observations of Occultations of Stars by the Moon made at the Royal Observatory, Greenwich, in the year 1903.

(*Communicated by the Astronomer-Royal.*)

Day. 1903.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
May 2	Disapp. 68 Geminorum	Great Equatorial	670	Dark	7 23 38·18	B.
June 2	,, p^3 Leonis	Merz Refractor	250	,,	9 43 47·44	C. D.
Sept. 3 (*a*)	,, B.A.C. 7063	Sheepshanks Equatorial	100	,,	7 4 29·96	W.
3	,, ,,	Astrographic Equatorial	225	,,	7 4 30·52	J. S.
Oct. 30 (*b*)	,, ρ Aquarii	,, ,,	225	,,	7 10 8·60	H.
30	,, ,,	Great Equatorial	670	,,	7 10 9 15	H. F.
30	,, ,,	Sheepshanks Equatorial	100	,,	7 10 10·08	W.
30	,, ,,	Great Equatorial (Corbett)	120	,,	7 10 9·35	V.
30	Reapp. ,,	Astrographic Equatorial	225	Bright	7 46 42·54	H.
30 (*c*)	,, ,,	Sheepshanks Equatorial	100	,,	7 46 43·39	W.
Nov. 4 (*d*) (*e*)	Disapp. ξ Arietis	Astrographic Equatorial	225	Dark	6 36 18·32	H.
7 (*f*) (*g*)	,, 111 Tauri	Sheepshanks Equatorial	100	Bright	11 9(58·52)	W.
7 (*f*) (*g*)	,, ,,	Astrographic Equatorial	225	,,	11 10 (1·59)	W. S.
9 (*b*)	Reapp. λ Geminorum	,, ,,	225	Dark	9 28 1·63	H.
9	,, ,,	Old Altazimuth	100	,,	9 28 2·76	H. F.

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation.	Observer.
1903.					h m s	
Nov. 9	Reapp. A Geminorum	Sheepshanks Equatorial	100	Dark	9 28 3·06	W. S.
Dec. 2 (*d*)	Disapp. W. B. H. 1033	,, ,,	100	,,	12 18 33·44	A. C.
2 (*d*)	,, ,,	Hodgson Telescope	165	,,	12 18 34·34	F. M.
2 (*d*)	,, ,,	Astrographic Equatorial	100	,,	12 18 33·89	W. S.
4 (*h*)	,, Bradley 686	,, ,,	225	Bright	8 52 44·20	H.
4	,, ,,	Great Equatorial	670	,,	8 52 47·24	R.
4	Reapp. ,,	Astrographic Equatorial	225	,,	9 50 49·43	H.
4	,, ,,	Great Equatorial	670	,,	9 50 50·40	R.
7 (*e*)	Disapp. B. F. 1146	Astrographic Equatorial	225	,,	13 44 39·64	J. S.
7	Reapp. ,,	,, ,,	225	Dark	14 31 47·87	J. S
10 (*i*)	Disapp. *d* Leonis	Sheepshanks Equatorial	100	Bright	13 13 50·25	W.
10	Reapp. ,,	Old Altazimuth	100	Dark	13 46 12·60	W.
31 (*b*)	Disapp. 75 Tauri	Astrographic Equatorial	225	,,	8 23 40·13	H.
31	,, ,,	Great Equatorial	670	,,	8 23 40·00	H. F.
31	,, ,,	Sheepshanks Equatorial	100	,,	8 23 39·50	W.
31 (*b*) (*j*)	Reapp. ,,	Astrographic Equatorial	225	Bright	9 6 41·34	H.
31 (*g*)	,, ,,	Sheepshanks Equatorial	100	,,	9 6(45·16)	W.

Notes.

(*a*) Not quite instantaneous. (*b*) Instantaneous.
(*c*) The observer noted "not more than 1 sec. late."
(*d*) The limb of the Moon was only slightly defective.
(*e*) The star was seen right up to the limb.
(*f*) Clouds passing. (*g*) Observation doubtful.
(*h*) The star appeared to glide up to and under the limb.
(*i*) Definition not good.
(*j*) The star appeared from a shallow indentation in the limb.

The apertures of the telescopes used are as follows :—

	inches.		inches
Great Equatorial	28	Great Equatorial (Corbett Telescope)... ...	6½
Merz Refractor	12¾	Hodgson Telescope	6
Astrographic Equatorial (Guiding Telescope)...	10	Old Altazimuth	4
Sheepshanks Equatorial	6¾		

The initals H., A. C., B., C. D., H. F., W., J. S., P. M., W. S., V., are those of Mr. Hollis, Mr. Crommelin, Mr. Bryant, Mr. Davidson, Mr. Furner, Mr. Witchell, Mr. Storey, Mr. Melotte, Mr. Stevens, and Mr. Vagg respectively.

Royal Observatory, Greenwich : 1904 *January* 5.

The "Great" Magnetic Storms, 1875 to 1903, and their association with Sun-spots, as recorded at the Royal Observatory, Greenwich. By E. Walter Maunder.

(*Communicated by the Astronomer-Royal.*)

In a paper "On the Relation between Magnetic Disturbance and the Period of Solar Spot Frequency," communicated by Mr. William Ellis to the Royal Astronomical Society in 1899, and appearing in the *Monthly Notices*, vol. lx. No. 2, pp. 142–157, Mr. Ellis has classified magnetic movement or disturbance in the following manner, Class I. including all days free from magnetic irregularity, and showing only the ordinary diurnal movement :—

Class.	Degree of Disturbance.	Greatest recorded Magnetic Movement. Declination.	Horizontal Force c.g.s. value × 10⁵.
I.	None	No magnetic irregularity in either element	
II.	Minor	Less than 10′	Less than 50
III.	Moderate	Greater than 10′ Less than 30′	Greater than 50 Less than 150
IV.	Active	Greater than 30′ Less than 60′	Greater than 150 Less than 300
V.	Great	Greater than 60′	Greater than 300

Adopting the same classification, there were nineteen "great" storms in the twenty-nine years from 1875 January 1 to 1903 December 31. The extreme amplitude of the movements in declination exceeded one degree in all these cases but two. In these two cases it exceeded 55′ : but as the movement in horizontal force exceeded 350 on the c.g.s. scale, they have also been ranked as "great" storms, in accordance with Mr. Ellis's definition. The nineteen are thus distinctly the most important "storms" of the entire period of twenty-nine years.

All the nineteen without exception took place when there was present on the Sun a group of spots with projected area of over 1000 millionths of the area of the visible disc ; *or* when a group, at one time very large, had returned in a diminished form to the central meridian.

With one exception—No. 18—all the storms began with an exceedingly characteristic movement of the magnets, generally only moderate in amount, but distinguished by its suddenness. This appeared simultaneously in the registers of declination, horizontal force, vertical force, and earth currents, the remarkable feature of the movement being its instantaneousness. This characteristic movement is referred to in Tables I. and II. by the word "sharp" in the column "Character of Beginning." Taking the time of this swift characteristic movement as that of the commencement of the storm, it would appear that these

nineteen storms did not begin indifferently when a great spot was anywhere on the Sun's disc, but that the most important group visible always lay within a certain region, the position of which, relative to the central meridian of the Sun, was as follows :—

				D.	H.
Extreme easterly position,	19° E.	; in time		1	11
„ westerly	„	47° W.	„	3	14
Mean	„	14° W.	„	1	2

The greatest of these nineteen storms were the following :—

Storms 4 *and* 5.—These occurred in a disturbed period lasting from 1882 April 13 to April 24 ; *i.e.* synchronous with the passage of Spot-group 729, which was visible April 13 to April 25.

Storm 4 was synchronous with a sudden outburst in Group 726 and a smaller outburst in Group 729.

Storm 5 with Group 729 at the normal position, *i.e.* one day past the central meridian.

Storm 7.—A very intense and long-continued disturbance, lasting in all from 1882 November 11 to November 26, synchronising with Group 885—the largest group of the cycle—in its entire passage across the visible disc, viz. from November 12 to November 25. The storm was the most violent of any from the beginning of 1875 up to the time of Storm 19.

Storm 9.—Synchronous with Group 2421, the greatest spot-group of the entire twenty-nine years.

Storm 10.—Synchronous with the return of the last-named group, the disturbance lasting during its entire passage across the disc—1892 March 4 to 17. A very long-continued rather than a violent disturbance.

Group 2421 at its return in 1892 March was numbered Group 2440.

Storm 14.—Disturbance lasted from 1894 February 20 to March 1. Synchronous with the passage across the disc of Group 3412, from February 16 to March 1.

Storm 19.—The greatest storm of the nineteen as regards the magnitude of the movements, but by no means the most long-continued.

The connection between the magnetic "storm" and a great sun-spot was especially clear in the cases of the first six storms noted above. The spots were amongst the most remarkable even of the "nineteen largest" groups, and the magnets were disturbed during nearly the whole time of the passage of the spots across the disc, the disturbance becoming a "great" storm near the time of central transit.

In five cases out of the 19 two considerable groups were on the disc, and it is therefore uncertain which should be associated with the storm. In the case of *Storms* 12 and 13 one of the two groups (Group 2583) was near the central meridian at the time of the beginning of both the storms. The other group (Group 2581)

seen during Storm 12 was not seen during Storm 13. Group 2581 is, therefore, probably to be associated rather with the "active" disturbance four days earlier, when it likewise had just passed the central meridian. This "active" disturbance is therefore associated with it in Table II.

Four points appear clear from the tables:—

1. There is a real connection between large Sun-spots and great magnetic storms. This is shown (*a*) by the fact that in every case a "great" storm was synchronous with the passage of a large spot or with its return; (*b*) by the correspondence of the very greatest storms to the time of visibility of the very greatest spots; (*c*) by the fact that there were no considerable storms in the interval between the two great spot-groups of 1898 September 3–15 and 1903 October 4–18. The appearance after an interval of more than five years of a "very large" group of spots was answered by a magnetic disturbance greater than any which had occurred in the interval.

2. There is a real, but only rough, connection between the size of the spot and the intensity of the storm. This is shown by the correspondence of seven out of the nineteen greatest storms to seven out of the nineteen greatest spots in a period of over twenty-nine years. The proportion of correspondences in so long a period is far too high to be due to accident. Of the nineteen spot-groups shown in Table II. the nine largest all synchronised with a "great" or "active" storm.

3. The area of the spot-group is not by any means an exact index of the degree or intensity of the magnetic disturbance. This is shown by the fact that the nineteen largest sun-spots corresponded in time to magnetic disturbances as follows:—

To "great"...	7
" "active"	7
" "moderate"	2
" "minor"	2
" "none"	1
	19

4. The "great" storms did not begin indifferently at any time of the passage of a great spot across the disc of the Sun, but during the period of five days beginning thirty-four hours before the centre of the spot-group reached the central meridian, and ending eighty-six hours after that time, the mean time being twenty-six hours after the spot reached the central line.

At the same time, as mentioned above, in some of the greatest storms a distinct disturbance began with the entrance of the group on the disc and lasted until its disappearance at the west limb. In these cases, however, a great storm of an altogether distinct character began with a typical sharp movement after the spot had reached the central region of the disc.

Details of the "storms" and spot-groups follow.

The "Great"

The following nineteen magnetic storms comprise *all* the storms since the enc of arc. The table therefore includes *all* the most violent storms in the last tw

Ref. No.	Magnetic Storm. Greenwich Civil Time. From			To			Duration.	Character of Beginning.	Extreme Amplitude of Movement. Dec.	H.F.	V.F.	Larg No. of Group.	Tim
		d	h		d	h	h		′				
1	1880 Aug.	12	12	Aug.	14	8	44	Sharp	65	·016	·008	343/5	A
2	1881 Jan.	31	12	Feb.	1	3	15	„	75	18	8	412	J
3	Sept.	12	12	Sept.	14	2	38	„	60	17	8	566	S
4	1882 Apr.	16	23	Apr.	17	23	24	„	60+	30+	22+	726	A
5		20	3		21	8	29	„	70+	20+	8	729	
6	Oct.	2	10	Oct.	3	3	17	„	60	14	No regis.	846	S
7	Nov.	17	10	Nov.	21	4	90	„	110	50+	·025	885	N
8	1886 Mar.	30	8	Apr.	1	3	43	„	65	20+	7	1860	M
9	1892 Feb.	13	5	Feb.	14	18	37	„	70+	29+	15+	2421	F
10	Mar.	11	22½	Mar.	13	5	30	„	75	26	6±	2440	M
11	May	18	8	May	19	8	24	„	73	16	11	2515	M
12	July	16	12½	July	17	20	32	„	75	36+	7+	2583	J
13	Aug.	12	12	Aug.	13	2	14	„	65	34	12	2615	A
14	1894 Feb.	22	22	Feb.	26	1	75	„	55	21	7+	3412	F
15	July	20	6	July	21	2	20	„	60	36	14	3624	J
16	Aug.	20	2½	Aug.	20	18	15	„	65	22+	12+	3668	A
17	1898 Mar.	15	1	Mar.	16	8	31	„	75	18+	10+	4702	M
18	Sept.	9	14	Sept.	10	11	21	Grad.	55	20+	10	4781	S
19	1903 Oct.	31	6	Nov.	1	5	23	Sharp	121	64	32	...	O

Of these nineteen storms—

7 synchronised with one of the nineteen greatest spots.
1 „ „ the return of one of the nineteen greatest spots.
9 „ „ a "large" group of spots.
2 „ „ the return of a "large" group of spots.

19

A group with a projected area of $\frac{1}{1000}$th of the visible disc of the Sun is ranked as "large."

…ns, 1875–1903.

…which the extreme amplitude of movement in declination has amounted to one degr…
…n.

…rval between and Transit	Date and Place on Disc when*				Mean Coord. of Group.		Area of Group on Day of Storm.		Total Projected Area.	Total Corrected Area.	Proportion to Mean for Year.
	First seen		Last seen		Long.	Lat.	Projected.	Corrected.			
					°	″					
11	July 20 D		Sept. 18 W		235·0	+ 24·5	1191	708	1850	1388	3·3
2	Jan. 29 D		Feb. 27 D		165·8	− 13·0	1083	581	1544	1097	1·5
12	Sept. 6 D		Oct. 5 D		100·7	+ 13·6	1238	777	2332	1770	2·4
1	Mar. 23 D		May 10 D		93·0	− 19·3	1693	880	6750	4667	4·0
6	Apr. 13 E		May 24 W		64·8	28·5	3343	1908	6809	4314	4·3
10	Sept. 25 E		Dec. 1 W		50·2	21·7	1647	1046	3561	2101	2·1
10	Oct. 20 D		Dec. 20 D		120·7	+ 19·2	4194	2352	4640	2771	2·7
14	Mar. 26 E		May 3 W		116·0	− 16·8	1008	518	1641	1028	2·7
7	Nov. 15 E		Mar. 17 W		255·7	− 28·3	5252	2909	7137	4905	4·0
9	Nov. 15 E		Mar. 17 W		250·3	− 28·4	1111	602	1313	793	0·7
14	Apr. 17 D		May 22 W		84·3	− 14·0	1055	580	1503	1195	1·0
…	Jan. 13 D		Aug. 16 D		33·1	− 31·3	1031	728	1074	1546	1·3
…	Jan. 13 D		Aug. 16 D		16·5	33·4	337	214	1304	942	0·8
…	Jan. 25 D		Mar. 1 W		186·9	− 32·0	2203	1275	3416	1941	1·5
3	Jan. …	14 E	Aug. …	19 W	76·6	12·4	700	488	2790	1793	1·4
3	Aug. …	11 E	Sept.	14 D	26·4	+ 0·4	1773	1230	2148	1466	1·1
14	Mar. 6 E		May 1 D		110·5	13·2	1514	1210	1974	1559	4·1
4	Sept. 3 E		Nov. 6 D		230·0	− 12·1	3742	2011	3750	2021	5·4
3			…		208·0	+ 22·0	1216	970	1737	1368	…

Of these nineteen storms
5 took place with two "large" groups on the disc.
3 " " a "large" group east of the C.M.
11 " " a "large" group west of the C.M.

19

Taking the mean "Interval between Storm and Transit" for the 14 cases where there was only one "large" group, the result is + 1 ^{d} 2^{h}, corresponding to a position of 14°·3 west of the C.M.

* It means that the spot formed or dissipated on the disc i.e. in the … hemisphere. E, that it was first seen at the east limb. W, that it was last seen at the west limb.

The Nineteen

These include *all* the groups of which the projected area exceeded on any d
area exceeded the 500th part of the

No. of Group.	Limiting Dates. From	To	No. of Appearance.	Total Duration of Group. Rotation.	Days.	Means for Apparition. Area.	Long.	Lat.	Day of Greatest Area Projected. Day.	Area.	Corr Day.
726	1882 Apr. 10	Apr. 23	2nd	3	49	833	93°0	−19°3	Apr. 20	2517	Apr. 2
729	Apr. 13	Apr. 25	1st	2	56±	1744	64·8	−28·5	Apr. 18	3693	Apr. 2
885	Nov. 12	Nov. 25	2nd	3	62	1711	120·7	+19·2	Nov. 18	4667	Nov. 1
1062	1883 June 25	July 7	1st	2	53±	1353	36·2	+10·5	June 30	3573	June 3
1080	July 19	July 31	3rd	4	80	1321	73·9	− 9·5	July 25	3115	July 2
1343	1884 Mar. 31	Apr. 7	1st	4	94±	1132	19·1	−10·3	Apr. 3	2486	Apr.
1706	1885 June 15	June 27	2nd	4	106±	1363	241·0	+10·6	June 21	3767	June 2
2293	1891 Aug. 29	Sept. 10	4th	8	185	1212	221·5	+21·8	Sept. 5	2846	Sept.
2421	1892 Feb. 5	Feb. 18	4th	5	138±	2402	255·7	−28·3	Feb. 11	5359	Feb. 1
2581	July 4	July 16	2nd	5	135±	1591	83·2	+11·0	July 10	3994	July
3106	1893 Aug. 2	Aug. 13	1st	2	53±	1594	292·6	−17·6	Aug. 7	4771	Aug.
3281	Nov. 16	Nov. 28	1st	3	74±	1234	322·1	− 5·1	Nov. 20	2992	Nov. 2
3412	1894 Feb. 16	Mar. 1	2nd	2	43±	1208	186·9	−32·0	Feb. 21	3038	Feb. 2
3668	Aug. 11	Aug. 23	1st	2	42±	1246	26·4	+ 6·4	Aug. 16	3741	Aug. 1
3731	Oct. 2	Oct. 14	1st	1	27±	1453	59·9	−12·4	Oct. 6	4123	Oct.
4428	1896 Sept. 10	Sept. 22	1st	1	27±	1731	55·7	+13·1	Sept. 17	4254	Sept. 1
4497	1897 Jan. 3	Jan. 15	1st	3	81±	2132	342·8	− 6·9	Jan. 8	5239	Jan.
4781	1898 Sept. 3	Sept. 15	2nd	4	82	1556	239·0	−12·1	Sept. 10	4033	Sept. 1
	1903 Oct. 4	Oct. 18	1st	2	56±	1700	201·0	−22·0	Oct. 11	3800	Oct. 1

The figures for the spot-group of 1903 October 4–18 are estimated only.

Of the nineteen spot-groups—

7 were in the Northern Hemisphere.
12 „ „ Southern „
19

Nine spot-groups had a mean corrected area for the apparition of over 1500 millionths.

Groups, 1875 1903.

part of the visible disc of the Sun—2500 millionths, or of which the corrected **phere** of the Sun—2000 millionths.

of Transit across C.M.	Total Projected.	Area Corrected.	Time of Beginning G. Civil T.	Magnetic Storm. Class.	Duration.	Character of Beginning.	Extreme Amplitude of Movement. Dec.	H.F.	V.F.	Interval between Storm & Transit
h			d h		h		′			d
7 0	6750	4667	Apr. 16 23	Great	24	Sharp	60 +	·030 +	·022 +	·0
5 3	6537	3934	Apr. 20 3	„	29	„	70 +	20 +	8	+ 1
5 20	5422	2937	Nov. 17 10	„	90	„	110	50 +	25	1 1
1 13	4111	2229	June 30 5	Moder.	47	Grad.	20	8	2	1
5 21	4548	2843	July 30 0	„	84	Sharp	25	8	3	+ 4
1 18	614	423	...	None	...	..	...	...	...	...
1 12	4483	2328	June 24 22	Active	36	Sharp	30	12	4	+ 3 1
5 15	3432	2172	Sept. 9 10	„	70	„	33	6	5	+ 5 1
22	5863	3800	Feb. 13 5	Great	37	„	70 ·	29 +	15 +	+ 1
1 6	6391	4275	July 12 18	Active	42	„	33	22	6	+ 2 1
7 20	5499	5128	Aug. 6 4	„	44	„	30	15	4	1
2 9	3840	2132	...	Minor	...	...	...	...	...	...
2 11	4101	2707	Feb. 22 22	Great	75	Sharp	55	21	7 +	·0 1
7 5	3809	2288	Aug. 20 21	„	15	„	65	22 ·	12 +	·2 2
1 3	4667	2587	...	Minor	...	...	...	...	...	...
1 17	4110	2021	Sept. 18 2	Active	22	Grad.	33	14	3	·1
1 10	4739	2441	Jan. 2 13	„	14	„	39	4	4	6 2
1 6	3759	2021	Sept. 9 14	Great	21	„	55	20 ·	10	·0
25	4800	2800	Oct. 12 18	Active	10	Sharp	35	...	...	+ 1

Mean + 0 2

Of these 4 were synchronous with "great" storms.

„ „ 5 „ „ "active" „

9

Ten spot groups had a mean corrected area for the apparition of over 800 millionths and less than 1500 millionths.

Of these 3 were synchronous with "great" disturbances.

„ „ 2 „ „ „ "active" „

„ „ 2 „ „ „ "moderate" „

„ „ 2 „ „ „ "minor" „

„ „ 1 „ „ „ no disturbance.

10

Storm No. 1, 1880 *August* 12–14.—Preceded by disturbance from August 11^d 12^h–August 12^d 6^h. Extreme amplitude in Dec.=23′. Followed by very quiet period.

Sun-spots on 1880 *August* $12^d{\cdot}485$.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
342	321	459	324·2	+12·5	+70·4
343	795	444	234·5	+24·6	−19·3
344	211	131	225·6	−16·9	−28·2
345	396	264	213·5	+21·0	−40·3
346	127	90	216·0	−18·9	−37·8
Totals	1850	1388			

These five groups should probably be reckoned as three, since in the following rotation Nos. 343 and 345 form one almost continuous stream, and 344 and 346 another.

There were thus two important groups on the Sun at the time of the storm :—

Nos. 343–345, with total projected area 1191 and total corrected 708. This was its greatest appearance. It formed on the disc July 20, and was last seen at the west limb September 18.

Nos. 344–346, with total projected area 338, and total corrected 221. In the next rotation it was six times as large ; a very great group, with a mean corrected area for the time of appearance, September 4–16, of 1314. It was first seen, at the east limb, August 9, and last seen, at the west limb, October 14.

Storm No. 2, 1881 *January* 31–*February* 1.—Preceded by a moderate movement from January 30^d 23^h–January 31^d 7^h. Extreme amplitude in Dec.=16′. Followed by very quiet period.

Sun-spots on 1881 *January* $31^d{\cdot}452$.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
411	202	360	168·8	+15·2	+22·6
412	1083	581	165·9	−12·9	+19·7
413	96	56	116·2	−16·6	−30·0
414	13	8	107·1	−14·5	−39·1
415	150	92	70·5	+18·8	−75·7
Totals	1544	1097			

Only one of these groups was seen during any other rotation.

No. 412 formed on the disc on January 29 ; passed off at the west limb, February 4 ; reappeared, east limb, on February 20 ; disappeared on the disc, February 27. January 31 was the day of its greatest area. Its rise, therefore, was very rapid.

Storm No. 3, 1881 *September* 12–14.—Preceded by two moderate movements, one from September $8^d\ 22^h$ to September $9^d\ 3^h$; extreme amplitude in Dec.$=15'$; the other from September $9^d\ 21^h$–September $10^d\ 3^h$; extreme amplitude in Dec.$=14'$. Followed by an active movement, September $14^d\ 13^h$–September $15^d\ 1^h$; extreme amplitude in Dec.$=30'$.

Sun-spots on 1881 *September* 13^d·583.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
566	1238	777	101°·6	+13°·7	+49°·4
568	342	216	88·4	− 3·3	+36·2
569	23	13	27·4	+17·2	−24·8
570	51	30	21·5	+19·6	−30·7
571	228	224	351·9	+17·0	−60·3
572	421	473	348·0	+22·1	−64·2
573	29	37	344·2	+17·8	−68·0
Totals	2332	1770			

Of these groups none were seen in the previous rotation, and only two in the following, viz. Nos. 566 and 572. No. 566 formed on the disc, September 6, and disappeared likewise on the disc, October 5. No. 572 was not nearly so large a group as No. 566, and was first seen, September 12, at the east limb, disappearing on the disc, October 14.

Storms Nos. 4 and 5, 1882 *April* 17 and *April* 20–21. These two storms occurred in a disturbed period. Its history is as follows:—

April $13^d\ 23^h$–$14^d\ 7^h$. Fluctuations in Dec. ($\pm 3'$), with wave at commencement ($-10'$), and wave at $14^d\ 4^h$ ($+10'$).

April $14^d\ 12^h$–$15^d\ 1^h$. Fluctuations in Dec. ($\pm 3'$) with wave at $18\frac{1}{2}^h$ ($-8'$).

April $15^d\ 23^h$–$16^d\ 4^h$. Fluctuations in Dec. ($\pm 3'$).

April $16^d\ 13\frac{1}{2}^h$–19^h. Fluctuations in H.F. ($\pm$·0015).

April $16^d\ 23\frac{1}{2}^h$. Storm No. 4 begins.

April $17^d\ 23^h$. Storm No. 4 ends.

April $18^d\ 20\frac{1}{2}^h$. Sharp wave in Dec. ($-6'$).

April $20^d\ 3^h$. Storm No. 5 begins. Oscillations sometimes 14 or 15 an hour.

April $20^d\ 12^h$. Very rapid oscillations end.

April $21^d\ 8^h$. Storm No. 5 ends.

April $21^d\ 22^h$. Wave in Dec. ($+8'$).

April $23^d\ 20\frac{1}{2}^h$–$24^d\ 9^h$. Fluctuations in Dec. ($\pm 5'$).

Sun-spots on 1882 *April* $17^{d}\cdot587$.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
			°	°	°
722	115	285	160·3	+21·1	+75·2
728	970	940	144·0	−17·8	+58·9
726	1693	880	92·5	−18·3	+ 7·4
732	35	19	72·2	−21·9	−12·9
729	3388	1963	65·3	−28·7	−19·8
733	317	177	60·4	−13·8	−24·7
734	160	200	20·6	+10·9	−64·5
735	72	203	4·4	−11·6	−80·7
Totals	6750	4667			

Sun-spots on 1882 *April* $21^{d}\cdot409$.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
			°	°	°
731	28	70	113·9	−18·7	+79·3
726	2402	2258	92·3	−17·5	+57·7
729	3396	2109	64·0	−28·7	+29·4
733	221	125	62·2	−13·2	+27·6
734	444	237	22·7	+11·4	−11·9
735	339	193	6·6	−10·2	−28·0
738	68	124	321·3	+ 7·1	−73·3
Totals	6898	5116			

Only three of these groups were considerable.

No. 726 was seen in the preceding and also in the following rotation. It formed and dissipated in the visible hemisphere, being first seen March 23, and last seen May 10. It appeared at the east limb April 10, and underwent a sudden and great development on April 17, increasing from an area of 156 on April 16 to an area of 1371 on April 18. This was synchronous with Storm No. 4.

No. 729 came on at the east limb on April 13 with the first slight fluctuations of the magnetic needle, and passed the C.M. on April 19. It passed off at the west limb on April 25, returned in the next rotation, and was last seen at the west limb, May 24.

No. 728 developed from an area of 769 on April 16 to an area of 940 on April 17, and diminished afterwards. This increase seems to have been coincident with the much greater increase of No. 726. It was first seen, April 11, on the disc, and last seen, May 5, also on the disc.

All three groups were at their greatest development during the present apparition.

Storm No. 6, 1882 October 2–3.—Preceded by a quiet time. Followed by fluctuations, October 4^d 16^h–23^h, in Dec. ($\pm 5'$), and by an "active" disturbance, October 5^d 18^h–October 6^d 16^h (Dec. $\pm 16'$). Fluctuations in Dec. ($\pm 5'$) from October 8^d $19\frac{1}{2}^h$ to October 9^d 1^h and October 9^d $19\frac{1}{2}^h$ to October 10^d 2^h. October 10^d $18\frac{1}{2}^h$–$20\frac{1}{2}^h$; wave in Dec. ($-20'$).

Sun-spots on 1882 October 2^d·560.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
845	16	11	66°·3	+21°·0	+42°·1
846	1647	1046	50·0	−21·7	+25·8
847	119	64	40·4	+20·3	+16·2
848	911	534	34·6	−22·9	+10·4
849	823	418	32·7	+ 8·0	+ 8·5
850	45	28	348·3	+ 6·6	−35·9
Totals	3561	2101			

Three of the groups were considerable, and two of them, Nos. 846 and 848, were in the same latitude, and only about 12° apart in longitude. These, therefore, probably formed only a single disturbance. In the following rotation the two groups appear to have coalesced. This double group, 846+848, was by far the most important visible on October 2. It was seen during three rotations, being first seen, September 25, at the east limb, and last seen, December 1, at the west limb. It attained its greatest area during the present apparition.

Storm No. 7, 1882 November 17–21.—A very long-continued disturbance, beginning about November 11^d 21^h and lasting until November 26^d 22^h. Characteristic sharp swings occurred November 14^d 14^h, November 15^d 8^h, November 16^d 8^h, November 17^d 10^h, November 19^d 13^h, November 25^d $16\frac{1}{2}^h$. The great storm lasted November 17^d 10^h–November 21^d 4^h; but from November 11^d 21^h–November 13^d 23^h was an "active" disturbance, of extreme amplitude 50′ in Dec. Two other "active" disturbances followed, the one November 21^d 16^h–24^h; amplitude 40′ in Dec.; the other November 25^d $16\frac{1}{2}^h$–November 26^d 3^h; amplitude 37′.

Sun-spots on 1882 November 17^d·277.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
877	42	100	216°·9	−24°·4	+75°·6
887	56	35	173·3	+19·2	+32·0
884	86	44	148·1	− 7·8	+ 6·8
885	4194	2352	120·1	+19·1	−21·2
888	124	104	88·5	− 6·3	−52·8
889	137	136	81·5	+10·9	−59·8
Totals	4639	2771			

There was only one considerable group, and that one, No. 885, was the largest in the whole cycle, 1878 to 1889, as Storm No. 7 was the most violent magnetic storm in the same period.

This group formed on the disc, October 20; passed off at the west limb, October 28; returned to the east limb, November 12; passed off at the west limb, November 25; returned to the east limb, December 10; disappeared on the disc, December 20. Its mean area during the apparition November 12–25, the period of its greatest development, was 1711.

Storm No. 8, 1886 March 30 to April 1.—Preceding the storm there was a double wave in Dec. (+8′ to −8′), March 29^d $0\frac{3}{4}^h$–$3\frac{1}{2}^h$. There were also fluctuations of about ±3′ in Dec. every day from March 15 to March 28.

The storm was followed by fluctuations in Dec. (±4′), April 1^d 6^h–11^h, and a wave (−10′) $18\frac{1}{2}^h$–19^h.

The storm itself was divided into two short but sharp disturbances, the one March 30^d 8^h–March 31^d 5^h, the other March 31^d 11^h–April 1^d 3^h. In both the oscillations attained a frequency of 20 per hour.

Sun-spots on 1886 *March* 30^d·232.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
1856	57	172	200·2	+ 5·5	+76·3
1859	525	273	136·5	− 1·2	+12·6
1860	1008	518	115·3	−16·8	− 8·6
1861	34	47	53·7	−11·9	−70·2
1862	14	18	56·1	− 8·7	−67·8
Totals	1638	1028			

Three of these groups were seen in other rotations, viz. Nos. 1856, 1859, and 1860. Of these, Group No. 1860 was the most considerable. It was first seen, March 26, at the east limb, and last seen, May 3, at the west limb. Its mean area during the present apparition, in which it attained its greatest development, was 487.

Storm No. 9, 1892 February 13–14.—Preceded by small daily fluctuations (±3′), with occasional waves of 7′ to 10′ in Dec. from January 28 to February 12.

Followed by a moderate disturbance (17′) February 15^d 13^h–24^h; and by small daily fluctuations until February 20^d 18^h–February 21^d 4^h, when another moderate disturbance took place (24′). This began with a characteristic sharp movement.

Sun-spots on 1892 February 13^{d}·408.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
			°	°	°
2421	5252	2999	255·4	−28·6	+19·6
2425	1023	529	239·5	−19·4	+ 3·7
2426	134	100	193·2	+15·9	−42·6
2427	203	191	180·2	+ 9·6	−55·6
2428	273	374	169·5	+12·8	−66·3
2429	253	712	158·4	+15·4	−77·4
Totals	7138	4905			

Of these six groups No. 2421 was much the largest. It was first seen on the east limb on 1891 November 15. The present appearance was therefore during its fourth rotation. It appeared on the east limb on February 5, passed off at the west limb on February 18, and its mean area for the apparition was 2402, the largest group in the eleven-year cycle 1879–1901.

Storm No. 10, 1892 March 11–13.—A long-continued but not very intense disturbance preceded the storm. It commenced February 29^{d} 21^{h} and continued until March 9^{d} 5^{h}. Extreme amplitude 45′ in Dec.

After the storm there were fluctuations in Dec. (±3′) March 15^{d} 20^{h}–March 16^{d} 3^{h}, with wave (−15′) March 15^{d} 22^{h}–16^{d} 0½^{h}.

Sun-spots on 1892 March 11^{d}·152.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
			°	°	°
2443	68	36	260·6	−15·7	+17·0
2440	1111	602	250·5	−28·5	+ 6·9
2442	103	110	181·1	−23·4	−62·5
2444	31	45	173·0	−26·0	−70·6
Totals	1313	793			

The only important group was No. 2440, the return of group No. 2421, the premier group of the cycle. No. 2440 was seen on the east limb, March 4, and reached the west limb, March 17. Its mean area was 386. It did not return.

Storm No. 11, 1892 May 18–19.—Preceded by moderate fluctuations and waves from May 16^{d} 22^{h}–May 17^{d} 24^{h}. Extreme amplitude in Dec. 20′. Followed by small sharp movements from May 19^{d} 15^{h}–May 20^{d} 8^{h} (±3′).

Sun-spots on 1892 *May* $18^{h}{\cdot}439$.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
2515	1055	589	84·4	−14·2	+22·6
2519	284	149	50·4	+10·0	−11·4
2522	14	7	41·9	+ 6·2	−19·9
2523	4	4	7·1	−25·5	−54·7
2524	147	446	342·1	+14·9	+79·7
Totals	1504	1195			

Three of these were considerable groups.

Group No. 2515 was the largest of the three, with a mean area of 742 for the rotation, and had passed the central meridian about a day and a half at the time of the storm. No. 2524 was actually on the east limb at the time. No. 2519, the smallest of the three, was one day short of reaching the central meridian.

No. 2515 formed on the disc, April 17; made its second appearance, May 10, at the east limb; and disappeared, May 22, at the west limb. No. 2519 appeared first at the east limb, May 14, and disappeared on the disc, June 17. No. 2524 formed on the disc, April 23, and was last seen, June 26, at the west limb.

Storm No. 12, 1892 *July* 16–17.—Preceded by "active" disturbances: one from July $12^{d}\ 18^{h}$–July $13^{h}\ 12^{d}$, the other from July $13^{d}\ 18^{h}$–July $14^{d}\ 11^{h}$. Followed by small rapid fluctuations in Dec. July $21^{d}\ 0^{h}$–8^{h}, with wave $2\frac{1}{2}^{h}$–$4\frac{1}{2}^{h}$ ($+20'$).

Sun-spots on 1892 *July* $16^{h}{\cdot}240$.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
2581	177	393	81·1	+10·4	+77·5
2583	1031	728	31·7	−31·3	+28·1
2584	183	108	25·7	+28·4	+22·1
2586	283	317	301·6	− 9·5	−62·0
Totals	1674	1546			

All four groups were considerable, and all were seen during more than one rotation.

Of the four groups No. 2581 was much the largest and longest lived, but it was passing off at the west limb when the storm began. It attained its greatest area, 1591, during this apparition.

No. 2583 was the second in size, and was at its largest during this apparition. Mean area 829.

It seems doubtful which group should be associated with the spot, probably No. 2583. The groups were first seen:—No. 2581, June 7, east limb; No. 2583, June 13, on disc; No. 2584, June 12, on disc; and No. 2586, July 15, east limb. They were

last seen :—No. 2581, Oct. 5, west limb ; No. 2583, August 16, on disc ; No. 2584, July 20, west limb ; and No. 2586, August 22, west limb.

Storm No. 13, 1892 *August* 12–13.—Preceded by two "moderate" disturbances, viz. August 3^d 14^h–23^h, and August 4^d $14\frac{1}{2}^h$–August 5^d 1^h. A quiet period followed the storm.

Sun-spots on 1892 *Aug.* 12^d·280.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
			°	°	°
2615	337	214	15·4	−30·5	+ 9·4
2620	144	73	11·4	+14·5	+ 5·4
2621	212	112	13·7	+25·0	+ 7·7
2617	127	66	11·3	− 8·4	+ 5·3
2618	106	57	354·0	−14·2	−12·0
2622	367	299	313·3	+10·4	− 52·7
2623	104	121	302·8	− 7·4	−63·2
Totals	1397	942			

None of the groups were large, but two were returns of groups seen on the disc during Storm No. 12, viz. No. 2615, a return of No. 2583 ; and No. 2623, a return of No. 2586. This is important, as it renders it probable that the other two groups seen during Storm No. 12, viz. Nos. 2581 and 2584, were not connected with that disturbance. The group No. 2615, the return of No. 2583, appears the most likely group on this occasion. Its mean area during the rotation when it attained its greatest development was 829 millionths, July 7–19 ; whilst No. 2622, the next most important group, was only 426 during the rotation Sept. 7–18.

Storm No. 14, 1894 *February* 22–26.—Preceded by "active" disturbance, amplitude 36′ in Dec., beginning with characteristic sharp movement at February 20^d $20\frac{1}{2}^h$ and lasting till February 22^d 6^h. Followed by "active" disturbance, amplitude 50′ in Dec., beginning with characteristic sharp movement at February 23^d 15^h, and lasting till March 1^d 3^h. Minor fluctuations followed till March 2^d 20^h.

Sun-spots on 1894 *February* 23^d·210.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
			°	°	°
3418	94	53	200·3	−26·4	+21·8
3415	2	1	193·5	−27·7	+15·0
3412	2263	1275	187·5	−31·8	+ 9·0
3420	241	123	174·5	−15·7	− 4·0
3413	366	214	152·7	−25·8	−25·8
3416	155	90	151·0	+ 3·7	−27·5
3417	295	185	142·1	−16·1	−36·4
Totals	3416	1941			

Group No. 3412 was the only large one. It formed on the disc, January 25, passed off at the west limb, January 29; reappeared at the east limb, February 16; and passed off at the west limb, March 1. It did not return again. Mean area during the present rotation 1208, its greatest development.

Storm No. 15, 1894 *July* 20–21. Preceded and followed merely by the small fluctuations common throughout the year.

Sun-spots on 1894 *July* 20^{d}·176.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
3624	700	488	77·0	−12·5	+40·5
3628	25	14	64·6	−11·5	+28·1
3629	559	299	57·3	+ 6·9	+20·8
3630	542	298	53·7	−11·2	+17·2
3635	14	7	44·5	−16·6	+ 8·0
3637	542	351	357·3	+12·8	−39·2
3638	403	326	344·7	+14·7	−51·8
3640	5	10	320·5	−10·1	−76·0
Totals	2790	1793			

Groups No. 3624 and No. 3637 were the largest and longest lived, both having passed their greatest area during an earlier rotation. Of the two No. 3624 was not only the larger now, but was much the larger at greatest development, and was now in an earlier phase of its decline. It was first seen, June 14, at the east limb, and last seen, August 19, at the west limb. It attained its greatest area during the previous apparition, 1894 June 14–25.

Storm No. 16, 1894 *August* 20.—Preceded and followed merely by the small fluctuations common throughout the year.

Sun-spots on 1894 *August* 20^{d}·398.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Longitude from C.M.
			°	°	°
3681	5	5	41·7	+ 5·2	+58·1
3668	1773	1239	28·4	+ 6·9	+44·8
3682	6	4	19·1	−17·6	+35·5
3683	17	8	349·8	+ 7·1	+ 6·2
3672	24	13	346·9	−19·3	+ 3·3
3674	162	83	336·3	− 5·8	− 7·3
3675	25	17	321·0	−26·7	−22·6
3678	29	21	299·7	−12·5	−43·9
3679	106	76	297·7	+17·8	−45·9
Totals	2147	1466			

Only three of these were considerable or long-lived—viz. Nos. 3668, 3674, and 3679; and of these Group No. 3668 was by far the most important. It was first seen, August 11, at the east limb, and last seen, September 14, on the disc. It attained its greatest area during the present apparition, 1894 August 11–23.

Storm No. 17, 1898 *March* 15–16.—Preceded by an "active" disturbance, March 11^d 14^h–March 12^d 6^h; amplitude in Dec. = 30′. The movements, however, were slow and not numerous.

The maximum range of movement in the storm itself was over 60′, thus ranking it as a "great" storm; but the storm was not a violent one in other characteristics, the oscillations being comparatively few and slow. Only the fluctuations ordinary to the year followed.

Sun-spots on 1898 *March* 15^d·372.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Longitude from C.M.
			°	°	°
4702	1514	1256	122·7	−13·3	+53·2
4703	131	102	116·3	+11·0	+46·8
4704	322	197	104·4	− 8·2	+34·9
4706	7	4	94·2	− 3·7	+24·7
Totals	1974	1559			

Of these groups No. 4702 was much the largest. No. 4706 was very small and short-lived, and the other two groups were only about one-fourth or one-fifth the area of No. 4702, even when they were largest. Group No. 4702 was first seen, March 6, at the east limb, and last seen, May 1, on the disc. It attained its greatest area during the present apparition, 1898 March 6–17.

Storm No. 18, 1898 *September* 9–10.—Preceded by two successive waves in Dec. (−7′ and −6′) on September 8^d 16^h–20^h.

The storm scarcely amounted to a "great" one. The extreme amplitude in Dec. was not quite a degree, and the oscillations were not very frequent.

Followed by movements lasting until September 11^d 12^h, about 18′ in Dec.

Sun-spots on 1898 *September* 9^d·624.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Longitude from C.M.
			°	°	°
4782	10	6	248·4	− 7·2	+15·3
4781	3742	2011	238·1	−11·7	+ 5·0
4783	7	4	230·9	−18·4	− 2·2
Totals	3759	2021			

The only important group was No. 4781, the other two being very small and seen only on the one day.

No. 4781 was seen during three rotations—viz. September 3–15, September 30–October 12, and October 28–November 6. Its first appearance was at the east limb on September 3; its last appearance, on the disc, on November 6. It attained its greatest area during the present apparition, 1898 September 3–15.

Storm No. 19, 1903 *October* 31–November 1.—Preceded by a wave in Dec. and H.F. at October $30^d\ 21^h$, and then by small movements up to the time of the characteristic instantaneous movement.

Sun-spots on 1903 *October* 31^d·426.

Letter for Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Longitude from C.M.
			°	°	°
a.	1216	670	296·0	−20·0	− 5·8
b.	520	698	232·0	+17·5	−69·8
Totals	1736	1368			

Group *a* appeared at the east limb on October 25, and reached the west limb November 6. The measures for October 31 are approximate only; not definitive.

Further Note on the "Great" Magnetic Storms, 1875–1903, and their association with Sun-spots. By E. Walter Maunder.

The facts brought out in the preceding paper are, so far as they go, in entire agreement with the facts tabulated in the important paper by the Rev. W. Sidgreaves read before the Society 1900 December 14. The difficulty is in their interpretation. It has been urged that if there be a real direct connexion between a particular great sun-spot and a great magnetic storm, then that connexion should always show itself. Every great spot on the Sun should be answered by a great magnetic storm here; every great magnetic storm should be synchronous with the appearance of some great spot; and, further, the violence of the storm should bear a definite relation to the extent of the spot. This relation does indeed hold good if we consider only the average of a number of instances; it does not hold good universally in every individual case.

An example is in the immediate recollection of everyone. The sun-spot which crossed the central meridian on 1903 October 12 was one of the six or seven largest groups seen during thirty years. The magnetic storm which occurred at this

time was an "active" one, not a "great" one. The sun-spot of October 31 was a large one, but not of extraordinary dimensions—less than one-third the size of that of October 12. Synchronous with its passage over the central meridian was the greatest magnetic storm that has been recorded for thirty years—perhaps the greatest that has yet been recorded at Greenwich. The natural expectation would have been that the reverse should have been the case, that the greater spot should have corresponded to the greater storm, and the smaller to the feebler one.

Yet such an expectation, however natural, involves a number of assumptions, for some of which we have very little warrant. It assumes, for example, that the efficiency of a spot is greatest at the moment of its attaining its greatest area. More important still, it assumes that its influence is equally great in all directions; that there is nothing like *direction* in the forces or emanations which proceed from that disturbed region of the Sun of which the spot is the visible indication. Yet this assumption is founded upon no sufficient evidence, even if it be not a wholly gratuitous one.

If we consider the solar corona we recognise at once that it is not symmetrically distributed round the Sun, evenly thinning out to exactly the same extent in all directions. On the contrary it is highly structured. Whatever conception we form of its nature, we are obliged to consider the lines which compose it as essentially lines of force: they indicate regions where action is greater than in the neighbouring darker areas. The eclipses of the last eight years have been specially instructive on this point. The corona of 1896, as photographed both by Sir George Baden-Powell's expedition to Nova Zembla and by MM. Kostinsky and Hansky of the Russian expedition, showed one great lobe whose outlines curved together till they united to form one long straight ray. In the eclipse of 1898 no fewer than four such synclinal regions were seen, one of which was triple, all of which terminated in straight rays of prodigious length. The longest was actually photographed for a distance from the Sun's centre which could not have been less than five millions of miles. In the two following eclipses, 1900 and 1901, the same features were perceived, though the long rays were not photographed to the same distance as in 1898; and there is no reason to doubt that they are features of every eclipse. That these extensions were not photographed before 1896 is no doubt due simply to the circumstance that a sufficient exposure and extent of field was not employed in order to secure them.

If we suppose that the effect of a solar disturbance travels outwards in somewhat the same manner as these long coronal rays—in other words, that the effect is greatest in some one direction which need not be truly radial, since the great coronal rays are not necessarily so—this will remove the ground of the difficulty before us. The intensity of any magnetic storm due to a solar disturbance would then depend upon two factors:

first, the actual magnitude of the disturbance itself, and next, upon the distance of the Earth from the direction of maximum effect. We should find, as we actually do, that when the average was taken of a large number of cases the frequency of magnetic storms and their intensity would correspond to the size of the solar spots; but at the same time we should also find, as we do, that there would be a wide margin of irregularity in special instances. It is in perfect accord with this suggestion that we actually find at the moment of commencement of the nineteen great storms examined that the most important spot on the Sun was always found within a restricted area on the surface. If the influence of the spot were exactly equal over the whole sphere of which it was the centre it is difficult to understand why this relation should have shown itself.

In the foregoing remarks I have confined myself entirely to the spots. We have at present no sufficient material for a similar discussion in the case of faculæ, prominences, or flocculi. In the ordinary way we see prominences only round the limb, faculæ only near it; of flocculi we have not yet enough observations; spots, on the other hand, we see wherever they exist in any part of the hemisphere turned towards us, and our knowledge of them may be said to be fairly complete. Further, the four different orders of phenomena are not independent, but interdependent; and concerning the first three we know that they go through their variations in the course of a solar cycle in substantial accord. At the present time, whatever may be the case in the future, spots are the most easily observed and most fully observed of all the various phenomena which can afford us any index of the solar activity.

Suggested Connection between Sun-spot Activity and the Secular Change in Magnetic Declination. By Mrs. Walter Maunder.

(*Communicated by E. Walter Maunder.*)

A connexion between the Sun and terrestrial magnetism has been recognised as existing in the following relations :—

(1) Daily.—The diurnal range of magnetic declination, dip, and intensity, according to the hours of local mean time, when by means of the Earth's rotation different parts of its surface are exposed to the action of the Sun's rays.

(2) Yearly.—The annual variation in the amount of the diurnal range corresponding to the variation in the presentation of any particular locality to the Sun's rays in the course of the year.

(3) Cyclical: "Eleven-year" Period.—Variation in the

amount of the diurnal range synchronous with the variation of the spotted area of the Sun.

In terrestrial magnetism this cycle is shown in the variation of the diurnal range, both of magnetic variation, dip, and intensity, and is evidenced, moreover, both (*a*) by the frequency of storms and (*b*) by the variation of the diurnal range when cleared of storms.*

On the Sun the "eleven-year" cycle is shown by sun-spots, faculæ, prominences, and corona, and in the case of sun-spots is evidenced both (*a*) by the frequency of giant spots alone and (*b*) by the variation in the spotted area of the Sun when cleared of giant spots.

These three periodic variations in the Earth's magnetism, which are thus known to vary in sympathy with the Sun, form, according to Mr. L. A. Bauer, but a small part of the whole magnetic force of the Earth—less than 5 per cent.—and they are generally ascribed to electric currents in the upper regions of the atmosphere. At least 95 per cent. of the Earth's magnetism is to be referred to causes within the crust, largely to a system of electric currents imbedded deep within the interior of the Earth. In particular the secular change in the three elements of terrestrial magnetism (declination, dip, and intensity) should probably be referred largely to the effect of secondary electric currents generated within the Earth by its rotation round an axis not coincident with its magnetic axis.

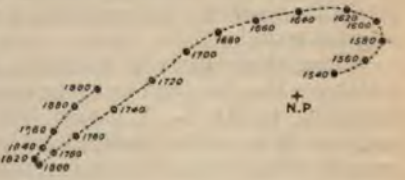
It being thus the case that the secular change in the Earth's magnetism seems primarily to be due to internal causes, it is all the more interesting to note that certain critical changes in the Earth's magnetism appear to have synchronised with certain critical changes in the activity of the Sun.

The following tables show the secular change in the magnetic declination at London (long. 0°., lat. 51½°) for every twenty years from 1540 to 1900, and for Baltimore (long. 76½° W., lat. 39½° N.) from 1640 to 1900 :—

Year.	London.	Baltimore.	Year.	London.	Baltimore.
	°	°		°	°
1540	7·2 E.	...	1740	15·3 W.	3·2 W.
1560	9·6 E.	...	1760	19·6 W.	2·0 W.
1580	10·9 E.	...	1780	22·7 W.	1·0 W.
1600	10·1 E.	...	1800	24·1 W.	0·7 W.
1620	7·3 E.	...	1820	24·1 W.	0·9 W.
1640	3·3 E.	5·3 W.	1840	23·2 W.	1·8 W.
1660	0·6 W.	6·0 W.	1860	21·6 W.	3·0 W.
1680	3·9 W.	6·1 W.	1880	18·7 W.	4·3 W.
1700	7·1 W.	5·5 W.	1900	16·5 W.	5·4 W.
1720	11·0 W.	4·5 W.			

* "On the Relation between Magnetic Disturbance and the Period of Solar Spot Frequency," by Mr. W. Ellis, F.R.S., *Monthly Notices*, vol. lx. pp. 142–157.

In the table for London it will be noted that there is a maximum declination E. about 1580, zero declination about 1660, maximum declination W. about 1810, since which date it has been returning eastward. For Baltimore there was a maximum declination W. about 1680, a minimum declination about 1800, since which date it has been again increasing westward. The diagram (fig. 1) will perhaps show more clearly the path of the point of intersection of the two magnetic meridians. The curve for the period between 1540 and 1640 is drawn



+ Baltimore

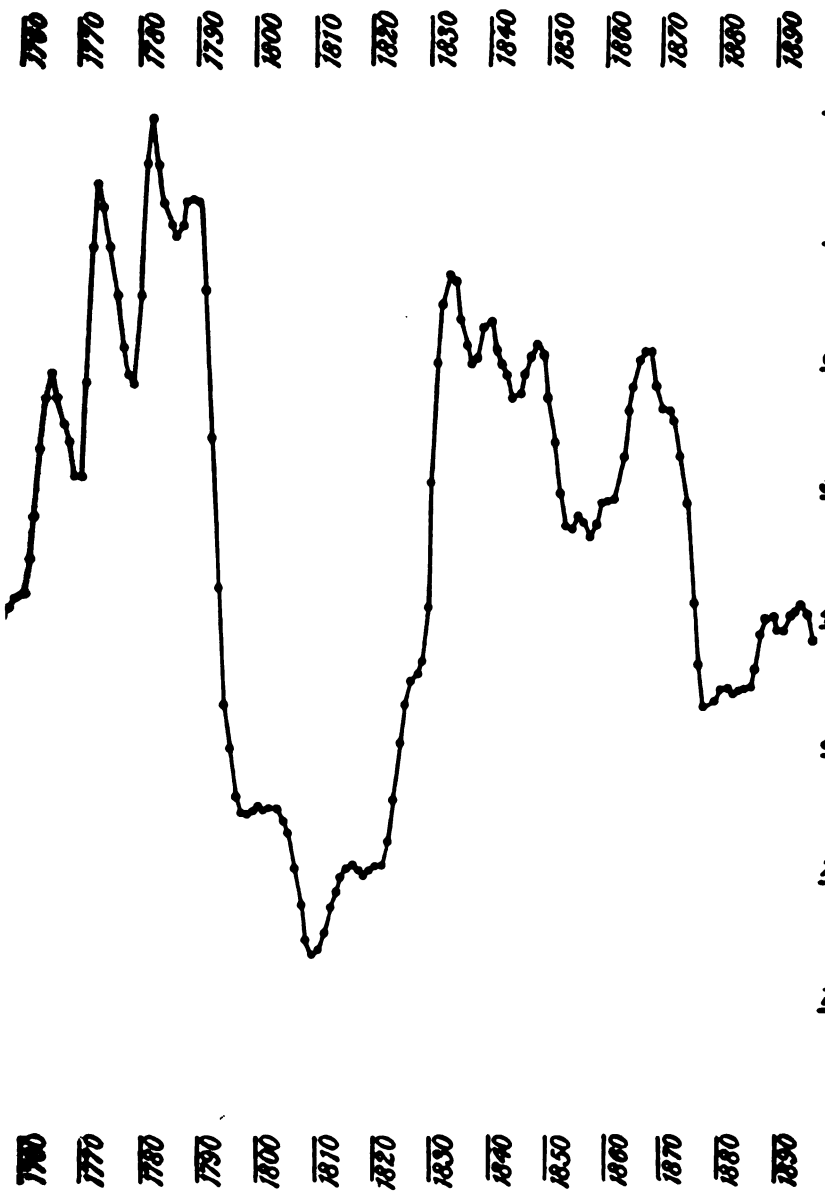
+ London

FIG. 1.

on the supposition that the needle at Baltimore varied then in the same manner as it did during the period of observation.

The path of the magnetic pole, as this intersection of the magnetic meridian may perhaps be called, would seem not to be a mere revolution round the geographical pole, but to be more like the swing of a pendulum. Evidently there are nodes or points where the direction of its motion changed about 1580; between the years 1640 and 1680, when it was moving rapidly and was at the same time near the north pole of the Earth; and about 1800.

When other pairs of stations in Europe and America are



PROF. WOLFER'S "RELATIVE SUN-SPOT NUMBERS" SMOOTHED FOR THE "*ELEVEN-YEAR*" PERIOD.

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taken, the curves traced by the intersection of their magnetic meridians is of the same character as that shown by the two selected.

Sun-spots were first discovered in the year 1610, and since that period we know of two striking cases of prolonged solar quiescence. The first was the most pronounced, and lasted from about the year 1640 to about the end of the century, during which period but a single spot now and then appeared even at times when the Sun should have been most active. The "eleven-year" cycle seemed obliterated, and an almost unbroken solar calm prevailed. Auroræ were notedly absent.

The second period of solar quiescence was remarkable, but not so complete. The "eleven-year" cycle could still be plainly traced, but for several succeeding maxima the solar activity was much below the normal; and this was emphasised by the great activity that preceded and followed the period of semi-quiescence. The accompanying diagram, which gives Wolfer's relative spot numbers cleared of the "eleven-year" cycle, will show this plainly (Plate 13). During this period of solar quiescence auroræ were also deficient.

We have, of course, no means of saying what was the state of the Sun's activity earlier than 1610. At that period spots were abundant, but the stationary position eastward of the magnetic declination needle in the years 1580–1600 may also have been marked by a period of quiescence in solar activity and in auroræ.

Prior to 1600 there seems to have been a change in the law of secular change. Mr. L. A. Bauer (in his "Magnetic Tables and Isogonic Charts for 1902") has found both for Rome and for Fayal Island, in the Azores, that there is a marked difference for the fifteenth and sixteenth centuries between the curve as given by the present apparent law of secular change and the full curve as deduced from observations obtained by the aid of early "compass charts."

There is no material for discussing the secular change of any of the magnetic elements in the southern hemisphere, neither are there observations of dip or magnetic intensity for a sufficient length of time in the northern; and these are necessary for a full discussion of the question. As it is, I have been obliged to assume for London and Baltimore that they are uninfluenced by local magnetism, or at least that such local magnetism does not affect the law of secular change. Of this there is no evidence for or against.

To sum up. The ordinary sun-spot cycle has twice been disturbed within the period of observation by long-continued calms. The first and most remarkable took place in the latter half of the seventeenth century; the second at the commencement of the nineteenth. These two periods of solar calm appear to have been answered on the Earth by corresponding periods of absence of auroræ. There was thus sympathy between solar

quiescence and terrestrial magnetism. When the movement of the north "magnetic pole" is considered it appears that at these two very epochs it was passing through two critical points of its path—the first when it was at its nearest approach to the geographical pole, the second when it was at its greatest elongation from it.

The Aurora and Magnetic Disturbance.

By William Ellis, F.R.S.

In a paper that appeared in the *Monthly Notices* of the Society for 1899 December, on the relation between magnetic disturbance and the period of solar-spot frequency, I showed from the observations of the fifty years 1848 to 1897 at the Royal Observatory, Greenwich, the general relation existing between the period of solar-spot frequency and the frequency of magnetic disturbance in the progression from Sun-spot maximum to Sun-spot minimum, and again from Sun spot minimum to Sun-spot maximum; and further pointed out that, in addition there existed in the frequency of magnetic disturbance an annual inequality that has no counterpart in the march of Sun-spot frequency. In a following paper appearing in the *Monthly Notices* for 1901 June, I compared this seasonal variation in the frequency of magnetic disturbance at Greenwich with the variation in frequency of the *Aurora* in the same locality, showing that in both phenomena there existed maximum epochs at or near the equinoxes, and minimum epochs at or near the solstices. I now desire to pursue the question a little further.

As regards the *Aurora*, although in our latitude there is, as mentioned, maximum of frequency at the equinoxes and minimum of frequency at the solstices, this condition undergoes modification in higher latitudes, the winter minimum becoming less pronounced as higher latitudes are approached, until it altogether disappears. In the recently published *Catalog der in Norwegen bis Juni* 1878 *beobachteten Nordlichter*, J. Fr. Schroeter has combined the work of Tromholt for Norway and Rubenson for Sweden, and formed abstracts that give, separately for each one of five latitudinal regions over Scandinavia, the monthly frequency of the *Aurora* as found from observations, many thousands in number, made during the years 1761 to 1877. And from a paper by Mr. R. C. Mossman "On the Aurora Borealis in London," contained in the *Journal of the Scottish Meteorological Society*, third series, vol. xi. p. 58, corresponding information is to be found for other positions south of the above. To these I have added the monthly frequency of magnetic disturbance at Greenwich and Paris. The various numerical results are given in the annexed table, in each case in percentage of the total frequency.

Month.	Frequency of the Aurora. Scandinavia, 1761–1877. Latitude North.					North East of Scotland, 120 years.	Edinburgh, 81 yrs.	London, 189 years.	Frequency of Magnetic Disturbances. Royal Observatory, Greenwich, 50 years.	Parc St.-Maur, Paris, 5 years.
	North of 62½°.	62½° to 65°.	65° to 62½°.	62½° to 58°.	South of 58°.	57½°.	56°.	52½°.	52½°.	49°.
July	0·0	0·0	0·0	0·4	0·6	0·4	1·8	1·9	7·0	7·9
Aug.	0·4	1·1	2·8	5·7	4·9	4·4	6·4	5·6	7·5	10·7
Sept.	7·8	9·7	13·1	13·6	14·9	12·9	12·3	14·5	9·9	12·1
Oct.	15·1	14·6	14·2	13·8	13·5	15·8	13·9	16·9	10·4	9·3
Nov.	14·4	14·0	12·8	10·4	10·3	12·0	11·7	9·6	8·4	8·7
Dec.	15·7	14·1	11·5	9·6	8·2	9·6	4·8	6·4	7·1	5·6
Jan.	16·4	15·3	13·2	9·5	8·2	10·9	10·0	8·6	8·0	5·9
Feb.	13·8	14·6	12·3	11·2	11·9	12·7	12·3	10·5	9·9	6·7
Mar.	14·8	13·7	14·5	13·5	12·6	12·0	13·9	10·2	10·3	10·2
Apr.	1·6	2·9	5·4	10·9	13·3	7·1	9·0	10·7	8·9	7·7
May	0·0	0·0	0·2	1·3	1·5	2·2	3·6	4·0	7·1	8·2
June	0·0	0·0	0·0	0·1	0·1	0·0	0·3	1·1	5·5	7·0

The numbers for the five Scandinavian regions are taken from page 420 of the before-mentioned *Catalog*, and those for the North-east of Scotland, Edinburgh, and London from the paper by Mr. Mossman. These are all percentage values. As regards magnetic disturbance at Greenwich, the days of moderate, active, and great disturbance, appearing in Table I. of my paper (*Monthly Notices*, 1899 December), have been combined and re-arranged for each month respectively on the aggregate of the fifty years employed. The days thus included are those on which disturbance occurred in declination greater than 10′, or in horizontal force greater than ·00050 c.g.s. The resulting values (monthly number of days) commencing with July are 282, 301, 401, 420, 339, 285, 322, 400, 416, 359, 289, and 223, which were converted into percentage values for insertion in the table. For Paris, M. Moureaux, in a paper *Sur la périodicité des perturbations de l'aiguille aimantée horizontale à l'Observatoire du Parc Saint-Maur*, has given, for the five years 1883 to 1887* the number of disturbances exceeding in declination 3′ and in horizontal force ·00020 c.g.s. for each month on the aggregate of the whole five years. But as the variation in the monthly values is very similar in both elements, I have for the present purpose combined them in one series, giving for the several months commencing with July the numbers 622, 846, 955, 729, 687, 444, 468, 527, 801, 607, 643, and 554 respectively, which, similarly

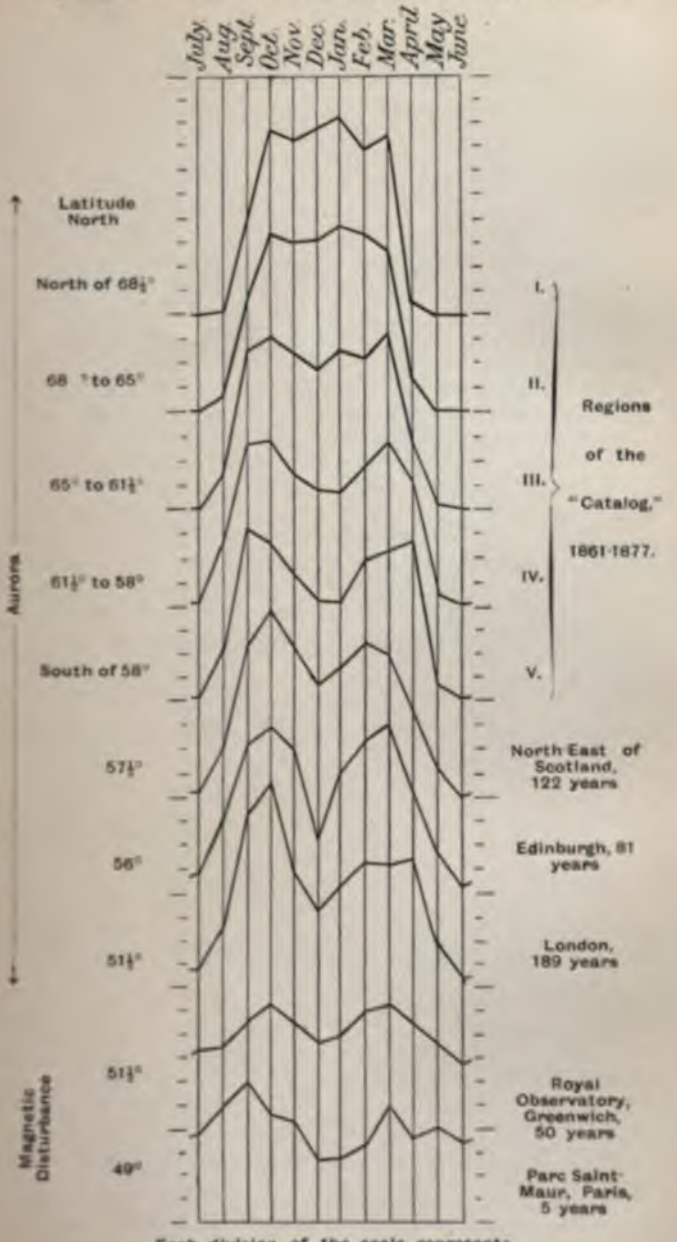
* *Annales du Bureau Central Météorologique de France*, Année 1887. *Mémoires*, p. B. 35. (In a later volume for 1897 M. Moureaux has given similar information for the fifteen years 1883 to 1897.)

converted into percentage values, are the numbers appearing in the table, the contents of which are graphically represented in fig. 1 (Plate 14).

It will be remarked that the curves of frequency of magnetic disturbance at Greenwich and Paris are very similar, showing maxima at or near the equinoxes and minima at or near the solstices, and similar also to that of the aurora at London, excepting that the autumn maximum of the latter is of more pronounced character than that of spring, as is also the case in the north-east of Scotland. It will be further observed that the strongly marked winter minimum of the aurora in lower latitudes becomes less and less marked as more northern latitudes are approached; in regions IV. and III. it is distinctly less marked, and in regions II. and I. it disappears. The autumn maximum visibly tends to become later, and the spring maximum somewhat earlier as the higher latitudes are approached, the winter depression at the same time diminishing until eventually there remains only a mid-winter maximum at or near the winter solstice. The mid-winter minimum of lower latitudes being thus converted in higher latitudes into a mid-winter maximum, it becomes a question, remarking the similarity of the auroral and magnetic curves in lower latitudes, as to what happens in higher latitudes as regards magnetic disturbance. Does it still run with the aurora? That is, does the winter depression of frequency in higher latitudes become similarly converted into a mid-winter maximum. By analogy this might be expected to be the case. But is it so? The summer effect is not easily discussed as regards aurora, because of the daylight towards the north; the winter effect is, however, readily followed, and the condition of things is clear, but information appears to be wanting as regards what happens in the case of magnetic disturbance.

It may be well to give some further indication of the really persistent character of the seasonal variation in frequency of magnetic disturbance in our latitude. The agreement between the curves for Greenwich and Paris shown in fig. 1 is sufficient evidence of the existence of the inequality, but the question may be somewhat further illustrated. For this purpose the number of days of magnetic disturbance at Greenwich during four years about each of the Sun-spot maxima of 1848, 1860, 1870, 1884, and 1894 were grouped together as indicating periods of greater frequency of disturbance, and similarly for four years about each of the Sun-spot minima of 1856, 1867, 1879, and 1890, as indicating periods of lesser frequency of disturbance, the two groups so formed containing twenty and sixteen years respectively, as follows, the comparision showing here, in addition to the law of frequency, also the absolute number of days involved.

Fig. 1.—Annual Inequality of Aurora and of Magnetic Disturbance.



Each division of the scale represents 2·0 of the percentage values of the Table. The longer lines of the scale indicate the zeros of the several curves.

Number of days in the twenty years of greater frequency of magnetic disturbance (near Sun-spot maximum).

	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
48–1851	22	13	32	36	25	24	25	35	22	21	22	13
58–1861	24	33	38	44	14	29	22	25	44	33	20	24
69–1872	32	41	45	39	31	26	30	39	50	54	34	27
82–1885	28	27	29	29	35	20	20	32	30	34	23	24
92–1895	41	22	48	49	36	32	35	53	56	30	32	36
tal for 20 years ...	147	136	192	197	141	131	132	184	202	172	131	124
onthly Mean ...	7·4	6·8	9·6	9·9	7·0	6·5	6·6	9·2	10·1	8·6	6·5	6·2

Yearly average = 94·4 days.

Number of days in the sixteen years of lesser frequency of magnetic disturbance (mean Sun-spot minimum).

	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
54–1857	9	13	21	18	16	11	17	30	25	24	14	6
65–1868	20	26	43	50	30	6	25	41	30	24	18	12
76–1879	4	3	7	4	12	8	7	4	7	5	9	3
87–1890	20	27	25	29	35	17	23	37	22	22	22	10
tal for 16 years ...	53	69	96	101	93	42	72	112	84	75	63	31
onthly Mean ...	3·3	4·3	6·0	6·3	5·8	2·6	4·5	7·0	5·3	4·7	3·9	1·9

Yearly average = 55·6 days.

The monthly means of the two groups are graphically represented in fig. 2. We see for the years about maximum of Sun-spots that, on the average, magnetic disturbance occurred on some ninety-four days of the year, and about minimum of Sun-spots on some fifty-five days. But the point to which attention is to be drawn (see fig. 2) is the circumstance that the annual inequality is shown not only in periods of greater frequency of magnetic disturbance near to Sun-spot maximum, but that it is also strongly marked in periods of lesser frequency near Sun-spot minimum. It will be seen also that with some few irregularities the inequality can be very well followed in the separate four yearly groups of years about Sun-spot maximum and Sun-spot minimum. It may be added that in the fourteen years of the fifty years employed that remain intermediate between the epochs of maximum and minimum and of minimum and maximum of Sun-spots, the winter minimum is much less pronounced, the summer minimum remaining much the same, on the whole producing a flatter curve, as indicated by the thin line in fig. 2, for which (without giving all the corresponding figures) it may be added that the monthly means (number of days) commencing with July are 5·9, 6·9, 8·1, 8·7, 7·5, 8·0, 8·4, 7·4, 9·3, 8·0, 6·8, and 4·9. The annual inequality is thus most strongly developed near epochs of maximum and minimum of Sun-spots.

It may be that the flatter intermediate curve is in some measure due to accidental circumstances.

The magnetic disturbance that has been here dealt with is, as before mentioned, that taken as including days of moderate, active, and great disturbance at Greenwich, according to the convention of the previously mentioned paper of 1899 December. But it may be further pointed out that considering separately the frequency of these three different degrees of disturbance, it is found that the excess of the equinoctial frequency over the solstitial frequency is greater, the greater the degree of disturbance. The three degrees are thus defined :—(1) moderate, indicating disturbance in declination greater than 10′ but less than 30′, and in

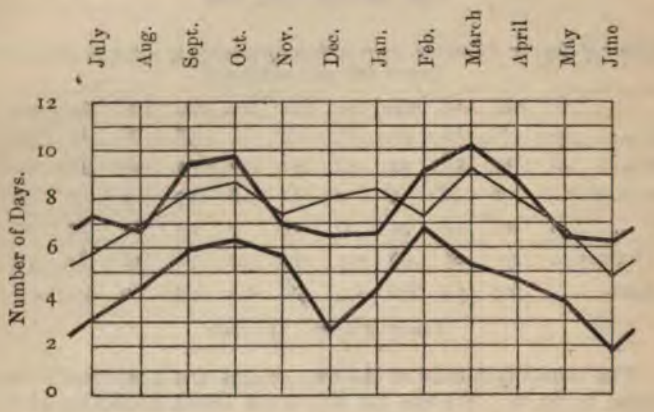


FIG. 2.—Annual Inequality of Magnetic Disturbance at Greenwich.

The upper thick line indicates the annual inequality in frequency of disturbance near maximum of Sun-spots, and the lower thick line that near minimum of Sun-spots; the thin line represents the inequality in years intermediate between these extreme epochs.

horizontal force greater than ·00050 but less than ·00150 c.g.s.; (2) active, indicating disturbance in declination greater than 30′ but less than 60′, and in horizontal force greater than ·00150 but less than ·00300 c.g.s.; and (3) great, indicating disturbance in declination greater than 60′, and in horizontal force greater than ·00300 c.g.s., the three months of February to April, and from August to October, being taken as equinoctial, and the remaining six months as solstitial.

Degree of Disturbance.	Number of Days.	Frequency of Disturbance (Percentage). Equinoctial.	Solstitial.	Excess of Former.
Moderate	3,663	56	44	12
Active	299	61	39	22
Great	75	71	29	42

In this table no regard is paid to Sun-spot frequency, the whole fifty years being alike employed. It thus finally appears:

(1) That the annual inequality in frequency of magnetic disturbance at Greenwich is strongly marked, not only at periods of greater frequency and correspondingly greater magnitude of disturbance near Sun-spot maximum, but also at periods of lesser frequency and correspondingly lesser magnitude of disturbance near Sun-spot minimum.

(2) Also that the annual inequality in frequency of disturbance is greater as the degree of disturbance becomes increased.

These conclusions may appear to be in some sense contradictory. But it is to be remarked (fig. 2) that although the lower thick line includes a less number of days—that is, on the whole, a lesser numerical frequency of disturbance—than does the upper thick line, the inequality in frequency is of similar character in both. For although the frequency of active and great disturbance influences mainly the upper thick line, it also affects in no inconsiderable degree some of the years that become included in the formation of the lower thick line.

Now, if disturbance frequency progresses as does the 11± year Sun-spot frequency—that is, on the whole, with no annual period,* the mean monthly number of days of disturbance taken through a series of years should similarly, as the number of years employed is increased, more and more approximate to equality of value in all months of the year, by gradual elimination of the transient fluctuations of value, instead of which the combination of years really brings out a definite annual inequality in frequency to be observed, as already mentioned, in the four yearly groups of years, but distinctly shown in the aggregate, alike in the years of maximum frequency and minimum frequency of disturbance, diminished in the latter case as to the absolute number of days of disturbance in the various months of the year, but still showing a most marked annual inequality, which can only be understood as an undoubted physical effect.

The annual inequality in the frequency of magnetic disturbance in our latitude has thus a real existence. Our knowledge of the relations subsisting between solar and other phenomena, so far as our own geographical position is concerned, has also reached a certain definite stage—that is to say, as regards the 11± year Sun-spot cycle, a period of great solar activity is also one of great magnetic activity with frequent displays of the aurora,† and a period of solar quiet is similarly also one of magnetic quiet, with no exhibitions of the aurora. It is thus apparent:—

(1) That in addition to the progression in frequency of magnetic disturbance and of the aurora in harmony with the 11± year Sun-spot period, there exists in both phenomena, in

* *Monthly Notices*, 1899 December, p. 152.

† At Greenwich magnetic disturbance is not necessarily accompanied by aurora, but when aurora does appear there is also magnetic disturbance.

our latitude, an annual inequality having maxima at or near the equinoxes and minima at or near the solstices to which there is no counterpart in the progression of solar-spot frequency.*

(2) That the equinoctial maxima of frequency in the case of the aurora disappear in higher latitudes, becoming merged into a single midwinter maximum, with entire disappearance of the winter minimum.

Thus, viewing the relation that exists in our latitude between the annual inequality of magnetic disturbance and of the aurora, and the modification that the inequality undergoes in higher latitudes in the case of the aurora, it becomes of interest to ask whether in the case of magnetic disturbance any similar modification of the inequality occurs? As already remarked, I do not know of any available information bearing thereon.

To turn now from the question of the annual inequality in frequency of the aurora to another matter, the relation of the aurora to the $11\pm$ year Sun-spot period, it appears from observations made at Ivigtut, in Greenland,† in latitude 61° north, at Godthaab in 64° north, and at Jacobshavn in 69° north, that years of greater frequency of the aurora were years of lesser frequency of Sun-spots. But Rubenson‡ having discussed the Swedish observations made between the years 1721 and 1877, has represented them by formulæ, n being the number of years after the initial epoch, as follows:—

$$1728{\cdot}2 + 11{\cdot}07 \times n, \text{ for maximum,}$$
$$1734{\cdot}2 + 11{\cdot}03 \times n, \text{ for minimum,}$$

and found that the observed epochs of maximum and minimum frequency of the aurora during the period mentioned were, with a few exceptions, in close agreement with corresponding epochs calculated from the formulæ, indicating also a near agreement with the corresponding maximum and minimum epochs of Sun-spots. Rubenson suspects that there may be long period variations in the aurora, but as to the existence of such variations in these and related phenomena there appears at present, I think, to be a good deal of uncertainty. But as to the $11\pm$ year period his results appear to be opposed to those found at stations in Greenland as above given, which indicate greater frequency of aurora with lesser frequency of Sun-spots, although situated in the same latitudinal zone. It may be of interest to examine a little more closely this point. In Rubenson's work (p. 298) a table is given showing the epochs of maximum and minimum frequency of the aurora as determined from the Swedish observa-

* The annual inequality in the diurnal range of the magnetic elements is similarly a terrestrial effect that has no counterpart in the progression of Sun-spot frequency.

† *Exploration Internationale des Régions Arctiques*, 1882–3, *Expédition Danoise*, tome i. p. 19.

‡ *Catalogue des Aurores Boréales observées en Suède depuis le xvi^me^ siècle jusqu'à l'année* 1877.

tions, those epochs which are supposed to be more or less doubtful being placed in parentheses. Omitting these latter, and comparing the remaining values with the corresponding Sun-spot epochs taken from Professor Wolfer's paper in the *Meteorologische Zeitschrift* for 1902 May, we have the annexed table.

Observed Epochs of Maximum and Minimum Frequency of the Aurora and of Sun-spots Compared.

Maximum Epochs			Minimum Epochs.		
Rubenson. (Aurora.)	Wolfer. (Sun-spot.)	Excess of Former.	Rubenson. (Aurora.)	Wolfer. (Sun-spot.)	Excess of Former.
1729·9	1727·5	+2·4	1736·6	1734·0	+2·6
1741·2	1738·7	+2·5	1744·3	1745·0	−0·7
1749·9	1750·3	−0·4	1756·0	1755·2	+0·8
1761·0	1761·5	−0·5	1777·8	1775·5	+2·3
1788·3	1788·1	+0·2	1798·6	1798·3	+0·3
1804·6	1805·2	−0·6	1811·8	1810·6	+1·2
1819·6	1816·4	+3·2	1824·4	1823·3	+1·1
1831·7	1829·9	+1·8	1834·2	1833·9	+0·3
1839·0	1837·2	+1·8	1844·7	1843·5	+1·2
1851·0	1848·1	+2·9	1856·3	1856·0	+0·3
1871·3	1870·6	+0·7			
Mean		+1·3	Mean		+0·9

The epochs of maximum and minimum frequency of the aurora thus appear to fall respectively at or near to maximum and minimum epochs of Sun-spots as in our latitude, but following the Sun-spot epochs a little later in time, and on the whole by a longer interval at maximum epoch than at minimum epoch. There are irregularities, however, in the progression of auroral frequency as well as in that of Sun-spot frequency. Thus the 1788·3 observed maximum epoch of aurora was a greatly accelerated one, arriving 6·3 years earlier than the calculated time by the formula for maximum above given, which is 1794·6. And Professor Wolfer, who gives the observed Sun-spot maximum as 1788·1, has also added formulæ to represent the observed values, as follows:

$$1749{\cdot}37 + 11{\cdot}091 \times n, \text{ for maximum,}$$

$$1744{\cdot}21 + 11{\cdot}141 \times n, \text{ for minimum,}$$

from which the calculated time of maximum is found to be 1793·7, the observed time being thus 5·6 years earlier. That acceleration of the auroral maximum by 6·3 years should be accompanied by an acceleration of the Sun-spot maximum by 5·6 years is a striking proof of the interrelation of the two phenomena, since they thus remain in close accord, the auroral maximum exceeding the Sun-spot maximum (see the preceding table)

by only 0·2 year. We may also compare the formulæ of Rubenson for aurora with those of Wolfer for Sun-spots by reducing them to two epochs, one near the beginning and one near the end of the series for aurora employed, as follows:—

Maximum Epochs.			Minimum Epochs.		
of Aurora (Rubenson.)	of Sun-spots (Wolfer.)	Excess of Former.	of Aurora (Rubenson.)	of Sun-spots (Wolfer.)	Excess of Former.
1750·3	1749·4	+0·9	1745·2	1744·2	+1·0
1872·1	1871·4	+0·7	1866·6	1866·8	−0·2

giving mean differences of +0·8 year and +0·4 year respectively as compared with +1·3 year and +0·9 year, found by comparison of the epochs as observed, in which latter (as before mentioned) a few epochs of aurora understood to be somewhat doubtful were not included. These results indicate that throughout Sweden, during the period covered by the observations discussed, the times of maximum and minimum frequency of the aurora were, on the whole, in close relation with those of maximum and minimum frequency of Sun-spots, both phenomena being, with the 11± year periodical variation of magnetic diurnal range and of magnetic disturbance, all alike the effect of some common unknown cause.

What is already known as regards the relations existing between solar-spots, terrestrial magnetism, and the aurora in middle latitudes may be but an outer fringe of the subject, but corresponding information in regard to polar regions is not to the same extent available. As regards magnetism, observations in those regions have hitherto been more or less of a temporary nature. But to obtain any sufficient knowledge of the physical constitution of the globe, observatories, not of temporary but of permanent character, are necessary in all regions, as well as in the more habitable portions of the globe, and the north polar regions offer opportunities for placing such observatories as do not exist in the same way in the south.

Our Earth is more and more found to be not an isolated body simply receiving light and heat from the Sun, as formerly commonly conceived, but one subject to other external influence made manifest to us in various ways with which we have only in later times for the most part become acquainted, but of the precise nature of which little more is known than that, complex in action, it seems to be one that may pervade the whole solar system, involving in one common field of action all the bodies thereof. If we could plant observatories on some of the other planets of our system, and communicate therewith, how enlarged might be our knowledge of the action of the forces that surround us.

Note on Mr. Ritchey's Photographs of the Andromeda Nebula.

By W. H. Wesley.

(*Communicated by the Secretaries.*)

At the meeting of the Society in 1903 March there were exhibited a series of photographs of nebulæ taken by Mr. G. W. Ritchey with the 24-inch reflector of the Yerkes Observatory. The photographs were remarkable as showing in the *Andromeda* and *Orion* nebulæ the detail both in the bright central portions and in the faint outlying parts, and the question was raised as to whether some selective process of development or printing had been employed.

In the *Monthly Notices* for 1903 May (vol. lxiii. p. 395) appeared a letter from Mr. Ritchey fully explaining the process he adopted in developing astronomical negatives. Mr. Ritchey further says that "in the case of such objects as the *Orion* and *Andromeda* nebulæ something more is needed if the detail in the bright central parts and the faint extensions are both to be shown on the same positive. One negative is reserved precisely as developed by the above method. For transparencies and lantern-slides a second negative is exposed in the telescope, developed as above described, and the parts which are so dense that they cannot be printed are reduced locally by the use of a very weak reducing solution. Much time and care are given to this, and an attempt is made to keep the relative brightness of the various parts the same in kind, though not in degree, as in the untouched negative, which is constantly used for comparison during the process."

The two beautiful transparencies, therefore, which Mr. Ritchey presented to the Society, and which were shown at the March meeting, were taken from negatives which had been locally reduced. The advantages of showing on the same plate the detail of all parts of the nebula are sufficiently obvious. At the same time, while the utmost care was evidently taken with the reduction, it seemed very desirable to compare these photographs with entirely untouched negatives to find if any false effects or spurious detail had been introduced by the process.

A short time ago Mr. Ritchey sent, through Mr. Hinks, two untouched negatives of the *Andromeda* nebula, so that, with regard to this nebula the opportunity was afforded for such a comparison. The negative from which the enlarged transparency was made had an exposure of four and a half hours; and as one of the untouched negatives had been exposed four hours it was particularly suitable for the comparison. The detail of the nebula is so complex that I principally concentrated my attention upon the finer detail immediately surrounding the great central condensation, as this was most liable to be affected by the local reduction. I examined with special care the edges of

the dark rifts, where the bright bands are broken up into very minute flocculent patches. The result of my examination is that I find nothing on the transparency that is not on the negative. I specially looked for any *edge* of greater brightness which might mark the limit which the reducing solution reached, but can find nothing of the kind. I can therefore only conclude that the process of local reduction has been so carefully carried out that no spurious detail has been produced in the negative from which this transparency was made.

Since the prolonged examination of small details renders one extremely liable to overlook larger and more general effects, I have repeatedly broken off my examination and resumed it on other occasions; but the other results have always been the same. Mr. Hinks and I have also together compared the photographs, but we both came to the conclusion that whatever was in the transparency could be found in the untouched negatives.

New Double Stars detected with the $17\frac{1}{2}$ *in. Reflector during the year* 1903. By T. E. Espin, M.A.

The following stars have been found to be double during the year 1903. As they are so few, and the measures are for the most part incomplete, I have not numbered them.

B.D.	R.A. 1880	Decl.	P.	D.	Mags.
°	h m	° ′	°	″	
53·234	1 2·5	+53 59	215·7	5·1	8·6 11·8
63·1346	17 21·4	63 51	19·1	6·5	9·0 11·5
64·1256	18 15·6	64 1	332·7	8·6	8·2 12·0
51·2372	20·9	51 35	198·7	2·7	8·6 8·7
60·1844	42·4	60 32	103·6	4·3	9·1 11·1
61·1816	19 5·2	61 4	243·9	6·1	9·1 9·8
59·1979	12·7	59 33	116·3	7·4	9·0 11·6
59·1981	13·3	59 34	113·7	8·1	8·8 11·7
64·1346	20·2	64 18	216·3	4·4	8·8 9·9
64·1364	35·6	64 47	19·3	8·9	8·5 10·5
64·1369	37·8	64 39	313·7	2·7	8·8 9·4
64·1386	46·3	64 23	70·7	6·5	8·0 10·5
59·2160	57·3	59 25	145·0	4·1	9·0 11·5
52·2883	21 8·7	52 48	—	4±	9·0 12
63·1814	22 5·8	63 31	—	4±	9 11

The Rotation Period of Saturn in 1903. By W. F. Denning.

The extensive disturbance observed on Saturn in 1903 offered an excellent opportunity for determining the rotation period of the north-temperate region of the planet. No doubt, however, the rate of the spots merely represented that of an atmospheric current, not nearly conformable with the motion of the globe.

To ascertain the rotation period reliably and within small limits of error it is by no means necessary to depend upon the transit times of spots derived by micrometric measurement. The method of eye estimation may not be quite so exact, but, as in the case of *Jupiter's* markings, it is capable of furnishing excellent results. There are certainly more important factors affecting such investigations than the manner of taking transits. The question of *identification* is a serious one. The mis-identification of planetary features has proved a fruitful source of large errors, and is scarcely avoidable unless observations are obtained at short intervals and a fairly numerous list of transits accumulated.

Large irregular markings, light and dark, were pretty abundant in the northern hemisphere of *Saturn* during the past year, and these underwent certain variations in aspect. The form, size, and brilliancy of the luminous spots appeared notably inconstant, and there were changes in the rate of motion. Several of the objects alluded to were compound, consisting of two or three parts wholly or partly divided by dusky masses or wisps similarly to those sometimes seen crossing the bright equatorial region of *Jupiter*. To follow the same individual marks on *Saturn*, and safely single them out one from another after various intervals, formed the most critical and delicate work in the recent telescopic study of the planet. As a test of the correctness of my own identifications I have collected together all the observed transits of spots I could find by various observers, and the comparisons made have, I hope, been the means of eliminating serious mistakes. I here append the transit times from observations secured between June 23 and September 22 of Barnard's spot, and of two smaller spots following it marked B, C, and D respectively. The times are compared with an adopted, uncorrected rate of $10^h\ 38^m$ and the residuals given.

Observed Transit Times of Barnard's White Spot on Saturn and of two other White Spots following it. 1903, *June 23–September 22.*

Observer.	Date. 1903.	Transit G.M.T. h m	Computed Period $10^h\ 38^m$. h m	Residual O−C. h m	Spot.	Notes.
Barnard ...	June 23	21 42	21 42	± 0	B	b., e., o.
Barnard ...	24	18 58	18 58	± 0	B	b., e., o.
Hartwig ...	26	13 36	13 30	+ 6	B	
Massow and Maluka	26	13 38	13 30	+ 8	B	

Observer.		Date. 1901.	Transit G.M.T. h m	Computed Period 27d 3m. h m		Residual O—C. h m		Spot.	Notes.
Sola...	...	June 26	13 20	13 30	−	11		B	d.
Brough	..	27	21 28·7	21 24	−	5·3		B	m.m.
Wilson	...	July 1	21 26	21 6	−	12		B	b, e.
Graff	...	4	13 19	12 54	−	25	B or C?		
Brough	...	4	18 40·4	18 2	−	45·4	C		m.m.
Sola...	...	8	12 40	12 36	−	4	B		
Sola...	...	9	20 6	9 32	−	8	B		
Sola...	...	11	24 45	25 2	−	17	B		
Perth	..	11	13 20	25 2	−	8	B		
[illegible]	...	12	12 18	12 18	=	0	B		m.m.
[illegible]	...	12	12 30	12 48	+	30	C		
Phillips	...	12	12 30	12 18	−	12	B		
Kibbler	...	12	12 30	12 18	−	12	B		
Brough	..	15	20 [illegible]	20 12	−	[illegible]	B		m.m.
[illegible]	...	15	20 [illegible]	20 12	−	[illegible]	B		[illegible]
[illegible]	...	16	23 30	23 24	−	6	B		[illegible]
[illegible]	..	17	12 [illegible]	12 [illegible]	−	[illegible]	B		[illegible]
[illegible]	..	17	12 [illegible]	12 [illegible]	=	[illegible]	B		m.m.
[illegible]	..	17	12 [illegible]	12 [illegible]	=	[illegible]	B		m.m.
[illegible]	..	18	15 [illegible]	15 [illegible]	−	[illegible]	B		m.m.
Sola...	..	20	13 [illegible]	13 [illegible]	−	[illegible]	B		
Sola...	..	21	14 [illegible]	14 [illegible]		[illegible]	B		
[illegible]	..	22	15 [illegible]	15 [illegible]	−	[illegible]	B		
Sola...	..	23	16 [illegible]	16 [illegible]	−		B		
[illegible]	..	24	17 [illegible]	17 [illegible]	−	[illegible]	B		[illegible]
[illegible]		24	17 [illegible]	17 [illegible]	−	[illegible]	B		[illegible]
[illegible]		24	17 [illegible]	17 [illegible]		[illegible]	B		[illegible]
[illegible]		24	17 [illegible]	17 [illegible]	−	[illegible]	B		[illegible]
Sola		25	18 [illegible]	18 [illegible]		[illegible]	B		
[illegible]		26	19 [illegible]	19		[illegible]	B		[illegible]
[illegible]		26	19 [illegible]	19 [illegible]		[illegible]	B		m.m.
[illegible]		27	20 [illegible]	20 [illegible]		[illegible]	B		m.m.
[illegible]		28	21 [illegible]	21 [illegible]		[illegible]	B		
[illegible]		29	22 [illegible]	22 [illegible]		[illegible]	B		
[illegible]		30	23 [illegible]	23 [illegible]		[illegible]	B		
[illegible]		31	24 [illegible]	24 [illegible]		[illegible]	B		
[illegible]		[illegible]	[illegible]	[illegible]		[illegible]			

Observer.	Date. 1903.	Transit G.M.T. h m	Computed Period $10^h\ 38^m$. h m	Residual O—C. h m	Spot.	Notes.
D.	Aug. 5	11 35	10 30	+1 5	D	
D.	12	22 24	12 38	— 14	B	b., o.
Williams ...	13	10 14	9 54	+ 20	B	s.
Brenner ...	17	9 36	9 36	± 0	B	
Burnham ...	19	15 31·2	14 46	+ 45·2	C	f.
Brenner ...	21	9 18	9 18	± 0	B	s., m.m.
D.	21	10 9	9 18	+ 51	C	r. s., o.
D.	25	9 12	9 0	+ 12	B	
D.	25	10 18	9 0	+1 18	D	
D.	29	8 37	8 42	— 5	B	b., o.
D.	29	10 3	8 42	+1 21	D	
D.	Sept. 2	8 28	8 24	+ 4	B	
D.	2	9 36	8 24	+1 12	D	
D.	5	10 44	10 50	— 6	B	o.
Williams ...	5	11 5	10 50	+ 15	B	s.
D.	6	7 50	8 6	— 16	B	f.
D.	6	8 40	8 6	+ 34	C	
Williams ...	9	10 41	10 32	+ 9	B	b.
Williams ...	13	10 24	10 14	+ 10	B	d.
D.	14	8 8	7 30	+ 38	C	
Williams ...	17	10 7	9 56	+ 11	B	s., b., o.
D.	17	10 9	9 56	+ 13	B	o.
D.	18	7 25	7 12	+ 13	B	
D.	22	6 49	6 54	— 5	B	o.
D.	22	8 12	6 54	+1 18	D	

In first column transits by D. = Denning.

In last column: b., bright; f., faint; d., double; e., extended; l., large; s., small; o., obvious; m.m., micrometric measure; v., very; r., rather; p., preceding; f., following.

The discussion of a considerable number of transits of bright spots recorded here and elsewhere show that the corrected rotation period was as nearly as possible

$$10^h\ 38^m\ 3^s$$

between the latter part of June and middle of September. Then a marked acceleration apparently occurred, and from the observations made at Bristol, July to December, of fifteen bright and dark spots the mean rate of rotation was about

$$10^h\ 37^m\ 56^s.$$

I shall be glad if Professor Hough will look into the foregoing table of transits, for I am bound to conclude that in his interesting paper in *Monthly Notices* for 1903 December, p. 122, he has incorrectly identified the very few (though doubtless accurate) observations taken by himself at the Dearborn Observatory and by Professor Burnham at the Yerkes Observatory. The rotation periods he has deduced are therefore too great. The observation of August 19, $15^h\ 31^m{\cdot}2$ G.M.T., was certainly not of Barnard's spot at all, but of a smaller object further south, and following it about three-quarters of an hour. Professor Barnard (*Astronomical Journal*, No. 547, p. 180) specially alludes to a smaller spot, following on July 13 and 14, the principal one which he was the first to discover. The same object also came under observation on several occasions at Bristol. Professor Hough, in his paper alluded to, also identifies spots seen on July 6, 22, and 30 (period $22^h\ 38^m\ 30^s{\cdot}5$), but the first of these was different, and very probably the same as that of August 29, marked C in the foregoing table. The transits of July 22 and 30 undoubtedly referred to the same marking (marked D in the table), and this object was well observed by Barnard on August 2, and many times at Bristol in July and subsequent months, following Barnard's spot about an hour and a quarter.

I entirely differ from Professor Hough in his depreciation of eye-estimated transits of markings on *Jupiter* and *Saturn*. Professor Hall, in his valuable and accurate observations of the white equatorial spot which he discovered on *Saturn* in 1876, December, relied upon eye estimations, and concluded the probable error of determining the time when the spot was at the centre of the disc to be 2·3 min. on a good night and 3·5 min. on a bad one. Professor Barnard's two pairs of eye-estimated transits of his recent spot clearly prove two things—viz. that its period was just about $10^h\ 35^m$, and that the four observations were very accurate and comparable with the best micrometrical measures:—

	h. m.	h. m.	m.
June 27	21 42	21 42	+0
28	8 18	8 18	+0
July 25	20 11	20 11	−1
26	7 46	7 46	−2

In the table giving the combined results of observers the residuals are certainly larger than they ought to be, and this is due to the fact that the observations were frequently made when the spot was near the limb, where the foreshortening of the disc rendered the estimation of the position of the spot very difficult. It is probable that the observations of Jupiter's spots, made at the centre of the disc, will be found to agree to within a minute or two with the above values.

It will be noticed that Herr Brenner's six observations of Barnard's spot agree to the very minute with the uncorrected rotation period of $10^h\ 38^m$. I cannot explain this singular circumstance.

With reference to the transits of minor spots obtained by Professors Hough and Burnham, I may add that I made eight observations of each of Burnham's spots of July 23, $17^h\ 28^m$ and $17^h\ 57^m$ G.M.T. Hough's transit of August 20, $15^h\ 32^m{\cdot}3$ G.M.T. seems to be of the former marking of the pair. It followed Barnard's spot about $3^h\ 24^m = 115°$. I have twelve observations of Burnham's spot of August 9, $17^h\ 32^m{\cdot}6$ G.M.T., which preceded Barnard's spot about $3^h\ 10^m = 107°$.

Observations of *Saturn* were made here on seventy-eight nights between July 1 and December 11. The last good observation of Barnard's spot was on November 24, $4^h\ 17^m$, when it preceded the zero meridian (rate $10^h\ 38^m$ uncorrected) 27 minutes. The spot C in the table I appear to have lost after September, but D I retained in view until December 10, when it was in transit at $4^h\ 30^m$, and followed the z.m. 52 minutes.

This z.m. requires to be corrected chiefly for differences in the longitude and distance of *Saturn*. Between the end of June and middle of September (the period comprised in the table) there would be a *plus* correction of more than 2 seconds per rotation for the relative displacement in longitude. Between the end of June and beginning of December there was a slight *minus* correction of about 1 second, the far greater distance of *Saturn* more than compensating for the variation in longitude.

I hope to place before a later meeting of the Society some further details of my observational results, with a determination of the mean rotation period.

Bishopston, Bristol: 1904 *January* 4.

Ephemeris for Physical Observations of

		☿	L. ☿.	R.	Apparent Diameter. Equat. Diam.	Excess over Polar.	Defect of Diam.	☿	☿
May	1	333°50′	239°070	2·442	34″22	2″20	0″07	5″12	245°01
	3	333 0?3	239 986	2·474	34·39	2·21	0·08	5·67	245·33
	5	333 [illegible]	240 863	2·506	34·57	2·22	0·10	6·20	245·63
	13	333 927	241·768	2·538	34·77	2·23	0·12	6·71	245·91
	17	330·048	242·635	2·569	34·99	2·25	0·14	7·21	246·18
	21	330 172	243·485	2·600	35·23	2·26	0·16	7·70	246·43
	25	330 300	244·317	2·630	35·48	2·28	0·18	8·16	246·66
	29	330·428	245·128	2·661	35·75	2·30	0·20	8·61	246·88
June	2	330·558	245·918	2·692	36·05	2·32	0·22	9·03	247·10
	6	330·688	246·686	2·726	36·36	2·34	0·24	9·43	247·31
	10	330·817	247·429	2·758	36·69	2·36	0·26	9·81	247·52
	14	330·945	248·145	2·789	37·04	2·38	0·28	10·16	247·71
	18	331·072	248·834	2·819	37·41	2·41	0·31	10·48	247·90
	22	331 196	249·492	2·845	37·80	2·43	0·33	10·77	248·06
	26	331 318	250·121	2·870	38·20	2·46	0·35	11·03	248·22
	30	331·436	250·716	2·895	38·63	2·49	0·37	11·26	248·37
July	4	331 548	251·277	2·920	39·07	2·52	0·39	11·45	248·51
	8	331·655	251·790	2·947	39·53	2·55	0·41	11·61	248·67
	12	331·756	252·284	2·974	40·00	2·58	0·42	11·73	248·84
	16	331·849	252·728	3·000	40·50	2·61	0·43	11·80	249·00
	20	331·930	253·120	3·023	41·01	2·64	0·44	11·84	249·16
	24	331·999	253·457	3·046	41·52	2·67	0·44	11·83	249·32
	28	332·053	253·800	3·060	42·05	2·71	0·45	11·78	249·48
Aug.	1	332·104	254·000	3·001	42·50	2·74	0·44	11·67	249·63
	5	332·100	254·083	3·113	43·15	2·78	0·44	11·52	249·78
	9	332·130	254·340	3·133	43·70	2·81	0·43	11·33	249·91
	13	332·122	254·563	3·152	44·21	2·85	0·41	11·07	250·04
	17	332·100	254·752	3·170	44·83	2·88	0·39	10·76	250·17
	21	332·072	254·923	3·188	45·36	2·92	0·37	10·41	250·30
	25	332·038	255·080	3·20	45·93	2·95	0·35	10·00	250·43
	29	331·999	255·220	3·221	46·47	2·98	0·32	9·54	250·56
Sept.	2	331·96	255·341	3·239	47·00	3·01	0·30	9·02	250·68
	6	331·92	255·45	3·256	47·53	3·04	0·28	8·48	250·81
	10	331·88	255·55	3·272	48·06	3·07	0·25	7·88	250·93
	14	331·84	255·64	3·288	48·59	3·10	0·23	7·25	251·05
	18	331·80	255·72	3·303	49·12	3·13	0·21	6·58	251·17
	22	331·76	255·79	3·318	49·65	3·16	0·19	5·88	251·29
	26	331·72	255·85	3·332	50·18	3·19	0·17	5·15	251·41
	30	331·68	255·90	3·346	50·71	3·22	0·15	4·39	251·53

Jupiter, 1904–5. By A. C. D. Crommelin.

Greenwich Noon.		Longitude of ♃'s Central Meridian. $177^{\circ}\cdot90$ I.	$250^{\circ}\cdot27$ II.	corr. for Phase.	Light-time.	A—O.	B.
1904.		°	°	°	m	°	°
May	1	60·13	251·74	+0·11	48·542	233·96	+2·48
	5	330·96	132·05	·14	48·312		
	9	241·81	12·38	·17	48·059	234·68	2·51
	13	152·70	252·75	·19	47·783		
	17	63·61	133·14	·23	47·484	235·42	2·53
	21	334·55	13·56	·26	47·164		
	25	245·53	254·02	·29	46·824	236·16	2·55
	29	156·54	134·50	·32	46·464		
June	2	67·58	15·02	·35	46·086	236·89	2·57
	6	338·65	255·57	·39	45·690		
	10	249·76	136·16	·42	45·277	237·62	2·59
	14	160·91	16·78	·45	44·848		
	18	72·09	257·44	·48	44·405	238·35	2·61
	22	343·31	138·14	·50	43·949		
	26	254·56	18·87	·53	43·481	239·09	2·63
	30	165·86	259·64	·55	43·004		
July	4	77·20	140·45	·57	42·518	239·83	2·65
	8	348·58	21·30	·59	42·023		
	12	260·00	262·20	·60	41·523	240·55	2·67
	16	171·46	143·14	·61	41·019		
	20	82·97	24·13	·61	40·512	241·29	2·69
	24	354·52	265·16	·61	40·003		
	28	266·12	146·23	·60	39·496	242·02	2·71
Aug.	1	177·76	27·35	·59	38·993		
	5	89·45	268·52	·58	38·494	242·76	2·73
	9	1·18	149·73	·56	38·002		
	13	272·96	30·99	·53	37·521	243·49	2·75
	17	184·78	272·29	·50	37·051		
	21	96·65	153·63	·47	36·596	244·22	2·77
	25	8·56	35·02	·44	36·157		
	29	280·51	276·46	·40	35·736	244·95	2·78
Sept.	2	192·51	157·93	·35	35·337		
	6	104·54	39·44	·31	34·959	245·68	2·80
	10	16·61	280·99	·27	34·609		
	14	288·71	162·56	·22	34·286	246·42	2·81

Greenwich Noon. 1904.		P.	L−O.	B.	Apparent Diameter. Equat. Diam.	Excess over Polar.	Defect of Illum.	*d.*	Q.
		°	°	°	″	″	″	°	°
Sept.	18	337·971	253·242	3·252	48·85	3·15	0·16	6·46	251·64
	22	337·893	252·855	3·253	49·24	3·17	0·13	5·72	251·98
	26	337·803	252·429	3·251	49·56	3·19	0·09	4·93	252·46
	30	337·708	251·959	3·246	49·84	3·21	0·06	4·10	253·17
Oct.	4	337·608	251·481	3·240	50·07	3·23	0·04	3·25	254·30
	8	337·506	250·969	3·231	50·26	3·24	0·02	2·38	256·69
	12	337·402	250·441	3·219	50·37	3·24	0·01	1·49	260·99
	16	337·296	249·903	3·204	50·42	3·25	0·00	0·64	
	20	337·192	249·361	3·188	50·41	3·25	0·00	0·47	
	24	337·091	248·822	3·170	50·34	3·24	0·01	1·29	54·44
	28	336·994	248·293	3·149	50·20	3·23	0·02	2·16	58·70
Nov.	1	336·901	247·780	3·126	50·00	3·22	0·04	3·04	61·65
	5	336·813	247·289	3·103	49·75	3·20	0·06	3·89	64·00
	9	336·731	246·826	3·079	49·45	3·18	0·08	4·71	64·88
	13	336·657	246·399	3·054	49·09	3·16	0·11	5·51	65·46
	17	336·591	246·010	3·028	48·70	3·14	0·14	6·26	65·90
	21	336·533	245·664	3·002	48·26	3·11	0·18	6·97	66·16
	25	336·482	245·365	2·977	47·77	3·08	0·21	7·64	66·40
	29	336·441	245·116	2·951	47·25	3·05	0·24	8·25	66·62
Dec.	3	336·409	244·918	2·926	46·72	3·01	0·27	8·81	66·80
	7	336·385	244·774	2·902	46·17	2·98	0·30	9·32	66·96
	11	336·370	244·685	2·879	45·61	2·94	0·33	9·77	67·10
	15	336·364	244·653	2·858	45·04	2·91	0·35	10·17	67·22
	19	336·366	244·678	2·838	44·46	2·87	0·37	10·51	67 34
	23	336·378	244·757	2·819	43·87	2·83	0·39	10·80	67·45
	27	336·398	244·890	2·802	43·28	2·79	0·40	11·03	67·56
	31	336·428	245·078	2·786	42·70	2·75	0·41	11·20	67·68
1905. Jan.	4	336·466	245·318	2·772	42·12	2·71	0·41	11·33	67·81
	8	336·513	245·610	2·760	41·56	2·68	0·41	11·40	67·94
	12	336·568	245·951	2·749	41·02	2·64	0·40	11·42	68·08
	16	336·632	246·340	2·740	40·49	2·61	0·40	11·40	68·22
	20	336·705	246·775	2·733	39·97	2·57	0·39	11·33	68·37
	24	336·786	247·252	2·727	39·47	2·54	0·37	11·22	68·52
	28	336·877	247·770	2·723	38·99	2·51	0·36	11·07	68·67
Feb.	1	336·977	248·328	2·722	38·53	2·48	0·34	10·87	68·82
	5	337·085	248·923	2·720	38·09	2·45	0·33	10·64	68·97
	9	337·202	249·555	2·720	37·66	2·42	0·31	10·37	69·14

Greenwich Noon.	Longitude of ☿'s Central Meridian. 877°90 I.	870°27 II.	corr. for Phase.	Light-time.	A—O.	B.
1904.	°	°	°	m	″	″
Sept. 18	200·84	44·16	·18	33·994		
22	112·98	285·78	·14	33·734	247·14	2·83
26	25·14	167·42	·11	33·510		
30	297·32	49·08	·07	33·320	247·87	2·84
Oct. 4	209·50	290·74	·05	33·167		
8	121·68	172·40	·03	33·051	248·60	2·86
12	33·85	54·05	+0·01	32·978		
16	306·01	295·69	·00	32·946	249·33	2·87
20	218·15	177·31	·00	32·953		
24	130·26	58·90	−0·01	33·000	250·07	2·89
28	42·33	300·45	·02	33·091		
Nov. 1	314·36	181·96	·04	33·220	250·80	2·90
5	226·35	63·43	·06	33·388		
9	138·29	304·85	·10	33·593	251·54	2·91
13	50·16	186·21	·13	33·839		
17	321·98	67·51	·17	34·118	252·27	2·92
21	233·74	308·75	·21	34·430		
25	145·43	189·92	·26	34·775	253·00	2·94
29	57·05	71·02	·30	35·149		
Dec. 3	328·60	312·05	·34	35·550	253·73	2·95
7	240·09	193·02	·38	35·976		
11	151·51	73·92	·41	36·424	254·45	2·96
15	62·86	314·75	·45	36·891		
19	334·14	195·51	·48	37·374	255·19	2·97
23	245·36	76·21	·51	37·871		
27	156·51	316·85	·53	38·383	255·92	2·98
31	67·60	197·43	·55	38·903		
1905.						
Jan. 4	338·64	77·95	·56	39·430	256·65	2·99
8	249·62	318·42	·56	39·960		
12	160·56	198·84	·57	40·493	257·37	2·99
16	71·45	79·21	·56	41·027		
20	342·29	319·53	·56	41·558	258·10	3·00
24	253·09	199·81	·55	42·085		
28	163·85	80·06	·53	42·606	258·84	3·01
Feb. 1	74·58	320·27	·51	43·119		
5	345·28	200·45	·49	43·621	259·56	3·02
9	255·95	80·60	·47	44·112		

Greenwich Noon. 1904.	P.	L−O.	B.	Apparent Diameter. Equat. Diam.	Excess over Polar.	Defect of Illum.	d.	Q.
	°	°	°	″	″	″	°	°
Feb. 13	337·329	250·219	2·721	37·26	2·39	0·29	10·07	69·31
17	337·465	250·913	2·724	36·87	2·37	0·27	9·74	69·50
21	337·610	251·636	2·727	36·51	2·35	0·25	9·38	69·70
25	337·763	252·386	2·731	36·16	2·33	0·23	9·00	69·92
Mar. 1	337·926	253·161	2·736	35·84	2·31	0·21	8·59	70·15
5	338·098	253·960	2·742	35·54	2·29	0·18	8·15	70·39
9	338·279	254·781	2·749	35·26	2·27	0·16	7·69	70·65
13	338·469	255·621	2·757	34·99	2·25	0·14	7·22	70·93
17	338·668	256·480	2·765	34·75	2·23	0·12	6·73	71·25
21	338·875	257·353	2·774	34·53	2·22	0·10	6·22	71·59
25	339·091	258·241	2·783	34·33	2·21	0·09	5·69	71·97
29	339·315	259·142	2·792	34·15	2·20	0·07	5·15	72·38
Apr. 2	339·547	260·055	+2·802	33·98	2·19	0·06	4·61	72·84

The following is a list of the Greenwich Mean Times when th the illuminated disc, and the intervals between successive passage

Date. 1904.	Passage of Zero Meridian. System I.		System II.		Intervals between Passages. I. 9^h+	II. 9^h+
May	h	m	h	m	m	m
1	8	11·80	2	58·99	50·623	55·802
3	9	24·91	4	38·00		
5	0	47·39	6	17·00		
7	2	0·49	7	56·00		
9	3	13·58	9	34·98		
11	4	26·68	1	18·17		
13	5	39·75	2	57·14		
15	6	52·81	4	36·11		
17	8	5·87	6	15·07	50·608	55·787
19	9	18·91	7	54·00		
21	0	41·32	9	32·92		
23	1	54·31	1	16·04		
25	3	7·30	2	54·93		
27	4	20·28	4	33·80		
29	5	33·24	6	12·66		
31	6	46·19	7	51·50		
June						
2	7	59·14	9	30·33	50·585	55·765
4	9	12·05	1	13·38		
6	0	24·96	2	52·18		

Date. 1904.	Passages of Zero Meridian. System I.		System II.		Intervals between Passages. I. 9^h+	II. 9^h+
June	h	m	h	m	m	m
8	1	47·24	4	30·96		
10	3	0·11	6	9·73		
12	4	12·98	7	48·49		
14	5	25·83	0	27·23		
16	6	38·67	1	10·21		
18	7	51·51	2	48·93	50·563	55·7[illegible]
20	9	4·32	4	27·63		
22	0	26·55	6	6·32		
24	1	39·32	7	44·99		
26	2	52·07	9	23·63		
28	4	4·81	1	6·54		
30	5	17·54	2	45·16		
July						
2	6	30·26	4	23·77		
4	7	42·96	6	2·36	50·537	55·7
6	8	55·64	7	40·94		
8	0	17·77	9	19·49		
10	1	30·41	1	2·32		
12	2	43·01	2	40·84		
14	3	55·66	4	19·34		

nwich Noon.	Longitude of Σ's Central Meridian. 877°90 I.	870°27 II.	corr. for Phase.	Light-time.	A—O.	B.
	°	°	°	m	°	°
eb. 13	166·59	320·73	·44	44·591	260·29	3·02
17	77·22	200·83	·41	45·056		
21	347·82	80·92	·38	45·504	261·02	3·03
25	258·41	320·99	·35	45·936		
Iar. 1	168·98	201·05	·32	46·350	261·75	3·04
5	79·54	81·09	·29	46·744		
9	350·09	321·12	·26	47·118	262·47	3·04
13	260·64	201·15	·23	47·470		
17	171·18	81·17	·19	47·798	263·21	3·05
21	81·72	321·19	·17	48·103		
25	352·26	201·21	·14	48·385	263·93	3·05
29	262·80	81·23	·12	48·644		
pr. 2	173·34	321 26	−0·09	48·880	264·66	+3·05

ed zero-meridians of the two systems will pass the middle of ilitate the determination of intermediate ones :

Passage of Zero Meridian.		Intervals between Passages.		Date.	Passage of Zero Meridian.		Intervals between Passages.	
ystem I.	System II.	I. 9^h+	II. 9^h+	1904.	System I.	System II.	I. 9^h+	II. 9^h+
h m	h m	m	m	Aug.	h m	h m	m	m
5 8·27	5 57·83			23	8 23·44	7 18·83		
6 20 85	7 36·30			25	9 35 70	8 56·97		
7 33·41	9 14·75	50·510	55·689	27	0 57·50	0 39·47		
8 45 95	0 57·50			29	2 9·73	2 17·58		
0 7·97	2 35·93			31	3 21·94	3 55·66		
1 20 49	4 14·33			Sept.				
2 32 98	5 52·71			2	4 34·13	5 33·73		
3 45·46	7 31·10			4	5 46·31	7 11·79		
				6	6 58·47	8 49·84	50·431	55·608
4 57·92	9 9·42			8	8 10·62	0 32·27		
6 10 37	0 52 09			10	9 22·76	2 10·28		
7 22 81	2 30·40	50 483	55·661	12	0 44·40	3 48·29		
8 35·22	4 8·70			14	1 56·52	5 26·28		
9 47 61	5 46·99			16	3 8·63	7 4·26		
1 9 51	7 25·25			18	4 20·73	8 42·23		
2 21 88	9 3 50			20	5 32·82	0 24·60		
3 34 23	0 46·09			22	6 44·89	2 2·56	50·413	55·590
4 46·56	2 24·30			24	7 56·95	3 40·50		
5 58·87	4 2·49			26	9 9·00	5 18·43		
7 11·16	5 40·67	50·457	55·634	28	0 30·64	6 56·36		

Date. 1904.	Passage of Zero Meridian. System I.		System II.		Intervals between Passages. I. 9^h+	II. 9^h+
Sept.	h	m	h	m	m	m
30	1	42·68	8	34·29		
Oct.						
2	2	54·72	0	16·61		
4	4	6·75	1	54·51		
6	5	18·77	3	32·41		
8	6	30·80	5	10·32	50·406	55·583
10	7	42·83	6	48·23		
12	8	54·87	8	26·15		
14	0	16·51	0	8·48		
16	1	28·55	1	46·40		
18	2	40·60	3	24·33		
20	3	52·66	5	2·26		
22	5	4·72	6	40·20		
24	6	16·80	8	18·17	50·419	55·596
26	7	28·91	0	0·56		
28	8	41·04	1	38·56		
30	0	2·73	3	16·58		
Nov.						
1	1	14·92	4	54·63		
3	2	27·10	6	32·69		
5	3	39·31	8	10·78		
7	4	51·54	9	48·90		
9	6	3·80	1	31·41	50·455	55·631
11	7	16·09	3	9·58		
13	8	28·41	4	47·78		
15	9	40·76	6	26·00		
17	1	2·66	8	4·25		
19	2	15·05	9	42·52		
21	3	27·46	1	25·16		
23	4	39·90	3	3·40		
25	5	52·38	4	41·85	50·498	55·676
27	7	4·88	6	20·24		
29	8	17·41	7	58·66		
Dec.						
1	9	29·98	9	37·10		
3	0	52·05	1	19·88		
5	2	4·67	2	58·39		
7	3	17·31	4	36·92		

Date. 1904.	Passage of Zero Meridian. System I.		System II.		Int between I. 9^h+
Dec.	h	m	h	m	m
9	4	29·98	6	15·49	
11	5	42·68	7	54·08	50·542
13	6	55·41	9	32·69	
15	8	8·17	1	15·61	
17	9	20·96	2	54·29	
19	0	43·21	4	32·98	
21	1	56·02	6	11·65	
23	3	8·88	7	50·46	
25	4	21·77	9	29·24	
27	5	34·70	1	12·28	50·584
29	6	47·63	2	51·10	
31	8	0·58	4	29·95	
1905. Jan.					
2	9	13·55	6	8·80	
4	0	35·95	7	47·70	
6	1	48·97	9	26·60	
8	3	2·01	1	9·74	
10	4	15·07	2	48·69	
12	5	28·14	4	27·66	50·615
14	6	41·22	6	6·64	
16	7	54·32	7	45·65	
18	9	7·44	9	24·66	
20	0	29·95	1	7·90	
22	1	43·12	2	46·95	
24	2	56·30	4	26·02	
26	4	9·49	6	5·10	
28	5	22·69	7	44·19	50·641
30	9	35·90	9	23·29	
Feb.					
1	7	49·13	1	6·60	
3	9	2·37	2	45·74	
5	0	24·96	4	24·88	
7	1	38·22	6	4·04	
9	2	51·48	7	43·21	
11	4	4·76	9	22·39	
13	5	18·05	1	5·72	50·657
15	6	31·35	2	44·90	
17	7	44·65	4	24·09	

Date. 1904	Passage of Zero Meridian. System I.		System II.		Intervals between Passages. I. 9^h+	II. 9^h+
	h	m	h	m	m	m
Feb. 19	8	57·96	6	3·38		
21	0	20·61	7	42·58		
23	1	33·94	9	21·79		
25	2	47·26	1	5·15		
27	4	0·60	2	44·37		
Mar. 1	5	13·94	4	23·61	50·669	55·848
3	6	27·29	6	2·85		
5	7	40·64	7	42·10		
7	8	53·99	9	21·35		
9	0	16·68	1	4·76		
11	1	30·04	2	44·02		
13	2	43·40	4	23·28		
15	3	56·76	6	2·55		
17	5	10·12	7	41·82	50·673	55·854
19	6	23·49	9	21·08		
21	7	36·86	1	4·50		
23	8	50·23	2	43·77		
25	0	12·93	4	23·04		
27	1	26·30	6	2·31		
29	2	39·67	7	41·58		
31	3	53·04	9	20·85		
Apr. 2	5	6·41	1	4·27	50·674	55·854

The quantities in the ephemeris are to be interpolated directly for the times for which they are required, the equation of light having been already applied.

The position of *Jupiter's* North Pole is assumed to be R.A. $17^h\ 51^m\ 58^s{\cdot}89$, N.P.D. $25°\ 26'\ 13''{\cdot}5$ at the beginning of 1904, and R.A. $17^h\ 51^m\ 59^s{\cdot}13$, N.P.D. $25°\ 26'\ 14''{\cdot}1$ at the beginning of 1905.

P denotes the position-angle of the northern extremity of *Jupiter's* axis, reckoned eastward from the northernmost point of the disc.

L—O + 180°, A—O + 180° are the jovicentric right ascensions of the Earth and Sun respectively, reckoned in the plane of the planet's equator from O, the point of the vernal equinox of *Jupiter's* northern hemisphere; B, *B* are the jovicentric declinations of the Earth and Sun above the planet's equator.

The adopted values of the diameters at distance 5·20 are: Equatorial $38''{\cdot}419$; Polar $35''{\cdot}945$.

The assumed time for light to traverse the unit distance is $498^s{\cdot}92$, this being the same value as that used by Mr. Marth.

d denotes the jovicentric angle between the Earth and Sun.

Q denotes the position-angle of the point of greatest phase, and is reckoned eastward from the northernmost point of the disc. It also gives the position-angle of the shadows of the satellites measured from the satellites themselves.

If we call B″ the jovigraphical latitude of the centre of the disc, then we can find B″ by the formula:

$$B'' = \sec^2 \epsilon_0\, B, \text{ where } \sec \epsilon_0 = \frac{a}{b} = \frac{15{\cdot}53}{14{\cdot}53}.$$

The longitudes of *Jupiter's* central meridian are computed with unaltered values of the rates of rotation and of the zero-meridians in the two adopted systems. The addition of the

"Corr. for Phase" gives the longitudes of the meridians which bisect the illuminated disc.

The sidereal periods of rotation corresponding to the two adopted systems are $9^h\ 50^m\ 30^s{\cdot}004$, $9^h\ 55^m\ 40^s{\cdot}632$.

The ephemeris has been somewhat abbreviated as compared with those in recent years, being for every four days instead of every two days. Instead of giving every transit of the zero meridian, only one in every two days is given; but any intermediate transit may be readily found by applying to the nearest transit in the table once or twice the interval between successive passages, which interval is tabulated every sixteen days.

I have received several observations of the longitude of the Great Red Spot in System II. from Mr. W. F. Denning and Rev. T. E. R. Phillips, from which I have deduced the following mean values.

Date. 1903.	Longitude. °	No. of Observations.	Date. 1903.	Longitude. °	No. of Observations.
May 28	30·0	2	Sept. 8	32·9	5
June 26	32·0	3	Oct. 23	34·6	4
July 11	31·4	5	Nov. 19	34·4	6
Aug. 14	32·7	8	Dec. 7	34·4	2*

It appears that the rapid diminution in the longitude which took place in 1902 has ceased and been followed by a slow recovery. Rev. T. E. R. Phillips considers that the spot was more conspicuous in 1903 than it has been for some years; it should therefore be carefully watched in case of a revival of activity.

A list of times of elongation of the fifth satellite is given in the *Connaissance des Temps*.

It may be mentioned here that the *Connaissance des Temps* for 1899 and following years gives ephemerides for the satellites of *Mars*, *Saturn*, *Uranus*, and *Neptune* in the same form as those formerly contributed to the *Monthly Notices* by Mr. Marth.

* To these may be added the mean of two observations just received from Mr. Denning, which give 35°·3 on 1904 January 8.

Benvenue, 55 *Ulundi Road*, *Blackheath*, *S.E.*,
1904 *January* 2.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXIV. FEBRUARY 12, 1904. No. 4

ANNUAL GENERAL MEETING.

Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

The Report of the Auditors of the Treasurer's accounts for the year 1903 was read, and is given on p. 258.

The Annual Report of the Council was partly read; see pp. 255 to 387.

The Address was delivered by the President, after which the Gold Medal was handed to His Excellency the American Ambassador, for transmission to Professor G. E. Hale, to whom the Medal had been awarded for his method of photographing the Solar Surface and other astronomical work.

The Society then proceeded to the ballot for Officers and Council for the ensuing year, the names of those elected being given on p. 402.

The thanks of the Meeting were given to the retiring Members of Council, and also to the Auditors of the Treasurer's Accounts and the Scrutineers of the Ballot.

J. D. Bharda, New High School, Bombay, India;
Tyson Crawford, 35 Ludgate Hill, E.C.;
E. Vincent Heward, Hereward Cottage, Westgate-on-Sea, Kent;
Eber Jachin Sharpe, 164 Colum Road, Cardiff, South Wales;

Ernst Spiegel, 27 Fitzjohn's Avenue, Hampstead, N.W. ; and
Philip Edward Vizard, Belsize Lodge, Belsize Lane, Hampstead, N.W.,

were balloted for and duly elected Fellows of the Society.

The following Candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

John Borthwick Dale, M.A., Assistant Professor of Mathematics, King's College, London, "Myosotis," New Malden, Surrey (proposed by Major P. A. MacMahon) ;
Rev. William Charles Eppstein, M.A., Head Master, Reading School, Berks (proposed by Rev. J. M. Bacon) ;
Walter Nuttall, B.A., Schoolmaster, Hallfold, Whitworth, Rochdale (proposed by Rev. E. Wainwright) ; and
William Edward Rolston, Solar Physics Observatory, South Kensington, S.W. (proposed by W. J. S. Lockyer).

Report of the Council to the Eighty-Fourth Annual General Meeting of the Society.

The following table shows the progress and present state of the Society :—

	Compounders	Annual Subscribers	Total Fellows	Associates	Patron and Hon. Members	Grand Total
1902 December 31	257	383	640	44	1	685
Since elected	+ 3	+ 26	...	+ 4	+ 2	...
Deceased	- 8	- 10	...	- 1	...	...
Resigned	...	- 8	...	...	...	...
Removals	- 14	- 14	...	...	...	..
Expelled	...	...	...	...	...	...
1903 December 31	266	377	643	47	3	693

RECEIVED.

	£	s.	d.	£
Balances, 1903 January 1:—				
At Bankers', as per Pass-book	231	16	9	
Country Cheque not credited till 1903 ...	8	10	6	
At Bankers', on deposit	300	0	0	
				540
Dividends on £1,250 Metropolitan 3-per-cent. Stock	35	12	7	
Dividends on £932 19 0 Metropolitan 2½-per-cent. Stock	22	3	1	
Dividends on £3,400 East Indian Railway 3-per-cent. Debenture Stock	96	17	11	
Dividends on £3,200 London and North-Western Railway 3-per-cent. Debenture Stock	90	8	0	
Dividends on £4,000 Midland Railway 2½-per-cent. Debenture Stock	94	3	4	
Dividends on £1,860 Gas Light and Coke Co. 3-per-cent. Debenture Stock	52	10	11	
Dividends on £1,650 Commercial Gas Co. 3-per-cent. Debenture Stock	47	0	6	
Dividends on £500 Lancashire and Yorkshire Railway 3-per-cent. Preference Stock... ...	7	1	10	
Interest on £300 on Deposit at Bankers'	2	0	3	
				447 18
Received on account of Subscriptions:—				
Arrears	134	8	0	
Annual Contributions for 1903	554	8	0	
„ „ in Advance	10	10	0	
Admission Fees	56	14	0	
First Contributions	39	18	0	
				795 18
Composition Fees				260 8
Sales of Publications, &c.:—				
At Williams and Norgate's, 1902	7	13	11	
At Society's Rooms, 1903	58	0	6	
Sales of Photographs	38	11	0	
				104 5
Income Tax refunded by Commissioners of Inland Revenue				28 13
Due to Assistant Secretary on account of Turnor and Horrox Fund				2 8
				£2,179 18

Examined and found correct, 1904 Jan. 5:
W. J. S. Lockyer,
C. Thwaites,
A. C. D. Crommelin.

Astronomical Society, from 1903 *January* 1 *to December* 31.

PAID.

	£	s.	d.	£	s.	d.
Assistant Secretary: Salary	250	0	0			
" " for assistance in editing Society's Publications ...	50	0	0			
				300	0	0
House Duty	2	12	6			
Fire Insurance	9	9	6			
				12	2	0
Printing, plates, &c., *Monthly Notices* (Spottiswoode & Co.)	513	18	10			
Photo-engraving, *Monthly Notices* (Dent & Co.) ...	6	2	9			
Printing, &c., Appendix to *Memoirs* (Harrison & Sons)	16	13	6			
Plates for Appendix to *Memoirs* (London Stereoscopic Co.)	10	9	6			
Printing, &c., Appendix to *Monthly Notices* (Harrison & Sons)	3	3	6			
Printing, &c., List of Fellows and Miscellaneous (Spottiswoode & Co.)	23	9	6			
				573	17	7
Computation of Ephemerides in *Monthly Notices* ...				15	0	0
Turnor and Horrox Funds: Purchases for Library	22	3	8			
Binding books in Library	28	19	10			
				51	3	6
Reproduction of Photographs, Hinton & Co. ...				30	12	8
Cataloguing astronomical literature for the International Catalogue of Scientific Literature ...				30	0	0
Clerk's Wages	52	0	0			
Postage and Telegrams	83	15	5			
Carriage of Parcels, &c.	2	14	11			
Stationery (Spottiswoode & Co.)	9	0	6			
Sundry Stationery and Office Expenses	3	15	5			
				151	6	3
Expenses of Meetings	19	14	0			
Lantern Expenses	7	12	6			
Time Signal, Rental of Wire	5	0	0			
				32	6	6
Special Allowance to Assistant Secretary during progress of work carried out by H.M. Office of Works				36	0	0
House Expenses	61	18	5			
Coals and Gas	52	12	2			
Electric Light Expenses	5	16	11			
Sundry Fittings and Repairs	19	5	5			
Sundries	5	12	8			
				145	5	7
Partition in Instrument Room	17	10	0			
Decorating	9	19	6			
Restoring Oil Painting	2	18	6			
				30	8	0
Lee and Janson Fund: grant to Mrs. Martin				20	0	0
Purchase of £500 Lancashire and Yorkshire Railway 3-per-cent. Preference Stock at 93¼, including brokerage, &c.... ...				471	8	0
Cheque-book, Deductions on Cheques, &c. ...				1	1	4
Cheques outstanding, 1902 Dec. 31				16	19	6
Repayment to Assistant Secretary of amounts due 1902 Dec. 31 on Account of Petty Cash and Turnor and Horrox Fund				2	1	9
Balances, 1903 December 31:—						
At Bankers', as per Pass-book	246	9	5			
Country Cheque not credited till 1904 ...	9	16	0			
In hand of Assistant Secretary on Petty Cash Account	4	10	10			
				260	16	3
				£2,179	18	11

Report of the Auditors.

We have examined the Treasurer's accounts of receipts and expenditure for the year 1903, and have found and certified the same to be correct. The cash in hand on December 31, 1903, including the balance at the bankers', &c., amounted to 260*l.* 16*s.* 3*d.*

The invested property of the Society has been increased by the purchase of 500*l.* Lancashire and Yorkshire Railway 3-per-cent. Consolidated Preference Stock.

The books, instruments, and other effects in the possession of the Society have been examined, and they appear to be in a satisfactory condition.

We have laid on the table a list of the names of those Fellows who are in arrear for sums due at the last Annual General Meeting, with the amount due against each Fellow's name.

(*Signed*) W. J. S. Lockyer.
C. Thwaites.
A. C. D. Crommelin.

1904 *January* 5.

Trust Funds.

The Turnor Fund: A sum of £464 18*s.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be used in the purchase of books for the Library.

The Horrox Memorial Fund: A sum of £103 6*s.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be used in the purchase of books for the Library.

The Lee and Janson Fund: A sum of £334 10*s.* 9*d.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be given by the Council to the widow or orphan of any deceased Fellow of the Society who may stand in need of it.

The Hannah Jackson (née Gwilt) Fund: A sum of £309 18*s.* 6*d.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be given in Medals or other awards, in accordance with the terms of the Trust.

Assets and Present Property of the Society, 1904 January 1.

	£	s.	d.	£	s.	d.
Balances, 1903 December 31:—						
At Bankers', as per Pass-book	246	9	5			
Country Cheque not credited	9	16	0			
In hand of Assistant Secretary on Petty Cash Account	4	10	10			
	260	16	3			
Less due to Assistant Secretary on account of Turnor and Horrox Fund...	2	8	3			
				258	8	0
Due on account of Subscriptions:—						
3 Contributions of 5 years' standing	31	10	0			
3 „ 4 „	25	4	0			
6 „ 3 „	37	16	0			
32 „ 2 „	134	8	0			
64 „ 1 „	134	8	0			
	363	6	0			
Less 5 Contributions paid in advance ...	10	10	0			
				352	16	0
Due for Photographs sold				2	11	0
Due from Messrs. Williams and Norgate for sales of Publications during 1903				9	4	5

£3,400 East Indian Railway 3-per-cent. Debenture Stock, including the Turnor Fund, the Horrox Memorial Fund, the Lee and Janson Fund, and the Hannah Jackson (née Gwilt) Fund.

£3,200 London and North-Western Railway 3-per-cent. Debenture Stock.

£4,000 Midland Railway 2½-per-cent. Debenture Stock.

£1,860 Gas Light and Coke Co. 3-per-cent. Debenture Stock.

£1,650 Commercial Gas Company 3-per-cent. Debenture Stock.

£500 Lancashire and Yorkshire Railway 3-per-cent. Consolidated Preference Stock.

£1,250 Metropolitan 3-per-cent. Stock.

£932 19 0 Metropolitan 2½-per-cent. Stock.

Astronomical and other Manuscripts, Books, Prints, and Instruments.

Furniture, &c.

Stock of Publications of the Society.

Three Gold Medals.

Stock in hand of volumes of the *Memoirs* :—

Vol.	At Society's Rooms	At Williams & Norgate's	Vol.	At Society's Rooms	At Williams & Norgate's
I. Part 1	8	...	XXXIII.	83	...
I. Part 2	42	...	XXXIV.	88	...
II. Part 1	51	3	XXXV.	52	...
II. Part 2	16	3	XXXVI.	125	8
III. Part 1	65	1	XXXVII. Part 1	268	7
III. Part 2	81	1	XXXVII. Part 2	214	8
IV. Part 1	76	3	XXXVIII.	200	1
IV. Part 2	89	3	XXXIX. Part 1	164	2
V.	62	3	XXXIX. Part 2	171	2
VI.	50	6	XL.	182	1
VII.	89	3	XLI.	317	1
VIII.	61	3	XLII.	158	3
IX.	63	3	XLIII.	157	...
X.	63	...	XLIV.	136	1
XI.	71	...	XLV.	172	...
XII.	77	...	XLVI.	147	2
XIII.	68	...	XLVII. Part 1	2	...
XIV.	292	...	XLVII. Part 2	18	...
XV.	56	...	XLVII. Part 3	2	...
XVI.	78	1	XLVII. Part 4	8	...
XVII.	60	1	XLVII. Part 5	8	...
XVIII.	60	...	XLVII. Part 6	9	...
XIX.	66	...	XLVII.	190	1
XX.	61	...	XLVIII. Pt. 1	222	...
XXI. Part 1	182	...	XLVIII. Pt. 2	223	1
XXI. Part 2	49	...	XLIX. Part 1	347	1
XXI.	27	...	XLIX. Part 2	234	2
XXII.	83	...	L.	220	2
XXIII.	67	...	LI.	243	...
XXIV.	75	1	LII.	274	2
XXV.	80	...	LIII.	274	...
XXVI.	90	1	LIV. App. I.	577	...
XXVII.	341	1	App. II.	591	...
XXVIII.	291	...	App. III.	630	...
XXIX.	333	1	App. IV.	678	...
XXX.	90	1	App. V.	682	...
XXXI.	66	...	Index to *Memoirs*	597	2
XXXII.	70	...			

Stock in hand of volumes of the *Monthly Notices*:—

Vol.	At Society's Rooms	At Williams & Norgate's	Vol.	At Society's Rooms	At Williams & Norgate's
I.	51	...	XXXIV.	65	1
II.	53	...	XXXV.	51	...
III.	...	...	XXXVI.	25	1
IV.	...	...	XXXVII.	30	3
V.	...	...	XXXVIII.	95	1
VI.	38	...	XXXIX.	95	...
VII.	2	...	XL.	103	2
VIII.	149	1	XLI.	103	5
IX.	23	3	XLII.	111	1
X.	170	1	XLIII.	108	2
XI.	181	...	XLIV.	110	2
XII.	104	1	XLV.	113	1
XIII.	176	2	XLVI.	107	...
XIV.	175	3	XLVII.	121	2
XV.	167	1	XLVIII.	117	...
XVI.	152	1	XLIX.	108	7
XVII.	164	...	L.	108	9
XVIII.	241	...	LI.	109	6
XIX.	51	...	LII.	106	10
XX.	31	...	LIII.	108	13
XXI.	16	...	LIV.	108	13
XXII.	30	...	LV.	119	...
XXIII.	17	...	LVI.	119	2
XXIV.	22	...	LVII.	122	2
XXV.	13	...	LVIII.	120	...
XXVI.	9	...	LIX.	128	2
XXVII.	3	...	LX.	131	3
XXVIII.	70	...	LXI.	128	3
XXIX.	50	...	LXII.	132	4
XXX.	61	2	LXIII.	137	4
XXXI.	90	...	1st Index ...	540	1
XXXII.	106	5	2nd „ ...	788	...
XXXIII.	86	...			

Library Catalogue	538	2
„ „ Supplement	420	...

In addition to the above volumes of the *Monthly Notices*, the

Society has a considerable stock of separate numbers of nearly all the volumes. With the exception, however, of Vols. XXXVI. to LXIII., no complete volumes can be formed from the separate numbers in stock.

Celestial Photographs.

The following is a list of reproductions of Celestial Photographs published by the Royal Astronomical Society for sale to the Fellows :—

R.A.S. Ref. No.	Subject.	Photographed by
1	Total Solar Eclipse, 1889 January 1	W. H. Pickering
2	Total Solar Eclipse, 1893 April 16	J. M. Schaeberle
3	Total Solar Eclipse, 1886 August 29	A. Schuster
4	Nebulæ in the *Pleiades*	Isaac Roberts
5	Nebula M 74 *Piscium* (N.G.C. 628)	Isaac Roberts
6	Great Nebula in *Orion*	Isaac Roberts
7	Milky Way near M 11	E. E. Barnard
8	Milky Way near Cluster in *Perseus*	E. E. Barnard
9	Comet *c* 1893 IV. (Brooks), 1893 October 21	E. E. Barnard
10	Comet *a* 1892 I. (Swift), 1892 April 7	E. E. Barnard
11	Nebula about η *Argûs*	David Gill
12	Portion of Moon (Hyginus-Albategnius)	Lœwy and Puiseux
13	Comet *c* 1893 IV. (Brooks), 1893 October 22	E. E. Barnard
14	Comet *c* 1893 IV. (Brooks), 1893 October 20	E. E. Barnard
15	Comet *c* 1893 IV. (Brooks), 1893 November 10	E. E. Barnard
16	Comet *a* 1892 I. (Swift), 1892 April 26	E. E. Barnard
17	Comet *f* 1892 III. (Holmes), 1892 November 10	E. E. Barnard
18	Comet *a* 1892 I. (Swift), 1892 April 18	E. E. Barnard
19	Portion of Moon (Alps, Apennines, &c.)	Lœwy and Puiseux
20	Nebula in *Andromeda*	Isaac Roberts
21	*Jupiter*, 1892 September 26	Lick Observatory
22	Cluster M 13 *Herculis* (N.G.C. 6205)	W. E. Wilson
23	Total Solar Eclipse, 1893 April 16 (5 sec.)	J. Kearney
24	Total Solar Eclipse, 1893 April 16 (20 sec.)	J. Kearney
25	The Moon (Age $7^d\ 3^h$)	Lick Observatory
26	The Moon (Age $12^d\ 6\frac{1}{2}^h$)	Lick Observatory
27	The Moon (Age $16^d\ 18^h$)	Lick Observatory
28	The Moon (Age $23^d\ 8^h$)	Lick Observatory

R.A.S. Ref. No.	Subject.	Photographed by
29	The Sun, 1892 February 13	Roy. Obs., Greenwich
30	The Sun, 1892 July 8	Roy. Obs., Greenwich
31	Portion of Moon (Region of Maginus)	Lœwy and Puiseux
32	The Moon (Age 14^d 1^h)	Lick Observatory
33	Portion of Moon (Ptolemæus, &c.)	Lick Observatory
34	Portion of Moon (Mare Serenitatis)	Lick Observatory
35	Portion of Moon (Clavius, Licetus, &c.)	Lick Observatory
36	Portion of Moon (Regiomontanus, &c.)	Lick Observatory
37	Portion of Moon (Tycho, Thebit, &c.)	Lick Observatory
38	Portion of Moon (Theophilus, &c.)	Lick Observatory
39	Total Solar Eclipse, 1896 August 9 (3 sec.)	S. Kostinsky
40	Total Solar Eclipse, 1896 August 9 (26 sec.)	A. Hansky
41	Cluster M 56 *Lyræ* (N.G.C. 6779)	
42	Nebulæ M 81, 82 *Ursæ Majoris* (N.G.C. 3031, 3034)	
43	Cluster M 56 *Lyræ* (enlarged) (N.G.C. 6779)	
44	Solar Corona, 1871 December 12, Baikul	H. Davis
45	Solar Corona, 1875 April 6, Siam	Lockyer and Schuster
46	Solar Corona, 1878 July 29, Wyoming	W. Harkness
47	Solar Corona, 1882 May 17, Egypt	Abney and Schuster
48	Solar Corona, 1883 May 6, Caroline Island	Lawrance and Woods
49	Solar Corona, 1885 September 9, Wellington, N.Z.	Radford
50	Solar Corona, 1886 August 29, Grenada, W.I.	A. Schuster
51	Solar Corona, 1887 August 19, Japan	M. Sugiyama
52	Solar Corona, 1889 January 1, California	W. H. Pickering
53	Solar Corona, 1889 December 22, Cayenne	J. M. Schaeberle
54	Solar Corona, 1893 April 16, Fundium	J. Kearney
55	Solar Corona, 1893 April 16, Brasil	A. Taylor
56	Great Nebula in *Orion*	W. E. Wilson
57	Dumb-bell Nebula, *Vulpeculæ* (N.G.C. 6853)	W. E. Wilson
58	Spiral Nebula, *Canes Venatici* (N.G.C. 5194)	W. E. Wilson
59	Ditto (enlarged) (N.G.C. 5194)	W. E. Wilson
60	Annular Nebula, *Lyræ* (N.G.C. 6720)	W. E. Wilson
61	Meteor Trail and Comet Brooks, 1893 November 13	E. E. Barnard
62	Total Solar Eclipse, 1898 January 22 (5 sec.)	W. H. M. Christie
63	Total Solar Eclipse, 1898 January 22 (20 sec.)	W. H. M. Christie
64	Solar Corona, 1896 August 9, Novaya Zemlya	G. Baden-Powell
65	Solar Corona, 1898 January 22, Pulgaon, India	E. H. Hills
66	Nebula in *Andromeda*	Roy. Obs., Greenwich
67	Spectrum of Sun's limb, 1898 January 22	E. H. Hills

R.A.S. Ref. No.	Subject.	Photographed by
68	Annular Nebula, *Lyra* (N.G.C. 6720)	Lick Observatory
69	Dumb-bell Nebula, *Vulpecula* (N.G.C. 6853)	Lick Observatory
70	Spiral Nebula, *Canes Venatici* (N.G.C. 5194-5)	Lick Observatory
71	Spiral Nebula, *Ursa Major* (N.G.C. 5457)	Lick Observatory
72	Trifid Nebula, *Sagittarius* (N.G.C. 6514)	Lick Observatory
73	Great Nebula in *Orion*	Lick Observatory
74	Cluster M 13 *Herculis* (N.G.C. 6205)	Lick Observatory
75	Solar Surface with Faculæ	G. E. Hale
76	Faculæ and Prominences	G. E. Hale
77	Total Solar Eclipse, 1898 Jan. 22 ($\frac{3}{4}$ sec.)	W. H. M. Christie
78	Nebula H V. 14 *Cygni* (N.G.C. 6992)	W. E. Wilson
79	Portion of Moon (Theophilus, &c.)	Yerkes Observatory
80	Total Solar Eclipse, 1900 May 28 (30 sec.)	E. E. Barnard
81	Comet 1901 I., 1901 May 4	Roy. Obs., Cape of G. H.
82	Comet 1901 I., 1901 May 6	Roy. Obs., Cape of G. H.
83	Comet 1901 I., 1901 May 9	Perth Obs., W. Australia
84	Solar Surface with Faculæ	H. Deslandres
85	Solar Prominences	H. Deslandres
86	Nebula about Nova *Persei*, 1901 September 20	G. W. Ritchey
87	Nebula about Nova *Persei*, 1901 November 13	G. W. Ritchey
88	Total Solar Eclipse, 1901 May 18 (10 sec.)	F. W. Dyson
89	Total Solar Eclipse, 1901 May 18 (40 sec.)	F. W. Dyson
90	Comet *b* 1902 III. (Perrine), 1902 Sept. 29	Roy. Obs., Greenwich
91	Portion of Moon (Mare Serenitatis, &c.)	Yerkes Observatory
92	Portion of Moon (Rough Crater Region, Mare Nubium)	Yerkes Observatory
93	Portion of Moon (Tycho, Theophilus, &c.)	Yerkes Observatory
94	Portion of Moon (Bullialdus to Copernicus)	Yerkes Observatory
95	Portion of Moon (Copernicus)	Yerkes Observatory
96	Great Nebula in *Orion*	Yerkes Observatory
97	Great Nebula in *Orion* (Central portion)	Yerkes Observatory
98	Nebula in *Andromeda*	Yerkes Observatory
99	Nebula in *Cygnus* (N.G.C. 6960)	Yerkes Observatory
100	Nebula in *Cygnus* (N.G.C. 6992)	Yerkes Observatory
101	Cluster M 13 *Herculis* (N.G.C. 6205)	Yerkes Observatory
102	Cluster M 13 *Pegasi* (N.G.C. 7078)	Yerkes Observatory
103	Solar Surface with Faculæ	Yerkes Observatory
104	The Moon, 1900 April 5	P. Puiseux
105	The Moon, 1902 November 13	P. Puiseux
106	The Moon, 1903 February 6	P. Puiseux
107	The Moon, 1903 September 12	P. Puiseux

Nos. 44–55 and Nos. 64 and 65 form a series of corona photographs, oriented and reduced to the same scale.

The above photographs are now on sale to Fellows as prints, either platinotype or aristotype, mounted on sunk cut-out mounts, measuring 12 inches by 10 inches, and also as lantern slides. Nos. 44–55 and Nos. 64 and 65 are also supplied as transparencies, 6¼ inches square.

Price of prints, 1*s.* 6*d.* each; lantern slides, 1*s.* each; packing and postage extra.

Unmounted prints, 1*s.* each, can be obtained to order.

Transparencies, 6¼ inches square (Nos. 44–55 and Nos. 64 and 65), 3*s.* 6*d.* each.

Orders to be addressed to W. H. Wesley, Burlington House, London, W. In ordering prints or slides the R.A.S. Reference No. only need be quoted, but in the case of prints it should be stated whether platinotypes or aristotypes are required.

Instruments belonging to the Society.

A brief description of the chief instruments and other particulars relating to them will be found in *Monthly Notices*, vol. xxxvi. p. 126.

No. 1. The *Harrison* clock.
„ 2. The *Owen* portable circles, by Jones.
„ 3. The *Beaufoy* circle.
„ 4. The *Beaufoy* transit instrument.
„ 5. The *Herschel* 7-foot telescope.
„ 6. The *Greig* universal instrument, by Reichenbach and Ertel. The transit telescope, by Utzschneider and Fraunhofer, of Munich.
„ 7. The *Smeaton* equatorial.
„ 8. The *Cavendish* apparatus.
„ 9. The 7-foot Gregorian telescope (late Mr. Shearman's).
„ 10. The variation transit instrument (late Mr. Shearman's).
„ 11. The universal quadrat, by Abraham Sharp.
„ 12. The *Fuller* theodolite.
„ 13. The standard scale, by Troughton and Simms.
„ 14. The *Beaufoy* clock, No. 1.
„ 15. The *Beaufoy* clock, No. 2.
„ 16. The *Wollaston* telescope.
„ 17. The *Lee* circle.
„ 18. The *Sharpe* reflecting circle.
„ 19. The *Brisbane* circle.
„ 20. The *Baker* universal equatorial.
„ 21. The *Reade* transit.
„ 22. The *Matthew* equatorial, by Cooke.

No. 23. The *Matthew* transit instrument.
„ 24. The *South* transit instrument.
„ 25. A sextant, by Bird (formerly belonging to Captain Cook).
„ 26. A globe showing the precession of the equinoxes.

The *Sheepshanks* collection :—

„ 27. (1) 30-inch transit instrument, by Simms, with level and two iron stands.
„ 28. (2) 6-inch transit theodolite, with circles divided on silver; reading microscopes, both for altitude and azimuth; cross and siding levels; magnetic needle; plumb-line; portable clamping foot and tripod stand.
„ 29. (3) Equatorial stand and clock movement for $4\frac{6}{10}$-inch telescope (telescope lost); double-image micrometer; two wire micrometers; object-glass micrometer.
„ 30. (4) $3\frac{1}{4}$-inch achromatic telescope, with equatorial stand; double-image micrometer; one terrestrial and three astronomical eyepieces.
„ 31. (5) $2\frac{3}{4}$-inch achromatic telescope of $28\frac{1}{4}$-inch focal length, with stand; one terrestrial and three astronomical eyepieces.
„ 33. (7) 2-foot navy telescope.
„ 34. (8) Transit instrument of 45 inches focal length, with iron stand and also Y's for fixing to stone piers; two axis levels.
„ 35. (9) Repeating theodolite, by Ertel, with folding tripod stand.
„ 36. (10) 8-inch pillar sextant, by Troughton, divided on platinum, with counterpoise stand and artificial horizon.
„ 37. (11) Portable zenith telescope and stand, $2\frac{3}{4}$-inch aperture and 45 inches focal length; 10-inch horizontal circle and 8-inch vertical circle, reading to 10″ by two verniers to each circle.
„ 38. (12) 18-inch Borda repeating circle, by Troughton, $2\frac{1}{8}$-inch aperture and 24 inches focal length; the circles divided on silver, the horizontal circle being read by four verniers, and the vertical circle by three verniers, each to 10″.
„ 39. (13) 8-inch vertical repeating circle, with diagonal telescope, by Troughton and Simms; circle divided on silver, reading to 10″; a 5-inch circle at eye-end, reading to single minutes; horizontal circle 9 inches diameter in brass to single minutes.
„ 40. (14) A set of surveying instruments, consisting of a 12-inch theodolite for horizontal angles only, reading to 10″; two sets of adjusting plates; tripod stand with enclosed telescope; heavy stand for theodolite; Y-piece of level; two large and three small ground-glass bubbles divided; level collimator, object-glass $1\frac{5}{8}$-inch diameter

and 16 inches focal length; micrometer eyepiece, comb, and wires; mercury bottle and trough.

No. 41. (15) Level collimator, with object-glass 1¾-inch diameter and 16 inches focal length; stand, rider-level, and fittings.

„ 42. (16) 10-inch reflecting circle by Troughton, reading by three verniers to 20″; counterpoise stand; artificial horizon, with mercury; two tripod stands.

„ 43. (17) Hassler's reflecting circle, by Troughton, with counterpoise stand.

„ 44. (18) 6-inch reflecting and repeating circle, by Troughton and Simms, contained in three boxes, two of which form stands. Circle divided on silver, reading to single minutes; two inside arcs divided to single degrees, 150 degrees on each side; artificial horizon and mercury.

„ 45. (19) 5-inch reflecting and repeating circle, by Lenoir, of Paris.

„ 46. (20) Reflecting circle, by Jecker, of Paris, 11 inches in diameter, with one vernier reading to 15″.

„ 47. (21) Box sextant; reflecting plane and level.

„ 48. (22) Prismatic compass, by Troughton and Simms.

„ 49. (23) Mountain barometer.

„ 50. (24) Prismatic compass, by Thomas Jones, mounted with a cylindrical lens.

„ 51. (25) Ordinary 4½-inch compass with needle.

„ 52. (26) Dipping needle, by Robinson.

„ 53. (27) Compass needle, mounted for variation.

„ 54. (28) Magnetic intensity needle, by Meyerstein, of Göttingen; a strongly fitted brass box with heavy magnet; filar suspension.

„ 55. (29) Box of magnetic apparatus.

„ 56. (30) Hassler's reflecting circle, by Troughton; a 10½-inch reflecting and repeating circle, with stand and counterpoise, divided on platinum with two movable and two fixed indices; four verniers reading to 10″.

„ 57. (31) Box sextant and glass plane artificial horizon, by Troughton and Simms.

„ 58. (32) Plane 2½-inch speculum, artificial horizon and stand.

„ 59. (33) 2½-inch circular level horizon, by Dollond.

„ 60. (34) Artificial horizon, roof, and trough; the trough 8½ by 4½ inches; tripod stand.

„ 61. (35) Set of drawing instruments, consisting of 6-inch circular protractor and common protractor, T-square; one beam compass.

„ 62. (36) A pantograph.

„ 63. (37) A noddy.

„ 64. (38) A small Galilean telescope with object-glass of rock crystal.

„ 65. (39) Five levels.

No. 66. (40) 18-inch celestial globe.
„ 67. (41) Varley stand for telescope.
„ 69. (43) Telescope, with object-glass of rock crystal.
„ 71. Portable altazimuth tripod.
„ 72. Four polarimeters.
„ 74. Registering spectroscope, with one large prism.
„ 76. Two five-prism direct-vision spectroscopes.
„ 78. $9\frac{1}{4}$-inch silvered-glass reflector and stand, by Browning.
„ 79. Spectroscope.
„ 80. A small box, containing three square-headed Nicol's prisms ; two Babinet's compensators ; two double-image prisms ; three Savarts ; one positive eyepiece, with Nicol's prism ; one dark wedge.
„ 81. A back-staff, or Davis' quadrant.
„ 82. A nocturnal or star dial.
„ 83. An early non-achromatic telescope, of about 3 feet focal length, in oak tube, by Samuel Scatliffe, London.
„ 84. A Hollis observing chair.
„ 85. Double-image micrometer, by Troughton and Simms.
„ 86. $4\frac{1}{2}$-inch Gregorian reflecting telescope, by Short, with altazimuth stand and 6-inch altitude and azimuth circles and two eyepieces.
„ 87. $3\frac{1}{4}$-inch Gregorian reflecting telescope with wooden tripod stand.
„ 88. Pendulum, with 5-foot brass suspension rod, working on knife-edges, by Thomas Jones.
„ 89. A Rhabdological Abacus. A contrivance invented by Mr. H. Goodwyn, consisting of a box filled with compartments, in which are square rods covered with numbers, which can be arranged so as to facilitate the labour of multiplying high numbers.
„ 90. An Arabic celestial globe of bronze, $5\frac{3}{4}$ inches in diameter.
„ 91. Astronomical time watch-case, by Professor Chevallier.
„ 92. 2-foot protractor, with two movable arms, and vernier.
„ 93. Beam compass, in box.
„ 94. 2-foot navigation scale.
„ 95. Stand for testing measures of length.
„ 96. Artificial planet and star, for testing the measurement of a fixed distance at different position-angles.
„ 97. 12-cell Leclanché battery.
„ 98. 2-foot 6-inch navy telescope, with object-glass $2\frac{1}{2}$ inches, by Cooke, with portable wooden tripod stand.
„ 99. 12-inch transit instrument, by Fayrer and Son, with level and portable stand.
„ 100. 9-inch transit instrument, with level and iron stand.
„ 101. Small equatorial sight instrument, by G. Adams, London.
„ 102. Sun-dial, by Troughton.
„ 103. Sun-dial, by Casella.

No. 104. Sun-dial.
„ 105. Box sextant, by Troughton and Simms.
„ 106. Prismatic compass, by Schmalcalder, London.
„ 107. Compass, by C. Earle, Melbourne.
„ 108. Prismatic compass, by Negretti and Zambra.
„ 109. Dipleidoscope, by E. Dent.
„ 110. Abney level, by Elliott.
„ 111. Pocket spectroscope, by Browning.
„ 112. Universal sun-dial.
„ 113. Double sextant, by Jones.
„ 114. Two models, illustrating the effects of circular motions.
„ 115. A cometarium.
„ 117. Two old sun-dials.
„ 118. A 10½-inch sixteenth-century celestial globe, on bronze tripod stand.
„ 119. Specimens of diffraction gratings, by Prof. W. A. Rogers.
„ 120. A 6-prism spectroscope, by Browning.
„ 121. Spitta's improved maximum and minimum thermometer.
„ 122. A 6-inch speculum, with flat; the speculum said to be by Sir W. Herschel, and re-figured by Sir J. Herschel.
„ 123. A 6-inch refracting telescope, by Grubb, with 3 eyepieces.
„ 124. Position micrometer, by Cooke.
„ 125. A 6-inch refracting telescope, by Simms, with eyepieces and solar diagonal.
„ 126. 3½-inch portable refracting telescope, by Tulley, with tripod stand.
„ 127. Globe representing the visible surface of the Moon, by John Russell, R.A. (1797).
„ 128. Bichromate battery and Ruhmkorff coil.
„ 129. Slater's improved armillary sphere.
„ 130. 10-inch brass pillar sextant with counterpoise stand, by Troughton.
„ 131. Double box sextant, by Cary.
„ 132. Equatorially mounted camera with 2½-inch portrait lens and telephotographic enlarging lens by Dallmeyer; iron pillar.
„ 133. 3½-inch equatorial by Ross, with tall tripod stand, equatorial mounting, eyepieces, and micrometer.
„ 134. Old transit instrument, 2-inch aperture and 3 feet focal length (without stand), formerly belonging to Dr. Longfield, of Cork.
„ 135. Globe of Mars, by E. M. Antoniadi.
„ 136. A small universal instrument by W. and S. Jones, London; the telescope 1½-inch aperture and 15 inches focal length. [Presented by Miss Moore.]
„ 137. Polar siderostat by Hilger, with 4½-inch mirrors. [Presented by Mr. Alexr. Foote.]

No. 138. Transit instrument, aperture $2\frac{3}{4}$-inch, with collimator and stand. [Presented by Mrs. Cross.]

„ 139. Transit instrument, aperture $1\frac{1}{2}$-inch, with portable stand. [Presented by Mrs. Cross.]

„ 140. $3\frac{1}{2}$-inch object-glass and tube. [Presented by Mrs. Cross.]

„ 141. 9-inch Newtonian reflector and equatorial stand. [Presented by Mrs. Cross.]

Besides the above, there is the following apparatus available for eclipse work :—

4 Slits for Spectroscope.
Abney doublet lens used in photographing the corona.
2 Dallmeyer negative enlarging lenses.
Cœlostat with 16-inch plane mirror.

The following instruments are lent, during the pleasure of the Council, to the undermentioned persons :—

No. 6. The Greig universal instrument, to Mr. W. Heath.

„ 23. The *Matthew* transit, to Captain W. Noble.

„ 29. (3) Equatorial mounting, clock, &c., to the Rev. C. D. P. Davies.

„ „ Wire micrometer (No. 2), to the Rev. C. D. P. Davies.

„ 30. (4) $3\frac{1}{4}$-inch equatorial and stand, to Mr. M. E. J. Gheury.

„ „ Double-image micrometer, to the Rev. W. J. B. Roome.

„ 31. (15) $2\frac{3}{4}$-inch telescope (object-glass only), to the Rev. C. D. P. Davies.

„ 36. (10) Sextant, stand, &c., to Mr. Stanley Williams.

„ 37. (11) Zenith telescope (object-glass only), to the Rev. C. D. P. Davies.

„ 45. (19) Horizon, roof and mercury bottle, to Mr. Stanley Williams.

„ 69. (43) Telescope with rock-crystal object-glass, to Sir W. Huggins.

„ 98. 2-foot 6-inch navy telescope, to the Rev. J. M. Bacon.

„ 119. Specimens of diffraction gratings, to Dr. Jas. Hunter.

„ 120. 6-prism spectroscope, to Dr. Jas. Hunter.

„ 123. 6-inch telescope, by Grubb (object-glass only), to Mr. W. E. Wilson.

„ 126. $3\frac{1}{2}$-inch portable telescope, by Tulley, to Mr. J. H. Bell.

„ 130. 10-inch pillar sextant, to Mr. F. Robbins.

„ 133. $3\frac{1}{4}$-inch equatorial, by Ross, to Dr. A. W. Roberts.

„ 137. $4\frac{1}{8}$-inch polar siderostat, by Hilger, to Dr. Jas. Hunter.

„ 138. $2\frac{3}{4}$-inch transit and stand, to the Rev. E. Goetz.

No. 139. Transit instrument and stand, to the Rev. C. L. Tweedale.

„ 140. $3\frac{1}{2}$-inch object-glass and tube, to the Rev. C. L. Tweedale.

The Gold Medal.

The Council have awarded the Society's Gold Medal to Professor G. E. Hale for his method of photographing the solar surface and other astronomical work. The President will lay before the Society the grounds upon which the award has been founded.

Publications of the Society.

During the past year vol. lxiii. of the *Monthly Notices* has been issued.

In accordance with the arrangement made with the Royal Society, mentioned in previous Annual Reports, two Appendices to vol. lxiii. of the *Monthly Notices* have been issued.

The following Appendices to the *Memoirs* have also been issued :—

Appendix IV. to vol. liv. Lockyer and Baxendall, The Spectrum of γ *Cygni*.

Appendix V. to vol. liv. Evershed, Solar Eclipse of 1900 May 28.

Appendix I. to vol. lv. Poynting, Radiation in the Solar System.

Vol. liv. of the *Memoirs* will shortly be published.

Obituary.

The Council regret that they have to record the loss by death of the following Fellows and Associate during the past year :—

Fellows :—J. H. Brown.
E. T. Carter.
A. A. Common.
* William Francis.
C. H. Gatty.
James Glaisher.
William Harnett.
W. A. Kibbler.
Samuel Kinns.
Thomas Mackenzie.
A. B. Martin.
F. M. Newton.
W. I. Page.
F. C. Penrose.
Lord Pirbright.
Harold Seward.
Washington Teasdale.
Major-Gen. W. H. Wardell.
W. L. Watson.

Associate :—Proser Henry.

John Henry Brown was born at Woolwich on the 4th of September 1836. His father was a dentist, who practised at Woolwich and afterwards at Brighton. From childhood he was fond of scientific pursuits, and while at school made himself a photographic camera. He became a dentist and joined his father in practice in the Old Steyn, Brighton. Later he moved to Hove, where he continued to practise till seven or eight years ago, when he retired.

He was a clever mechanician, and about 1867 invented the now well known Iris Diaphragm. He refused to patent this invention, preferring to publish it at once for the free use of all. He possessed a small telescope, of which he made constant use. He became a Fellow of the Society on the 10th of February 1888.

Mr. Brown married in 1883, and leaves a widow and two

* Although Dr. Francis died subsequently to 1903 December 31 an account of his life is given in this year's Report.

step-children. His death took place at his residence at Brighton on the 19th of December 1903. [For these particulars the Council are indebted to Mr. Arthur F. Griffith.]

Edward Tremlett Carter was born in Calcutta in 1866. He was the eldest of ten surviving children. Brought to England at an early age, he was privately educated at Bristol, and later at the Merchant Venturers' College in that city and at Bristol University College.

Completing his studies in physics and engineering under Professor Hele Shaw and Professor Silvanus P. Thompson, Mr. Carter remained for a short time at the College as demonstrator, until he obtained an appointment at the School of Electrical Engineering and Submarine Telegraphy, Hanover Square, W., first as assistant to the late Mr. Lant Carpenter and afterwards as a lecturer at the school. Concurrently with this appointment he practised as a consulting engineer, and was a frequent contributor to the technical press. On the closing of the School of Electrical Engineering in 1893 he joined the permanent staff of the *Electrician* under Mr. Trotter, who was then editor, and upon whose retirement in 1895 Mr. Carter was appointed assistant editor under Mr. W. G. Bond. In 1897 Mr. Carter went over to Montreal as representative of the *Electrician* at the meeting of the British Association there, and made also an extended tour in America on behalf of the paper. Returning to England, he soon after succeeded Mr. Bond as editor-in-chief of the *Electrician*.

He was to a small extent an author of fiction as well as a writer on engineering subjects. His chief work in book form was *Motive Power and Gearing for Electrical Machinery*.

He had a delicate constitution, and in the winter of 1899, after a severe attack of pleurisy and bronchitis following upon influenza, he was obliged to give up work. A tour in the Mediterranean and Egypt did him good, but the improvement was not a permanent one, and in October 1902 it was discovered that his lungs were seriously affected. He underwent the open-air treatment for some time, with, however, no useful result, and died on the 16th of April at Clevedon, Somerset, where he had been devotedly nursed by his wife after it had been found that the sanatorium treatment was of no avail.

Mr. Carter interested himself deeply in astronomy, more especially in astrophysics and the bearing of astronomical phenomena and history on the evolution theory of the universe.

Mr. Carter was a member of the Institution of Electrical Engineers and of the Société des Ingénieurs Civils de France, and of the Physical Society of London. He became a Fellow of this Society on the 9th of January 1891. He leaves a widow and three sons.

[The Council are indebted to Mr. G. Thomas-Davies for this notice.]

Andrew Ainslie Common was born at Newcastle-on-Tyne on the 7th of August 1841. He was the son of Thomas Common, a well-known surgeon of the north, who gained celebrity by his treatment of cataract. The family is of Scottish border stock and came originally from Dumfriesshire, and the name Common is one of the many variants of Comyn.

While Dr. Common was still in infancy his father died, and the means of the family were straitened by pecuniary misfortunes. In consequence he had to go into the world to seek his living at an early age. He owed his attainments and position entirely to his own character and energy. He had to rely on himself, and no friend advised him in his course of study; it may be that these were contributory causes to the freshness and freedom from prejudice he brought to bear on his work.

He married in 1867, a few years after his association with his uncle in the firm of Matthew Hall & Co., of London, sanitary engineers, a firm which he conducted with success till 1890, when he practically retired from business.

Before he was ten years old he showed a constant interest in a telescope his mother borrowed for him from Dr. Bates, of Morpeth, but he had no opportunity for some years of following his inclination towards astronomical studies. In 1874 he set up in London a 5½-inch refractor mounted equatorially, and made with it his first attempts at astronomical photography. Two years later he became a Fellow of the Royal Astronomical Society. About the same time he moved to Ealing, where he lived till his death in 1903.

He decided to improve his instrumental equipment, and obtained two glass discs of 17 inches diameter, with the intention of grinding a mirror and mounting a reflecting telescope. He, however, abandoned this idea and ordered an 18-inch silvered glass reflector from Calver. The mounting he designed and carried out himself. He commenced with this instrument, which was at the time a powerful one, in 1877, and in January 1878 he communicated to the Astronomical Society observations of the outer satellite of *Mars* and of the satellites of *Saturn*.

He was alive to the possibilities of photography in astronomy, and possibly the comparative slowness of the plates of that time was one reason which led him to obtain a larger reflector and to devote so much care to its mounting. In the *Monthly Notices* of April 1879 he has a "Note on Large Telescopes, with Suggestions for Mounting Reflectors." He carried these into practice shortly afterwards by obtaining a 36-inch mirror from Calver and mounting it in accordance with his own ideas. Particulars of the mounting are given in vol. xlvi. of the *Memoirs*. The mounting, which was a departure from recognised forms, showed great engineering skill, and particular attention may be drawn to the relief of the friction of the polar axis by partly floating it in mercury, and

to the plan projected but not carried out of electric control by the sidereal clock for very prolonged photographic exposures.

With the 3-foot reflector Dr. Common made visual observations of the satellites of *Mars* and *Saturn*, and the nebulæ in the *Pleiades*. He also obtained a photograph of Comet *b* 1881 on the 24th of June 1881, on the same night that it was photographed by Draper in America, the earliest successful photographs of a comet.

But Dr. Common's energy was mainly devoted to the photography of the *Orion* Nebula. His first attempt was on the 20th of January 1880, and was a total failure, but he patiently improved the driving of his clock and took advantage of each increase of sensitiveness in photographic plates till on the 17th of March 1882 he obtained a photograph "which excited the admiration of all the astronomers who had an opportunity of inspecting it." *

He still further perfected the guiding of his telescope, and obtained on the 30th of January 1883, with an exposure of 37 minutes, the splendid photograph with which all astronomers are familiar. Of the merits of this photograph he modestly remarked: "Although some of the finer details are lost in the enlargement, sufficient remains to show that we are approaching a time when photography will give us the means of recording, in its own inimitable way, the shape of a nebula and the relative brightness of the different parts in a better manner than the most careful hand-drawings."

The Gold Medal of the Society was awarded to him in 1884 for the great success which had attended his efforts in celestial photography.

Shortly after this Dr. Common sold his 3-foot telescope to Mr. Edward Crossley, of Halifax, who presented it later to the Lick Observatory. He took a year's rest from astronomy before commencing the great work of his life, the construction of his 5-foot reflector. Into this he put his whole heart, and devoted months and years of patient labour to the difficult task he had set himself. In the construction of his large mirror he had to face a number of unexpected problems and to put up with wearying disappointments. He overcame the difficulties by accurate observation of the effect of tiny and apparently quite trivial details, and by his perseverance in correcting and improving his work by the experience he gained. The Council reports of his Observatory for the years 1887–1892 are interesting records of his progress.

In February 1887 he reports that the year has been entirely devoted to the construction of the 5-foot reflector. The machine for grinding was completed in September 1886, and great progress made with the mirror. Permanent photographic records of the progressive state of the surface were obtained by

* Address by the President (E. J. Stone) on presenting the Gold Medal, *Monthly Notices*, vol. xliv. p. 211.

Foucault's test. The heavy work of the mounting was in hand. By February 1888 the mirror had been polished and figured several times. Evidence of internal strain in the glass was found, and, as it was uncertain to what extent this would affect the image, he contemplated ordering another disc. The mounting was in a forward state, the telescope tube being connected to the polar axis, which was a wrought-iron cylinder of 8 feet diameter, floating in a tank of water to relieve the friction. The reflector was practically completed in September 1888, and was ready for use in February 1889, but a slight ellipticity was found in the star images. It was accordingly refigured and resilvered in the spring of that year ; the images were much improved, but at the same time a new disc was ordered. Between March 1890 and February 1891 the telescope was used on every available night, but, owing to unfavourable weather, very few nights were suitable for photography of nebulæ. Photographs of the *Orion*, *Dumb-bell*, *Pleiades*, and other nebulæ were obtained. The new 5-foot disc had been received, and such skill had Dr. Common acquired that while "it took two years of worry and anxiety to make the first mirror, the second disc was made into an almost perfect mirror in three months from the time it was first put on the machine."* A detailed account, full of interest, is given in the *Memoirs*, vol. I., "On the Construction of a Five-foot Equatorial Reflecting Telescope." This paper contains a description of the grinding machine, the manufacture of the tools for grinding and polishing, the methods of figuring and testing, of silvering, and of supporting the mirror, and of the equatorial mounting of the telescope. Of the performance of the 5-foot he says at the close of his paper : "As far as I am able to judge from the use of other mirrors of considerable size the gain has been maintained, and the power of the 5-foot over the 18-inch and the 3-foot is proportionate to the size. On nebulæ this is seen to great advantage, both visually and photographically. Such an object as *Mimas*, which the 18-inch would, under the most favourable circumstances, just render visible and the 3-foot show fairly well, can with the 5-foot be seen close to the end of the ring, and away from it could not be overlooked." The pleasure he derived from the construction of his mirror may be gathered from a sentence in his address when chairman of the Astronomical Section of the British Association in 1900 : "The invention of the telescope is to me the most beautiful ever made. Familiarity both in making and in using has only increased my admiration."

Owing to various causes very little use was made of the telescope. A few excellent experimental photographs were taken. One night, while using the instrument as a Newtonian

* The actual time of work was from the 25th of March 1891 to the 24th of June. The machine was in use for 273½ hours, and at 25 strokes a minute this gives 410,250 strokes to make a flat disc into a nearly perfect mirror.—*Memoirs*, vol. I. p. 147.

reflector, he narrowly escaped a fall from the high staging. This decided him to devise, if possible, means of working the telescope from the eye end. The difficulty and risk of boring a hole in the centre of the 5-foot led him to attempt a modification of Cassegrain's form, the second mirror being inclined, so that the image should be clear of the large mirror, but the results, though promising at first, were not satisfactory. No further observations were made with the instrument, as Dr. Common's energies were about this time diverted to new problems in connection with gun-sights and telescopes for the army and navy. In this field his great knowledge of optics and practical skill were happily combined and very successful results obtained, to the great benefit of the country. He was engaged in this work to the time of his death. As regards its national importance the following words of Captain Percy Scott, R.N., spoken at a dinner at the Savage Club on 22nd of November 1902, may be put on record here. He said "that the nation owed a deep debt of gratitude to Dr. Common for the great improvements that he had made in gun-sights. It mattered not how good the gun was, nor how good a man there was behind it; unless the sight was perfect good firing could not be made. The great stride by the British Navy lately in that direction was entirely due to Dr. Common. . . . He had produced a telescope gun-sight which would, when properly used, quadruple the fighting efficiency of our battle-ships."

In the manufacture of this 5-foot reflector Common acquired so intimate an acquaintance with the mechanical properties of glass that the grinding and figuring of smaller mirrors became a matter of ease. In connection with the peculiar difficulties of constructing flat mirrors he devised a specially sensitive form of spherometer. He was extremely generous both with his time and money, and when a mirror was wanted for astronomical work Dr. Common invariably undertook to supply one. For the eclipse of 1889 he made two mirrors of 20 inches aperture and 4·5 inches focus, presenting them to the Royal Society. He made a 30-inch mirror for the Solar Physics Observatory. When Sir Henry Thompson presented an equatorial to the Royal Observatory Dr. Common generously supervised the construction of the 30-inch mirror carried on one side of the declination axis. In 1900 he presented a beautiful flat mirror of 20 inches to the National Physical Laboratory.

The 16-inch cœlostats which Dr. Common designed and made for the Eclipse Expeditions of 1896 bear witness to his mechanical as well as to his optical skill. Attention was directed by M. Lippmann to the fact that there would be no movement in the sky reflected by a mirror which turned at half the Earth's rate about an axis parallel to itself and passing through the pole. The advantages of this arrangement for eclipse photography were further enforced in England by Dr. Stoney and Professor Turner. Dr. Common immediately offered to make such an instrument and

have it tried. The beautiful instrument in which he embodied what was till then merely an interesting geometrical property seems now almost indispensable for eclipse work.

The interest he took in developing new forms of astronomical instruments may be illustrated by the Sheepshanks telescope of the Cambridge University, to which he presented a 16-inch flat mirror, in order that the polar siderostat form of instrument might be tried. The Durham Almucantar is another instrument in which the design was largely indebted to the engineering skill of Dr. Common, whose assistance was gratefully acknowledged by Professor Sampson.

He attended the conferences at which the Astrographic Chart was organised, and though he had shown what could be done with reflectors he unhesitatingly recognised the advantages possessed by refractors for this work, and had great admiration for the beautiful results attained by the brothers Henry. At the conference of 1887 he and M. Janssen were appointed a committee for dealing with astronomical photography outside the work of the Chart.

He was interested in eclipse work, and allowed his observer, Mr. Taylor, to go to the eclipse of December 1889 rather than have a station unoccupied, though Mr. Taylor's assistance had only just been obtained for help with the new 5-foot mirror. He himself went to Norway in 1896 to observe the eclipse of that year, but had cloudy weather.

He joined the Society in 1876, and was elected on the Council in 1879. He received the Gold Medal in 1884. He was treasurer of the Society from 1884 to 1895, and President in 1895 and 1896. He was elected a Fellow of the Royal Society in 1885, and served on its Council from 1893 to 1895. From 1894 he was one of the representatives of the Royal Society on the Board of Visitors of the Royal Observatory, Greenwich. In 1891 the honorary degree of LL.D. was conferred on him by the University of St. Andrew's. In 1900 he was chairman of the Section of Astronomy at the British Association, when he delivered an address on the development of astronomical instruments and the application of photography to astronomy.

He died suddenly on 2nd of June 1903, from heart failure, while at work in his study. He leaves a widow, one son, and three daughters.

The energy he threw into his work and the pleasure his successful results gave him have been already referred to. There was in his character an absence of self-seeking and a disposition to work for the general good of astronomy. He had his reward in the esteem and regard of his fellow astronomers.

F. W. D.

[The Council are indebted to Mr. Drower for some particulars of Dr. Common's life, and also to a notice published by the Royal Society.]

William Francis was born in London in February 1817. He was educated partly at University College School, but chiefly in France and Germany, at St. Omer, Cravelt, and Gera. He studied at University College, London; then proceeded to the University of Berlin, and afterwards to Giessen, where he took his degree of Doctor of Philosophy in 1842. His long residence and frequent travels abroad enabled him to acquire an accurate knowledge of French and German, and brought him into relations with many of the leading men of science on the Continent.

In 1842 he established the *Chemical Gazette*, which he edited until the end of 1859, when it was merged in the *Chemical News*. By means of this publication and by the abstracts and translations which he for many years contributed to the *Philosophical Magazine* he did good service in making known in England the work of Continental chemists. He published in *Taylor's Scientific Memoirs* translations of important foreign papers, including Ohm's *Die galvanische Kette mathematisch bearbeitet*, and Helmholtz's treatise *Die Erhaltung der Kraft*. From 1851 until his death he was one of the editors of the *Philosophical Magazine*, and from 1859 of the *Annals and Magazine of Natural History*. His wide acquaintance with different branches of science, as well as with leading scientific men in England and abroad, made him well fitted for these functions, and the sound judgment with which he discharged them is well known.

Dr. Francis was an original member of the Chemical Society, of which he was elected an Associate in 1841, and soon afterwards a Fellow. He was elected a Fellow of the Linnean Society in 1844, and of the Royal Astronomical Society in 1851, being with one exception the oldest surviving Fellow of the latter Society. He was also an original member of the Physical Society. For a great part of his life he was actively engaged in business in the firm of Taylor and Francis, printers and publishers, of which he had been a partner since 1852.

Dr. Francis married in 1862 Miss Isabella Gray Taunton, by whom he had two sons and six daughters, all of whom survive him. He died at his residence, the Manor House, Richmond, on the 18th of January 1904.

Dr. C. H. Gatty was born at Crowhurst, in Sussex, on the 6th of March 1836, and was the younger son of Mr. George Gatty, who was a taxing master in Chancery. A keen sportsman and typical English country gentleman, Mr. George Gatty acquired the Felbridge property, one of the finest sporting estates in the neighbourhood, with its beautiful park and surroundings, by purchase from the late Lord Liverpool; and on the premature death of his elder brother Mr. C. H. Gatty became heir to the property. He was educated at Trinity College, Cambridge, and possessed very different tastes from those of either his father or his brother, giving up his whole time to scientific and literary study. St. Andrew's University conferred the

degree of LL.D. upon him in recognition of services he rendered to it, and he was also a Fellow of the Royal Society of Edinburgh. After the death of his father, in 1864, he lived with his mother at Felbridge, until she died in 1876, and subsequently his life became akin to that of a recluse and almost a hermit. Like his father he was most munificent in his contributions to anything and everything for the public benefit or advantage. The present Felbridge Church was built by the elder Mr. Gatty, and possesses the remarkable peculiarity of standing in the two Surrey parishes of Godstone and Tandridge, within a quarter of a mile of the Sussex border. Dr. Gatty was chairman of the East Grinstead bench of magistrates in Sussex, and for years attended the bench at Godstone, in Surrey, of which county he was also a justice of the peace. He had no near relations and very few intimate friends, leading, as has been said, the life of a recluse and shutting himself up with his fine and valuable library and instruments. He possessed an elaborate and probably almost unique orrery.

He was elected a Fellow of the Society on the 13th of May 1870, and was at one time a regular attendant at our meetings, but never contributed any paper. His scientific interests were largely divided between astronomy and botany. He died at Felbridge on the 12th of December 1903.

W. N.

James Glaisher, although in early life attached to astronomical pursuits, will be hereafter remembered principally as a meteorologist, not only as having for so many years held the position of Superintendent of the Magnetical and Meteorological Department at the Royal Observatory, Greenwich, but also, in a great degree, on account of his independent work in many directions in the cause of meteorological science. A pioneer in meteorology, he very early turned his attention to the organisation of the many amateur meteorological observers dispersed throughout the country, who, hitherto working in an isolated manner, and too often with inferior instruments, became by his vigorous action federated, as it were, into a system having a common object, thus making it possible, with improved instruments and methods, to produce a reliable body of information on the meteorology of the country—an early type of the more extended international systems that became in after years established. He was also largely instrumental in founding the British Meteorological Society, now known as the Royal Meteorological Society, in the early years of which it was by his own personal exertions that the Society was placed on that secure and stable footing the fruits of which the existing Society now enjoys.

It may be convenient, in the first instance, to speak of his astronomical work. Born on the 7th of April 1809 at Rotherhithe, London, Mr. Glaisher was first heard of as receiving

instruction in the use of astronomical instruments at the Royal Observatory, Greenwich, in the year 1829, in which year he was appointed an assistant upon the Grand Triangulation of the Ordnance Survey of Ireland. He was engaged upon this work, in which he took the keenest interest, for two years, but was then disabled by a very serious illness, brought on by exposure on the mountains, and was obliged to leave the Survey in consequence. On his recovery he became an assistant at the Cambridge Observatory, then under the direction of Professor Airy. A large mural circle, provided for this Observatory, having been mounted in October 1832, and divided on its own pier by Mr. Simms in November, Mr. Glaisher commenced work with this instrument on the 9th of January 1833, the observations during this and the two following years being nearly entirely made by him. A fine series of equatorial observations of Halley's Comet, extending from the 2nd of September 1835 to the 16th of January 1836, was also his work. After the then Professor Airy had taken up the position of Astronomer Royal at Greenwich Mr. Glaisher, who had meanwhile been appointed assistant at Greenwich, was allowed leave of absence to remain for a time at Cambridge, entrusted in a great degree with the task of clearing up the 1835 work. He joined the staff of assistants at Greenwich about the middle of the month of February 1836. Since the year 1834 Mr. Glaisher had been employed, first at Cambridge and afterwards for a time at Greenwich, on the "Reduction of the Greenwich Observations of Planets from 1750 to 1830," but for this work a special staff of computers at Greenwich was soon afterwards formed. He became further employed on the reduction of the Cambridge observations of the Sun, Moon, and planets, and also in the collection of the Cambridge annual results of star places for the formation of the Cambridge Catalogue of 726 stars, reduced to the 1st of January 1830. The Rev. R. Main, then First Assistant at Greenwich, having commenced an investigation into the correction of the elements of the orbit of the planet *Venus*, employing the Cambridge observations of 1833 to 1835 and the Greenwich observations of 1836, had written two papers thereon, but not having time to pursue the subject this work was continued by Mr. Glaisher in two further papers, extending the investigation to include also the Greenwich observations of 1837 to 1839. These papers, under the title "Continuation of the Investigation for the Correction of the Elements of the Orbit of *Venus*," were read before the Royal Astronomical Society on the 12th of June 1840 and the 12th of March 1841 respectively. In April 1858 he communicated to the Society a paper on the observation of the annular solar eclipse of the 14th–15th of March of that year, made near Oundle, Northamptonshire, including observations by other observers; and at the meeting of April 1863 one on "Lines in the Solar Spectrum as observed in the Balloon Ascent of the 31st of March." And at the meeting of

December 1865 he gave an account of "The November Meteoric Shower" of that year, when a considerable number of meteors were seen, but the celebrated display occurred in the following year, 1866.

The Astronomer Royal, in his report to the Board of Visitors in 1836, referred to the proposed construction of a magnetic observatory at Greenwich, and the building was erected at the beginning of the year 1838. The instrumental equipment was sufficiently advanced to allow observations of magnetic declination to be made on one day in each of the months of February, May, August, and November of the years 1839 and 1840 at each fifth minute throughout the twenty-four hours. But in the latter part of the year 1840 the magnetical and meteorological observatory became fully constituted, the Astronomer Royal giving the position of superintendent to Mr. Glaisher, who at once applied himself energetically to the duties of his new post. The first intention had been to carry on the observatory for three years, a period that was afterwards extended to five years, as for the various colonial observatories at that time temporarily established. Finally the observatory was made permanent. Observations of the magnetical and meteorological instruments were made at intervals of two hours, day and night, unless abnormal change or movement was remarked, when extra eye observations, involving much labour, were taken. On the 25th of September 1841 there occurred a remarkable magnetic storm, which, well observed, revealed a new class of magnetic phenomena. An account of this storm being printed and circulated, it was found that at such distant places as Toronto, Trevandrum, and the Cape of Good Hope disturbances of similar character had occurred on the same day, appearing also to have commenced at Toronto "nearly at the same absolute time as at Greenwich." Again on the 24th of September 1847 there occurred another very remarkable magnetic storm, also well observed, "to which there had been nothing comparable before." But from the year 1848 the labour of taking at such times long series of eye observations became unnecessary, since the variations of the principal magnetic and meteorological instruments were afterwards recorded by photography, on a plan arranged by Mr. Charles Brooke, the continuous registration thus obtained giving an insight into the character of the instrumental changes, especially those of the magnets, that has been of the greatest value. It became thus possible to diminish the number of eye observations and avoid so much work at night.

As soon as the duties of his new position permitted Mr. Glaisher undertook an important investigation, that of the question of radiation from the earth at night, and with the assistance of the members of his staff he carried out an extensive series of observations on this interesting branch of meteorological research, employing for the purpose a large number of specially constructed, accurate and very sensitive thermometers, having their scales

engraved on the tube. The observations extended from September 1843 to May 1844. They were very numerous, and the results were incorporated in a paper "On the Amount of the Radiation of Heat at Night from the Earth, and from Various Bodies placed on or near the Surface of the Earth," communicated to, and read before, the Royal Society on the 4th of February 1847. In the course of these experiments he frequently passed the night in the open air, watching for the formation of the dew, and lying on the damp grass. This produced a severe attack of rheumatism, from which he did not recover for several years.

In another paper Mr. Glaisher collected the two-hourly meteorological observations of the barometer and thermometer made at Greenwich during the five years ending 1845, employing them to form, for each month of the year, corrections applicable to observations taken at any hour to obtain mean values, for pressure of the atmosphere, temperature of the air, and the various meteorological elements depending thereon. As regards temperature of the air, corrections obtained from this paper were employed to reduce to mean values the observations of temperature made at the apartments of the Royal Society. The results were contained in two papers, the second of which gave monthly temperatures of the air reduced to Greenwich, from 1771 to 1849, in all of which years, excepting from 1781 to 1786 and from 1844 to 1849, the results depended on the Royal Society records. The three papers here described were communicated to the Royal Society in the years 1848, 1849, and 1850 respectively.

For many years Mr. Glaisher communicated to the Registrar-General of Marriages, Births, and Deaths a quarterly report on the meteorology of the kingdom. This work had an origin in part accidental. Mr. Glaisher, having remarked that the Registrar-General had published the mean temperature of York as being five degrees higher than that of London, wrote to point out to him that this was a physical impossibility, when it appeared that there was no person in the office of the Registrar-General who understood matters of the kind. Mr. Glaisher, in his endeavour to bring into general use instruments of more perfect character than had been hitherto used, was at this time in correspondence with a large number of observers, and persuaded them to report their observations regularly to him, which, reduced on a uniform plan, formed the basis of the report which he commenced about the year 1846 (the writer of this being at that time one of the contributors) to supply quarterly to the Registrar-General with his own special "Remarks on the Weather during the Quarter." His hygrometrical tables, adapted to the use of the dry and wet bulb thermometer, published in 1847, have proved of great service to meteorologists. Many successive editions of these tables have been published, and they have been incorporated in foreign collections of meteorological tables.

The increasing attention that became paid to meteorological science suggested the desirability of forming a society which

should bring into closer union the meteorological workers of our country, by affording them means for the publication of meteorological observations and results, or papers of general meteorological interest, and in the year 1850 Mr. Glaisher, in conjunction with a few others similarly interested, founded the British Meteorological Society, afterwards known as the Meteorological Society, and later as the Royal Meteorological Society. He further undertook the post (so important in a new society) of secretary, a position which he occupied for some twenty years, with the result that, at the time of retiring therefrom, the society had become well established.

At the Great Exhibition of 1851 Mr. Glaisher was appointed Reporter to the Section of Philosophical Instruments, Class X., and produced an able and comprehensive report. Of astronomical instruments there was apparently not a large collection, but one was an interesting exhibit, the historical Westbury Circle made by Troughton at the beginning of the last century for Mr. Pond, afterwards Astronomer Royal. The two circles, it may be remarked, took nearly twelve weeks to graduate, working each week day on an average of eight hours daily, an operation at the present day a question of hours only. Of thermometers only Mr. Simms and Messrs. Negretti and Zambra in the British portion of the exhibition showed instruments graduated on the stem.

The following detached sentences extracted from a lecture given by Mr. Glaisher at the Society of Arts in 1852 (one of a series of lectures on the Great Exhibition) well show what had been the condition of matters, and the efforts that he was making to improve meteorological instruments and methods :—

"The collection of meteorological instruments in the exhibition would lead us to the conclusion that the conditions of good instruments are better understood and fulfilled by makers abroad than at home. That this will speedily cease to be the case I feel assured. . . . That the want of good instruments is experienced I can myself testify. For years I have pursued the subject of meteorology, and have long been convinced that a widely spread and universal system of simultaneous observation, uniformly reduced, must be the groundwork of its establishment as a science. For the sake of this I six years ago reduced my system to practice, and introducing it first among the meteorological observers, contributors to the reports of the Registrar-General, in a short time perceived with satisfaction some of the good results. . . . I have been gradually increasing the number of observers, and have now established upwards of fifty meteorological stations in different parts of the country, all of which contribute observations made and reduced upon my system. . . . I now come to that which had nearly caused the subversion of my scheme, the difficulty of obtaining good instruments at a fair and moderate price. . . . That I have been enabled to do. . . . All the barometers, and the greater part of the thermometers,

scattered among the observers have been made under my direction by him [Mr. Barrow], and constitute, with few exceptions, the most efficient instruments at their command."

In a paper by Mr. Glaisher "On the Severe Weather at the Beginning of the Year 1855, and on Snow and Snow Crystals," published in the *Proceedings of the British Meteorological Society*, he states that the primary figure or base of each crystal appeared to be "a star of six radii, or a hexagon of laminæ, and the compound varieties to include combinations of spiculæ, prisms, cubes, and rhomboids, aggregated upon and around the central figure." There are detailed descriptions of many of the forms of crystals, and also appended are 151 pictures of the different varieties of crystals, from drawings by Mrs. Glaisher based on his sketches. A further paper "On the Meteorological and Physical Effects of the Solar [Annular] Eclipse of the 15th of March 1858" contains meteorological observations made at very frequent intervals during the eclipse, at twenty-nine different stations, with remarks on the effects produced on birds and animals.

Papers on meteorological subjects become henceforward very numerous, amongst which may be found reports on meteorology, including one in relation to the health of the troops in India, discussions of collected observations of meteorological elements, as well as communications on special atmospheric phenomena. In one paper "On the Secular Increase of Mean Temperature" (*Proceedings of the British Meteorological Society*, 1865) Mr. Glaisher concluded from comparison with earlier records, instrumental and otherwise, that our climate during the preceding 100 years had altered, that the temperature of the year was 2° warmer, the winter months all much warmer, and every month of the year somewhat warmer than before. But the last sixty years show no definite change, and it might be desirable to rediscuss the older observations.

Another considerable and important work has yet to be spoken of, the balloon ascents undertaken for scientific purposes. A committee of the British Association having been appointed to consider the question of making observations in the higher strata of the atmosphere, after some preliminary trials with an insufficient balloon, Mr. Glaisher himself undertook to make the necessary ascents, with the aid of the aeronaut Mr. Henry Coxwell, in a large balloon made by the latter at his own cost for this special purpose. The observations proposed to be made were those of atmospheric pressure, temperature, and the hygrometric state of the atmosphere, the height and motions of clouds, and other phenomena. The first ascent was made on the 17th of July 1862, and the last on the 26th of May 1866, in all twenty-eight, some of them in the evening. On five occasions an altitude of between 10,000 and 20,000 feet was reached; on five occasions also an altitude of between 20,000 and 30,000 feet; and on one occasion (the 5th of September 1862) an altitude of 37,300 feet is said to have been reached, a pheno-

menal height, the experiences and sensations of the voyagers at which altitude are narrated in the article on "Aeronautics" in the *Encyclopædia Britannica*. It appeared as one of the results of the experiments that the rate of decrease of temperature with elevation near the earth was very different when the sky was clear from that when it was cloudy, and the equality of temperature at sunset and increase with height after sunset were remarkable facts that were not anticipated. Complete accounts of the observations made in these ascents are to be found in the volumes of the *British Association Reports* between the years 1862 and 1866. Other ascents were afterwards made at Chelsea in a captive balloon on the offer of the owner, M. Giffard, for the purpose of making further observations at low altitudes.

Mr. Glaisher retired from his position at the Royal Observatory at the end of the year 1874, but he continued to lead an active life, still communicating to the Registrar-General his quarterly weather report, one of his early organisations, which indeed he continued to supply until 1901—that is, altogether for a period of some fifty-five years. He took great interest in the Palestine Exploration work, and was chairman of the Executive Committee from 1880 almost up to the time of his death. He discussed observations of atmospheric pressure, temperature, wind, rain, and cloud at points in Palestine in fifteen papers communicated between the years 1891 and 1902 to the publications of the Palestine Exploration Fund.

Another important work that occupied a considerable part of the leisure time of Mr. Glaisher was the calculation of certain factor tables. The Committee of the British Association on Mathematical Tables, of which both Mr. Glaisher and his son Dr. J. W. L. Glaisher were members, in considering the question of factor tables, found that although Burckhardt had calculated tables giving the least factor of all numbers not divisible by 2, 3, or 5 for the first, second, and third millions, and for a small part of the fourth million, and that Dase and Rosenberg had calculated similar factors for the seventh, eighth, and ninth millions, there thus remained three intermediate millions for which factors were yet required. Crelle, previous to 1850, had presented to the Berlin Academy the manuscript of factor tables for the intervening fourth, fifth, and sixth millions, but on inquiry being afterwards made as to whether there were any chance of publication of the manuscript, the Secretary of the Physical and Mathematical Section of the Academy reported that the manuscript had been examined on a former occasion and found to be so inaccurate that "the Academy was convinced that the publication would never be advisable." The Committee of the British Association, considering, however, that the completion of the factor tables for these three millions was a matter of great importance, made a grant of 250*l.* towards the cost of doing the work, which, being one that required careful supervision,

was undertaken by Mr. Glaisher with the assistance of two computers, the results for these fourth, fifth, and sixth millions, with an explanatory introduction by Mr. Glaisher, being published (by Taylor and Francis) in three separate quarto volumes.

Mr. Glaisher was elected a Fellow of the Royal Society in the year 1849, and previously, in the year 1841, had been elected a Fellow of the Royal Astronomical Society, and at the time of his death was its oldest Fellow. Of the Royal Meteorological Society he was, as before mentioned, mainly its founder. He was connected also with many other scientific societies—the Royal Microscopical (of which he was President in 1865–68), the Photographic (of which he was President for the twenty-three years 1869–92), and others—and he served on various British Association Committees, being for many years a member of that on Luminous Meteors. In various industrial undertakings he took a lively interest to the end of his life. After his retirement from the Royal Observatory he continued to reside for a number of years at Blackheath, but eventually he removed to Croydon, where after a very active life he died at "The Shola," on the 7th of February 1903, in the 94th year of his age. He was buried at Shirley, near Croydon. W. E.

William Harnett was born at Tralee, co. Kerry, on the 26th of March 1830. He was educated at Oscott College, Warwickshire, and was for some time a corn broker in Liverpool.

Several years ago he retired from business and went to live at Wimbledon. He died on the 2nd of January 1903, leaving a widow, one son, and three daughters.

Mr. Harnett had long been interested in astronomy, though he only recently became a Fellow of the Society, being elected on the 11th of January 1895.

W. Ambrose Kibbler was born in 1848 at Hackney, where his father long practised as a doctor. The son was a student at the London Hospital, where he afterwards held for a short time the posts of house surgeon and house physician. In 1876 he took the degree of M.B. at Marshall College, Aberdeen. He joined his father at Hackney and continued to practise there till the end of his life. During the last few years Dr. Kibbler experimented with X-rays in cases of cancer. Though no permanent cure was effected he was able to alleviate suffering to some extent by this treatment.

Besides being a skilful physician Dr. Kibbler was an artist of considerable merit. He was a member of the Langham Sketching Club, and had several pictures hung at the Institute of British Artists. For many years he was an enthusiastic astronomical observer. About eighteen years ago he commenced observations of the Moon with a 9½-inch reflector. Some trouble with his eyes obliged him to discontinue this work

for a time. About five years ago he obtained a 12½-inch reflector and made observations of *Jupiter*, *Mars*, and *Saturn*. His artistic talent enabled him to make good drawings at the telescope. At the time of his death he had been a Fellow of the Society for little more than a year, having been elected on the 14th of February 1902. Dr. Kibbler married in 1879 and leaves two sons. He died on the 10th of September 1903.

Samuel Kinns was born on the 14th of November 1826, at Colchester, where his father, the Rev. John Osborne Kinns, was a Congregational minister. He obtained the degree of Ph.D. from the University of Jena in 1859, and was elected a Fellow of the Royal Astronomical Society in the same year. For about a quarter of a century he was principal of a school, called "The College," at Highbury New Park, and while there he published (in 1882) a work under the title *Moses and Geology*, the object of which was to reconcile scientific facts in geology and astronomy with the Biblical account of creation. This led to much controversy, but passed through several editions. Dr. Kinns was ordained by the Bishop of London (the late Dr. Temple, afterwards Primate) in 1885. Four years later he was presented by the Lord Chancellor to the living of Holy Trinity, Minories, which he held for ten years. In 1891 appeared his *Graven in the Rock*, the purpose of which is to show the harmony between the Biblical narratives and modern discoveries in Egyptian and Babylonian archæology; this work is profusely illustrated by photographs of the slabs and statues in the British Museum. Dr. Kinns also lectured very extensively on the above subjects. His last work, *Six Hundred Years*, gives an interesting sketch of the persons who had been in any way connected with the church in the Minories of which he had been vicar; the most interesting of these relates to the doings of Sir Isaac Newton whilst Master of the Mint. Dr. Kinns was never married, and resided latterly with friends at Haverstock Hill, Hampstead. About two years ago a paralytic seizure incapacitated him for work. He died on the 14th of July 1903, in the seventy-seventh year of his age. W. T. L.

Thomas Mackenzie, eldest son of Roderick Mackenzie, of Muir Houses, West Calder, was born on the 6th of May 1844. He was at school at Aberdeen, and at the age of 13 went to sea. For many years he was in the service of the Royal Mail Steam Packet Company. In 1892 he opened a school of navigation in Fenchurch Street, which he carried on to within a few months of his death. He was the author of *Practical Mechanics for Sailors*, and contributed numerous articles to the *Nautical Magazine* on Seamanship, Stability, Great Circle Sailing, and other questions connected with navigation. Captain Mackenzie was elected a Fellow of the Royal Astronomical Society on the 14th of January 1887. In November 1885 a paper written by him "On

the Application of a Nicol's Prism to Sextant Observations" was published in the *Monthly Notices*, in which he advocated the use of a Nicol's prism to get rid of the glare from the horizon in preference to coloured shades.

Captain Mackenzie died on the 24th of January 1903, leaving a widow.

Arthur Burnett Martin was the son of Captain K. B. Martin, R.N., harbour master at Ramsgate. Born on the 10th of May 1832, he entered the navy in 1845, but was invalided home from the Mediterranean in 1852. From 1852 to 1901 he was a teacher of navigation and nautical astronomy at Norie's Nautical Academy. During this period he prepared large numbers of pupils for the Board of Trade examinations. Most of his pupils belonged to the merchant service, with a small number of yachting men. He appears to have been an excellent teacher, with a faculty for putting things in such a way that they could not be forgotten, and with the ability to make his points clear to his dullest pupils. He wrote a work on navigation and revised *Norie's Epitome*. Mr. Martin was a Fellow of the Society for many years, being elected on the 10th of February 1854. He took great interest in astronomy and often gave lectures locally.

Owing to deafness he rarely attended the meetings of the Society in late years.

Mr. Martin died suddenly on the 5th of March 1903, leaving a widow.

Francis Murray Newton was born at Barton Grange, Taunton, in 1852. He was educated at Eton and University College, Oxford. Soon after leaving Oxford Mr. Newton went with Captain Orde Browne to Egypt as an observer of the Transit of *Venus*, the party to the Mokattam Hills, near Cairo, consisting of Captain and Mrs. Orde Browne and Mr. Newton and his sister, Miss Emily M. Newton. Their camp at Mokattam was formed on the 21st of October 1894 and struck on the 25th of December. The weather seems to have caused the observers a good deal of anxiety, and on the 8th of December, the day of the transit, the clouds did not break till $12^h\ 10^m$, while the expected time of egress was $13^h\ 22^m$. About 10 minutes before contact a thick cloud swept over the Sun and completely obscured it till within 10" of the contact, when it happily cleared off. Captain Orde Browne with a 6-inch, Mr. Newton with a 4½-inch, lent him by Mr. Warren de la Rue, and Miss Newton with a 3-inch, belonging to her brother, all made satisfactory observations of the time of internal contact at egress and later of external contact.

The volume of *Transit of Venus*, 1874, *British Observations*, contains (Plate X.) drawings by Mr. and Miss Newton relating to the egress of *Venus* as seen by them. There is also an

interesting drawing of *Venus* as seen at 13^h on the 5th of December, about three days before the transit, in which the crescent is 37° more than a semicircle, pointing, Mr. Newton suggested, to an atmosphere on *Venus*. After his return from Egypt Mr. Newton set up his small telescope at his house at Taunton, but his interest in practical astronomy ceased after a few years. In 1879 he started electrical engineering works at Taunton, which are still carried on.

Mr. Newton married in 1879, and leaves a widow, three daughters, and one son.

He became a Fellow of the Society in December 1875.

William Irving Page was born on the 19th of November 1840, at Ulster Place, Regent's Park. His father was an artist of considerable repute, who gained the Academy Gold Medal and Travelling Scholarship, and painted an interesting series of pictures in Greece. Mr. Page was a student at St. George's Hospital, where he was for some time house surgeon. He settled at Wimbledon, where he became very well known and respected.

Mr. Page was a Fellow of the Royal Geographical as well as of the Royal Astronomical Society. He was fond of travelling and had visited nearly all parts of the globe. He had an observatory at his house at Wimbledon, and spent considerable time with his telescope and spectroscope. He became a Fellow of the Society on the 8th of January 1886.

Mr. Page married in 1894, and leaves a widow and one daughter. His death, which took place quite unexpectedly on the 6th of September, was due to pneumonia, which developed from a cold caught while yachting.

Francis Cranmer Penrose was born in 1818, at Bracebridge, near Lincoln, of which parish his father, the Rev. John Penrose, was vicar. After four years at the Bedford Grammar School he obtained a scholarship at Winchester College. After leaving school he was for some time a pupil of Mr. Edward Blore, the architect. He then entered Magdalene College, Cambridge, and was a Senior Optime in the year 1842, the year in which Cayley was Senior Wrangler. While at Cambridge he rowed three times in the boat race against Oxford. After his three years at Cambridge he was appointed "Worts' Travelling Bachelor," a sum of money having been left by Mr. Worts "for the promotion or encouragement of investigations in foreign countries respecting the religion, learning, laws, politics, customs, manners of those countries . . . and for purposes of antiquarian and scientific researches in those countries." In 1851 Mr. Penrose brought out a treatise on *The Principles of Athenian Architecture*. In 1852 he was appointed Surveyor of the Fabric of St. Paul's Cathedral, a post he retained till 1897.

Mr. Penrose became a Fellow of the Royal Astronomical Society in 1867. He had a small telescope at his house at

Wimbledon, with which he frequently made observations. In 1868 he communicated to the Society observations of the transit of *Mercury*, and in 1870 observations of *Algol*, of Sun-spots, and of Occultations. Most of his papers in the *Monthly Notices* are, however, concerned with graphical methods of computation. In 1876 he describes an instrument for solving spherical triangles. The very ingenious method by which he predicted graphically the times of occultations, eclipses, &c., is developed in several papers; for instance, in 1873 he applied it to predict the times of the transit of *Venus* at various stations, and in 1900 to the times of commencement and end of the solar eclipse of that year.* He published an account of his method separately in 1869, and brought out a second edition of it last year. Another paper by Mr. Penrose, which is somewhat of a *tour de force* in the use of graphical methods, was published in the *Monthly Notices* in 1882, "On a Method for Finding the Elements of the Orbit of a Comet by a Graphical Process."

Another subject on which Mr. Penrose wrote was the Orientation of Greek Temples. His first paper is published in the *Philosophical Transactions* for 1893. He proceeded on the theory that in any temple oriented within the solstitial limits of its latitude the axis was so directed that, on the great festival of the year, the first beam of the rising Sun should fall upon the statue centrally placed in the temple, or on the incense altar in front of it; and, further, that the orientation (and consequently day of festival) would be so arranged that the approach of Sunrise would be indicated to the priest by a bright star visible from the *adytum*, and therefore of the same declination as the Sun and right ascension one hour or so earlier than the Sun. If a bright star is identified which satisfies this condition the date of the temple may be determined from the precessional motion. Mr. Penrose found suitable stars for twenty-eight temples, whose orientations ranged from 21° N. to 18° S., and assigned dates to the building of them. He supplemented this paper in 1897, and in 1901, with Sir Norman Lockyer, endeavoured to fix the date of Stonehenge.

Mr. Penrose received the Royal Gold Medal of the Institute of British Architects in 1883. He was elected an Honorary Fellow of Magdalene College, Cambridge, in 1885. In 1886 he was appointed director of the British Archæological Society at Athens, and in 1890 Honorary Antiquary to the Royal Academy. He was elected a Fellow of the Royal Society in 1894, and had been a Fellow of the Royal Astronomical Society since the 8th of February 1867.

Mr. Penrose died at his residence, Colebyfield, Wimbledon, on the 17th of February 1903.

Henry de Worms, first Baron Pirbright, was born in London in 1840. He was the third son of S. B. de Worms,

* See *Monthly Notices*, vol. lx. p. 484.

hereditary Baron of the Austrian Empire, by his wife Henrietta, daughter of Samuel Moses Samuel. Both on his father's and mother's sides he was descended from great mercantile families. Henry de Worms was educated at King's College, London, of which he became a Fellow in 1863. He was first intended for the medical profession, but was ultimately entered as a student of law in the Inner Temple, in 1860, and was called to the Bar in 1863. In 1862 he published a book, *The Earth and its Mechanism; being an Account of the various proofs of the Rotation of the Earth.* In 1874 his family were permitted by the late Queen to use their Austrian titles in England. After other attempts to enter Parliament he was in 1880 returned as one of the Conservative members for Greenwich, and on the reorganisation of that constituency was elected member for the Toxteth division of Liverpool. In 1872 he had published a book on the Austro-Hungarian Empire, and in 1877 another on England's policy in the East; ten years later he translated the *Memoirs of Count Beust.* From 1873 to 1886 he was president of the Anglo-Jewish Association.

Under Lord Salisbury's administration he was appointed in 1885 Parliamentary Secretary to the Board of Trade, and in 1888 Under-Secretary for the Colonies. In 1888 he was raised to the Privy Council, and named a plenipotentiary to the International Conference on Sugar Bounties. He presided at the Conference in London, and with Lord Salisbury signed for Great Britain the treaty for the abolition of the Bounties, but economic and political influences prevented the Convention coming into force. The West Indian trade continued to decline, and De Worms, who had been created Lord Pirbright in 1895, continued the agitation. A further conference on sugar bounties was held in Brussels, resulting in the Convention of 1902, which, however, was materially different from the Convention of London, and Lord Pirbright opposed its ratification by the English Government.

He lived in his later years at Henley Park, Guildford, where he was well known for his numerous charities. He married first the eldest daughter of Baron Todesco, of Vienna, by whom he had three daughters, and secondly Sarah, sister of Sir G. Faudel Phillips. Lord Pirbright leaves no heir.

He was elected a Fellow of the Society on the 13th of December 1861. He died at his London residence in Grosvenor Place on the 9th of January 1903.

HAROLD SEWARD was a son of the late Mr. James Seward, of Liverpool and Wandsworth. He was born in 1862 and educated at the Liverpool Institute, where he was Tate Scholar. He obtained a Mathematical Scholarship at Balliol, which he held from 1880 to 1884. He was also a Gilchrist Scholar of the London University. At Oxford he obtained a First Class in Mathematical Moderations in 1882, a Second Class in Natural Science in 1883, and a First Class in Final Honours Mathematics

in 1884. While at Oxford he studied Astronomy with Professor Pritchard and Mr. Plummer.

In 1884 Mr. Seward obtained a post in H.M. Patent Office as Deputy Examiner, which he held till his death. Apart from his work at the Patent Office, in which he was thoroughly interested, Mr. Seward's special interests were orchestral music and astronomy. He joined the Society in December 1884, immediately after his appointment at the Patent Office, and was a regular attendant at the meetings. He joined the British Astronomical Association soon after its foundation, and was for several years on the Council of that body. He contributed to the Society, in January 1896, a "Note on the Indexing of Scientific Papers." In this note, written at a time when the importance of indexing was strongly felt, and the methods to be adopted widely discussed, Mr. Seward offered some suggestions based on his wide experience at the Patent Office. It is interesting to note that the *International Catalogue of Scientific Literature* is being carried out on lines very similar to those indicated by Mr. Seward.

Mr. Seward married Florence, daughter of Mr. Borders, of 10 Balham Park Road. He died on October 14, 1903, after a year's suffering from a painful illness.

Washington Teasdale was born at Leeds, Yorkshire, on the 8th of August 1830. His father died whilst Washington was still a youth, and in consequence his younger brother, his sister and himself were brought up under the chief guardianship of his maternal grandfather, the late Mr. Christopher Heaps, one of the most prominent citizens of Leeds in the middle of the last century. He was educated at Mr. Richard Hiley's Academy, then a well-known school standing near the junction of Queen Square and Woodhouse Lane, and even in boyhood displayed a special interest in scientific pursuits. This appears to have been further fostered by his later training, the profession for which he was designed being that of a civil engineer. When the railway system of India was being constructed, he went out to that country to take part in its development. Here he remained for several years, acquiring such a complete mastery over the native language that it is said he preserved the habit of thinking in Hindustani till the very close of his life. On his return to England he settled in Leeds, and possessing the means, and having ample leisure, he devoted himself to the cultivation of his scientific tastes. These were many and varied, and there was hardly any local scientific society with which he was not connected. In particular he actively interested himself in the Leeds Astronomical Society, the Leeds Naturalist Club, the Scientific Association, the Philosophical and Literary Society, and the Institute of Science, Art, and Literature, throwing himself into whatever subject he took up with most delightful enthusiasm. Photography was an art in which he took especial interest; indeed, he was amongst the earliest workers

in this field, and in conjunction with a few friends, he founded a photographic society before he went to India, when the art was quite in its infancy, and he continued his active connection to the very last with the organisation which had succeeded the original society. He watched the development of the art with the most lively attention, and familiarised himself with every new process as it was introduced. He hardly ever went on a journey, or took a holiday, without his camera, and was continually adding to his rich collection of views. He was one of the first to use the lantern for lecture purposes, and was skilful in the design and preparation of slides for this work. He lectured hundreds of times in connection with various scientific and literary societies; his unaffected delight and interest in the subjects of which he was speaking rendering him in public, as in private, always most attractive.

Astronomy had, however, the chief fascination for him. He was elected a Fellow of this Society in 1886; he was an original member of the British Astronomical Association; and in his native town he had a large share in the resuscitation and the development of the Leeds Astronomical Society, of which he was President from 1893 to 1897. He contributed frequently to its *Journal*, and his services to it cannot be over-estimated.

His love of science amounted to a passion. It had for him an absorbing interest, and for the greater part of his life it was almost his sole occupation, and a hobby which gave a joy to existence. His house in Hyde Park Road, Leeds, was quite a treasury of scientific apparatus, works of art, and interesting curios, and other subjects which denoted the wide range of his studies and sympathies. Outside his more serious pursuits he had many scientific hobbies that afforded himself and his friends amusement. Amongst these may be mentioned, his pendulum writing, an ingenious method of procuring peculiar curved designs of which he claimed to be the originator; then it was the collection of finger prints of eminent men; more recently it was the mixing of inks by which through the medium of a few splashes he obtained the most curious gradations of colour. At one time he made a series of experiments on the problem of the cultivation of trees in large towns; at another he devoted a good deal of study to the monoliths at Stonehenge.

Mr. Teasdale married and had one child, but he survived both his wife and child by many years.

His death took place during the meeting of the British Association at Southport. He had gone thither to attend the meetings, and was visiting a friend, when he was seized with illness, and remained in a state of coma for some time. Hopes were, however, entertained of his recovery, but a second attack followed the next day, and he died on Saturday, the 19th of September 1903, at the age of 72. E. W. M.

Major-General William Henry Wardell, R.A., was the eldest son of Major W. H. Wardell, of the 66th (Berkshire) Regiment and the 93rd Highlanders. He was born in 1838 at Halifax, Nova Scotia. In 1855 he obtained a commission in the Royal Artillery. While a subaltern he served with a detachment of Royal Artillery, which landed Colonel M'Crea for the protection of Europeans at Port au Prince during the revolution in Hayti in January 1859, which ended the reign of the Emperor Faustin. He was for some time an instructor in mathematics at the Royal Military Academy, Woolwich. Later he held the post of assistant-superintendent of the Royal Gunpowder Factory at Waltham Abbey. In 1882, while colonel commanding the artillery in the Windward Islands, he was wrecked with some troops off Barbados. For his coolness and courage on this occasion, which materially assisted in the safe landing of all on board, he received the thanks of the Commander-in-Chief. In 1886 he retired from the service with the honorary rank of Major-General.

General Wardell was the author of *Notes on Gunpowder* and of the article on "Gunpowder" in the *Encyclopædia Britannica*, where he gave an interesting historical account as well as a technical treatment of the subject. He occasionally contributed articles on kindred subjects to scientific reviews. In 1874 he was chosen to observe the transit of *Venus* from the island of Rodriguez, but was prevented from doing so by his military duties. He became a Fellow of the Society on the 14th of January 1876.

General Wardell died on the 24th of July 1903, at his residence, Sparkford Lodge, Winchester, leaving a widow and several children.

William Livingstone Watson was born at Kinross in 1835, and was educated at the Universities of Edinburgh and Glasgow. He entered on a mercantile career in Glasgow in 1855, joining Messrs. James Finlay & Co., a firm of East India merchants. He was a partner in this firm for twenty years, retiring in 1890. From 1876 to 1890 he represented the firm in London. He was for fifteen years chairman of the London Board of the Royal Insurance Company, for some time the Merchants' Marine Insurance Company, and of the Agra Bank, a director of the Indo-China Steam Navigation Company, the Assam-Bengal Railway, the Clan Line of Steamers, and several Indian Sea Companies.

Mr. Watson was a man of very varied interests, and was a Fellow of the Royal Geographical Society and the Society of Antiquaries, as well as of the Royal Astronomical Society. He had at his Scottish residence at Ayton, Abernethy, Perthshire, a 12-inch refractor. This instrument was exhibited at the Great Exhibition in 1851, and was then one of the largest in Great Britain.

Mr. Watson also found time to take an active part in the local

affairs of Perthshire, being a J.P., a member of the County Council, and chairman of two School Boards. He was interested in the working of the Gothenburg system and endeavoured to introduce it into this country.

Mr. Watson was elected a Fellow of this Society on the 8th of January 1892. He married in 1878, and leaves one son. He died on the 19th of May 1903 at his house in Scotland.

Mathieu-Prosper Henry was born at Nancy on the 10th of December 1849. He and his brother Paul went to Paris in 1864, where they became assistants in the meteorological service of the Observatory, Paul in 1864 and Prosper in 1865. They were appointed assistant astronomers in 1868. In their lodgings at Neuilly they succeeded in making a 12-inch mirror and began the construction of an ecliptic chart. When this was brought to Delaunay's knowledge he transferred them from the meteorological to the astronomical side of the Observatory, and on the 1st of July 1871 put them in charge of the "Equatorial du Jardin." They took up the work begun by Chacornac in 1852, interrupted by his death, of constructing charts which should give all the stars down to the thirteenth magnitude in the zone of the ecliptic. Each chart is thirteen inches square, and covers a field of five degrees square, so that seventy-two are required to complete the belt. Chacornac had completed thirty-six of the charts, containing about 60,000 stars. By 1884* the brothers Henry had completed sixteen charts, with 36,000 stars, and nearly finished four more with 15,000 stars. The annual reports of the Director of the Paris Observatory show the progress of the work, and give an idea of the arduous toil and the perseverance of the two brothers. In the course of their work they discovered no less than fourteen minor planets, the first of these, *Liberatrix* (125), being discovered in September 1872. They also made observations of the satellites of *Uranus* and *Neptune*, and sketches of the planets at each opposition. In the winter of 1882 they went to the Pic du Midi to observe the transit of *Venus* from a height of near 10,000 feet at considerable risk to their lives, owing to avalanches. On the day of transit, the 6th of December, the sky was covered with clouds.

As the region of the charts approached the Milky Way the difficulties arising from the large number of stars (15,000 to 18,000 on one chart) made visual observations almost impossible even for observers of their skill and experience. They accordingly determined to try photography, with which Rutherford in America and Common in England had already obtained excellent results, and especially in view of the large number of faint stars shown on Dr. Gill's photograph of the Great Comet of 1882. They first made an object-glass of six and a half inches aperture, and seven feet focal length, achromatised for photographic rays.

* *Comptes Rendus*, t. xcix. p. 305.

In August 1884 Admiral Mouchez presented on their behalf to the Academy of Sciences a photograph of six square degrees of the sky, containing 1500 stars from the sixth to the twelfth magnitude. The exposure given was forty-five minutes. A parallel guiding telescope was used. These results were so satisfactory that MM. Henry, with the cordial support of Admiral Mouchez, commenced the construction of a 12·8-inch photographic objective. It was arranged that a parallel 10-inch guiding telescope should be attached to the photographic telescope. This instrument is the type of those now employed for the international photographic chart of the heavens. It was felt that the difficulties of making large photographic objectives would be overcome. "It is known," M. Mouchez said to the Academy, "that MM. Paul and Prosper Henry are not only excellent observers, but also the most skilful constructors of objectives in France; we can, therefore, have the greatest confidence in the results of their labours, for the history of science shows that it is always to astronomers who have made their own apparatus that we owe the most remarkable improvements in the instruments of astronomy and the most remarkable discoveries in the study of the sky."

The anticipations of Admiral Mouchez and MM. Henry were speedily realised, and on the 11th of May 1885 it was announced that the instrument was completed and that a field of 3° diameter was very sharply covered. A photograph with an hour's exposure was found to show stars of the fourteenth and some of the fifteenth magnitudes. By January 1886 MM. Henry had obtained a beautiful series of photographs illustrating the power of their instrument, among these a photograph of the *Pleiades*, showing the nebulosities, photographs of the field round *ε Lyræ*, showing the faintest stars recorded by Herschel, and photographs of the clusters in *Hercules* and *Perseus*.

By their persevering labours and great optical skill MM. Henry had made a great step in astronomical studies, of which the importance was soon realised throughout the world. In the course of a few years the project for the complete photographic chart of the sky was commenced, and is now well on its way towards completion.

During the progress of the astrographic chart MM. Henry have contributed several papers to elucidate minor points which arose as the work was being carried out. They have taken the photographs in the Paris zone and were largely responsible for the calculations and reductions. Again, more than half of the instruments engaged in the International Chart and Catalogue of the heavens have come from their workshop.

As illustrating the extent and character of the optical work carried out by MM. Henry it is only necessary to mention some of the larger objectives and mirrors made by them. The list includes the 32-inch photographic refractor, 24-inch visual refractor, and 40-inch mirror of the Meudon Observatory, the

24-inch equatorial coudé of the Observatory of Paris, the 30-inch refractor of the Nice Observatory, and the 31½-inch reflector of the Toulouse Observatory.

In writing of Prosper Henry it has been impossible to separate his work from that of his brother Paul. M. Callandreau, their colleague in the National Observatory, writes* of them: "So united was their friendship and collaboration that at the Observatory we seemed to see but one person; so forgetful were they of giving prominence to their respective merits that it is impossible to decide what may belong to each of their common work." The astronomers who attended the conferences at Paris, and thus had an opportunity of meeting Prosper Henry, no less than his colleagues at the Paris Observatory, admired equally the modesty of his nature and the variety and extent of his scientific attainments.

He died suddenly on the 25th of July 1903, while taking a holiday at Pralognan, in Savoy.

M. Henry was a Chevalier of the Legion of Honour, and officer of Public Instruction. He was elected an Associate of the Royal Astronomical Society on the 8th of November 1889.

F. W. D.

* *Popular Astronomy*, December 1903.

Proceedings of Observatories.

The following reports of the proceedings of Observatories during the past year have been received from the Directors of the several Observatories, who are alone responsible for the same.

Royal Observatory, Greenwich.
(*Dr. Christie, Astronomer Royal.*)

Transit Circle.—More observations were obtained with the transit circle during the year than in any previous year since its erection, in spite of 1903 having been the wettest of all these years. The number of transits observed was 14,236, and of zenith distances 12,190. Such good progress has been made with the observation of the 10,000 stars within 26° of the pole which will be used as reference stars for the astrographic plates, that it has been decided to terminate these observations at the end of 1905, and form a nine-year catalogue instead of ten-year as originally proposed. The Sun has been observed on the meridian 180 times, its horizontal diameter 139 times, and its vertical diameter 157 times. The Moon has been observed 118 times. The mean error in R.A. of Hansen's Lunar Tables with Newcomb's corrections is $-0^s\cdot19$ for 1903. The R−D discordance is exactly the same as last year, the correction to the D observation being

$$+0''\cdot05+0''\cdot26 \sin \text{Z.D.}$$

Great progress has been made in the re-reduction of Groombridge's observations. The revised observations have all been brought to the mean equinox 1810·0. Precessions and secular variations have been computed. The observations have all been compared with modern Greenwich observations, and proper motions derived, and a comparison for check with the Radcliffe Catalogue (1845·0) is in progress. Copy for press of the ledgers and of the catalogue is in progress.

It is proposed to re-examine the comparison of the observations of the Moon made at Greenwich since 1751. Several papers by Mr. Cowell in connection with this work have appeared in the *Monthly Notices* during the year.

The Altazimuth.—This instrument has been used as a reversible transit circle in all four positions. During the year 1461

observations of right ascension and 1084 of north polar distance have been made. Reflexion and direct observations of zenith distance have been obtained in all positions. Eighty-nine observations of the Moon have been made, 37 of these being in the meridian, and 52 extra-meridian. Of the latter 30 were obtained in the first quarter and 22 in the last. Taking the transit circle and altazimuth together, a very large number of observations of the Moon have been secured, distributed with considerable uniformity throughout the lunation. The following table gives the number of days on which the Moon was observed at a given age, and the number of observations with the two instruments :

Moon's Age in Days.	No. of Days Observed.	No. of Observations. T.C.	Altaz.	Moon's Age in Days.	No. of Days Observed.	No. of Observations. T.C.	Altaz.	Moon's Age in Days.	No. of Days Observed.	No. of Observations. T.C.	Altaz.
2–3	2	0	2	11–12	4	4	3	20–21	7	7	0
3–4	2	1	2	12–13	6	6	5	21–22	7	4	4
4–5	6	2	6	13–14	8	7	5	22–23	6	3	4
5–6	8	7	7	14–15	7	7	3	23–24	3	3	3
6–7	5	4	5	15–16	9	9	4	24–25	6	3	4
7–8	8	5	8	16–17	5	5	2*	25–26	6	5	3
8–9	6	6	5	17–18	7	7	0	26–27	5	0	5
9–10	7	6	3	18–19	5	5	0				
10–11	8	8	6	19–20	4	4	0				

Reflex Zenith Tube.—During the year 781 complete observations besides 60 observations without reversal were obtained. Attempts have been made to observe the same star for as long a period throughout the year as possible. γ *Draconis* has been observed 65 times, seven other stars at least 20 times, and twenty-seven have been observed 10 times or more.

Equatorials.—Occultations of seventeen stars by the Moon have been observed by one or more observers during the year.

28-*inch Refractor.*—The weather has been very unfavourable for the observation of close and difficult double-stars. In consequence of this, although there is an increase in the number of stars measured, it is almost entirely made up of stars whose distance apart is more than 2″·0. An analysis of the observations gives :

98	of stars of	distance		$>0''\cdot5$
100	,,	,,	,,	$0''\cdot5-1''\cdot0$
93	,,	,,	,,	$1''\cdot0-2''\cdot0$
153	,,	,,	,,	$>2''\cdot0$

On the average, stars whose distance apart is less than 1″·0 were measured on 2·6 nights, the remainder on 2·1 nights.

The following list contains some of the most interesting stars measured during the year :

Star.	Mags.		Dist.	No. of Nights.	Star.	Mags.		Dist.	No. of Nights.
	m	m	"			m	m	"	
ε Pegasi	3·9	5·0	0·1	13	Σ 1629	6·7	7·9	0·3	5
δ Equulei	4·5	5·0	0·3	18	Σ 1728	6·0	6·0	0·3	4
70 Ophiuchi	4·1	6·1	1·6	14	β 612	6·3	6·5	0·3	5
Σ 3121	7·5	7·8	0·6	6	ζ Boötis	3·5	3·9	0·3	6
7ª Androm.	5·0	6·2	0·4	2	β 80	8·0	8·8	0·2	2

Thompson Equatorial.—The dome after twenty years of use was found to require extensive renewal of the papier-mâché covering. This was carried out in the autumn, and afterwards the 30-inch mirror was resilvered on November 27 without removing it from the dome, apparatus for inverting the mirror and holding it when inverted in the silvering solution having been constructed. The telescope was out of use from September 14 to December 2 during the repair of the dome and the resilvering of the mirror.

With the 26-inch refractor 70 photographs of *Neptune* and its satellite were taken on 31 nights, between January 1 and May 4, of which 53 have been measured. From December 2 to December 31 15 photographs were obtained on ten nights ; 21 photographs of double stars, 5 of *Nova Geminorum*, and 12 for adjustments, were also obtained.

With the 30-inch reflector the following photographs were obtained :

Comet *d*, 1902 : 35 photographs on 30 nights.
Comet *a*, 1903 : 18 " " 14 "
Comet *b*, 1902 : 6 unsuccessful attempts.
Comet *c*, 1903 : 27 photographs on 27 nights.

In addition, eight long-exposure photographs were obtained of Comet *c*, 1903 (see *Monthly Notices*, 1904 January).

Photographs of 21 minor planets were obtained on from one to six nights each, the average number of nights being three for each planet.

The measurement of the photographs of comets and minor planets is deferred till the photographs of *Eros* of 1900 are measured. For the measurement of the *Eros* photographs a new micrometer, having a glass diaphragm in the focus of the microscope carried on cross-slides by micrometer screws, was obtained from Messrs. Troughton and Simms early in the year. The object-glass of the microscope is so mounted that a magnification of 1 or 2 in the body of the microscope can be used at pleasure to adapt it to photographs with the Thompson and Astrographic telescopes respectively. At the beginning of the year the screws were examined, the division errors of the diaphragm determined, and the distortion of the microscope object-glass investigated. About the same time the errors of the diaphragms belonging to the duplex micrometer were determined. The errors of the

réseau employed in the *Eros* photographs were also investigated. The actual measurement of the photographs was not begun till October, and since then has been pushed on energetically.

Astrographic Equatorial.—During the year ninety-one photographs were taken on forty-six nights. These include eleven for the catalogue, twenty for the chart, twenty of *Nova Persei*, a few miscellaneous photographs and nine for adjustment of the instrument. The number of plates measured (direct and reversed) in the year was 104. The measurement of the Greenwich plates is completed from Dec. 79° to Dec. 82°. Altogether 1002 plates out of 1149 containing 1879 square degrees out of 2088 (90 per cent. of the total amount) have been measured. The numbers of stars measured in the three zones are :

Zone.	No. of Stars.	Area in Sq. Degrees.	No. per Sq. Degree.
79°	7380	65·6	112·5
80°	5980	59·4	100·7
81°	4750	53·2	89·3

The Introduction to Vol. 1 of the *Astrographic Catalogue* is in type. This volume contains the measures of rectangular coordinates and diameters from 64° to 72° N. Dec., together with plate constants and other data. These measures will be continued to the pole in Vol. 2. It is intended to give in Vol. 3 a catalogue of R.A. and N.P.D. (1900·0) of all the reference stars and all the stars found in the catalogues of the *Astronomische Gesellschaft* (1875), and in Carrington's *Circumpolar Catalogue* (1855).

Copy for press has been prepared for Zones 72°, 73°, and 74°.

During the year the stars visible on 108 chart plates have been counted, completing the area from Dec. 64° to Dec. 71°.

The reproduction of the chart plates by purely photographic processes has just been begun. The number of copies made in the first instance will be fifty of each chart plate. These will be on bromide card and on double the scale of the original negatives.

Photoheliograph.—Photographs of the Sun have been taken on 176 days with the Thompson (9-inch) photoheliograph, and on 59 days with the Dallmeyer (4-inch), during the repair of the Thompson dome. Of the photographs 545 have been selected for preservation. Photographs have been received from India up to 1903 October 7 ; for the year ending 1903 October 7 photographs are available for 334 days out of 365. Photographs from Mauritius have not been received since the date of the last annual report, the last negative received at Greenwich being for 1902 June 20, and it is hoped that some of the numerous gaps will be filled when these photographs are received. The Greenwich photographs are measured to 1903 August 8, the Indian ones to October 7. The increase of solar activity during the year has been very marked, the group of spots from October 4 to October 18 being an especially large one.

The reductions of the observations by the Greenwich

observers for the determination of the Paris-Greenwich longitude were completed in May.

The printing of the Greenwich-Waterville-Canso-Montreal longitude is in progress.

Professor Albrecht and Dr. Wanach made a determination of the longitude Potsdam-Greenwich from May to July.

In July and October Major Burrard and Major Lenox-Conyngham swung the half-seconds pendulums, which they are to use for the Indian Survey, in the Record room of the Observatory.

The volume of Greenwich Observations for 1901 is printed, and will be distributed shortly.

Mr. W. C. Nash retired on December 31, after a long and honourable connection with the Observatory of nearly forty-eight years. Mr. Bryant has succeeded him in the Superintendence of the Magnetic and Meteorological Department.

Royal Observatory, Cape of Good Hope.
(Sir David Gill, H.M. Astronomer.)

The weather generally throughout the year has been exceptionally unfavourable for astronomical observation, and much of the time of H.M. Astronomer has been occupied with the installation, adjustment of, and experimentation with new apparatus.

The pits which give access to the underground azimuth marks of the new transit circle, described in former reports, have at last been lined with iron. The houses covering the collimators and marks have been erected, the piers built, the collimators erected, and the wires laid and fittings installed for electric illumination of the marks and pits.

The object-glasses of long focus for adjusting the upper marks vertically over the underground marks have not yet been received from Mr. Simms, but he promises their early delivery.

A very thorough investigation of the division errors of the new transit circle has been in progress during the year, but is not yet complete. It is intended to investigate the error of every division line on the fixed circle (5′ to 5′), and on the movable circle only the lines marking whole degrees. The results, so far as they go, are most satisfactory in every way. The probable error of a single pointing, derived from the residuals from interwoven series made by the same observer, is $\pm 0''{\cdot}18$; and the same probable error, $\pm 0''{\cdot}18$, is derived whether we group the observations so as to include or avoid the effects of flexure of the circle, or obtain it from comparison of the division errors as independently determined by different observers. The fact that the circles are solid cast-iron discs without arms accounts for their rigidity, and the general stability of the instrument and the clearness of the division under the new illumination account for the freedom of the investigation from unforeseen systematic error or personality in reading the divisions. Mr. Hough has communicated a paper to the Society on the methods employed.

An eye-end has recently been received from Repsolds which permits the observation of transits with a moving wire—that is to say, the wire is caused to travel at a rate approximately equal to that of the star either solely by the hands of the observer himself, or with the aid of clockwork which can be set to run at nearly the required speed. In both cases the eyepiece travels with the wire, so that, when properly managed, the star appears at rest in the centre of the field. The observer has thus merely to maintain the bisection of a star on a wire which is apparently fixed in the centre of the field, and the chronograph records are automatically made by electric contacts with alternate conducting and non-conducting pieces on the drum of the screw which causes the wire to travel.

Much time has been spent on plans for perfecting this system of observation, which seems to be the only one by which we can get rid of personality in R.A. depending on magnitude and on declination.

The spectroscope of the Victoria telescope, which had been in the hands of the Cambridge Scientific Instrument Company since May 1901, arrived at the Cape on May 27 last. It is now fitted with four new very transparent prisms, made of lighter flint glass than that of the three original prisms, which were rather highly coloured and absorbed too much of the violet rays. In addition the spectroscope is now enclosed in a double aluminium cover stuffed with feathers, and with exquisite automatic means for maintaining the included air at uniform temperature by means of automatically regulated electric currents passing through coils of manganin wire. An electric fan inside the aluminium cover circulates the air, to preserve uniform temperature throughout. The temperature inside the prism box is registered continuously on a drum by a Callendar recorder, giving 1° F. = one inch, and can be maintained for days together within $\pm 0°{\cdot}1$ F.

The new sidereal clock arrived at the same time with the spectroscope, and it has been mounted in the now disused magnetic house.

This clock is certainly the most elaborate, and probably by far the most perfect, yet constructed. It consists essentially of a pendulum swinging in a nearly air-tight glass enclosure, the air in which is maintained automatically at uniform temperature and low uniform pressure. The impulse given to the pendulum depends on gravity only, and is entirely independent of the effects of any sticking or repulsion at points of electric contact. The pendulum of the slave clock is rather shorter than the main pendulum ; it is "held up" by a spring catch for about one-tenth of a second at each alternate beat, and this catch is discharged by a trigger, the electric magnet of which is short-circuited at each alternate second by the fall of the light gravity arm of the main pendulum on a fixed platinum anvil. The slave clock pendulum performs all the contacts necessary, *i.e.* for raising and

lowering the gravity arm of the main pendulum at the proper instants, and for making the contacts for the chronograph; it also indicates the minutes and seconds ; it periodically actuates the feeler connected with the temperature control, and makes the contacts connected with the automatic control of the pressure. A detailed description of the instrument is in course of preparation.

The progress in printing the catalogue of 8560 stars has been very slow ; only 120 pages (0^h to 13^h) have been received from the printer during the year.

Part i. vol. xi. of the *Cape Annals*, containing a discussion of the Heliometer Triangulation of Southern Circumpolar Stars within 2° of the South Pole, has been passed through the press and is ready for distribution.

Part ii. of the same volume, containing the results of a discussion of photographic plates covering the same region, is ready for press.

The work of the transit circle has been confined to the completion of a catalogue of 2798 selected zodiacal stars, besides all stars brighter than 8·1 magnitude which are not contained in Gould's General Catalogue (excepting those in the zone Decl. −40° to −52°, most of which are included in the Cape Catalogue for 1900), stars required for geodetic purposes and stars of which occultations have been observed at the Cape. A few observations, which will be obtained during the month of January, are only now required to complete the work. A final redetermination of the magnitude equation of the observers will complete the work of the old transit circle for the present. The completion of a catalogue necessarily involves considerable lacunæ in the observing list, so that the total number of observations is somewhat less than usual, viz. :

Number of transits ...	6262	Azimuth	304
Determinations of Z.D.	5711	Run	269
Collimation	98	Nadir	267
Level	268	Flexure	23

The reductions of the meridian observations to apparent place are complete to the end of 1903. The reductions to mean place are complete in R.A. to September 18, and in Declination to August 20.

Only sixteen separate phenomena of occultations have been observed, all of which were predicted by the *Nautical Almanac* Office.

Disappearances at the dark limb	12
Reappearances „ „	4

Five of these phenomena were observed by two observers.
One „ „ was „ three „

The following observations of comets have been made by Mr. Cox:

Comet *a* 1903 on 8 nights
b 1903 „ 11 „ } *Ast. Nach.* 3906.
c 1903 „ 6 „ forwarded to *Ast. Nach.* Nov. 18.

The star *C.P.D.* −62°, 6358, has been proved to be a long-period variable, 8·0 mag. to invisible. The positions of forty-five faint stars of which occultations have been observed were determined by differences of R.A. and Decl. from known stars with the 7-inch equatorial.

In February and March Mr. Innes made with the 18-inch object-glass of the Victoria telescope 161 sets of measures of 108 double stars, two of which were of new pairs, the series including ten sets of measures of *α Centauri*.

During Mr. Lunt's absence Mr. Goatcher has measured and discussed all the lines visible in the spectra of *Canopus* and *Sirius* taken with the three-prism spectrograph. Within the limits of λ4202 and λ4584 he finds the spectrum of *Canopus* to contain the lines of H, Fe, Ti, Cr, Sc, Ca, Mg, Ba, Sr, Y, C ?, V ?, Zr ?, and these elements account for all the lines visible. The spectrum of *Sirius* within the same limits shows the presence of H, Fe, Ti, Cr, Ca, Ba, Sr, Mg. He has also measured and reduced photographs of the spectra of α_1 and α_2 *Centauri*, *β Orionis*, *α Pavonis*, and *α Scorpii*.

Previous to July the Victoria telescope was employed by Mr. Goatcher in photographing star clusters and nebulæ.

Between July 22 and November 7 the instrument was dismounted for the purpose of adapting the remodelled spectroscope with its automatic temperature control and the Callendar recorder, which registers continuously the temperature of the interior of the prism box. The mounting of the numerous electric leads, the re-balancing of the telescope, the fitting and adjustment of all subsidiary apparatus and switches occupied some months. One hundred and four photographs were taken for the focussing and adjustments of the spectrograph—mainly by Newall's method. In this method one part of the slit is used, and the image of the spectrum is formed by rays passing through one half of the collimator objective and through the thinner half of the prisms; immediately afterwards another adjacent part of the slit is used, and another spectrum is photographed on the same plate by rays which have passed through the other half of the collimator object-glass and through the half of the prisms adjacent to their bases. With the present object-glass of the camera telescope it is impossible to obtain coincidence of the lines in two such photographs throughout the entire length of the spectrum. With proper adjustment the lines coincide for a certain distance and then deviate like those of a very fine vernier. In the remodelled spectroscope a screen is provided, which moves in a slide, so that

it can be pushed in front of either half of the object-glass or clear of it. This slide is worked by a rod coming outside the aluminium case, and a spring, dropping into proper notches, enables the observer to place the screen exactly in any one of the three requisite positions.

Experience with the original spectroscope showed the necessity for this, as a perceptible systematic difference in the apparent radial velocity, amounting to several kilometres per second, was obtained from measures of lines in different parts of the spectrum. This anomaly has its origin in the different exposure required for rays passing through a greater or less thickness of glass. If the spectrum is over-exposed the mean image of a line photographed with the full aperture would depend in greater proportion on the rays which pass through the half of the prism next to the base than would be the case in an under-exposed spectrum ; and it is practically impossible to maintain an exact relative density between the image of the comparison spectrum and that of the star. The remedy appears to be to use for determinations of radial velocity only those lines which are in perfect coincidence when separately photographed through the thick and thin halves of the prisms. For this purpose the rule has been laid down to take through both halves of the prisms a plate of the iron spectrum immediately before or after any photograph intended for determination of radial velocity, and to use for the velocity determinations only the part or parts of the spectrum in which the lines in the two adjacent spectra are in perfect coincidence. The whole of the experiments and adjustments were completed on November 7, but, in consequence of the exceptionally unfavourable weather, only twenty-two plates of spectra could be obtained on thirteen nights before the end of the year.

The most remarkable result obtained is the rapid radial velocity of *a Phœnicis*, which in December was receding at 105 kilometres per second from the Earth and 82 kilometres from the Sun.

The new objective prism of 24 inches aperture and $11\frac{1}{4}°$ of refracting angle in a new mounting, which permits its use alone or conjointly with the Grubb prism, was received from Messrs. Zeiss at the end of November, but has not yet been adapted to the telescope.

The following oppositions of major planets have been observed with the heliometer during the year :

	No. of Obs.	Nights.
Opposition of Mars	20	4
„ „ Uranus	24	3
„ „ Saturn	32	4
„ „ Jupiter	44	6
„ „ Neptune	30	6

The following observations have been made in connection with the triangulations connecting the comparison stars :

	Observations.
For Neptune, 1901–4	87
„ Mars, 1901	55
„ Uranus, 1901–2	154
„ Saturn and Jupiter, 1901	132
„ Saturn, 1903; Jupiter, 1902	57
„ Jupiter, 1903	25

Fifty observations have also been made connecting the position-angles and distances of the stars employed as standards for the photographic plates of *Jupiter's* satellites in the oppositions of 1902 and 1903, with pairs of stars contained in the triangulation of the *Victoria* comparison stars.

Comet *a* 1903 was also observed with the heliometer on seven nights.

The reduction of the fine series of heliometer observations of *Jupiter's* satellites made by Mr. Cookson in 1901 and 1902 is completed, and Mr. Cookson is now engaged in England in deriving the elements of the orbits, the mass of *Jupiter*, and the constants of the inequalities which involve the masses of the satellites.

With the astrographic telescope the following work has been accomplished :

	No. of Plates.	No. of Exposures.	Duration of each Exposure.
			m m m
Triple-exposure chart plates (Ilford S.R.) passed	22	66	30, 30, 30
„ „ „ „ „ „ rejected	18	54	30, 30, 30
„ „ „ „ (Ilford Monarch) passed	80	240	20, 20, 20
„ „ „ „ „ „ rejected	11	33	20, 20, 20
			m m s
Revision Catalogue plates (Ilford S.R.) passed	104	312	6, 3, 20
„ „ „ „ „ rejected	53	159	6, 3, 20
„ „ „ (Ilford Monarch) passed	110	330	3, 1½, 10
„ „ „ „ „ rejected	37	111	3, 1½, 10
Jupiter's satellites	33	297	5ˢ & trail
Adjustment plates	9	...	various
Comet *a* 1903	7	7	„

The following table gives the work done to the end of 1903 :

Mid. Dec. of Zone.	Triple Image Charts. Taken.	To be Taken.	Single Image Charts. Taken.	To be Taken.	Second Series. Catalogue Plates. Taken.	To be Taken.
°						
−41	130	14	...	...	143	1
42	...	...	126	18	142	2
43	124	20	...	...	140	4
44	...	...	123	21	114	30
45	106	38	...	...	111	33
46	...	...	124	20	109	35
47	108	36	...	...	104	40
48	...	...	123	21	83	61
49	84	36	...	...	59	61
50	...	...	104	16	52	68
51	68	52	...	...	50	70
−52	...	...	65	55	...	...
	620	196	665	151	1107	405

The first series of Catalogue Plates is completed.

In the measuring-room a good deal of time has been spent in training new observers and in revising old work.

During the year 1903 156 Catalogue Plates, containing 80,808 stars, have been measured in two positions of each plate—including 1866 Standard Stars, each of the latter being measured in two positions of the plate by both the measurers employed on each plate. Four plates measured in former years have since been rejected. The total number of measured plates is now 586, containing 328,000 stars. The actual state of the work is as follows :

Catalogue Plates.

Mid. Dec. of Zone.	No. of Plates Measured. Before 1903.	During 1903.	Out-standing.	No. of Plates Copied for Press. Before 1903.	During 1903.	Out-standing.
°						
−41	139	4	1	96	47	1
42	110	31	3	27	17	100
43	99	41	4	73	...	71
44	50	48	46	28	...	116
45	26	32	86	14	5	125
46	2	...	142	...	...	144
47	4	...	140	...	...	144
48	...	...	144	...	...	144
49	...	...	120	...	...	120
50	...	...	120	...	...	120
−51	...	...	120	...	...	120
	430	156	926	238	69	1205

The total number of separate star-discs measured on the Catalogue Plates of the Cape Zone will probably amount to about one million, corresponding with about 400,000 different stars.

The tabular coordinates of the reference stars (from the data of the *Cape Zone Catalogue* for 1900) have been computed for all the plates of all the zones, and the plate constants have been computed for all plates of the zone $-41°$, except of course for the single plate that is still wanting. For the same zone the coordinates of all the C.P.D. stars have been approximately computed, and these have been compared with the recent measures ; the corresponding C.P.D. numbers have been entered on the sheets for press.

The work of the Anglo-German Boundary Survey between German South-West Africa and British Bechuanaland has been completed.

The reports of the work of the Anglo-German Boundary Survey and of the Geodetic Survey in Rhodesia south of the Zambesi are practically complete, and will be ready for press within a few weeks.

The geodetic operations in the Transvaal and Orange River Colony were actively commenced in January 1903, under the superintendence of Colonel Morris, R.E., C.B.

The following represents the scope of the Survey:

I. *a.* Newcastle (Natal) to the Limpopo (along the 30th meridian), 415 miles.

b. Limpopo to Ottoshoop, 230 miles.

c. Belfast to Ottoshoop, 240 miles.

d. Newcastle to Christiana, 220 miles.

e. Ottoshoop to Kimberley, 245 miles.

f. Pretoria to Cala (through Kroonstad), 405 miles.

g. Hopetown to Natal (through Wepener), 320 miles.

h. Newcastle along 28th parallel to the sea, 140 miles, representing chains of triangulation of an approximate total length of 2215 miles.

Of this work there has been completed:

Reconnaissance	53 per cent.
Beaconing	13 ,,
Observation of angles	5 ,,

Only one 10-inch Repsold Theodolite was available during the year. Two similar instruments were shipped for the service by Messrs. Repsold on November 25.

Of the ten triangles measured the greatest triangular error is $+1''{\cdot}52$, and the probable error of a single angle derived from all the triangular errors is $\pm 0''{\cdot}327$.

II. *Base Lines—*

k. Belfast, 11·8 miles in length.
l. Ottoshoop, 10·8 „ „
m. Wepener, 13·5 „ „
n. Kroonstad, 12·5 „ „
r. Limpopo (not yet selected).

Of these *k* and *l* have each been independently measured three times, and *m* twice (the third measurement was being made at the date of report, December 29); *n* was selected and laid out by Colonel Morris in November, and will be measured on the completion of *m*. The site of the remaining base south of the intersection of the Limpopo by the 30th meridian has not yet been definitely selected, but it is expected that its measurement will be commenced in May 1904, when that region will probably be free from malaria and horse sickness.

These base lines have been measured with Jäderin wires of nickel steel. The experience now gained with that apparatus shows conclusively that when properly used all desirable accuracy can be secured.

A length of 480 feet was first laid down near the centre of each base and carefully measured with the standard 10-foot bars, of which the constants have been determined by comparison at the International Bureau of Weights and Measures (*Geodetic Survey of South Africa*, vol. i.). The lengths of the wires used in the reductions depend in all cases on numerous comparisons with the 480-foot ground standards.

The results of the measures are as follows :

	Belfast Base. Metres.	Ottoshoop Base. Metres.	Wepener Base. Metres.
Measure 1	18.994·027	17,438·188	21,655·265
2	·077	·182	·294
3	066	·163	...
	m.	m.	m.
Accidental prob. error of mean...	±0·0101 or ±0·0093(9)	±0·0050 or ±0·0046(8)	±0·0145 or ±0·0086(6)

Of these probable errors the first depends on the inter-agreement of the separate measures of the whole length of the base, the second on the inter-agreement of the measures of the separate sections of the base. The number of sections in each base is given in brackets. These probable errors do not include the probable error of absolute length of the 480-foot ground standards, which were measured with the steel bars at each comparison of the Jäderin wires, and this error does not amount to 1 : 1,000,000, including the error of the absolute length of the

bars, which is the largest part of the probable error. The mean of three measures with the Jäderin wires fully attains to a corresponding accuracy.

For the measurement of the arc of meridian from the Zambesi to Lake Tanganyika I have selected Dr. Tryggve Rubin, who was a member of the Swedish-Russian expedition for measurement of the Spitzbergen arc of meridian in the summer of 1901, and leader of the expedition which completed that work in the summer of 1902. After residence and work for three weeks at the Cape Observatory he sailed for Chinde on April 29, accompanied by Dr. F. O. Stoehr, M.B. (surgeon of the expedition), and Messrs. Edward Stroud and Paul Chapman as assistants. The party arrived at Chinde on May 12, where they were detained a week in landing and reshipping their instruments for Feira. The party proceeded to Fort Jameson, where the equipment of the expedition with native carriers was completed, and it finally reached its field of operations at Feira, on the Zambesi, on July 13. The season had been very dry; grass fires had begun, and any work beyond reconnaissance, beaconing and astronomical observation was rendered impossible by the smoke which completely obscured the horizon. On the way from Fort Jameson to Feira reconnaissances were made for a base line, and an excellent site was selected of nearly 10 miles in length on a plain alongside the river Loangwa, near its intersection with the parallel of $-15°$. The measurement of the base itself is deferred pending the arrival of a $2\frac{1}{2}$-metre standard, obtained on loan from the Russian Government at the request of the Royal Society.

A considerable amount of line-clearing and beaconing had been performed, and a number of astronomical determinations of latitude and azimuth had been made at the date of last report (September 3), but the dense smoke of the grass fires had greatly hindered progress. Dr. Rubin was, therefore, instructed to demarcate the Portuguese boundary running due south from the Zambesi near Zumbo, and to fix by astronomical observation the point where the 15th parallel of latitude crosses the river Loangwa.

These matters will facilitate the operations of our Anglo-Portuguese Commission (which is about to commence the work of definitive demarcation of the boundary between British and Portuguese territory), and will probably be completed before the end of the first rains (in October), when the sky will be free from the smoke of grass fires and geodetic operations may be resumed.

Mr. G. Tyrrell McCaw, who was appointed chief assistant to Dr. Rubin, left for Feira *via* Salisbury on October 5, having spent a month at the Observatory for training in astronomical work. He took with him the spare theodolite and nickel steel wires for the Jäderin apparatus.

The records of the seismograph have been regularly for-

warded to Professor Milne, Secretary to the Seismological Committee of the British Association.

The meteorological observations made during 1903 have been communicated to the Cape Meteorological Commission.

Mr. Franklin-Adams arrived at the Cape on July 28. His assistant, Mr. Kennedy, reached the Cape a month before him to erect the fine photographic equatorial with a triple lens, by Cooke, of 10 inches aperture and 45 inches focal length, with which he is making a star chart of the southern hemisphere on plates covering 15° × 15°. The work is now half completed, with satisfactory results.

Royal Observatory, Edinburgh.
(Dr. Copeland, Astronomer Royal for Scotland.)

The programme for meridian observations included the zodiacal stars and heliometer comparison stars in Sir David Gill's lists, and the *Berliner Jahrbuch* clock star-list. The total number of observations, all of which were made by Mr. Clark, was 1478, of which 995 were of stars in Sir D. Gill's lists, 396 of clock stars, 72 of azimuth stars, and 15 of planets. With the exception of a few clock stars, each of these observations included measures of both right ascension and declination. The instrumental errors were determined both at the beginning and end of each night's work. The instrument appears to have maintained its stability throughout the year. Owing to the fracture of one of the bevelled wheels used for raising the roof shutters of the meridian house, observations were interrupted for ten days in December.

The spectroscopic observations of the rotation of the Sun, commenced by Dr. Halm in 1901, were continued by him throughout the year, and measurements of the displacements of the Dunér iron lines on opposite limbs were secured on all favourable occasions. Rotational values for nearly 200 heliographic latitudes were measured, extending over the full quadrant from the equator to the pole. A comparison of the resulting curve with that obtained in the preceding years points conclusively to the fact that the vigorous initiation of the new cycle, manifested during the past year by considerable activity on the Sun's surface, was attended by a change in the Sun's rotation of a most pronounced character. Although a definite opinion as to this change cannot yet be expressed, its reality seems clearly established by the observations. A paper on the subject will shortly be laid before the Royal Society of Edinburgh. The instrument appears to work admirably; this is perhaps best shown by the fact that the probable error of a measurement is only one-half of that of an observation made by Professor Dunér.

The catalogue of the stars observed by the late Professor Henderson and Mr. Wallace during the years 1834–45 is now

completed. A thorough comparison has been made by Dr. Halm and Mr. Blackett with other catalogues, especially Lalande, Piazzi, and the catalogues of the *Astronomische Gesellschaft*. The differences Edinburgh-*Astronomische Gesellschaft* catalogues in both co-ordinates have been formed in the order of right ascension as well as in that of declination. One object of this investigation was to ascertain whether systematic differences exist between the first period, extending from 1834 to 1840, and the second, extending from 1841 to 1845. Such differences have, indeed, been found, especially in the declinations, where, however, they can be well accounted for by the assumption that there was a constant difference of about 2° F. in the indications of the thermometer in the two periods. No note has been found that would give a clue to the cause of such a difference in the thermometer readings.

Occasional observations of minor planets and comets and a determination of the screw value of the micrometer were made by Dr. Halm with the 15-inch refractor. With the same instrument micrometer observations of Comet 1903 *c* were made by Mr. Clark on eight nights, two or three comparison stars being observed each night.

The time service for the cities of Edinburgh and Dundee has been conducted as in former years, being controlled by the Molyneux mean time clock of this Observatory. On five days during the year the one o'clock gun was not fired. On one of these days both of the friction fuses used missed fire, a most unusual occurrence, as the misfire of a single fuse is expected to happen on an average only once in a thousand times ; on the other four days the apparatus was under repair. The time ball was not raised on February 7 on account of the violence of the gale of that day. It may be mentioned that in this gale a velocity of not less than eighty miles an hour was recorded by the anemometer.

The Milne seismograph has been again in operation during the year, and the photographic record shows traces of fifty-seven unfelt earthquakes, all of semi-amplitude not less than 0·25 mm. Of these five only were of large amplitude, the most remarkable of them being that of January 14, of which the semi-amplitude amounted to 11 mm. Copies of the records of this instrument are forwarded regularly to Professor Milne, Secretary to the Seismological Committee of the British Association for the Advancement of Science.

The lunar eclipse of April 11 was observed by the staff. A few photographs were obtained by Mr. Heath, and the first and last contacts, two immersions and four emersions of faint stars, were observed by Mr. Clark.

Watch was kept, by request, for the *Lyrid* meteors on the night of April 19, and for *Leonids* on November 14. No meteors were seen on either of these occasions, both the nights being unfavourable for such observations.

The usual meteorological observations have been made by the

staff of the Observatory. The Robinson anemometer and the King's barograph have been kept in operation throughout the year and the records tabulated with regularity. The weekly readings of the deep rock thermometers on the Calton Hill have been continued.

Cambridge Observatory (Sir R. S. Ball).

After nearly forty years' work as chief assistant, Mr. Graham retired on a pension on June 24. At the same time Miss Walker resigned the position of computer, which she had held for twenty-one years.

Mr. A. R. Hinks, M.A., Trinity College, has been promoted to be chief assistant, and took up residence at the Observatory in September.

Meridian Circle.—In the first four months of 1903 the meridian circle was used by Miss Walker on thirty-six nights in observation of Gill's zodiacal stars.

With the resignations above referred to a complete break has been made in the work of the meridian circle. Of the zodiacal stars about half are fully observed, and the reductions are complete.

In July Mr. W. E. Hartley, M.A., Trinity College, Cambridge, was appointed second assistant, and took up the meridian work. After a thorough examination of the state of the instrument it was decided that a number of repairs and improvements should be made before regular observation was resumed; and these alterations have occupied the latter months of the year. The principal additions to the instrument are a printing arrangement for the declination micrometer, a large setting wheel and new pointer, and a mercury dot apparatus, as described by Sir David Gill, for the determination of pivot errors.

A copy of the results of the observation of Sir David Gill's first list of heliometer comparison stars has been sent to him. The observation of the second list is complete, and will be sent shortly.

Sheepshanks Photographic Equatorial.—With the aid of a grant from the Government Grant Fund of the Royal Society the reduction of photographs of *Eros* for the determination of the Solar Parallax has progressed steadily during the year. Mr. Hinks has been assisted by Miss Julia Bell, Girton College, during the whole year, and by Miss Anne Malden, Newnham College, since October. The present state of the work is as follows:—

Eighty-nine exposures made at Cambridge are completely measured. To combine with these there are measures of 144 exposures kindly communicated by the Directors of the Paris, Algiers, Lick, Minneapolis, Northfield, Tacubaya, and San Fernando Observatories.

A system of standard places of comparison stars has been deduced from measures made at Paris and Cambridge; at least ten of these stars are measured upon nearly all the plates. The reduction of all the measures of the planet to this standard system is in progress and will shortly be finished. All the interpolations from the ephemeris and calculations of parallax factors, &c., are completed, ready for the formation of the final equations of condition.

A copy of the adopted places of comparison stars has been sent to the Oxford University Observatory at the request of the Director.

Mr. H. N. Russell, formerly Fellow of Princeton, and recently appointed a Research Assistant to the Carnegie Institution, has joined the staff as an honorary assistant, and has been actively at work with the photographic equatorial. The working list of stars selected for parallax determination has been greatly enlarged, to serve as a programme of joint work by Mr. Hinks and Mr. Russell. In the course of the year 183 exposures for measurement have been made, in addition to a large number of plates made in the trial of partial colour screens, used in contact with the plate, for reducing to measurable dimensions the photographic images of bright stars. The division errors of the standard réseau, Gautier, No. 86, have been determined.

Meteorological Observations.—At the end of the year the climatological observations were discontinued at the Observatory, the Cambridge "station of the second order" having been transferred to the University Botanic Garden.

It is proposed to devote the energy thus set free from routine work to the gradual establishment at the Observatory of experimental researches in atmospheric physics. As a beginning the following instruments are now being set up at the Observatory:

1. A Bendorf electrograph, with radium radiator, lent by Dr. W. N. Shaw, F.R.S.

2. A Dines self-recording barometer, lent by the Meteorological Council.

3. An equatorially mounted Callendar sunshine receiver, lent by Mr. W. E. Wilson, F.R.S.

Mr. C. T. R. Wilson, F.R.S., is kindly supervising the erection and standardisation of the instruments.

The records will be deposited at the Meteorological Office.

The Newall Telescope, Cambridge Observatory.
(*Mr. H. F. Newall.*)

The Newall telescope was used for observations on eighty-three nights in the course of the year 1903.

The work throughout the year has been a continuation of the spectroscopic studies carried on in recent years, principally in the

determination of velocity in the line of sight. Special attention has been given to the spectra of *Sirius*, *Procyon*, *α Boötis*, and of certain stars in Taurus. Observations of chosen "velocity-reference-stars" have been continued in co-operation with other observatories; and a note giving results relating to some of these stars as observed in 1902 was published in the *Monthly Notices*, March 1903, lxiii. 296.

One hundred and sixty-eight (168) photographs of stellar spectra have been obtained, including a few in which pairs of spectra of different stars are set side by side on the same plate for purposes of rapid comparison. Many other photographs have been rejected for various reasons. The adjustments of the instrument have been regularly tested by means of special photographs.

Dunsink Observatory.
(Prof. C. J. Joly, Royal Astronomer of Ireland.)

Systematic work with the meridian circle was suspended during the year 1903 in order to devote all possible attention to the reduction of the observations of Sir David Gill's Zodiacal Stars made in 1900, 1901, and 1902, and completed in 1902. All these observations are now reduced to apparent place, and those taken in 1900 are reduced to mean place. The precessions and secular variations have also been computed for the observations of 1900.

Advantage was taken of the interruption of continued observation in order to overhaul the instrument. The mirrors for the illumination of the field were re-silvered, and the object-glass was thoroughly cleaned and some traces of fungus removed. On readjustment the definition was found to be considerably improved.

About 200 observations were made with the meridian instrument during the year for the determination of time, the time-service to Dublin being continued as usual. An investigation was also made into the barometric and thermometric variations of the rate of the Dent sidereal clock over a long period, and tables were drawn up to facilitate the application of the necessary corrections. These tables have proved very satisfactory.

A large number of barograph records of the great storm of February last, which was especially severe in the neighbourhood of Dublin, were sent by request to the Observatory from various parts of Ireland. These were carefully compared and discussed, and a short account of the results was published in *Symons's Meteorological Magazine*.

A number of photographs were taken with the Roberts equatorial, and the South equatorial was chiefly used for showing objects of interest to visitors on the monthly open nights.

University of Durham Observatory. (*Prof. R. A. Sampson.*)

Work with the almucantar during the year has been confined to the preparation of an account of the observations taken up to 1902 December. This was published in *Monthly Notices*, April. The severe storm of February 26 blew the dome off the almucantar house; the telescope did not suffer, but advantage was taken of the interruption to overhaul it, to re-silver the mirror and fix it more securely in its cell.

During the year a memoir describing Adams's MSS. on the perturbations of *Uranus* has been prepared for the press by Professor Sampson; it is being published in the R.A.S. *Memoirs*, vol. liv.

Apart from this all available time has been devoted to pushing on the *Jupiter's Satellites* computations; these are now well forward and a solution of the normal equations is being made. The work has been assisted by a grant from the Government Grant Committee. A considerable body of completed MS. has been despatched during the year to Harvard College Observatory for publication.

Glasgow Observatory. (*Prof. L. Becker.*)

Owing to the continued unfavourable weather in the past year little progress was made with the observations of the stars in the immediate vicinity of the pole. These stars are measured during a night at several points of their daily orbits, with the transit circle, with the object of determining the position of the pole and the coordinates of the fundamental pole-stars. During the first and last quarters of the year, which, on account of the length of the nights, are most favourable for the research, there were only two nights on which the observations of the faint stars could be carried on for any length of time. The total number of nights devoted to these observations was but five.

The spectrograph attached to the Breadalbane reflector was, during the first two months of the year, reserved for Nova *Persei*. A plate which had already been exposed twenty-four hours in November of the preceding year was several times inserted again, with the outcome that another two hours were added to the time of exposure in January and February. Unfortunately two occasions in January could not be taken advantage of, because the temperature of the spectrograph differed too much from those on the previous occasions. This plate contains our last record of the spectrum of the new star. On March 3 the spectrum was inspected in the focal plane of the spectrograph without an eyepiece, and it appeared to be reduced to one spot, which, by means of the spark spectrum, was identified as the

principal nebular line. Another line in the neighbourhood of 4480 was noted as doubtful.

In April and May the red and yellow region of the spectrum of *Mars* and the Moon was photographed on ten nights; the plates have not yet been reduced. For the rest of the year work with the spectrograph was suspended; a new triple object-glass for the collimator, f 10, achromatised from wave-length 5000 to 3700, was supplied in December by Messrs. Steinheil & Sons, and provision was made for automatic heating of the instrument.

A paper containing the discussion of the spectrum of Nova *Persei* as photographed here is now ready for the press.

The time service and meteorological work has been carried on as in former years.

Liverpool Observatory. (*Mr. W. E. Plummer.*)

No change has been made in the staff or in the instrumental equipment of this Observatory during the past twelve months. In the absence of any photographic apparatus, efforts were made in the early part of the year to enable the staff to take some part in the measurement and the reduction of photographic plates taken at other observatories; but though the directors of some observatories were kind enough to send selections of plates for the purpose of measurement, the project fell through owing to the impossibility of securing a suitable measuring machine.

Under these circumstances the series of observations pursued in earlier years has been continued. These include the systematic observation of comets, while bright enough to be observed in the equatorial, the coordinates of a certain number of binary stars, whose choice is also limited by the aperture of the telescope, and other extra-meridional observations.

The Observatory is a meteorological and seismological station, and observations connected with these subjects are zealously pursued. In connection with the routine work may be mentioned the distribution of time signals, the testing and rating of chronometers and other philosophical apparatus for which the Mersey Docks and Harbour Board are prepared to grant certificates of test. With the office of Director of the Observatory is coupled that of Hon. Reader of Astronomy in the University of Liverpool, and under this arrangement lectures are regularly given in the Observatory.

Radcliffe Observatory, Oxford.
(*Dr. Rambaut, Radcliffe Observer.*)

The work with the transit circle during the year 1903 has been confined to observations required to complete the working list of stars and reflection observations for the determination of the R—D correction.

The reduction of the observations has been pushed on vigorously, and all the observations up to date, both in R.A. and N.P.D., have been reduced to mean place.

The preparation of the new catalogue, which will contain the places of stars observed during the ten years 1894–1903, is well in hand. All the observed places have been entered in the catalogue, and checked as far as 1899 February, and approximate mean places for the epoch 1900·0 have been computed. This new catalogue will contain the places of about 1770 stars, and of these the annual precessions and secular variations have been computed and examined for 1041. The remaining 729 are chiefly zodiacal stars.

The Barclay 10-inch equatorial has been used chiefly for observations of new or variable stars and comets. Nova *Aurigæ* was observed on January 2, Professor Wolf's variable in *Cygnus* on October 9, Nova *Persei* on November 25, and Professor Turner's Nova *Geminorum* on March 26 and 30, April 4, 13, 20, and 29, May 4 and 20, and November 25.

Search was made for the comets 1902 *b* (*Perrine*), 1896 V (*Giacobini*), 1903 *d* (*Brooks*), and the position of the first determined with the ring micrometer on February 21 and 23. This instrument was also employed in observing occultations of stars during the lunar eclipse of April 11 and for other miscellaneous observations. (See *Monthly Notices*, lxiii. 7.)

The work of testing and examining the new twin photographic and visual refractor and of getting all the parts into thorough working order has absorbed a large proportion of the time and energy of the Observatory staff during the year. This work has been very much hampered by the unfavourable weather which prevailed, especially during the autumn months. Within two months after being mounted the 24-inch and 18-inch objectives had twice to be taken out of their cells and cleaned in order to remove moisture which had penetrated between the lenses. To avoid a recurrence of this trouble two small condensing gas stoves were erected early in the year in the basement of the equatorial tower, from which the slightly heated air rises into the dome. These have been found to work very satisfactorily, even in cases of extreme humidity of the air.

Other unforeseen circumstances very much delayed the preliminary trials, and it was not till April that it was possible to obtain photographs with the 24-inch. A number of plates were exposed for determining the best separation for the crown and flint lenses and for testing the figure of the objective, but some of the minor fittings of the instrument were much delayed, so that regular work could not be commenced before the end of the year.

Among the objects photographed are the clusters, Pleiades, M 13 (*Herculis*), h VI, 33 and 34 (*Persei*), Præsepe (*Cancri*), h VIII. 8 (*Cygni*), M 38 (*Aurigæ*), and the Moon.

It is intended to commence an investigation of the parallax

of stars in selected regions, according to the method proposed by Professor Kapteyn (*Bulletin de la Carte du Ciel*, tome i. p. 262), and photographs of the fields surrounding B.D. 35°, 4013 and 61 *Cygni* were taken and sent to Professor Kapteyn to enable him to compare them with those already taken by Professor Donner with the astrographic telescope of the Helsingfors Observatory, and discussed by Professor Kapteyn in the *Publications of the Astronomical Laboratory at Groningen*, No. 1.

The focal length of the instrument is 22 feet 6 inches, and it is adapted to carry plates up to 12 inches square. The stellar images appear sensibly circular over a field of 1½ degrees in diameter.

A number of double stars were examined for the purpose of testing the 18-inch objective. Amongst those divided were O.Σ. 175, O.Σ. 234, B.D. 21°, 984, and γ^2 *Andromedæ*, but the bifilar micrometer was not received till the middle of December, and the seeing has been so bad since then that no measures have been attempted.

During the lunar eclipse of April 11 several occultations of stars were observed with this instrument.

Meteorological observations have been carried on as usual, as well as observations of underground temperature by means of platinum resistance thermometers, as in previous years, but it is intended to bring the latter to a close shortly.

University Observatory, Oxford. (*Prof. H. H. Turner.*)

The completion of the work for the Astrographic Catalogue has been prevented by a wholly unexpected failure in sensitiveness in the photographic plates supplied, which is still causing trouble. Only ten plates out of the 1180 are still required, and these have been taken, and could have been measured and reduced, several times over, but on no occasion did they show enough stars relatively to previous plates, owing to some cause not yet fully understood, which seems, however, to be certainly connected with the sensitiveness of the films, and not with either weather or developer. This fault first occurred in 1903 February, when advantage had been taken of a spell of fine weather to take forty or fifty plates, most of which were found on development to fall short of the proper standard. The makers did all they could to rectify the defect, and at once sent us some better plates in substitution; and on this occasion we were more than compensated for the loss of time and trouble by the fortunate discovery of Nova *Geminorum*, which appeared after the defective plates had been taken, but in time to be shown on those which replaced them. But it was no longer possible to get all the plates in the spring, and on July 31 we were in the position of having completely measured and reduced all the plates but twenty-three, which could not be taken till October. The night of October 30–31

being particularly fine, Mr. Bellamy and Mr. Plummer spent the whole night at the Observatory, and were able to report the last plate secured at 5.20 A.M., ten minutes before twilight would have put an end to the work. Unfortunately there was again an unexpected failure in sensitiveness in the plates. The best of them could just be passed for measurement, though none were really good, and ten had to be rejected. These ten are still (1904 January 1) required, for a spell of bad weather has since prevented further exposures. Meanwhile Mr. Bellamy has paid a special visit to the plate-makers (Messrs. Elliott & Son) and discussed the matter with them. It appears that our requirements were not fully comprehended; the film required for faint and scattered points of light, such as stars, would not be the same as that for bright diffused light, even though very great "rapidity" is required in both cases. It is accordingly hoped that such misadventures may be rarer in future; but it was a considerable disappointment to the whole staff that we were not able to announce the work complete on December 31. They had worked diligently and loyally to that end, and I should like to record here my recognition of this fact and my sympathy with their disappointment.

The discovery of Nova *Geminorum* naturally brought a little extra work with it. Mr. Bellamy took several plates of the region, one with 100 min. exposure. The positions of 166 surrounding stars were measured and published by him (*Monthly Notices*, vol. lxiii. p. 326, and vol. lxiii. p. 512), and he also measured the region of X *Geminorum*, which occurs on the same plate (*Monthly Notices*, vol. lxiii. p. 526). Photographs taken by Dr. Max Wolf and by Mr. Parkhurst before the outburst indicated a faint star very near the place of the Nova, with which it might be identical; but careful measures of copies of these plates, kindly sent to Oxford by the owners, threw doubt on the identity, and Professor Barnard has since seen the small star alongside the Nova (*Monthly Notices*, vol. lxiii. p. 565).

The examination of the differences between the places of stars deduced from our measures and the Cambridge meridian observations has been extended to the Declinations. The results are in fair accord with those deduced from the R.A.'s mentioned in the last Report, but an independent discussion threw some doubt on the reality of the systematic proper motions deduced by the former method. It seems probable that these systematic motions are too small to be detected by the indirect method of comparing plates of *different* regions, taken at intervals of five years, with the same meridian catalogue, and we must wait until plates of the same regions can be repeated at a suitable interval (*Monthly Notices*, lxiv. p. 3).

Mr. H. C. Plummer, M.A., in addition to his work in the Observatory, has published three papers in the department of mathematical astronomy; one on the determination of the elements of an orbit (*Monthly Notices*, vol. lxiii. p. 147), and two on

"Oscillating Satellites" (*Monthly Notices*, vol. lxiii. p. 436, and lxiv. p. 98).

A method of photographing the Moon among the stars was tried and found successful (*Monthly Notices*, vol. lxiv. p. 19), but no systematic work can at present be undertaken in this direction.

The work of reducing the Rousdon Observatory observations of variable stars has occupied much time during the year, but is now completed and nearly printed. Fortunately the discussion of the results has rewarded us by bringing out several points of great interest, which seem worthy of further investigation. By adopting a compendious method of presenting the results it is shown that the characteristic constants can be easily and accurately deduced, and that the formulæ given in Chandler's Third Catalogue can be confirmed or corrected; and it is hoped that a definite step has been taken towards the interpretation of these formulæ.

Seeing that the completion of our work on the Astrographic Catalogue would set free measuring apparatus and experienced measurers, the suggestion was made that we should measure the plates taken at the Perth Observatory, West Australia, for which it had been found impossible to obtain funds from the West Australian Government. But the suggestion was entirely conditional on the provision of funds from some source independent of the University. To make sure that no unforeseen difficulties would arise, Mr. Cooke sent fifty of his plates to this Observatory, and twenty-one of them were measured and compared with the places of the Argentine General Catalogue, the others not being measured at present because it seemed doubtful whether they showed enough stars. For this work a small grant was obtained from the Government Grant Committee of the Royal Society. But a Committee of the Royal Society appointed for the consideration of the whole scheme was not in favour of proceeding further, and the project has therefore been abandoned.

One of the Harvard sky charts of fifty-five plates has been purchased, and is being examined with a view to determine the optical distortion. The investigation, which is in the hands of Mr. Kr. Lows of Christiania, is not yet completed; but the distortion is clearly very small. Mr. J. C. W. Herschel, from counts of star images on the plates, finds that the field is satisfactorily uniform as regards star magnitude (*Monthly Notices*, vol. lxiv. p. 118). Mr. J. H. Metcalf, of Vermont, U.S.A., has photographed the regions of several variable stars and measured the positions of a large number of stars on the plates.

Towards the end of the year Mr. C. L. Brook, F.R.A.S., generously presented to the Observatory one of the beautiful Stereo-comparators made by Messrs. Zeiss, of Jena, from designs by Dr. Pulfrich. It is hoped that valuable results may be obtained by comparison with this instrument of plates of the same region taken at different dates. Mr. Metcalf has already acquired some experience with the instrument.

Temple Observatory, Rugby. (*Mr. G. M. Seabroke.*)

The interest in astronomy has increased in the schools during the past year. Besides the attendance on fine evenings last term, two regular evenings in the week have been given up for instruction in the use of instruments.

An astronomical branch of the Natural History Society has been formed, and they will have the use of the 12-inch reflector for photography of the Moon and micrometer measures.

This work, which is the primary object of the Observatory, prevents much other work being done. It is hoped that some at least of the boys will take away with them a taste for astronomy.

Solar Physics Observatory, South Kensington.
(*Sir Norman Lockyer.*)

Observations of Sun-Spot Spectra.—Although spots were recorded on the Sun's disc on 192 days during 1903, there were only 99 days on which it was found possible to observe their spectra. Altogether the spectra of 130 spots were observed in the F—*b*, *b*—D regions. An analysis of the records shows that the lines generally affected were due to vanadium, titanium, or the "unknown" substances. A paper containing a detailed discussion of the widened lines observed since 1879 will shortly be published.

In addition to observations of widened lines in Sun-spots at South Kensington, those made at the Maharajah Takhtasinghi Observatory at Poona are also supervised, and an analysis of the results obtained there leads to the same general conclusion mentioned above. A photographic daily record of the conditions obtaining on the Sun's visible disc is kept at South Kensington, daily photographs being received from Dehra Dûn (India) and Mauritius. These are mounted as soon as they have been received, and arranged and finally bound in half-yearly volumes. The Mauritian prints are primarily used to fill up the gaps in the Indian record.

In addition to this current record prints are being prepared from the excellent series of negatives obtained by Canon Selwyn, at Ely, between 1863 and 1874, which have been loaned for this purpose by the Royal Society. Of these 1048 prints have been made, of which 859 have been mounted.

The Spectro-heliograph.—After a long series of experiments with an improvised instrument a final form of spectro-heliograph was decided upon, and the parts not already in the possession of the Observatory were ordered from the different makers. Some of these were received and mounted in connection with the 18-inch siderostat at the western end of the Observatory grounds

during October 1902. Further experiments were then carried out to determine the best form for the secondary slit, and a curved slit devised in accordance with the results was ordered from Messrs. Hilger, and received here during May 1903. From May 19 to the end of October a series of good "K" light photographs of the solar disc was obtained. It was, however, found that towards the end of October the low altitude of the Sun, combined with the cloudy and hazy atmosphere and restricted horizon available, prevented photographs being obtained even with greatly lengthened exposures, and it was therefore decided to abandon the attempts until the early spring.

Considerable difficulty was experienced during the exposures from the fact that under the influence of the reflected beam of solar rays the focal length of the 6-inch Taylor photo-visual lens, which is used for producing the image on the first slit, was found to change rapidly. To obtain a photograph showing fine details it was necessary to refocus this lens at the commencement of each exposure. Given a clear sky, photographs showing excellent detail could be obtained with an exposure of thirty seconds; altogether some 200 successful photographs were obtained between May and October.

Stellar Spectra.—Owing to the abnormally bad meteorological conditions of 1903 for photographic work, the absence of the first assistant for some months through illness, and of the second, who has been loaned to the Indian Government for a period of sixteen months, for service at the Kodaikanal Observatory, the amount of night work performed was somewhat below the average. The instruments chiefly used were a prismatic camera, equipped with a 6-inch objective, and a 6-inch 45° objective prism (Henry), and attached to the 10-inch refractor; and a 2-inch prismatic camera fitted with a calcite prism and a quartz objective, and fixed to the side of the 6-inch Dallmeyer equatorially mounted. The spectra of nineteen stars were obtained with the former instrument, several being photographed repeatedly for special investigations. The latter instrument was used for obtaining on a small scale the complete (i.e. ultra-violet to red) spectra of pairs of stars, each member of a pair being situated on different horizons of the temperature classification curve. This work was undertaken in order to test the sequence of the stellar groups on the temperature curve previously obtained from the consideration of the various spectra from the chemical standpoint, and the result of the research is very satisfactory. The spectra of each pair of stars to be compared were photographed on the same plate under as similar conditions as possible of atmosphere, exposure, altitude, photographic treatment, &c., and are thus strictly comparable. These identical conditions are difficult to obtain, owing to the sudden atmospheric changes, the different magnitudes, effective actinic qualities, and declinations of the stars, and many exposures had to be made to secure the thirteen successful pairs obtained. Owing to the small staff (now

augmented), and the pressure of other work, the 30-inch reflector and the 9-inch prismatic camera were not in regular use throughout the year; the spectra of several faint stars were obtained with the last-named.

The 36-inch reflector has been undergoing modifications and additions at the hands of Messrs. T. Cooke & Sons, who have fitted it with motors for working the fine adjustments, and a new $3\frac{1}{2}$-inch following telescope with slipping plate and electric illumination for the field. During the readjustment of the instrument some photographs of the nebula in the *Pleiades* and the *Orion* nebula have been obtained.

Meteors.—Arrangements were made for observing the *Perseid* shower, but clouds prevented observations. No organised watch was kept for the *Leonids*, but about 18 were seen during the intervals in other observations on November 14, whilst on the 15th over 50 were seen, and some 30 were charted by one of the observers, Mr. W. E. Rolston, at his residence.

Laboratory Work.—Numerous spectra of terrestrial substances were obtained with the 3-inch Cooke spectroscope, which is mounted on cement piers and kept always in adjustment, ready for immediate use.

The photographs obtained included a number of spectra of the dust which fell in various parts of England during the great dust storm of February 19, 20, and 21.

During the storm on the early morning of May 31 three photographs of lightning spectra were secured by Dr. W. J. S. Lockyer, by means of a 5×4-inch Kodak and an improvised camera, each fitted with a Thorpe transmission grating. To understand these and to determine the origins of the lines a large number of laboratory spectra were obtained with the former instrument similarly adjusted.

To this end the spectra of a high-tension spark, from the large Apps coil, in air, and in air with large excess of water vapour, nitrogen, and oxygen were obtained.

Meteorological Reductions.—The marked accordance found to exist between solar and terrestrial changes rendered it imperative that as complete a record as possible of the meteorological data (*i.e.* more especially "pressure" and "rainfall") obtained from stations situated in every part of the world should be prepared.

For this reason it was found necessary to extract a great amount of data from the various official meteorological records—a great part of which was secured from the Meteorological Office—and to reduce them to convenient forms for the discussion. This work has occupied much of the time of the computing division during the past twelve months, and the great number of curves and tables thus prepared are now available for future work, and form a valuable collection of facts.

Publications.—Two papers dealing with the interrelation of solar, magnetic, and meteorological phenomena were communi-

cated to the Royal Society under the following titles: "On the Similarity of the Short Period Pressure Variation over Large Areas" (*Proc. Roy. Soc.* vol. 71, pp. 134, 135); "The Relation between Solar Prominences and Terrestrial Magnetism" (vol. 71, pp. 244–250).

Two other papers on solar phenomena, under the following titles, were also communicated: "Solar Prominence and Spot Circulation, 1872–1901" (*Proc. Roy. Soc.* vol. 71, pp. 446–452); "On a Probable Relationship between the Solar Prominences and Corona" (*Monthly Notices*, June 1903, pp. 481–488).

A fifth paper, entitled "The Spectrum of γ *Cygni*" (*Phil. Trans.*, Series A, vol. 201, pp. 205–222), was communicated to the Royal Society.

A number of papers dealing with the reduction of the spectra of various celestial and terrestrial elements, solar and meteorological changes, and the spectra of Sun-spots are nearly ready for publication, whilst further papers on subjects under these headings are being prepared.

Stonyhurst College Observatory. (*Rev. W. Sidgreaves.*)

The meteorological and magnetical continuous records have been carried on as usual, except that the vertical force magnet has been left out of running on account of its defective balance.

A few photographs of the spectra of Sun-spots in the region $\lambda 4958 - \lambda 5140$ have been taken with the large spectrometer, but larger mirrors are required to make the work more efficient.

The solar surface has been observed on 220 days, notwithstanding the unfavourable weather, and 194 drawings have been added to the series, with notes of 13 dates of clear surface. But progress with the stellar spectrographs has been slower than ever.

The spectroscopic study of β *Lyræ* was completed in October, and the results were presented to the Society at the November meeting; but the reproduction of the spectrograms has delayed the publication till the January number of the *Monthly Notices*.

The astronomical instruments of the late Colonel Cross's Observatory at Redscar have been presented to the Stonyhurst Observatory by his son, the present squire. The smaller of two equatorial polar axes, built for a 7-inch Newtonian reflector, has been mounted in a revolving shed, and is intended to carry the 4-inch prismatic camera as soon as some difficulties connected with the clock-driving have been overcome. The two spectrographs will then be in operation together, one on the Perry equatorial for the blue and yellow regions, and the prismatic camera for the violet and ultra-violet spectrum.

Mr. Edward Crossley's Observatory, Bermerside, Halifax.

Owing to the bad health of the observer, Mr. Gledhill, very little work was done at this Observatory in the year 1903 beyond the usual metereological observations.

Wolsingham Observatory (Rev. T. E. Espin).

The work of measuring double stars has been continued, as in the previous years. Fifteen new pairs detected during the year have been communicated to the Society.

Sir William Huggins's Observatory, Upper Tulse Hill.

The photography of the spectra of stars and nebulæ, which has been in progress for some years, has been continued during the past year.

Experimental work in the laboratory has included the photographing of the spectrum of the spontaneous radiation of radium-bromide, and its recognition as identical with the negative pole band spectrum of nitrogen.

Rousdon Observatory, Lyme Regis, Devon.
(Late Sir C. E. Peek's; C. Grover, Observer in Charge.)

The building and instruments are maintained in good working order. The year as a whole has been distinctly unfavourable for astronomical work. The six months April to September were the best half of the year. December was abnormally cloudy; during the sixteen days 9th to 25th the Sun by day and the stars by night were scarcely visible. Observations were made on 148 nights. The 6·4-inch Merz equatorial has been used in the observations of long-period variable stars, and 460 magnitude determinations have been made. Argelander's method has been followed, as during the previous seventeen years. At each observation the light of the variable is estimated relatively to five comparison stars in the same field of view, the mean result being assumed to be the magnitude on the date of observation. About twenty-five long-period variables are under regular observation, and, as most of these are circumpolar in this latitude, their light changes are continuously recorded.

Transits of stars are taken as often as required for the regulation of the sidereal clock and mean-time chronometer.

Considerable attention was given to the planet *Mars*, and seventeen sketches were made between March 10 and May 8. Power 310 was mostly used, and, as at former oppositions, many

of the well-known features of the planet as figured by Green and Knobel in the *Memoirs of the R.A.S.* were very well seen.

The eclipse of the Moon on April 11 was observed under very favourable weather conditions.

Borrelly's Comet (c 1903) was observed on thirteen nights between July 7 and August 22. Toward the end of July it was distinct to the naked eye, and in the telescope the tail was a long straight ray, with a fainter ray from each side, giving it a decidedly fan-shaped aspect.

The transit of *Jupiter's* fourth satellite was observed on November 16, when the satellite appeared on the disc of the planet as a perfectly black, well-defined, circular spot.

Dr. Isaac Roberts's Observatory, Crowborough Hill, Sussex.

The following list includes the nebulæ, clusters, &c., which have been photographed during the year 1903. In spite of the abnormally great rainfall the year showed a perceptible increase in the number of fine nights, as compared with its predecessor. The month of December, however, was a marked exception, the sky being persistently overcast during the greater part of it. Two photographs were obtained of Borrelly's comet, on July 24 and 26, which showed a remarkable change in the structure of the tail in so short an interval.

List of the Principal Photographs taken in 1903.

	Approx. R.A. h m	Decl. ° ′	Expos. h m
Neb. ⁂ III. 147 Andromedæ	0 5	+25 22	1 30
Neb. ⁂ V. 16 Andromedæ	0 13	+29 31	1 30 (two)
Index Cat. 11 Cassiopeiæ	0 15	+55 2	0 51
Neb. à 17 Piscium	0 16	+21 53	1 30
Neb. à 29 Andromedæ	0 28	+47 57	1 30
Neb. M 31 Andromedæ	0 37	+40 43	0 57 1 30 2 15
Neb. M 33 Trianguli	1 28	+30 9	1 30
Neb. ⁂ II. 282 Ceti	1 30	− 7 51	1 30
Neb. ⁂ II. 438 Ceti	2 16	− 5 59	1 30
Neb. ⁂ II. 619 Arietis	2 54	+24 50	1 30
Neb. à 2522 Eridani	3 15	−19 47	1 30
Nebulosity near the *Pleiades* ...	3 44	+25 16	2 58 4 0 (two)
Neb. ⁂ II. 514 Tauri...	4 31	− 0 21	1 30
Neb. ⁂ II. 457 Eridani	4 43	− 5 37	1 30
Neb. ⁂ III. 447 Orionis	5 23	− 5 24	1 1
Neb. à 356 Orionis	5 24	− 8 28	1 30

	Approx. R.A. h m	Decl. ° ′	Expos. h m
Index Cat. 423–4–6 Orionis ...	5 28	− 0 32	1 30 (two)
Neb. M 42 Orionis	5 30	− 5 27	0 15 1 30
Index Cat. 427–8 Orionis ...	5 31	− 6 38	1 30 (two)
,, ,, 430 ,, ...	5 33	− 7 8	1 30
H's Nebulous Region No. 27	5 43	+ 1 8	2 44 2 57
N.G.C. 2175 Geminorum ...	6 3	+20 30	1 30
Neb. H IV. 20 Monocerotis ...	6 6	− 6 11	1 30
Index Cat. 443 Geminorum ...	6 10	+22 30	1 30 (two)
Neb. H II. 726 Cancri ...	8 4	+34 15	1 30
Neb. H IV. 55 Lyncis ...	8 5	+46 18	2 51
Neb. H III. 710 Lyncis ...	8 7	+49 23	1 30
Neb. H II. 719 Lyncis ...	8 7	+36 34	1 30
Neb. H III. 599 Cancri ...	8 22	+21 49	1 30
Neb. H II. 727 Lyncis ...	8 38	+35 4	0 58
Neb. H II. 80 Cancri ...	8 43	+19 27	1 30
Neb. H II. 823 Ursæ Majoris	8 50	+51 44	1 19
Neb. H II. 490 Lyncis ...	9 3	+33 32	1 30
Neba. H I. 56–7 Leonis ...	9 26	+21 57	1 30
Neb. H I. 168 Ursæ Majoris	10 12	+41 55	1 30
Neb. H III. 700 Ursæ Majoris	10 33	+42 11	1 30 2 0
Neb. H II. 362 Leonis Minoris	10 46	+28 30	1 30
Neb. H III. 20 Leonis ...	10 47	+10 41	1 30
Nebæ. M 65, 66, &c., Leonis	11 14	+13 50	1 30
Nebæ. H I. 227–8 Ursæ Majoris	11 39	+56 43	1 30
Nebæ. H I. 248, 251 Ursæ Majoris	11 46	+60 36	1 30
Groombridge 1830 Ursæ Majoris	11 47	+38 27	0 15
Neb. H V. 51 Draconis ...	12 12	+70 1	1 30
Neb. M 99 Virginis	12 14	+14 59	1 30
Neb. H V. 43 Canum Venaticorum	12 14	+47 51	1 30
Neb. H I. 75 Comæ Berenicis	12 15	+30 10	1 30
Neb. H V. 29 Comæ Berenicis	12 21	+34 5	1 20 2 0
Neb. M 88 Virginis	12 27	+14 58	1 30
Nebæ. M 58, 59 Virginis ...	12 35	+12 22	1 30
Neb. H I. 84 Comæ Berenicis	12 45	+26 3	1 30
Neb. M 94 Canum Venaticorum	12 46	+41 39	1 0
Neb. H I. 134 Virginis ...	12 49	− 9 59	1 30
Cl. M 53 Comæ Berenicis ...	13 8	+18 42	0 30

	Approx. R.A. h m	Decl. ° ′	Expos. h m
Cl. M 3 Canum Venaticorum	... 13 37	+28 53	0 30
Neb. Hⱼ I. 181 Boötis... ...	... 13 53	+42 20	2 0
Neb. Hⱼ I. 190 Boötis... ...	... 13 54	+37 54	1 30
Cl. M 5 Libræ	... 15 13	+ 2 27	0 30
Cl. M 13 Herculis	... 16 38	+36 39	0 15 0 30
Cl. M 92 Herculis	... 17 14	+43 15	0 30
Neb. h 1989 Herculis... ...	... 17 47	+23 6	1 0
Cl. M 18 Clypei	... 18 14	−17 11	0 30
N.G.C. 6643 Draconis ...	... 18 22	+74 31	1 30
Cl. Hⱼ VIII. 12 Aquilæ ...	... 18 31	− 8 19	0 30
Cl. M 26 Aquilæ	... 18 40	− 9 30	1 30
Neb. M 57 Lyræ	... 18 50	+32 54	0 10 0 20
Cl. Hⱼ VII. 18 Vulpeculæ ...	... 19 39	+23 4	0 30
Cl. h 2048 Cygni	... 19 39	+39 57	1 0
Milky Way in Cygnus ...	... 19 45	+35 30	1 0
Neb. M 27 Vulpeculæ ...	... 19 55	+22 27	1 0 (two)
Cl. M 75 Sagittarii	... 20 0	−22 13	1 0
Cl. Hⱼ VII. 59 Cygni	... 20 0	+43 43	1 0
Cl. M 29 Cygni	... 20 20	+38 12	1 0
Neb. Hⱼ V. 15 Cygni	... 20 42	+30 21	2 55
Nebulosity in Cygnus... ...	... 20 45	+31 20	1 45
Cl. M 72 Aquarii	... 20 48	−12 55	1 0
Neb. Hⱼ V. 14 Cygni	... 20 52	+31 19	2 5
Neb. Hⱼ III. 858 Equulei ...	... 21 10	+ 2 25	2 0
Cl. Hⱼ VII. 50 Cygni	... 21 21	+47 36	0 50
Cl. M 15 Pegasi	... 21 25	+11 44	0 30 (two)
Neb. Hⱼ II. 247 Delphini ...	... 21 56	+17 16	1 30
Neb. Hⱼ II. 1 Aquarii... ...	... 21 57	−21 18	1 30
Cl. Hⱼ VI. 29 Cephei	... 22 11	+53 50	1 0
Neb. Hⱼ IV. 31 Aquarii ...	... 22 27	−14 38	1 0
Neb. Hⱼ II. 705 Cephei ...	... 22 36	+60 46	1 0
Neb. Hⱼ II. 429-30 Piscium ...	... 23 9	+ 3 48	1 30
Neb. Hⱼ II. 467 Piscium ...	... 23 11	+ 6 8	1 30
N.G.C. 7571 Pegasi	... 23 12	+18 26	1 30
Neb. Hⱼ II. 236 Aquarii ...	... 23 13	− 5 12	1 30
Neb. Hⱼ III. 473 Pegasi ...	... 23 19	+16 12	0 48
Neb. Hⱼ III. 213 Pegasi ...	... 23 27	+15 18	1 30

Comet *c* 1903 (Borrelly) 45ᵐ (July 24), and 60ᵐ (July 26).

Mr. Saunder's Observatory, Crowthorne, Berks.

With the assistance of Mr. J. A. Hardcastle, F.R.A.S., progress has been made in the measurement of some of M. Lœwy's negatives of the Moon. Four plates in all have now been measured, giving an average of about 400 points on each. The constants of three of these plates have been found, and the reductions are in a forward state, though one plate only has been completed. Standard positions are being gradually built up for the points most frequently measured, and the probable error in the value of a co-ordinate as derived from a single plate is thus found to be about 0″·13. This work is being accomplished with the aid of a grant made by the Government Grant Committee of the Royal Society.

With the telescope some progress has been made with the minute study of special formations on the Moon, but the atmospheric conditions have prevented much work in this direction. The lunar eclipse of 1903 April 11 was observed, and the diameter of the white spot surrounding Linné was measured immediately before and after its passage through the shadow. The diameter was found to have increased 0″·50±0″·195 during the interval, very closely agreeing with an increase of 0″·55 found by Professor W. H. Pickering at Harvard, although Professor Pickering's value of the final diameter was only 3″·34, whilst the measures made at Crowthorne gave 6″·05, showing a difference of 2″·71.

Daramona Observatory, Streete, Westmeath. (*Mr. W. E. Wilson.*)

The weather during the past year prevented any work being done in the Observatory. The rainfall amounted to 45·72 inches, which is about 10 inches above the average for the last twenty-five years. In the month of March, which is generally a good one for clear skies, rain fell on every day but one. In the laboratory some experiments were made on the effect of pressures up to 300 atmospheres on the absorption spectrum of certain liquids. The mirrors of the 2-foot reflector and also of the large heliostat were re-silvered.

Abbassia Observatory, Cairo. (*Capt. H. G. Lyons.*)

During 1903 the work of the Observatory has been carried on upon the same lines as during the preceding three years, the reorganisation having taken place from the beginning of 1900. The Observatory is under the general direction of Captain H. G. Lyons, the Director-General of the Survey Department, and the

superintendent is Mr. E. B. H. Wade, who is assisted by a staff of five.

The work of the Observatory is chiefly meteorological, in which section hourly observations have been made of the temperature, atmospheric pressure, relative humidity, vapour tension, wind velocity and direction, and sunshine. Of these the atmospheric pressure is registered by a Sprung-Fuess balance barograph, the wind velocity by Dines's pressure tube recorder, the wind direction (provisionally) by Denza's recorder, and the other elements by Professor Callendar's electrical recorder. Eye observations three times a day are also made of barometric pressure, temperature, relative humidity, vapour tension, state of the sky, direction and velocity of the wind, rainfall, evaporation, actinic intensity, and underground temperature.

Observations are also made with the transit instruments on every fine night for rating the standard clocks for the purposes of the daily time signal. The Observatory is also occupied with the standardisation of instruments for sending to meteorological stations in Egypt and in the Soudan, fourteen of which are now furnishing daily telegraphic reports of the weather to the office of the Survey Department, Giza. Magnetic observations are made weekly at Helwan, twenty-three kilometres from Cairo, with a Kew set of instruments, and determinations of atmospheric electricity with a portable electrometer are made at the same time. A continuous record of earth tremors is furnished by Milne's seismograph.

During the year twenty occultations have been observed with the 21-cm. Brünner equatorial. The Observatory is also responsible for the astronomical observations required for the triangulation of the country and their computation.

This Observatory is being transferred to a new site, about fifteen kilometres south of Cairo and on the desert plateau near the town of Helwan.

Kodaikanal Observatory. (*Prof. C. Michie Smith.*)

The director having left India on furlough on October 31, 1902, the Observatory was put under the charge of Mr. C. P. Butler, who arrived at the station on January 3, 1903. He had received special instructions to arrange for the routine observation of Sun-spot spectra as early as possible, and with this object a spectroscope was specially put together from parts of others at the Observatory, none of the original instruments being of sufficient power for the purpose. This was first mounted on the old Lerebours and Secretan equatorial of six inches aperture, and later on the six-inch Cooke refractor, the latter being preferred on account of its being provided with a position circle, which is necessary for the measurement of prominences at the Sun's limb.

It was necessary for some time to observe the spectra in duplicate, as the assistants had had very little experience in this class of work, but after becoming familiar with the difference between subjective and objective widening of the lines they quickly became efficient observers, and from the middle of the year they have been entirely responsible for the duty, although the supervision was kept up from time to time.

Sun-spot Spectra.—It appears from the results obtained that the spectrum of Sun-spots during the year has been practically identical with that recorded for the past few years at other institutions, and no evidence of variation has been detected. The lines observed are due chiefly to titanium, vanadium, scandium, and some unknown element or elements. The solar activity showed a marked increase during the year, and spectrum observations were obtained on 114 days.

Prominences.—Wherever the sky was clear enough the positions of all prominences visible at the limb were measured, and the position-angles recorded were at once reduced to solar latitudes.

Eight-inch Solar Pictures.—No. 4 Dallmeyer photo-heliograph was taken to pieces and thoroughly overhauled. After re-erection and adjustment several plates were taken for focussing, and afterwards a photograph has been taken on every day when the weather has permitted. These are almost always taken early in the morning, as the sky generally becomes hazy about 9·30 or 10 A.M., rendering further observational work impracticable.

Photographs of Sun in Monochromatic Light.—An instrument had been ordered for obtaining photographs of faculæ, &c., consisting of a spectro-heliograph on the Hale-Deslandres plan, with an 18-inch Foucault siderostat to supply the light to the spectroscope, a 12-inch Cooke photo-visual objective being provided for forming the image on the primary slit. The building for this was got ready, but the spectro-heliograph, which was promised in August, had not been received at the end of the year. The 18-inch siderostat was safely received and erected. This is working, and the work can be started as soon as the spectroscopic equipment is received from England.

Photographs of the Sun-spot Spectra.—A few plates were taken of the large spots in March, but the dispersion at the direct focus was much too small, and the work was discontinued until the arrival of the long focus lenses which have been ordered for the purpose.

Time Observations.—The transit instrument was got into use for this work, but the almost general clouding of the sky before noon rendered it impossible to use this method regularly, and during the greater part of the year the chronometers and clocks have been rated from sextant observations of the Sun in the early morning. As a check time signals are received from the Madras Observatory at 4 P.M. each day. For some time these have been quite useless, owing to their being retransmitted by hand

at intermediate stations. By the kindness of the telegraph authorities this has now been remedied, and a *through* circuit installed, making the signals in every way reliable.

Meteorology.—Complete records of meteorological observations have been made, as in previous years, and all the instruments have been in good working order throughout the year. The total rainfall for the year was 69·55 inches. The maximum daily wind was 971 miles during the storm of December 4.

Actinometry.—Whenever the sky has been quite free from haze measurements of the solar radiation have been taken with a Balfour-Stewart actinometer, but the number of such days has been very few during the present year.

Seismology.—The Milne seismograph has been in action throughout the year, and the data concerning the chief disturbances forwarded to various authorities, as in former years.

Magnetic Observations.—The underground magnetic vault has been repaired considerably, to exclude the damp, which had begun to seriously affect the instruments. The cases of the magnetographs have been hermetically sealed to the pillars and drying agents put inside. By these precautions the evil has been considerably lessened, but constant attention has been necessary. The records have been regularly obtained during the year, but the constant illness of the observers has caused inconvenience by the frequent changes.

Very good records were obtained of the great disturbances of October 12 and 31; in the latter case the deflection was so great that the spot of light went off the paper. The mean values of the observed elements are as follows :

Mean Westerly Declination	= 0° 22′·1 (W.)	1903.
Mean Inclination or Dip	= 3° 5′	
Mean Horizontal Intensity	= 0·37369 C.G.S.	

Melbourne Observatory. (*Mr. P. Baracchi.*)

Meridian observations were made with the 8-inch transit circle, and are as follows :

	R.A.	N.P.D.
Clock stars	476	...
Circumpolar stars	323	106
List stars	1300	1299
Heliometer comparison stars	549	552
Total	2648	1957

The list stars were, as in previous years, selected from the Photographic Catalogue plates to serve as fundamental stars for the reduction of these plates. The total number now completely

observed three times or more is 4481. The heliometer comparison stars were observed at the request of Sir David Gill, and completes his list published in April 1901. These observations have been fully reduced, and will shortly be forwarded to the Cape Observatory. The separate results and annual catalogue for 1902 have been prepared. The reduction of all other meridian observations are well advanced.

Astrographic Operations.—

Chart plates with triple exposure of 30^m each	146
Catalogue plates (duplicate series)	99
Test plates on south polar regions	40
Test plates on Oxford type regions	12
Test plates for centre, focus, trails, &c.	31

Eighteen chart plates and three catalogue plates were rejected as defective.

The state of the work to 1903 December 31 is as follows :

Chart plates with triple exposure of 30^m each, passed as satisfactory	372
Chart plates with single exposure of 60^m each ...	Complete
Catalogue plates	Complete
Catalogue plates (duplicate series)	235

Since using filtered water for developing, and drying in practically dustless air, the chart negatives obtained show a very decided improvement.

Terrestrial Magnetism.—In addition to the usual routine of photographic registration and absolute measurement of the magnetic elements, the full programme in regard to terrestrial magnetism prescribed by the Royal Society and Royal Geographical Society for co-operation with the Antarctic expeditions, which was to have terminated in March 1903, was, at the request of these societies, continued throughout the year, and will not be concluded till next March.

Further progress has been made in the measurement and reduction of the magnetic curves of the past thirty-five years. This work is now complete to January 1894.

The Milne seismograph has been kept constantly at work. Reprints of the more important seismograms have been prepared, and the whole of the results so far obtained are being prepared for transmission to the Seismological Committee of the British Association.

Other Observatory work which has been carried on as usual for public local requirements comprises—

The Time service.

The Tide service.

The Weather service, with control of some 800 recording stations.

The installation and routine of the State Standards of Weights and Measures service, now in charge of this Observatory.

Rating of chronometers for the shipping, testing, surveying, and meteorological instruments, also a large number of air-meters for mining inspectors.

Sydney Observatory. (*Mr. H. C. Russell.*)

The first portion of the year the Astronomical branch was under Mr. Lenehan, but during the latter part his time was taken up with administrative work and assisting Professor O. Klotz, of the Ottawa Observatory, Canada, in the work he was deputed to do in determining the longitudes of the stations touched by the Pacific cable and generally throughout the world.

Another astronomer of note, Professor Hussey, was credited to Mr. Russell, who had instructions from the New South Wales Government to do what he could to further his mission, *i.e.* to find for the committee under the presidency of Professor Lewis Boss, of Dudley Observatory, if they could establish an astronomical station in New South Wales, Professor Hussey to report if the conditions of the climate would be suitable, &c.

In this work the Government placed Red Hill Observatory, with its assistant, under Mr. Hussey's immediate control. The report of this visit will, of course, be first placed with the American authorities, and a copy later will be forwarded to the Observatory. It may be mentioned that both these American astronomers expressed their gratitude for the ready assistance given.

Unfortunately the director, Mr. H. C. Russell, C.M.G., F.R.S., was taken with a severe illness, and most of the duties devolved on Mr. Lenehan.

During the latter part of the year rain and cloud interfered with the astronomical work, and the computers during the time were busy reducing previous observations.

Number of	R.A. observations	...	...	2644 stars
,,	N.P.D. ,,	...	...	2431 ,,
,,	Observations of	R.A. of Sun	...	446

making a total of 3090 R.A. observations.

The 9 A.M. collimations, 310; level, 309; nadir, 306. 9 P.M. level, 143; nadir, 134; azimuth determinations, 133.

Mr. Raymond, the assistant observer, had charge of the reduction of stars to January 1, and has the work well in hand. 1901 is ready for the printer, and has gone to swell the mass of observations awaiting necessary funds for publication. A new catalogue for overlapping astrographic plates has been computed,

and the astrographic work under Mr. Short at the Red Hill Observatory is in the following state :

Number of plates required to complete two sets of catalogue plates, 3. Number on original list, 1400.

Plates required to complete three sets of chart plates, 59 ; number on original list, 654.

Plates required to complete one set of overlapping plates, 950 ; number on original list, 1078.

The equatorial has not been used for double-star work during the year, as the observers were taken up with transit observations for central stars for the astrographic plates, which are urgently required.

Much of the officials' time has been absorbed attending to visitors, many of whom are only sightseers and have no scientific interest in looking over the Observatory. The number of visitors who attended during the year was 792.

This Report must not be closed without the expression of thanks to those officers in the Observatory for the very hearty way they have fulfilled their duties, and so helped in the general welfare of the institution.

Especial credit is due to Mr. Masters, who has helped by his aptitude and great constructive ability the work of instrumental repairs and construction.

Measurement of the Plates of the Astrographic Catalogue. Joint Report for Sydney and Melbourne.

The measurement of the plates of the Photographic Catalogue for the Sydney and Melbourne zones was carried on at the Melbourne Observatory under the same conditions as stated in previous Reports.

After March 31 the actual measuring was continued on a reduced scale by three measures only, as it was considered desirable to check for a short interval the rapid accumulation of material, the amount of which had already assumed large proportions, in order to commence the preparation of the work for publication.

During the year 1903 the errors of three réseaux and methods for determining photographic magnitudes were investigated, and 219 plates, containing 63,861 stars, were completely measured. The total number of plates completely measured in two positions up to 1903 December 31 are :

224 Sydney plates, containing 131,061 stars.

512 Melbourne plates, containing 147,343 stars.

The new measuring machine, constructed at the Sydney Observatory, and described in *Monthly Notices*, vol. lxiii. p. 39, has been installed at the Melbourne Observatory for the use of the Measuring Bureau. It reached Melbourne only a few days ago, and has not yet been tested, but the results of a preliminary trial

made at Sydney appear satisfactory. The Measuring Bureau was installed since the beginning of the year 1903 in a new spacious room built especially for the requirements of this class of work and the convenience of the observers.

Perth Observatory, Western Australia. (*Mr. W. E. Cooke.*)

Astrograph.—This year was unfortunate in some respects. A legacy of uncertainty as to the intentions of the Government with respect to the astronomical work was bequeathed from the previous year, and it was not until February 24 that authority was obtained to order any fresh plates for the astrograph. A cablegram was immediately despatched to England for one gross, and another gross was indented in the usual manner. Unfortunately, owing to departmental delays, this second order was not sent until June 1, and the regular supply of plates did not recommence until September. The weather, too, was remarkably cloudy throughout the winter and spring months. On the whole, therefore, progress with the international work has not been so rapid as might have been wished. Altogether 281 catalogue plates were taken and passed as satisfactory. The present state of this undertaking is as follows :—

Chart plates (three exposures of 30^m each) ...	42
Catalogue plates	406

The work is now again in steady swing, and it is hoped that substantial progress may be made during the present year. The measurement of the plates cannot be commenced at the Observatory, owing to the manner in which the vote for purely astronomical purposes has been restricted ; but the Royal Society has made a grant of 100*l.*, and the plates are being sent to Oxford, where Professor Turner has kindly offered to do the best he can with this grant.

Transit Circle.—Steady progress has been made with the standard stars for the catalogue plates. The portion assigned to the Observatory includes 32°–40° South declination. In 1902 32° and 33° were practically completed, and in 1903, at the request of Sir David Gill, 39° and 40° were undertaken. This overlaps his zones, which will enable the results to be compared. During the present year this zone work will be discontinued, and the various constant errors, such as errors of graduation, variation of personal equation for magnitude, &c., will be investigated. As a result of a few observations already made it appears probable that the errors of graduation amount in some places to as much as 3″ for the mean of four microscopes, and a systematic determination is imperative.

During 1903 the following observations were made :

Level error	202
Azimuth error...	293
Collimation error	27
Meridian mark	29
Nadir point	137
Flexure	8
Observations of R.A. (zone stars)	4085
„ N.P.D. „	4094
„ R.A. (clock stars)	937
„ N.P.D. „	11

Field Work.—Hitherto there has been nothing in the nature of an astronomical survey of the State, but the Observatory has now commenced to cooperate with the Lands Department, and during the year the positions of Coolgardie and Albany were determined by the Government Astronomer. The only available field instrument was a 12-inch theodolite, and the method adopted for obtaining time and latitude was the one described in the *Monthly Notices, R.A.S.*, for January 1903. This has proved to be a very valuable method, and results of a high order of accuracy can be obtained with a comparatively small instrument.

In fact, whilst waiting for the erection of a pillar at Albany, a 6-inch theodolite, on which a sensitive bubble had been mounted, was used, and the results were practically as good as with the 12-inch. It is confidently believed that with a 6-inch theodolite, provided with a sensitive level and suitable reticle, and a portable chronograph (or chronometer), the result of one clear night's observation would not be likely to be 1″ in error in either coordinate.

Meteorological.—The ordinary work has been continued and the forecasts have been as successful as usual. Their general usefulness is very much restricted, owing to the lack of means for their distribution.

The forecast which appears in the daily morning paper, for instance, is issued at 4 P.M. on the previous day, and it may happen that an anticipated storm, &c., is already in progress when the public read their paper. This condition, however, will gradually improve as population increases and means of intercommunication become more rapid. The percentage of successful results was as follows :

	Correct.	Partially Correct.	Wrong.
General forecast for the State, issued at noon ...	94	4	2
„ „ „ 4 P.M. ...	93	6	1
Special for the Goldfields, issued at noon ...	94	5	1
„ „ „ „ 4 P.M. ...	91	6	3

	Correct.	Partially Correct.	Wrong.
Special for the Murchison, issued at noon ...	95	3	2
„ for Perth and neighbourhood, issued at 9 A.M.	95	4	1

In addition two special storm warnings were issued to North-West ports, and these were amply justified by subsequent events. There were no other storms in that district during the year.

Publication.—The annual meteorological report for 1902 has been delayed, but is now practically ready for publication, and will be issued very shortly. Funds will not at present permit of any astronomical printing, but the observations are all reduced, and will be finally checked during the present year; and it is hoped that some means will be found for publishing the results of zones 32°, 33°, 39°, and 40° before very long.

Lovedale Observatory, Cape Colony. (*Dr. A. W. Roberts.*)

As in previous years the work done in 1903 was strictly confined to the observation of variable stars south of −30° Declination.

The number of stars under observation, and the number of observations made during the year, are as follows:

	Stars.	Observations.
Algol variables	6	164
Rapid variables	2	185
Short-period variables	22	478
Long-period variables	68	1227
Suspected variables	6	18
Total	104	2072

The total number of observations, 2072, is considerably under the number usually secured—a result chiefly due to the unfavourable observing weather which prevailed during the greater part of the year.

The loan of the Oxford wedge photometers, received about the beginning of the year from Professor Turner, has to be gratefully acknowledged. They are being used to determine the brightness of the comparison stars, to which the various variable stars under observation are related.

Mr. Tebbutt's Observatory, the Peninsula, Windsor, N. S. Wales.

Very little work having been done during the last three months, the report here given really applies to the first nine months of the year. The following is the work in each department:

Meridian Department.—

Nights on which local time was determined	... ...	86
Transits of stars with a declination not exceeding 40°	... ...	374
Transits of stars of high declination for azimuth	... ...	73
Separate determinations of level error	... ...	238
Separate determinations of collimation error	... ...	25
Separate determinations of azimuth error	... ...	64

Extra-Meridian Department. — The following micrometer comparisons were made with the 8-inch equatorial :

Objects Compared.	Nights of Observation.	Number of Comparisons.	Number of Comparison Stars.
(17) Thetis	... 5	77	1
(29) Amphitrite	... 4	45	2
(324) Bamberga	... 23	236	10
Comet 1902 III. (Perrine)	... 13	159	13
Comet 1903 III. (Grigg)	... 16	138	18

The observations of Comet 1902 III. were a continuation of a series commenced in 1902, and referred to in the last Report. On April 21 a telegram appeared in the *Sydney Morning Herald* announcing the discovery of Comet 1903 III. by Mr. John Grigg, of Thames, New Zealand, but without furnishing any positions, and it was not till April 25 that I received a letter from the discoverer giving rough places for April 17 and 18. By these means I was enabled to observe the visitor. The first four positions secured were cabled to Kiel, from which an approximate orbit was computed by Dr. Kreutz and Herr Ebell.

In addition to the work reported above, the usual daily rainfall and maximum and minimum monthly temperature observations have been carried on throughout the year.

Notes on some Points connected with the Progress of Astronomy during the Past Year.

Discovery of Minor Planets in 1903.

Forty-one new planets were discovered, or first announced, in 1903, as follows :

Provisional Designation.	Permanent Number.	Date of Discovery.	Discoverer.	Place of Discovery.
LA	500	1903 Jan. 16	Wolf	Heidelberg
LB	501	,, ,, 18	,,	,,
LC	502	,, ,, 19	,,	,,
LD	...	,, ,, 18	,,	,,
LE	...	,, ,, 18	,,	,,
LH	...	,, ,, 31	Dugan	,,
LJ	...	,, Feb. 6	Wolf	,,
LK	504	1902 June 30	Bailey	Arequipa
LL	505	,, Aug. 21	Frost	,,
LM	...	,, Oct. 21	,,	,,
LN	506	1903 Feb. 17	Dugan	Heidelberg
LQ	508	,, Apr. 20	,,	,,
LR	509	,, ,, 28	Wolf	,,
LS	...	,, ,, 20	,,	,,
LT	510	,, May 20	Dugan	,,
LU	511	,, ,, 30	,,	,,
LV	512	,, June 23	Wolf	,,
LW	...	,, ,, 30	,,	,,
LX	...	,, July 1	,,	,,
LY	...	,, Aug. 24	,,	,,
LZ	...	,, ,, 24	,,	,,
MB	...	,, ,, 24	,,	,,
MC	...	,, Sept. 20	,,	,,
MD	...	,, ,, 20	,,	,,
ME	...	,, ,, 20	,,	,,

Provisional Designation.	Permanent Number.	Date of Discovery.	Discoverer.	Place of Discovery.
MF	...	1903 Sept. 20	Wolf	Heidelberg
MG	...	,, ,, 20	Dugan	,,
MH	...	,, ,, 22	,,	,,
MK	...	,, ,, 24	Wolf	,,
MM	...	,, ,, 30	,,	,,
MN	...	,, Oct. 20	Dugan	,,
MO	...	,, ,, 20	,,	,,
MP	...	,, ,, 20	,,	,,
MQ	...	,, ,, 20	Wolf	,,
MR	...	,, ,, 25	Dugan	,,
MS	...	,, ,, 27	Wolf	,,
MT	...	,, ,, 27	,,	,,
MU	...	,, ,, 27	,,	,,
MV	...	,, ,, 27	Wolf-Götz	,,
MW	...	,, ,, 27	Wolf	,,
MX	...	,, Nov. 14	,,	,,

The following planets, unnumbered at the date of the last Report, have since received permanent numbers: JG 488, JM 489, JP 490, JQ 491, JR 492, JS 493, JV 494, KG 495, KH 496, KJ 497, KU 498, KX 499.

The following planets do not receive permanent numbers, not having been sufficiently observed: JT, JW, JX, JY, JZ, KA, KB, KC, KF. KL, KM, KN, KO, KP, KQ, KR, KS, KT, KV, KW, KY, KZ, LD, LE, LH, LJ, LM, LS, LW, LX.

Planet JO, to which the number 489 was first assigned, proves to be identical with 470 *Kilia*. The number 489 has been transferred to JM.

The following identities have been established: 483 with CP, 488 with CR, KD with 66 *Maia*, 498 with FF, LF with EM (this has been numbered 503), LG with 360, LO with DE (this has been numbered 507), LP with 406, MA with 184 *Deiopeia*, ML with 214 *Aschera*. A planet photographed at Washington in December 1903, and subsequently observed visually there, is probably identical with DW. An object designated MI was subsequently found not to be a planet. MV may be identical with 316 *Goberta*.

The following planets have been named: 357 *Ninina*, 360 *Carlova*, 368 *Haidea*, 383 *Janina*, 395 *Delia*, 396 *Ælolia*, 456 *Abnoba*, 458 *Hercynia*, 462 *Eriphyla*, 482 *Petrina*, 483 *Seppina*, 484 *Pittsburghia*, 488 *Kreusa*, 491 *Carina*, 493 *Griseldis*, 503 *Evelyn*, 507 *Laodica*.

Numerous photometric observations of 433 *Eros* were obtained at Arequipa during last year's opposition, but the results are not yet published. This planet will next be in opposition in August

1905, when it will be of the 12th magnitude, in 12° S. Decl., the circumstances resembling those at discovery in 1898.

M. O. Callandreau published some statistics of the orbit elements of minor planets in *Comptes Rendus* for April 20. He arranged them in groups according to increasing aphelion distance, finding that for the inner group (mean aphelion distance 2·4) the mean eccentricity is 0·075 and inclination 4°, while the corresponding quantities for the outer group (mean aphelion distance 3·9) are 0·212 and 8°. A. C. D. C.

The Comets of 1903.

To the account of the comets given in the last Annual Report it may be added that Giacobini's Comet, discovered on December 2, and remarkable for its great perihelion distance, remained visible till June 26, on which date it was observed at Mount Hamilton. The comet had then so dwindled in brilliancy as to be compared to a 13th-magnitude star, with a nucleus which was estimated at the 16th magnitude. Comet 1902 III. (but during the earlier observations having the designation *b* 1902), on its return from great Southern Declination, was first reobserved on January 29, and followed with difficulty, on account of increasing faintness, till March 23. On April 27 no trace of the comet was visible in the 36-inch refractor of Mount Hamilton. The orbit is parabolic.

The first comet of 1903 was discovered on January 15 at Nice, by M. Giacobini, in *Pegasus*. The motion was north-east and the brilliancy increasing. Consequently the comet soon became a conspicuous object, and in the middle of February was described as bright as the *Andromeda* nebula. At the time of greatest brilliancy, about March 16, the comet was too near the Sun for observation. Photographs were taken at Greenwich and elsewhere, but the position was unfavourable. After perihelion passage the comet became visible in the Southern Hemisphere, and was observed at the Cape of Good Hope till May 4. This orbit does not differ sensibly from a parabola.

On April 16 a comet was discovered by Mr. Grigg, of Thames, New Zealand, which was observed by Mr. Tebbutt at Windsor, N.S.W. The comet had passed perihelion on March 25, and was growing fainter. The apparent path of the comet did not permit it to be observed in the Northern Hemisphere. Observations were secured at Windsor and at the Cape of Good Hope till May 28. The only elements published appear to be those of Drs. Kreutz and Ebell, and the material on which they are founded is too rough to decide the exact character of the orbit.

On June 21 M. Borrelly, of Marseilles, made the third comet discovery of the year. This comet, found in *Aquarius*, and tolerably bright, was moving northwards and increasing in brilliancy, so that it became readily visible to the naked eye. It was

repeatedly photographed, and the structure and behaviour of the tail have been adequately described (*Lick Observatory Bulletin*, No. 47). On the examination of a photograph showing the passage of the comet over a tolerably bright star, Dr. Max Wolf, of Heidelberg, satisfied himself that the constituents of the comet had exercised selective absorption on the star's light (*Ast. Nach.*, No. 3914). The perihelion passage happened towards the end of August, when the comet became invisible, being too near the Sun. Towards the middle of September, when the brilliancy had declined to about that which the comet exhibited at the time of discovery, it stood in the Southern Hemisphere, and was observed at the Cape up to October 22. An attempt was made to trace the comet on plates taken at Harvard on May 28, and also on some taken at Arequipa on the 4th of that month. The question of possible earlier observation does not seem to be decided, but a suspicious object noted on the Harvard plates was probably not the comet. The possibility, however, of earlier detection by the photographic method, when the previous course of the comet has become known by the determination of approximate elements, possesses a distinct interest.

These are all the comets that can be said to have been discovered, but on August 20 Mr. Aitken, of the Lick Observatory, found Brooks's (1896, VI.) Comet, for which Dr. Neugebauer had computed an accurate ephemeris from the observations made at previous returns. The correction to this ephemeris was +22 sec. in R.A., and +1′ 41″ in Declination, which could apparently be removed by a slight alteration in the time of perihelion passage. Mr. Aitken describes the comet as faint and small, irregular in outline, and with but feeble condensation. The greatest diameter did not exceed 3′, and the brightness about equalled that of a 14th-magnitude star. This comet was followed at Lick till October 24, growing continually feebler in illumination and increasingly difficult to observe. Other observations are not known.

This was the only periodic comet whose return was successfully observed. Several other comets have probably passed through perihelion without detection. This list includes Brooks's Comet (1886, IV.), which should have returned to the Sun about March 25, but, as the comet was not seen either in 1892 or 1897, the error in its assigned position may be considerable. The circumstances for its observation were slightly more favourable than in those years, but the uncertainty of its position was necessarily greater. Perrine's Comet (1896, VII.), according to the calculation of Dr. Ristenpart, passed through perihelion on April 26, but the angular distance of the comet from the Sun always remained small till about November, when its brightness would have dwindled to about one-sixteenth of that when last seen in 1897. No observations have, therefore, been possible, and this is the more to be regretted since an accurate determination of the mean motion was most desirable, in order to decide whether

this comet has had any connection in the past with the well-known comet of Biela. Spitaler's Comet (1890, VII.) was due at perihelion in August last, but the elements are now very uncertain, as it was not observed at its return in 1897. Giacobini's Comet (1896, V.), for which Dr. Ebell prepared sweeping ephemerides, based on a period of 6·647 years, has likewise passed undetected, though sought for in the great telescope of the Lick Observatory. Faye's Comet, that has been so regularly observed at each returning perihelion passage, has proved disappointing. Its observation would have been specially interesting, since the comet has in its last revolution made a close approach to *Jupiter*, with the result, according to Dr. Strömgren, that the mean motion has been altered from 468″ to 480″, and the date of perihelion consequently accelerated by four months. On June 3 it was near perihelion, but the earliest chance of detection was at the latter end of August. Dr. Aitken was, however, unable to see any trace of it. Of D'Arrest's Comet, the calculations concerning which are in the hands of M. Leveau, it may be mentioned that the brilliancy has passed its maximum, and that, considering the unfortunate position in which it is placed, its redetection is practically hopeless. Winnecke's Comet is not in perihelion till January 21 of this year, but the ephemeris prepared by Dr. C. Hillebrand has hitherto proved fruitless.

Recent Researches on the Theory of Comets' Tails.

The mystery of cometary emanations has given rise to many researches during the last few years. Most of the recent writers on this subject have made use in some way or other of Maxwell's idea of *radiation-pressure*; Maxwell * observed that if light be regarded as an electromagnetic phenomenon, the mere incidence of light on an absorbent material body ought to cause a pressure on the body, measured by the amount of electromagnetic energy resident in unit volume of the region just in front of the body. This fact of radiation-pressure has been discussed since from the theoretical side by Bartoli,† Boltzmann,‡ and Larmor §; while from the experimental side its existence has been established by Lebedew,‖ and Nichols and Hull.¶ A reflecting body, of course, experiences the pressure to an even greater extent than an absorbent one.

Although electricity has been in one way or other connected with cometary emanations by various writers, the first close approach to the view now so largely held seems to have been made in 1881 by Faye,** who, speaking of the repulsive power exercised by the Sun on comets, said "J'ai tenté, il y a longtemps,

* *Elect. and Mag.* §§ 792-3.
† *Exner's Rep.* xxi. (1885), p. 198.
‡ *Wied. Ann.* xxii. (1884), p. 31.
§ *Ether and Matter* (1900), p. 130.
‖ Paris Congress (1900), *Reports on Physics*, ii. p. 133.
¶ *Astroph. Journal*, xvii. (1903), p. 315.
** *C. R.* xciii. p. 362.

de la rattacher à l'état d'incandescence du Soleil." A year later a definite theory was formulated by Fitzgerald.*

Fitzgerald, starting from Maxwell's expression, suggests light-pressure as the cause of comets' tails, and determines the size of a molecule of gas of given density in order that it may be neither attracted nor repelled by the combined effect of the Sun's light-pressure and solar gravitational attraction; observing that each different kind of gas would give rise to a separate tail, since the size and density of the molecules of different gases would not be the same. He seems, however, to have taken the light-pressure as due solely to absorption, and notes † the difficulty caused by the fact that gases absorb only a very minute proportion of the radiations that fall on them, and so (on his assumptions) will experience only a slight light-pressure.

Fitzgerald's hypothesis was again brought forward by Lebedew ‡ in 1892, and by Arrhenius § in 1900. Arrhenius, however, makes the modification of supposing that the matter repelled by the light-pressure is not gaseous, but consists of fine particles which have condensed from the gaseous emanations of the comet.

The next step was taken by Schwarzschild,‖ who observed that Arrhenius's conclusions were insecure, inasmuch as the particles repelled must be so small as to have dimensions comparable with the wave-length of light, and the magnitude of the light-pressure would therefore be profoundly altered by the effects of diffraction. Accordingly he investigated the problem of the diffraction of an electromagnetic wave round a perfectly conducting sphere (the effect of radiation-pressure will, of course, be greatest when the repelled particles are perfectly reflecting); and then applying Maxwell's formula for the pressure in terms of the electric and magnetic forces, he obtained the total average pressure on the sphere. The result shows that when the dimensions of the particles are either small or large compared with the wave-length of the incident light, the pressure is small compared with the gravitational attraction; but for intermediate values of the radius of the sphere, the pressure may play an important part.

Assuming that the radiation-energy of the Sun is partitioned among the various wave-lengths according to the law given by Wien for a perfect radiator (which, as Langley has shown, is to a first approximation correct), Schwarzschild comes to the final conclusion that light-pressure is sufficient to account for a repulsive force possibly as much as twenty times that of gravity on cometary appendages, but is not a possible explanation of greater repulsive forces than this. This result is really a difficulty in the

* *Proc. Roy. Dub. Soc.* part vi. vol. iii. (1882).
† *Electrician*, December 14, 1900.
‡ *Wied. Ann.* xlv. p. 292.
§ *Öfversigt af Kongl. Vet.-Ak. Förhandlingar* (1900), p. 545.
‖ *Munich Sitzungsberichte*, 1901, p. 293.

way of the light-pressure theory, for W. H. Pickering,* from observations of Comet 1892 I., obtained a solar repulsive force 39·51 times the attraction of gravitation, and Hussey† found in the case of Comet 1893 II. a repulsion 36 times that of gravitation.

It only remains to notice three other theories of comets which have appeared in late years.

Fessenden‡ puts forward the view that the tail consists of material particles carrying charges of negative electricity, and moving under the influence partly of the Sun's gravitation and partly of electrostatic forces, due respectively to a negative charge supposed to exist on the Sun, a positive charge on the comet's nucleus, and the negative charges on the other particles of the tail.

J. J. Thomson§ remarks that if the comet's nucleus when heated by the Sun's rays gives off negative ions, these ions would be repelled by the Sun's radiation and appear as a luminous tail behind the comet. On performing the calculations, however, it appears that the repulsion exerted on an ion by ordinary light-rays would be very slight, and the effect must therefore be caused by Hertzian waves emitted by the Sun, if such exist. There are two difficulties in the way of this theory—namely, that it is doubtful whether the nucleus is likely to be heated to the temperature required for emission of the ions, and that no Hertzian radiation from the Sun has hitherto been detected, although search has been made for it. Thomson does not give his view on these matters; we may, however, get over the first of the difficulties by supposing that the emission of ions is due, not to the heating of the nucleus, but to the incidence of the Sun's ultra-violet light on it; and the second cannot be regarded as fatal until it is known whether Hertzian waves from the Sun are absorbed in the upper regions of the Earth's atmosphere.

Lastly, Boys‖ suggests that the presence of radio-active substances in the nucleus may be the cause of cometary appendages. If we further suppose the Sun to have an electric charge, the rays, after leaving the nucleus, would be turned back by the electrostatic field so as to form a tail, and indeed the different types of rays which radio-active substances are known to emit would give rise to tails of different curvatures.

E. T. W.

Progress of Meteoric Astronomy in 1903.

Quadrantids.—Very windy and stormy weather occurred at the period of these meteors, and at Bristol on January 3 there was frequent lightning during the night. The sky was, how-

* *Annals of Harvard Coll. Obs.* vol. 32. † *Publ. Ast. Soc. Pacific*, 1895.
‡ *Astroph. Journ.* iii. p. 36 (1896). § *Phil. Mag.* iv. (1902), pp. 253-62. Presidential Address to Section A of the British Association, 1903.

ever, clear at many places on January 2 and 3. On the former date there was little evidence of the *Quadrantids*, but on January 3 they formed a well-marked though not unusually rich shower. Maximum on morning of January 4. The radiant as determined by six observers was :

	α.	δ.	↓s.		α.	δ.	↓s.
Astbury	228°	+53½°	6	Denning	228°	+53°	7
Besley	228½°	+51½°	6	A. S. Herschel	229°	+52°	9
Brook	222°	+50°	8	King	228°	+52°	8

Lyrids.—The shower returned on April 21 and 22, but it was not strongly represented. On April 18, 19, and 20 there was scarcely any sign of its activity. On April 21 and 22 Professor Herschel at Slough watched during a total period of 7¾ hours and saw 47 meteors, of which 19 were *Lyrids*, radiant 270°+35°. On April 21 and 22 Mr. King at Leicester watched for 5¾ hours and noted 51 meteors, including 19 *Lyrids* with radiant 271½° +33°. On April 19 at Bristol observations were made between 11h and 14h 30m, but meteors were scarce, only 12 being seen, including 2 doubtful *Lyrids*. At 13h 40m a fine meteor, 3×4, with a thick yellow train, moved in 4½ secs. from 157°+60° to 73°+60½°, directed from a radiant at 218°−31°.

Mr. R. M. Dole, of Jamaica Plain, Mass., watched for *Lyrids* on April 18, 19, and 20. 53 appear to have been observed, and 20 other shooting stars. 44 *Lyrids* were recorded on April 20, 12h to 15h. There seemed to be 3 radiants, the richest near α and the others near ζ and β. It is, however, highly probable that many of the meteors supposed to have been *Lyrids* really belonged to contemporary radiants in the same region. April meteors shooting from *Lyra* or its vicinity are not always true *Lyrids*, any more than that November meteors directed from the "Sickle" are invariably real *Leonids*.

May Aquarids.—Mr. G. M. Knight, of Hampstead, observed about 25 of these bright, long-pathed meteors on April 30, May 1, 2, and 3. The radiant was at about 330°±0°. 18 of the meteors were equal to or surpassed stars of the 1st magnitude, and several of them were observed in daylight at about 16h. The motions were slow, colour generally orange, and fine trails were left along their extended flights. Another radiant of less brilliant objects was detected at 302°+23°.

Perseids.—Moonlight greatly interfered with this stream. The earliest meteors of the shower were detected on July 21 in America, and the display did not wholly cease until fully a month later. The maximum apparently occurred on August 12. Mr. R. M. Dole observed the shower at Jamaica Plain, Mass., and found it less numerous than in 1902. On August 12, 11h 20m to 15h 44m (local time), he counted 111 meteors, and thought the average about 29 per hour. The *Aquarids* were well seen by him

on July 28 from a radiant at 338°−14° (12 ↓s). Professor Herschel at Slough noted 25 meteors on August 12, $10^h\ 45^m$ to 14^h, and the radiant appeared to be double. On the same night at Bristol *Perseids* were seen to be fairly abundant and pretty bright, but the gibbous Moon hindered the display from being really conspicuous.

Mr. King at Leicester maintained watches on 18 nights between July 13 and August 21, the time employed being twenty-four hours, during which 130 meteors, including 25 *Perseids*, were registered. He saw the first *Perseid* on July 26, and the shower appeared to be extinct on August 19 and 21. As determined by various observers the radiant exhibited its normal easterly drift, the positions being:

	° °	↓s.	
July 21	24+50	8	C. P. Olivier.
23	26+52	6	,,
28	33+55	3	,,
28	36+56	4	G. F. Paddock.
Aug. 3	34+55½	6	A. King.
4	35+59	...	D. E. Packer.
4	36+57	7	A. King.
11	44+56	16	C. P. Olivier.
12	46+58	20	W. F. D.
12	41+58	8	A. S. Herschel.
	49+61	5	
17	52+56	11	G. M. Knight.
18	56+57	14	,,
19	57+60	9	,,
20	57+59	5	,,

Orionids.—On October 22, 23, and 24, in watches extending over nine hours, Mr. Denning at Bristol recorded 74 meteors, including 14 *ζ Geminids* (99°+13°) and 10 *ν Orionids* (92°+16°). These results corroborate those obtained in the few preceding years, and show that the *ζ Geminids* have apparently displaced the *ν Orionids* as the richest shower of the October epoch.

Leonids.—A tolerably brilliant and somewhat unexpected display of these meteors occurred on the night following November 15, and afforded a partial compensation for the failures in several recent years. Between about 17^h and 18^h bright *Leonids* appeared at the rate of about four per minute. The meteors were not quite so abundant as those forming the shower observed in America on 1901 November 14, when six or seven per minute were counted. The really active phase of the recent return was decidedly brief, for on November 14 and 16 only a few *Leonids*, such as are visible in an ordinary year, made their appearance

The maximum was attained just as the morning twilight (of November 16) interfered, and it was thought possible that at western stations a really brilliant exhibition might have been witnessed an hour or two later. Reports from America, however, indicate that the shower must have rapidly declined after daylight had interrupted observation by English observers. At the Leander McCormick Observatory in Virginia Mr. C. P. Olivier on November 15, 18^h to 19^h G.M.T., counted only 22 *Leonids*,* while from 20^h to 21^h G.M.T. 28 per hour were seen. The following are brief extracts from various observations as to the number of meteors visible on November 15 :

T. W. Backhouse, Sunderland.—17^h 44^m to 18^h 5^m. 83 meteors per hour.

C. L. Brook, Meltham.—12^h to 15^h 30^m. 52 *Leonids* seen, but many others escaped notice while the observer was registering paths.

H. Corder, Bridgwater.—17^h to 18^h. About 200 meteors per hour. Following numbers seen :

h	m		h	m	Min.	Meteors.	
16	45	to	17	0	12	24	Imperfect conditions.
17	0	„	17	23	23	61	Fairly clear.
17	23	„	17	33	10	20	„
17	33	„	17	43	10	30	Clear.
17	43	„	17	53	10	26	„
17	53	„	18	3	10	20	Clouding over and dawn breaking.

W. de Cruce, Exeter.—16^h 25^m to 17^h 10^m. 108 meteors observed.

W. F. Denning, Bristol.—Hourly numbers seen in a rather contracted field of observation :

h	h	Meteors.	h	h	Meteors.
12	to 13	16	15	to 16	52
13	„ 14	20	16	„ 17	120
14	„ 15	48	17	„ 18	140

42 seen 17^h 30^m to 17^h 45^m. Probably 4 per min. 17^h 30^m to 18^h 5^m from a good position. Maximum about 17^h 40^m.

W. F. A. Ellison, Enniscorthy.—

h	m		h	m		
12	10	to	13	10	5	*Leonids*
13	10	„	14	10	16	„
14	10	„	15	10	36	„
15	10	„	15	25	11	„
15	25	„	15	40	0	„

* About 80 per hour were counted in England during the same hour, but the far more favourable position of the radiant probably accounts for the difference.

Royal Observatory, Greenwich.—About 150 meteors seen. Most prolific time 18^h, about 100 per hour.

J. R. Henry, Dublin.—11^h 25^m to 15^h 25^m. 86 meteors registered, including 52 *Leonids*. Between 16^h 25^m and 17^h 55^m the rate of appearance seemed 200 to 300 per hour.

Professor A. S. Herschel, Slough.—16^h 15^m to 18^h 15^m. Two hours. 180 meteors mapped and counted, and probably as many more were missed. Estimated rate of apparition, 200 to 250 per hour. 9 meteors seen in the minute ending 17^h 45^m.

M. Horner, Taplow.—86 *Leonids* seen during the last hour of darkness.

Rev. S. J. Johnson, Bridport.—Several hundreds of *Leonids*, with the usual streaks and swift motions, passed across the sky.

G. M. Knight, Hampstead.—

Date.	Watch. h h	Meteors.	*Leonids.*	Max. h m
November 13	15 to 18	39	33	17 10
14	" "	53	45	17 15
15	" "	120	113	17 30
16	" "	31	26	17 0

A. King, Sheffield.—17^h 57^m to 18^h 3^m. 18 *Leonids*. Hourly rate about 200.

W. H. Milligan, county Down.—14^h 25^m to 16^h 25^m apparent maximum; hourly rate, 80 to 100.

John McHarg, Lisburn.—13^h 20^m to 14^h 20^m. 20 *Leonids* seen.

A. G. Moffatt, Swansea.—16^h 30^m to 18^h. A brilliant display of large meteors, some green-coloured, the majority, however, electric blue.

R. Service, Dumfries.—18^h 30^m to 19^h. 42 *Leonids* observed.

G. C. Thompson, Cardiff.—Watching with a friend for several hours, only about 25 *Leonids* seen. A number of other meteors radiated from *Auriga*.

F. H. Wright, Northampton.—15^h to 15^h 30^m: 30 meteors seen. 15^h 30^m to 16^h: 60 meteors seen. Afterwards counted about 3 or 4 per minute. Max. 17^h 15^m, near which time 8 or 10 were several times counted in one minute, and 5 or 6 visible in the sky at the same instant.

In America * Professor E. E. Barnard, at Williams Bay, Wisconsin, watched on November 13 in a clear sky from 15^h to daylight, but only a few meteors were observed, and several of these came from a radiant somewhere near *Orion*. On November 14 the sky was partly cloudy and *Leonids* scarce. At 15^h 52^m, however, 5 members of the shower instantaneously appeared within 20° of the radiant. November 15 was cloudy.

* The times given by the American observers are local, and have not been reduced to G.M.T.

C. P. Olivier, at the University of Virginia, obtained results as under :

Date.					Period.	Meteors.	*Leonids.*
	h	m		h m	h m		
November 12	14	35	to	16 18	1 43	14	1
14	12	28	„	17 28	5 0	80	55
15	12	39	„	16 58	4 19	93	78

A large proportion bright, colour orange, some green, and occasionally red.

J. Stebbins, at Illinois, watched on November 14, 13^h to 17^h and saw 29 meteors, of which 20 were *Leonids*.

The following notes prove the comparative rarity of *Leonids* on November 14 and 16 :

Professor Herschel at Slough watched from 12^h 10^m to 14^h 30^m on November 14, and only saw 3 *Leonids*.

Mr. Denning at Bristol on the same night, looking towards the Sickle of *Leo* during the period 17^h to 17^h 45^m, did not see a single *Leonid* Sky unfavourable on November 16, and very few *Leonids* seen.

Mr. King at Sheffield made observations on November 16, 12^h 16^m to 14^h, but only recorded 2 *Leonids*.

The position of the radiant was determined as follows :

			↓s.
		° °	
Blum, G., Paris... ...	...	151½+22½	48
Brook, C. L., Meltham...	...	150½+23	13
Corder, H., Bridgwater	...	149 +22	...
Denning, W. F., Bristol	...	151 +22	33
Herschel, A. S., Slough	...	150 +24	16
King, A., Sheffield ...	...	148 +22	10
Knight, G. M., Hampstead	...	149½+23	5
Milligan, W. H., co. Down	...	149 +22	...
Olivier, C. P., Virginia	...	151 +22	78

The *Leonids* were brilliant, though few fireballs were seen. Professor Herschel says that of the 180 meteors he saw, 10 or 15 surpassed fixed stars in brightness, while quite 30 more could be compared with *Sirius* and *Vega*. Mr. Backhouse found 46, out of 67 *Leonids* registered, as bright as, or brighter than, 1st mag. stars. Mr. King says, "The meteors were mostly brilliant, the average mag. being 1 or a little greater."

Geminids.—Mr. F. H. Wright, Northampton, observed 22 short swift meteors on December 14, 6^h to 7^h P.M. Later in the evening they were less frequent. Mr. J. P. Kenyon at Didsbury found the *Geminids* very abundant on the same date, 9^h 30^m to 10^h 30^m, but the number diminished rapidly after the

latter time. Mr. C. L. Brook, watching for five hours on December 11, saw 34 meteors, including about 6 *Geminids*. On December 14 during three hours he recorded 25 meteors, of which 7 were *Geminids*. The radiant seemed a duple one at the points 110°+31° (10 ↓s) and 114°+27½° (6 ↓s).

Detonating Fireball.—On October 3, in the afternoon, a ball of fire passed over Tain, in the N. of Scotland, and apparently descended on the Edderton Hills with a loud explosion, the noise travelling all along the Sutherland mountains, and being heard in the surrounding counties. In the Rogart valley a vibration similar to an earthquake shock was experienced. The fireball does not, however, appear to have been accurately observed, and details are wanting.

The following is a list of the real paths of several bright meteors observed in England during the past year :

Date. 1903.	G.M.T. h m	Brightness.	Height at First. Miles.	Height at End. Miles.	Length of Path. Miles	Velocity per Sec. Miles.	Radiant Point. α ° δ °	Observers.
Jan. 3	7 25	1 − ♃	60	47	2	16	228+52	2
3	12 5	1 − >1	65	49	41	26	228+53	2
3	12 59½	1 − < ♃	67	54	30	20	227+50	2
10	9 30½	☾ = ☽	63	31	62	18	116+ 2	10
13	6 15	♃	57	54	200	34	218+39	3
14	7 6	2 × ♀	57	54	53	21	120− 3	8
25	7 59	♃	95	16	125	15	99+13	4
28	11 44	5 × ♃	62	41	181	20	280+43	12
April 22	10 36	>1 − ♃	70	56	28	19	271+34	2
22	12 32½	>1 − ♀	78	43	49	39	269+32	2
Aug. 12	10 55	♄	67	65	50	32	32+ 4	2
Oct. 23	11 47	1 − ♃	75	37	48	21	40+16	2
Nov. 15	10 18	> ♄ − > ♀	79	42	48	36	56+22	2
15	13 42	> ♀	72	32	47	23	61+24	2
25	13 59	2 × ♀ / −3 × ♀	90	53	64	...	149+28	3
25	16 45	♄ − ♀	77	52	30	60	151+25	2
15	17 14½	♃	77	52	30	45	152+25	2
15	17 23	♃	84	63	27	67	149+19	2
15	18 7	>1 − ♃	81	60	24	44	151+23	2
18	12 44	♀ − ½ ☽	75	33	51	33	50+20	2

There were a number of *Taurid* fireballs observed at the middle of November.

In addition to the radiants already mentioned only a few appear to have been determined. At the April epoch Mr. King observed feeble showers from the points 216°—26° and 256½°

$+37°$, and on August 21 there was an active centre at $3° + 25\frac{1}{2}°$, near α *Andromedæ*. At Bristol several tenuous streams were detected during observations in January, May, and October, and the radiants were satisfactorily derived by combining the meteors recorded in previous years.

Years.	Date.	Radiant. α	Radiant. δ	Meteors.	Notes.
		°	°		
1877–1903	Jan. 17–25	143	+38	9	Rapid, white.
1877–1903	Jan. 22–Feb. 1	159	+27	5	Slow, yellow.
1876–1903	Feb. 7–23	75	+41	7	Slow, bright.
1903	May 18–26	248	+29	6	Slow.
1886–1903	May 25–June 4	280	+31	10	Rapid.
1903	June 22–July 1	245	+64	5	Slow.
1877–1903	Aug.–Sept.	47	−11	5	Rapid, streaks.
1900–1903	Oct. 22–23	91	+59	7	V. rapid.
1903	Oct. 22–24	117	+46	6	Rapid, streaks.
1903	,,	133	+77	5	Rapid.
1900–1903	Oct. 22–31	59	+49	6	Rapid.
,,	Oct. 22–27	32	+19	7	Slow.
,,	Oct. 22–23	117	+30	5	Rapid, streaks.
,,	Oct. 22–31	29	+36	5	Medium.

Meteorites.—There appear to have been no well-authenticated cases of stonefalls or ironfalls during the year. Several questionable incidents of the kind have been reported in the papers, but they have not survived the test of investigation. For instance an alarming shower of stones was reported as having taken place on 1903 April 29 near St. Clears, Carmarthenshire; but some of the stones were picked up, and on being submitted to suitable examination were found to be palpably of terrestrial origin. A meteorite was said to have fallen near Ellis, in Edwards County, Texas, on 1902 August 16, and to have struck a Mexican sheep-herder named Guise on the head and killed him. On proper inquiries being made, however, the story failed to receive corroboration, and it was probably only a newspaper canard. W. F. D.

Solar Activity in 1903.

Sun-spots and Faculæ.—The development of the solar activity, as evidenced by the number and area of Sun-spots, has been exceedingly striking throughout the past year. In 1901 the days without spots were 81 per cent. of the total number, and in 1902, after the minimum was past, they were still 72 per cent. But this number in 1903 fell to about 20 per cent., and these spotless days were chiefly confined to the first half of the year.

There was not a single day without spots in the month of July, which was the first month of which this could be stated since November 1898. Later on in the year there was a yet longer period of unbroken activity, namely, from October 1 to December 24. The development of the faculæ has been greater still, and the proportion of days on which there were no conspicuous groups has fallen considerably below 10 per cent.

The progress towards greater activity has been on the whole very steady, but has shown a certain pulse, due to the circumstance, always characteristic of a period of revival, that the increasing activity is at first manifested chiefly in certain longitudes. Periods of greater or less spot frequency, in length about a fortnight each, have therefore alternated with each other, according as the more disturbed or more quiescent hemisphere has been turned towards us.

Allowing for this apparent pulsation, there was a slow and steady increase in the numbers and area of spots, from the beginning of the year up to March 21, when a period of very considerable activity set in, lasting till April 14—a period of twenty-five days—during which the mean daily spotted area amounted to 500 millionths of the Sun's visible hemisphere. This was followed by a short time of quiescence and a gradual rise until the active month of July. August and September were relatively quiet months, but with the beginning of October a very different state of things commenced. A large regular spot was seen on the east limb on October 2 in the northern hemisphere. On October 4 a small regular spot appeared in the southern hemisphere, and proved to be the forerunner of an exceptionally fine group—larger even than the great group of September 1898, and therefore the largest seen since January 1897. This was much the largest group of the year, and was rendered still more interesting by the great and frequent changes which took place in it, and by the considerable magnetic disturbance which occurred when it was about one day past the central meridian. From this time to the end of the year there was a constant succession of interesting and important groups. Of these the most remarkable in one sense was the group seen from October 25 till November 5, which passed the central meridian fifteen hours after the commencement of the greatest magnetic storm experienced for thirty years. Other very striking groups were those occurring from October 30 to November 11, and from November 4 to November 17; the first a succession of regular spots in the northern hemisphere, the second a long stream in the southern hemisphere, which underwent a striking series of changes. Both groups returned to the visible hemisphere during December.

The distribution of the spots in latitude has been strictly that usual at this stage of the solar cycle, nearly all the groups lying between latitudes 15° and 25° in either hemisphere. During the first six months of the year the groups were nearly equally

divided between the two hemispheres. During the last six months, and especially during the last three, the southern groups have shown a decided predominance.

The distribution of faculæ in latitude has changed somewhat since 1902, there being a distinct falling off in the proportion of groups observed in higher latitudes than 40°. E. W. M.

The Prominences.—During the past year a very marked revival of activity has set in, the mean number of prominences observed at Kenley having risen from 5·38 per diem in 1902 to 7·55 per diem in 1903, the former figure being the lowest recorded since the last Sun-spot maximum in 1893.

From observations made on 89 days in 1902 and 88 days in 1903 the following figures are derived for the two hemispheres:

	1902. per Diem.	1903, per Diem.
North hemisphere	2·69	3·64
South ,, ...	2·69	3·91
Total mean number	5·38	7·55

A more detailed analysis of the observations shows that there were two well-marked zones of activity in each hemisphere, quite symmetrically situated with respect to the solar equator, one between the parallels 20° and 30° and the other between 50° and 60°.

This distribution is much like that of 1902, excepting that the zones between 20° and 30° north and south, which were barely indicated in that year, have increased over fourfold in activity; whereas of the high latitude zones only the northern one shows any noticeable increase, and both have remained precisely in the same latitude as before, viz. +50° to 55° and −55° to 60°. Not the smallest tendency is shown to approach the poles, and the region from 65° to the pole in each hemisphere has remained quite barren of prominences.

A rapid advance of the high latitude zones of activity towards the poles may be expected, however, during 1904 and 1905, judging by the course of events twelve years ago. The final closing in of the high latitude prominences over the poles took place last about the epoch 1894·7, or one year after the spot maximum had been passed.

If injections of sodium and magnesium into the upper chromosphere and prominences be taken as an index of solar activity the past year shows a very great advance over 1902, when the lines of these elements were observed on only two occasions during 89 observing days. Last year, with 88 days of observation, nineteen "metallic" prominences were noted, of which ten were seen north of the equator and nine south.

Of the ten northern metallic prominences, eight appeared between the limits +18° and +28° in latitude and 190° to 260° in longitude—a region characterised by the outbreak of spots

and of an extensive range of calcium "flocculi" during June; and later by the large northern spot and accompanying flocculi of October and November.

Most of the southern metallic prominences were also in this same region of longitude, and between 24° and 30° south latitude. The most active eruption observed was on November 16 at 9 A.M., over the great southern spot-group then on the west limb. The hydrogen lines in this prominence showed intensely bright needle-like projections on each side, indicating violent motions in the line of sight. J. E.

The Rotation of Saturn.

An important addition has been made during the past year to our knowledge of the rotation and, incidentally, the physical condition of *Saturn* through the apparition of a number of bright spots in a somewhat high northern latitude (+36°) of the planet. The history of these spots is not uninteresting. They were first detected by Professor E. E. Barnard with the 40-inch refractor of the Yerkes Observatory. One of them was seen as early as June 15, but it was not until June 23 that it could be reobserved and its position accurately determined, when the discovery was announced by telegraph. A second accurate observation was obtained on the following night. Quite independently, and before he had heard of any spot having been observed elsewhere, Mr. W. F. Denning, on the night of July 1, detected another bright spot, which was followed by a dark mass, with his 10-inch reflector at Bristol. The bright spot was also observed on the same night by Señor J. Comas Solá at Barcelona with a 6-inch refractor. The observations soon showed, in fact, that there were actually quite a number of these white spots, all situated in about the same latitude, though the one first discovered seems to have been more conspicuous than any other.

A comparison of his observations of June 23 and 24 satisfied Barnard that the rotation period must be decidedly longer than $10^h\ 14^m$ (*A. J.* 547), but the first published announcement respecting the rate of rotation appears to have been made by Dr. K. Graff, of Hamburg (*A. N.* 3883). This astronomer derived a rotation period of $10^h\ 39^m{\cdot}01$ from a comparison of observations of the Barnard spot made on the nights of June 23, June 26, and July 4, a period that at first seemed improbably long.

Nevertheless subsequent observations showed it to be substantially correct, and several observers soon published confirmatory results. Thus Comas Solá from his own observations of the same spot found a period of $10^h\ 38^m{\cdot}4$ (*A. N.* 3894). Denning published a preliminary list of observations of this spot in the *A. N.* 3900, from which he inferred that the rate of motion was consistent with a rotation period of $10^h\ 38^m$, the mean of

seven other spots giving $10^h\ 39^m\ 21^s{\cdot}1$; whilst Herr L. Brenner, from his own observations at Lussinpiccolo, stated that the period was exactly $10^h\ 38^m$ (*The Observatory*, 1903, p. 391).

All the foregoing results were more or less preliminary, but some more definitive determinations have since been announced. A long list of observations of the spot first discovered, extending to the end of August, was published by Dr. H. C. Wilson in *Popular Astronomy*, 1903, p. 445, and from these he derived a rotation period of $10^h\ 38^m\ 4^s{\cdot}8$, a result which would seem to be very near the truth. Professor G. W. Hough, from a discussion of a few micrometrical measures of the Barnard spot, finds a variable motion, the rotation period being $= 10^h\ 38^m\ 18^s + n \times 0^s{\cdot}1856$, where $n =$ the number of rotations since June 27 (*Monthly Notices*, lxiv. p. 124); whilst three observations of another spot gave a period of $10^h\ 38^m\ 30^s{\cdot}5$. But Hough's results have been disputed by Denning, in a paper published in the *Monthly Notices*, lxiv. p. 239, who strongly questions the correctness of some of the former's identifications. It is obvious that correct identification of the observations is of chief importance in work of this kind, and it is necessary to bear in mind that in the present case the spots were really quite numerous, and that none of them seems to have possessed any permanent characteristic feature by which it might be recognised by its appearance; whilst, to add to the difficulty, the spot originally detected by Barnard was of large area, was somewhat indefinite, and sometimes had the appearance of being double. Hence it is easily seen that mistakes in identification could easily be made where there are only a few observations available. In the paper last mentioned Denning enumerates nearly fifty observations of the Barnard spot, and the gaps are so slight that it seems difficult to suppose that any serious mistake in identification can have been made by him. From these observations a period of $10^h\ 38^m\ 3^s$ was found for the spot in question for the interval between the latter part of June and the middle of September, a result very close to that found by Wilson. The mean period from about fifteen bright and dark spots from July to December is stated to be $10^h\ 37^m\ 56^s$.

It is fortunate that there can be no doubt whatever as regards the most important fact resulting from the recent observations, notwithstanding any possible differences of opinion there may be on the subject of identification. The spot first detected by Barnard must, from all accounts, have been of extraordinary brilliancy during the first two or three weeks of its existence, so that there is no uncertainty as to the reality of the remarkably long rotation period of about $10^h\ 38^m$; and this fact is a very important one, since the previous determinations of this element, all relating to lower latitudes, are clustered about a period of $10^h\ 14^m$. Moreover in the *Monthly Notices*, xviii. p. 72, will be found two observations by Dawes of a bright spot in the southern hemisphere of *Saturn*, estimated to be in about 40° or

50° S. latitude. These observations, as has been pointed out by Denning (*Knowledge*, 1903, p. 271), indicate a rotation period of about $10^h\ 24^m$, or at any rate one considerably longer than that of the equatorial markings. It seems clear, therefore, that we have on *Saturn*, as on *Jupiter*, a great equatorial current having an enormous velocity in the direction of the planet's rotation relative to the extra-equatorial surface material. But the velocity of this current on *Saturn* is almost incomparably greater than it is on *Jupiter*, amounting as it does in miles per hour to between 800 and 900, as compared with the 250 miles an hour of the latter planet. The important bearing of the recent observations on our knowledge of the physical condition of *Saturn* can be judged from this. It may be inferred for one thing that the surface material of the planet is in a more mobile state than that of *Jupiter*, a circumstance that had already been foreshadowed by the earlier observations, and which is, perhaps, not unconnected with the lesser density of the planet. A. S. W.

Double Stars.

As in former years the work has been classified under the two heads "Observation" and "Calculation." Abbreviated titles have been used as follows:

M. N.: Monthly Notices, R.A.S.
A. J.: Astronomical Journal.
L. O. B.: Lick Observatory Bulletin.

Observations—

R. T. A. Innes. *M. N.* lxiii. 7. Measures of 250 southern pairs made with the 18-inch refractor at the Cape Observatory in the year 1902.

Royal Observatory. *M. N.* lxiii. 7. Measures of some 340 pairs made with the 28-inch refractor at the Royal Observatory, Greenwich, in the year 1902.

Rev. T. E. Espin. *M. N.* lxiii. 3. Measures of 38 new double stars. In most of these the larger star is 8th or 9th mag.; the comes 8th to 13th mag. The distances vary from 5″ to 18″. Reflector, 17½-inch.

S. W. Burnham. *M. N.* lxiii. 7. Discovery of a close comes to Σ 1594, position-angle 318°, distance 1″·6. There is a note on measures of Σ 1594, and also on the double 31 *Leonis*.

J. L. Scott. *M. N.* lxiv. 1. Measures made with a 5-inch refractor at Hong Kong in the years 1902–3. There are 26 stars under 2″ separation, and 40 between 2″ and 3″.

John Tebbutt. *M. N.* lxiv. 1. Measures of 60 southern pairs in 1902. Good series of *α Centauri*, *γ Centauri*, and *γ Coronæ Australis*.

Royal Observatory. *M. N.* lxiii. 3. Occultation of *ω Leonis*.

E. E. Barnard. *A. J.* 546. Measures and discussion of *Krueger* 60.

Miller and Cogshall. *A. J.* 546. The Berlin zone (20°–25°). Observers noted a number of doubles when taking the meridian observations. Messrs. Miller and Cogshall with the 24-inch Lowell refractor have measured 60 of these pairs, of which three are under 1″ separation.

Eric Doolittle. *A. J.* 547. Measures of 227 doubles, mainly Burnham stars, made at the Flower Observatory in the years 1900–1–2.

W. J. Hussey. *L. O. B.* 40. Measures of 32 Otto Struve stars. These are mostly difficult pairs, for which observations were required.

W. J. Hussey. *L. O. B.* 44. Measures of some miscellaneous stars in the catalogues of Herschel, Hough, Burnham, and others. The Burnham measures are already in Yerkes I. A set of *Sirius* gives 1903·16, 120°·0, 5·97.

Calculation—

W. Bowyer. *M. N.* lxiii. 6. Discussion of measures of Σ 2525, showing that the motion is orbital—a provisional orbit—period 306 years.

J. E. Gore. *M. N.* lxiii. 3. On the Sun's stellar magnitude and the parallax of binary stars. Results are very accordant. From *α Centauri* the Sun's magnitude comes to −26·31, from *η Cassiopeiæ* −26·65, from *ξ Ursæ Majoris* −26·34.

R. G. Aitken. *L. O. B.* 36. Orbit of *ε Hydræ* AB, the close Schiaparelli pair, period 15·7 years.

W. J. Hussey. *L. O. B.* 32. Parallax of *δ Equulei.*

J. E. Gore. *Knowledge*, 1903 September. "Stellar Satellites." A discussion of the magnitudes, motions, distances from each other of various systems.

T. Lewis. *Observatory*, No. 334. "The Size of Stellar Systems."

T. Lewis. *Observatory*, No. 328. "Some Interesting Double Stars." Σ 13, Σ 39, Σ 86, Σ 262. T. L.

Variable Stars.

The work of discovery of variable stars has gone on apace since last Note, at least sixty-three new variables having been announced in 1903. It would seem useless giving the particulars of every one of these, as they are almost all contained in the *Astronomische Nachrichten*, Nos. 3846 to 3920. The following remarks occur: No. 3, 1903, *Ursæ Majoris* has the extraordinarily short period of $4^h\ 0^m\ 12^s{\cdot}8$, the changes at minimum being very rapid. The variables of shortest period hitherto known are two stars in the cluster *ω Centauri* and *S Antliæ*. The periods of these all exceed 7^h. No. 18, 1903, *Ursæ Minoris*,

is the well known comes to *Polaris*. Variation stated by Jost to be from $8^m.5$ to $9^m.6$, but brought forward with reserve. The light curve of 19, 1903, *Lyræ*, discovered by Williams, resembles those of *Y Lyræ* and *UY Cygni*. Nos. 23 to 28 are "suspected" only. Nos. 32 to 54 in the great nebula in *Orion* were discovered by Professor Wolf with the "stereo-comparator." No. 55, 1903, *Cygni* is *Algol* type. No. 59, 1903, *Cygni* is, according to Professor Pickering, not a Nova, but a variable.

One of the most remarkable events of the year in this branch of astronomy was the discovery of Nova *Geminorum* (14, 1903), by Professor Turner, at Oxford, on March 16. The circumstances are peculiar. One of the few plates needed for the completion of the zone for the astrographic survey had been exposed on the region in question, and subsequently rejected on account of erroneous setting. The Savilian Professor chanced shortly afterwards to compare the plate with others, and it was found that the cause of the erroneous setting was that the telescope had been set on the Nova. The position is given as R.A. 6^h 37^m $48^s.86$, Dec. $+30°$ $2'$ $39''$ (1900·0); magnitude at time of discovery 6·8. Since then the star has faded steadily, it being $8^m.70$ on 1903 March 26, $9^m.42$ on May 20, and was found at the Radcliffe Observatory, Oxford to have fallen to $11^m.99$ at the end of the year.

The outburst of this star must have been fairly rapid, for according to the Harvard photographs it was, if present, less than $11^m.4$ on March 1 and less than $9^m.5$ on March 2. It first appears on March 6 as of $4^m.97$, falling to $8^m.08$ by March 25.

The spectrum of the star showed the hydrogen lines $H\alpha$ and $H\beta$ very bright, at the Potsdam and Yerkes Observatories. Professor Barnard observed that the focus did not differ appreciably from that of the comparison stars; but a beautiful crimson image, undoubtedly due to $H\alpha$, was found a little outside the proper focus for the Nova.

Nova *Persei*, 1901, seems to have been fading slowly but steadily. It is now (January) about 10^m.

A great extension of our knowledge of the spectra of faint stars is now rendered possible by the application of the Crossley reflector of the Lick Observatory to this purpose. With a spectroscope built specially for this telescope photographs can be taken of stars below the 15th mag. It is obvious a huge and unworked field of research lies before such high optical power, especially among the long-period variables.

The work of observation of variable stars at Harvard College Observatory has been carried on during the past year with the energy characteristic of the director, Professor E. C. Pickering, who, we learn, made his one-millionth photometric setting on 1903 May 25.

We must note the appearance of "A Provisional Catalogue of Variable Stars" issued from Harvard College Observatory. This contains 737 stars. Until the issue of the Catalogue of the

Astronomische Gesellschaft it will doubtless be the leading authority.

The "Fifth Report of the Section of the British Astronomical Association for the Observation of Variable Stars" has been published as vol. xi. part iv. of the *Memoirs*. It contains the results of some 7504 observations, made principally in the three years 1900–1902; also diagrams of the light curves of a number of stars of short and long period, and of *Algol* type, as deduced from the observations.

Mr. Gore has lately called attention to the possible *secular* variation of the light of the stars. When examining Al-Sufi's *Description of the Fixed Stars*, written in the tenth century, he noticed certain stars which seemed to have either increased or diminished in light. As the magnitudes given by Al-Sufi are probably those of Hipparchus this takes us back over 2000 years in time. It is an interesting subject; unfortunately mathematics cannot be brought to bear on it, as in the case of gravitation, and hence these possible changes can only be substantiated by an accurate photometric record extending over centuries.

The extensive series of observations of variable stars made at the late Sir C. Peek's observatory will be edited by Professor Turner, and published shortly in the *Memoirs* of this Society.

Mr. A. S. Williams has published in our *Monthly Notices* an interesting account of the light curve of *UY Cygni*. The maximum is pointed, the rise from minimum to maximum being extremely rapid, while after maximum the declension of light is slow.

Mr. A. W. Roberts has also an important paper on the light curve of a close binary system. The typical system may be considered as consisting of two stars, equal in size and brightness, spherical in figure, revolving round each other in, or nearly in, contact, in an orbit whose plane lies in the line of sight. Infinite variations may be introduced by changes in the relative size, shape, and albedo of the two bodies, as well as in the inclination of orbit to line of sight. The stars *β Lyræ* and *RR Centauri* are good examples of the type. E. E. M.

Stellar Spectroscopy in 1903.

Nebulæ.—Mr. Perrine (Lick Obs. *Bulletin*, No. 31) has made bold attempts to photograph the spectrum of the nebulosity round Nova *Persei*, and with an exposure of thirty-four hours has succeeded in getting traces enough to convince him that the spectrum is not that of a gaseous nebula, but resembles more closely the spectrum of the Nova in its earlier stages.

New Stars.—The spectrum of Turner's Nova *Geminorum* leaves no doubt as to the nature of the star. The following

notes relating to it have been published, of which the last three contain photographic illustrations:

Professor Pickering, H.C.O. *Circular*, No. 70, and *Astroph. Jour.* xvii. 305.
Professor Hale, *Astroph. Jour.* xvii. 303.
Mr. Perrine, Lick Obs. *Bull.* No. 37, *Astroph. Jour.* xviii. 297.
Messrs. Reese and Curtis, Lick Obs. *Bull.* No. 37, *Astroph. Jour.* xviii. 299.

On the theory of new stars Professor H. Ebert contributes an article to *Ast. Nach.* **164**, 66.

Classification of Stellar Spectra.—In the *Publications* of the Yerkes Observatory (vol. iii. pt. v.) Professor Hale and Messrs. Ellerman and Parkhurst give a full account of their studies of the spectra of stars of Secchi's Fourth Type; and it forms a remarkable record of a research admirably planned, and forced through to a successful issue in spite of the great difficulties with which it is beset. One hundred and fifty (150) photographs of spectra of twenty-five stars were taken, and in order to study an extended range (3930–6600 λ) the spectra were photographed in four sections with appropriately sensitised plates. Many of the stars studied are so faint that exposures of five or six hours were not uncommonly necessary, and in one case a photograph received twenty-four and a half hours' exposure (on three nights). Forty-three (43) photographs of eight stars were chosen for special detailed measurement, the others being used for comparative study. In all there were 537 lines dealt with, and for each line a wave-length is deduced, corrected for the Earth's orbital and the star's radial velocity. As a result of various processes of exclusion it comes about that forty-nine of the measured lines are common to all of the eight stars, and these give an idea of the precision of the measures and a final check upon the adopted values of motion in the line of sight.

The research shows that the spectrum of Type IV. contains, as McClean's photographs showed, very numerous lines in addition to the characteristic flutings; and these flutings are identified as dark reversals of the bright flutings of cyanogen and of Swan's candle-spectrum. Plate 7—to mention one of an admirable series of photographic illustrations—sums up this part of the research in a striking way; it is based on a careful discussion of the measured wave-lengths corresponding to the edges of the flutings. About 200 of the lines studied are regarded as bright lines, though none of them can be identified with certainty at present; whilst among the dark lines there are so many conspicuous lines corresponding in position with those observed as widened lines in Sun-spot spectra, that the writers base on the correspondence a suggestion that the general radiation of these stars may approximate more nearly to that of the spotted

region of the Sun's surface than to that of the unspotted part. The relation of the fourth-type spectrum with that of the Wolf-Rayet stars is considered in detail ; and in considering classification and evolution the writers take the view that the stars with spectra of Secchi's third and fourth types are coordinate branches in development from the Sun, thus agreeing with Vogel's classification, and disagreeing with Lockyer's contention that stars of these types are far removed from each other in point of development.

Radial Velocity of Stars.—In the course of the year two publications have appeared relating to the results obtained in the scheme of cooperation of certain observatories in determining the radial velocity of chosen "Standard Velocity Stars." Mr. Newall (*Monthly Notices*, lxiii. 296) gives the results obtained for three stars in 1902 at the Cambridge Observatory. Messrs. Frost and Adams (*Astroph. Jour.* xviii. 237) give results relating to thirteen stars observed 1902–3.

The following table summarises these results :

	Epoch.	Yerkes Obs.	Epoch.	Cambridge Obs.
α Arietis	1902·84	−13·7 km/sec.	1902·8	−14·3 km/sec.
α Persei	2·77	− 2·1	1902·8	− 2·6
β Leporis	2·85	−12·4		
β Geminorum	3·12	+ 3·4		
α Crateris	3·23	+47·4		
α Boötis	3·12	− 4·8	1902·4	− 5·8
β Ophiuchi	3·14	−11·1		
γ Aquilæ	2·98	− 1·8		
ε Pegasi	2·65	+ 6·2		
γ Piscium	2·73	−10·9		
ι Aurigæ	2·85	+19·0		
ε Leonis	3·20	+ 5·5		
γ Cephei	3·13	−41·2		

Studies of Special Stars.—

Sir Norman Lockyer and Mr. Baxandall on γ *Cygni*, *Phil. Trans.* A. **201**, 205.

Mr. Stebbins on ο *Ceti*, *Astroph. Jour.* xviii. 341.

Mr. Reese on ϕ_2 *Orionis*, *Astroph. Jour.* xvii. 310.

Mr. Adams on $61_{1 \text{ and } 2}$ *Cygni*, *Astroph. Jour.* xviii. 67.

Mr. Frost on { ξ *Persei*, ε *Persei* / 25 *Orionis*, β *Piscium* } *Astroph. Jour.* xviii. 388.

Orbits and Parallax of Spectroscopic Binaries.—Professor Nijland points out (*Ast. Nach.* **161**, 103) a small simplification in Dr. Schwarzschild's method (*Ast. Nach.* **152**, 68) of computing the orbit of a spectroscopic binary.

Mr. W. S. Adams has computed (*Astroph. Jour.* xvii. 68) an

orbit for the spectroscopic binary *η Orionis* (the brighter component of the fourth magnitude of the well-known visual pair).

Mr. W. J. Hussey has calculated (Lick Obs. *Bulletin*, No. 32, and *Astroph. Jour.* xvii. 378) the parallax of the binary *δ Equulei* (period 5·7 years) from spectroscopic observations.

Variable Radial Velocity.—The following stars have in the course of the year been found to exhibit signs of variable velocity in the line of sight :

	R.A. h m	Decl. ° ′	Mag. m		
*ν Andromedæ	0 32	+33 10	4·4	Yerkes	Frost and Adams, *Astroph. Jour.* xviii. 384
*ξ Cassiopeiæ	0 37	+49 58	4·8	,,	,, ,,
*ρ Andromedæ	0 44	+40 32	...	Lick	Reese, Lick Obs. *Bull.* 31 and *Astroph. Jour.* xvii. 308
β Arietis	1 49	+20 16	...	Potsdam	Vogel, *Ast. Nach.* **163**, 145
*ν_4 Orionis	4 46	+ 5 26	4·0	Lick	Reese, Lick Obs. *Bull.* 31 and *Astroph. Jour.* xvii. 309; Frost and Adams, *ib.* xvii. 153
β Ceti	4 31	− 0 6	4·1	Yerkes	Frost and Adams, *Astroph. Jour.* xvii. 150
*ρ Eridani	4 31	− 3 33	4·1	,,	,, ,,
*τ Tauri	4 36	+22 46	4·4	,,	Frost and Adams, *Astroph. Jour.* xvii. 246
*ν_1 Orionis	4 49	+ 2 17	3·9	,,	Frost and Adams, *Astroph. Jour.* xviii. 150
*ι Orionis	5 17	− 0 29	4·6	,,	Frost and Adams, *Astroph. Jour.* xviii. 385
*ϕ Orionis	5 22	+ 3 1	4·7	,,	Frost and Adams, *Astroph. Jour.* xvii. 246
χ Aurigæ	5 25	+32 8	5·0	,,	Frost and Adams, *Astroph. Jour.* xviii. 385
*ε Orionis	5 30	− 5 59	3·0	,,	Frost and Adams, *Astroph. Jour.* xviii. 386
ζ Tauri	5 32	+21 5	3·0	,,	Frost and Adams, *Astroph. Jour.* xvii. 150
*δ Orionis	6 2	+14 47	4·4	,,	Frost and Adams, *Astroph. Jour.* xviii. 386

	R.A. h m	Decl. ° ′	Mag. m		
σ Geminorum	7 37	29 7	...	Lick	Reese, Lick Obs. *Bull.* 31 and *Astroph. Jour.* xvii. 309
ι Argus	8 3	−24 1	...	„	Campbell, Lick Obs. *Bull.* 31 and *Astroph. Jour.* xvii. 309
ω Ursæ Majoris	10 48	+43 47	...	Potsdam	Vogel, *Ast. Nach.* **163**, 145
γ Corvi	12 11	−16 59	...	Lick	Campbell and Curtis, *Astroph. Jour.* xviii. 307
η Virginis	12 15	0 7	4·1	Yerkes	Frost and Adams, *Astroph. Jour.* xvii. 152; Campbell and Curtis, *ib.* xviii. 307
ε Ursæ Majoris	12 49	+56 33	...	„	Adams, *Astroph. Jour.* xviii. 68; Vogel, *Ast. Nach.* **163**, 146
α Draconis	14 2	+64 51	...	Lick	Campbell and Curtis, *Astroph. Jour.* xviii. 307
α Coronæ Bor.	15 30	+27 3	...	Potsdam	Hartmann, *Ast. Nach.* **163**, 31
β Scorpii	16 0	−19 32	...	Yerkes	Adams, *Astroph. Jour.* xviii. 69; Slipher. Lowell Obs. *Bull.* No. 1
ε Herculis	16 56	+31 5	...	„	Adams, *Astroph. Jour.* xviii. 69
μ Herculis	17 14	+33 12	4·9	„	Frost and Adams, *Astroph. Jour.* xvii. 381
λ Scorpii	17 27	−37 2	...	Lowell	Slipher. Lowell Obs. *Bull.* No. 4
18 Aquilæ	19 2	+10 50	5·1	Yerkes	Frost and Adams, *Astroph. Jour.* xviii. 387
δ Aquilæ	19 21	+ 2 55	...	Lick	Campbell and Curtis, *Astroph. Jour.* xviii. 308
57 Cygni	20 50	+44 0	4·6	Yerkes	Frost and Adams, *Astroph. Jour.* xvii. 381
2 Lacertæ	22 17	+46 2	4·8	„	Frost and Adams, *Astroph. Jour.* xviii. 387
6 Lacertæ	22 26	42 37	4·6	„	„ „
ι Hev. Cassiopeiæ	23 25	+58 0	...	„	Frost and Adams, *Astroph. Jour.* xviii. 388

M. Tiekhoff (*Ast. Nach.* **164**, 49) gives an account of observations of β *Aurigæ*.

Herr Ludendorff (*Ast. Nach.* **164**, 81) discusses observations of the brightness of ϵ *Aurigæ*, and arrives at the conclusion that it is an *Algol* variable of period 27 years.

Rowland's System of Wave-Lengths.—In the past year or two several important discussions of systems of wave-lengths have appeared. We may call attention to the following :

Messrs. Perot and Fabry.	*Compt. R.* **130**, 653 ; *Astroph. Jour.* xv. 270 and xvi. 36.
M. Hamy	*Compt. R.* **130**, 700.
Prof. Louis Bell... ...	*Astroph. Jour.* xv. 157 and xviii. 191.
Dr. Eberhard	*Astroph. Jour.* xvii. 141.
Dr. J. Hartmann ...	*Astroph. Jour.* xviii. 167, trans. from the October No. of *Zeitschrift f. wissensch. Photographie, Photophysik und Photochemie.*

H. F. N.

Boss's Standard Catalogue.

During the year Professor Lewis Boss has published in the *Astronomical Journal* a series of most important and interesting papers in connection with a standard catalogue of 627 stars distributed over both hemispheres. Such a catalogue is, as he justly remarks, of fundamental importance in the determination of the solar apex, and in all questions relating to the movements of the sidereal system. For the proper motion of a single star the systematic corrections applied to the past observations may be of little consequence, but when the proper motions of a large number of stars are computed these corrections acquire a high importance. For instance, the position of the solar apex is 10° further north if the systematic corrections to the various catalogues employed in the computation of proper motions are such as make them conform to Professor Boss's system rather than that of the *Berliner Jahrbuch*.

The earliest observations used in Professor Boss's catalogue are those made at Königsberg in 1820. The catalogues of Bradley, Mayer, Groombridge, Piazzi are not, in Professor Boss's opinion, valuable in the formation of a standard system, though they may be very useful for the determination of the proper motions of other stars when such a system is formed. In this respect the catalogue differs to some extent from those of Auwers and Newcomb.

Comparisons with the catalogues of Auwers and Newcomb

(*A. J.* 531–532) show a small periodic term in the right ascensions for 1900·0 and the proper motions of the equatorial stars, but much more marked differences when the stars are arranged in zones of declination. Roughly speaking the systems of Auwers and Newcomb agree in right ascension, with the exception of a constant equinox correction. Professor Boss remarks that he has carefully considered these differences, and, though there is some uncertainty with the southern stars, is satisfied that existing testimony is in favour of his own system. For the position and motion of the equinox Newcomb's values have been used. The uncertainty attaching to this can only be corrected by further observations, and "it is earnestly to be hoped that these will soon be offered not from Greenwich only, but also from many other observatories situated in latitudes more favourable for the purpose."

There are small terms of the form $a \sin \alpha + b \cos \alpha$ in the differences of declinations of the systems Boss-Newcomb and Boss-Auwers, but, as with the right ascensions, the greatest differences are seen when the stars are arranged in zones of declination. There are also considerable differences in the proper motions, particularly of the southern stars, between this catalogue and "Boss's declinations of 1875." The most important features of this comparison are the large discordances between the proper motions in Boss's and Auwers's system of the stars north and south of 45° N. Dec.—the discordance which produces a difference of 10° in the determination of the direction of the Sun's motion. Newcomb's system agrees more nearly with Boss's in this particular.

The catalogue is given in *A. J.* 531–532, the methods of forming it in *A. J.* 534 and *A. J.* 540–541. The systematic errors of right ascension of the form $a \sin \alpha + b \cos \alpha$, the discussion and elimination of which is due to Newcomb, do not offer much difficulty; incidentally Professor Boss remarks that in some modern observations sufficient care is not taken to eliminate this form of error, a matter which should be easy with our modern clocks, as Bessel and Struve were able to avoid it with ordinary clocks exposed to considerable variations of temperature. The most obvious causes of error of right ascension depending on the declinations are (i.) the collimation, (ii.) the pivots, (iii.) the polar deviation n. Other causes are illumination of the wires and any want of fixity in the object-glass. To determine the errors arising from these causes in various catalogues *Polaris* and 45 stars extending to −20° Dec. which had been systematically well observed were taken in the first instance, and standard right ascensions and proper motions formed for these stars. The residuals of these stars in the different catalogues, grouped in zones 10° wide, gave a preliminary system of corrections, and the right ascensions of 113 additional stars were found. The zone corrections were redetermined and the catalogue

further expanded. Special treatment was necessary for the stars south of $-20°$ and for the close polars north and south.

The declinations are similarly treated, using Boss's previous catalogues as a provisional system. The corrections depending on right ascension are first disposed of, and those depending on declination caused by flexures, division error, error of assumed latitude, error of adopted constant of refraction. The greatest difficulty experienced in this part of the work was in correcting McClear's Cape Catalogue of 1833. The importance of this catalogue arises from its being the earliest southern catalogue suitable for the formation of a standard system previous to 1830.

No. 536 of the *Astronomical Journal* is devoted to the consideration of the magnitude equation of the Standard Catalogue. Comparisons were made with a large number of catalogues, and the important result obtained that there is no appreciable difference between the average magnitude equations for chronographic and eye and ear observing, at any rate between the first and sixth magnitude. From the determinations made at various observatories the magnitude equation $-0''.0077\ (M-3.5)$ is derived as applicable to the Standard Catalogue. Professor Boss remarks: "The determination of absolute magnitude equation should be regarded as an indispensable requisite in all observations aiming at precision." He recommends the use of screens.

In *A. J.* 545 he compares the declinations of Auwers' Bradley with those of the Standard Catalogue. Proceeding on the assumption that the differences between the quadrant pointing north and south are due to a change in eccentricity, and can be represented by a formula $k+x\sin\zeta$, he finds that $k=0''.42$ and $x=1''.20$. He discards the corrections given by Auwers as applicable to the zenith distances, and finds corrections which make the declinations agree with his own standard, subject to the above relationship between the corrections to be applied to equal zenith distances north and south. He shows that the observation of stars at upper and lower culmination are in fair agreement with his formula. He tabulates the corrections to be applied to Auwers' Bradley, and proposes to apply them to extend his catalogue to include a large number of secondary standard stars.

In *A. J.* 549-550 are given the systematic corrections to reduce a large number of the more important catalogues to this system, and the weights to which they are entitled. The object of first importance as regards these corrections is to obtain positions and motions of stars, in large numbers, which shall be as far as possible free from systematic error. Professor Boss's work is a step of fundamental importance in the progress of our knowledge of the sidereal universe, and it is with the greatest satisfaction we note that Professor Boss is continuing his researches.

F. W. D.

Dr. Auwers' Re-reduction of Bradley's Observations.

Dr. Auwers commenced his great work of the re-reduction of Bradley's Observations in 1866, and completed it in 1876. The work was planned to consist of three volumes, the first containing the basis of the reduction, the second the mean right ascensions and declinations of the single observations for 1755·0, the third the catalogue and the proper motions of the stars. Vol. ii. was published in 1882, vol. iii. in 1888, and vol. i. in August 1903.

The volume contains a complete account of the method of reduction of the star-catalogue, and also of Bradley's Sun and planet observations, and a discussion of the observations of stars made with the zenith-sector at Wanstead and Greenwich.

The first section is devoted to the transits, the method employed for forming the standard right ascensions of the stars to be subsequently used for clock error being of special interest. Eleven stars were taken, α *Lyræ*, γ, α, β *Aquilæ*, and α *Cygni* forming one group, and *Rigel*, *Capella*, α *Orionis*, *Castor*, *Procyon*, *Pollux* forming another. The position of the stars of the first group relatively to α *Aquilæ* and of the second relatively to *Procyon* were first determined. The difference of right ascension of α *Aquilæ* and *Procyon* was then determined, using all the available comparisons between the stars of one group and the means of consecutive culminations of stars of the second group. Means were formed for each month, and a final mean difference obtained freed from errors which arose from diurnal variations of clock rate, &c. The right ascensions of the rest of Maskelyne's 36 fundamental stars were next determined. The very skilful determination of the right ascensions of the clock stars is the main cause of the freedom from periodic error in Auwers' Catalogue, a freedom which Bessel's *Fundamenta* did not possess.

The second section explains the method of reduction of the declinations. The determination of the zenith-point corrections is fully described. These depend on the sector observations made by Bradley and Maskelyne at Greenwich and by Bradley at Wanstead. The determination was made directly for some days, but on other days none of the sector stars were observed at the quadrant. Consequently it was necessary to connect other stars, particularly the 36 Maskelyne stars and 7 stars in *Gemini*, which Bradley frequently observed, with the sector stars on days when both were observed at the quadrant, and use the positions of the Maskelyne stars thus determined for days when no sector stars were observed.

The reduction of the sector observations is given in Section III., and this is—apart from its relation to the quadrant observations—a very important and considerable work. Bradley's observations

at Greenwich from 1749 to 1760 gave the zenith distances of 85 stars with 1427 observations. In the Wanstead series from 1727 to 1729 there are 1830 observations of 70 stars, distributed over a zone $12\frac{1}{2}°$ wide passing through the zenith, and 32 of these stars are additional to the Greenwich series. Three series of Maskelyne's sector observations, 1768–1769, 1776–1777, 1785–1786, giving the zenith distances of 36, 28, and 28 stars respectively, are next considered. The whole of the sector observations are compared and unified, and as a final result a catalogue of the declinations of 130 stars for 1755·0 is formed. Of these stars 114 occur in the quadrant observations; and the positions of these 114 stars are used to give a homogeneous and accurate series of zenith-point corrections.

Section IV. contains the observations of the Sun. Attention may be drawn to the personalities of the observers, which are fully discussed. The observations of the Sun give an equinox correction of $+^s·056$, but the probable error is at least $\pm 0^s·10$, and on this account Dr. Auwers did not use the proper motions he derived from Bradley's observations for a new determination of the precession.

The next sections V. and VI. contain Dr. Auwers' researches on the correction to be applied to the quadrant observations for division error. He assumed that the cause of the discrepancies was a bending of the quadrant—the centre of the cross-wires of the telescope being on the normal to the quadrant would be raised or lowered by an amount depending on the angle the quadrant at that point made with the vertical plane. A number of transits made with the quadrant are utilised to determine its departure from a plane at each point, and the correction required by the zenith-distances is readily deduced. The circumpolar observations are also employed for the quadrant north, and the Sun observations for zenith distances 29°–73° quadrant south. The determination of these division-corrections is a matter of great difficulty, and Dr. Auwers' treatment has received some criticism.

Section VII. contains the re-reduction of Bradley's planetary observations.

The publication of this volume may fittingly recall the words of Dr. Glaisher in presenting the Gold Medal of the Society to Dr. Auwers in 1888. "Dr. Huggins, in transmitting this medal to Dr. Auwers, and conveying to him our congratulations upon the completion of so great a work, may I ask you at the same time to express to him not only our admiration of the manner in which the most refined skill has been combined with the most patient care in its performance, but also our personal acknowledgments for the years of laborious industry in which, with untiring zeal, he has devoted his time and powers to the not unworthy task of applying to the advance of modern astronomy the concluding labours of Bradley's useful life†."

F. W. D.

The Astrographic Catalogue and Chart.

During the past year two* of the observatories taking part in making the Astrographic Catalogue have begun publication—viz. the observatories of the Vatican at Rome and of Helsingfors, and besides these the Potsdam Observatory has published a third volume. Remembering that Paris has already issued its first volume, that Greenwich is on the eve of publication, and that the computation of the Oxford zone is practically done, the present state of the work may be summarised by saying that the Catalogue from the North Pole to declination 18° N. is beginning to be published; that from 18° N. to 16° S. the work of measurement and reduction is well advanced; the next section, which was originally allotted to certain South American observatories, is scarcely begun; and that the plates of the remaining part of the sky—*i.e.* from 40° S. to the South Pole, are being rapidly measured by the observatories at the Cape and Melbourne.

In these examples of publication from the four observatories mentioned, in each case a plate is taken as a unit, and all the stars on the plates are measured, even though they fall beyond the lines where the plate intersects its neighbours in the same zone or of declination two degrees above or below, so that measures of the same star may occur three, four, or even five times in the Catalogue on different plates. A measure is distinguished by the number of a plate and its number on that plate, and a star may therefore have several designations; which does not seem to be a convenient arrangement, unless, as is the case in the Helsingfors and Potsdam volumes, the right ascension and declination (approximate only in the latter case) of the star is attached to each measure. The resolutions of the conferences which have determined the plan of the astrographic work require only that the original measures of the rectilinear coordinates should be printed, with sufficient data to convert these into equatorial coordinates of the epoch 1900·0, but the various observatories have interpreted these resolutions more or less generously. Its terms have been carried out exactly by the Paris and Vatican Observatories. In these two cases the rectilinear coordinates are given measured in millimetres, and it may be noted that the fourth decimal place is given with the central lines of the réseau as axes, and with origin at the centre of the plate. With each plate is given the plate constants, by help of which these measured coordinates can be transformed to others, referred to axes coinciding with and perpendicular to a celestial meridian, and having for origin the point in the sky adopted as the theoretical centre of the plate. The volumes of the Potsdam

* Since this note has been in type four fascicules of the Catalogue have been issued by the Algiers Observatory, to which the zone between declination $-1°$ and $+4°$ is allotted.

Catalogue hitherto published do not give the plate constants. The Helsingfors Catalogue not only gives the constants but corrections for divisions and scale errors also in much detail, applies them to find the "standard" coordinates above described, and then deduces from these the ordinary equatorial coordinates, right ascension and declination.

Professor Donner, the director of Helsingfors Observatory, considers the present publication as merely subsidiary. He proposes to make a final catalogue in which all these star-places will be collected, thus going a long way beyond the present scheme laid down at the conferences, but if all the cooperating institutions could command sufficient resources to follow his lead, the result would be worthy of the great work.

In other respects the four catalogues show differences in detail, and in none more so than in the arrangement of the plates in the books. In the Potsdam volumes the plates are arranged in groups of the same declination, and the plates of each group are arranged in order of R.A. ; but a group covers only a few hours of right ascension, and hiatus is frequent. The stars on each plate are printed in order of right ascension, which, as has been said, is approximately given in the same line. The Paris volume already issued contains the plates whose centres are at declination 24°, and cover the whole of that zone, the stars being arranged on the plates nearly in order of the x coordinate increasing. The book from the Vatican contains fifty-four plates, whose centres are either at declinations 60°, 61° or 62° N., arranged in a somewhat arbitrary manner, the stars being printed in order of increasing x coordinate. The Helsingfors volume is numbered IV., and contains all the plates from R.A. 9^h to 12^h in the zone allotted to that Observatory (centres from Dec. 40° to 46°), and these are numbered and arranged as here illustrated. No. 1 has R.A. $0^h\,0^m$, Dec. 40°. Nos. 2, 3 and 4 are in R.A. $0^h\,0^m$, Dec. +42°, 44°, 46° respectively. No. 5 has R.A. $0^h\,5^m$, Dec. +41°, and so on. These varieties of arrangement make it clear that if the Astrographic Catalogue is to be regarded as a homogeneous work, a final catalogue as proposed by Professor Donner is inevitable.

Of other differences in details of the four catalogues, a notable example is furnished by the references to other star catalogues. Potsdam gives the zone and number in the Bonn *Durchmusterung* of such stars as appear in that work. Rome and Paris the numbers in the *Astronomische Gesellschaft* of stars used for determining the plate constants. In the Helsingfors list the places of the stars in the *Gesellschaft* catalogues, brought up to epoch 1900·0, are added for comparison with the photographic places, but not the catalogue numbers. All the catalogues give a determination of the magnitude of each star.

The above may be sufficient description of the results at present derived from the astrographic plates. The casual inquirer may possibly find some difficulty in making ready use

of the catalogues in their present form. Failing reduction of at least some of the measures to equatorial coordinates, the insertion of reference numbers to stars in other catalogues is an advantage. The accuracy of the star places does not quite reach the standard originally expected, but does not fall far short of it. Father Rodriguez de Prada gives for the probable error of a determination of right ascension, derived from the mean of four plates taken at Rome, $\pm^s\cdot048$, the equivalent of $\pm0''\cdot35$, and in declination also $\pm0''\cdot35$. M. Lœwy gives the probable error of a right ascension or declination deduced from one plate $\pm0''\cdot31$. No evaluation of the probable error is given of the Helsingfors work, but a casual examination of this seems to show that the P.E. in this case will be somewhat smaller than these.

The distribution of the enlargements on paper of chart plates by the French observatories has been continued during the year.

H. P. H.

Geodesy and Universal Time.

In the report to the Board of Visitors for 1903 the Astronomer Royal gave $9^m\ 20^s\cdot974\pm\cdot0113$ as the result for the difference of longitude between the meridians of Paris and Greenwich as determined in the spring of 1902, and $9^m\ 20^s\cdot909\pm\cdot0047$ as the result from the determination in autumn of the same year. The definitive value has not yet been derived from these figures, but the matter waits for comparison with the results found by the French observers.

In the summer of 1903 the difference of longitude between Greenwich and the Prussian Geodetic Institute at Potsdam was determined by Professor Albrecht and Herr Wanach, using Repsold registering micrometers on a transit of the coudé form. The result of this work is not yet published, but it is known that using previously determined values of the arc, Potsdam–Paris, it is not inconsistent with the figures in the previous paragraph.

The longitude of the Great Trigonometrical Survey of India has been known for many years to be in error by $2'\ 30''$. A recent publication* of the survey states that the longitudes of the maps are now referable to the Greenwich meridian, taking the longitude of the Madras Observatory as $80°\ 14'\ 54''$. This value is based on the determination of the longitude of Madras by Major Burrard and Major Lenox-Conyngham in 1895, which gave $80°\ 14'\ 47''$; the difference of $7''$ being accounted for by the following reasons. Kalianpur is the origin of the survey, and this place was found to be $2°\ 35'\ 29''\cdot5$ west of Madras when the arc Madras–Kalianpur was measured by astronomical methods, but the geodetic measures gave $2°\ 35'\ 36''\cdot6$ as the value. The

* Survey of India, *Professional Papers*, No. 7, 1903.

astronomical value is now used for forming the longitude of Kalianpur, but the longitude of Madras on the maps is given as the most recent *geodetic* value.

It is proposed to make determinations of gravity in the Himalayas and other parts of India by half-second pendulums of the Sterneck pattern. The officers above-named made series of observations at Greenwich and Kew in 1903 for the purpose of standardising a set of these instruments.

From the beginning of the year 1903 the time-ball at Shanghai has been dropped each day exactly eight hours earlier than Greenwich mean noon, to bring the time of the port in harmony with the universal time system. East European time, which is the time of the meridian two hours east of Greenwich, has been adopted for all the British Colonies in South Africa. A Bill has been laid before the Parliament of Portugal with the object of making Greenwich time the standard in that country.

H. F. H.

Papers Read before the Society from March 1903 to January 1904.

1903.

Mar. 13. On a new and accurate method of determining time, latitude and azimuth with a theodolite. W. E. Cooke.

On the desirableness of a re-investigation of the problems growing out of the mean motion of the Moon. Simon Newcomb.

On three of Sir William Herschel's observed nebulous regions in *Orion*. Max Wolf.

Proposal for the establishment of a southern belt of latitude stations. S. C. Chandler.

On the period and light curve of 7514 *UY Cygni*. A. Stanley Williams.

On the nebula *h* 2302 (N.G.C. 7822) *Cassiopeiæ*; the region surrounding ♅ II. 457 (N.G.C. 1665) *Eridani*; with ten new nebulæ; and on ♅ III. 558 (N.G.C. 7492) *Aquarii*. Isaac Roberts.

Observations of Comet *b* 1902 (Perrine) from photographs taken at the Royal Observatory, Greenwich. Communicated by the Astronomer Royal.

Note on photographs of Comet *d* 1902 (Giacobini) obtained with the 30-inch reflector at the Royal Observatory, Greenwich. Communicated by the Astronomer Royal.

Note on photographs of Comet *a* 1903 (Giacobini) obtained with the 30-inch reflector at the Royal Observatory, Greenwich. Communicated by the Astronomer Royal.

Velocity in the line of sight. Selected Stars. Cambridge Observatory, I. 1902. H. F. Newall.

Apr. 8. Rotation period of the markings on *Jupiter*. W. F. Denning.

A standard scale for telescopic observation. Percival Lowell.

On the orbit of Σ 2525. W. Bowyer.

Note on a new star in the constellation of *Gemini*. F. A. Bellamy.

Transits observed with the Durham Almucantar between 1901 November 23 and 1902 December 15. R. A. Sampson.

1903.

Apr. 8. On the place of Nova *Geminorum*. Max Wolf.

May 8. A possible cause of the Moon's obscurity on April 11. Rev. S. J. Johnson.

Results of micrometric measures of double stars made with the 28-inch refractor at the Royal Observatory, Greenwich, in the year 1902. Communicated by the Astronomer Royal.

Observation of the partial eclipse of the Moon, 1903 April 11. E. M. Antoniadi.

Expressions correctes de l'heure et des coordonnées des étoiles dans le système de l'axe instantané. F. Folie.

Eclipse of the Moon of 1903 April 11, observed at the Royal Observatory, Greenwich. Communicated by the Astronomer Royal.

Mean daily area of Sun-spots for each degree of solar latitude for each year from 1874 to 1902, as measured on photographs taken at the Royal Observatory, Greenwich. Communicated by the Astronomer Royal.

Areas of faculæ and Sun-spots compared with diurnal ranges of magnetic declination, horizontal force and vertical force, as observed at the Royal Observatory, Greenwich, in the years 1873 to 1902. Communicated by the Astronomer Royal.

Observations of stars occulted by the Moon during the Eclipse of 1903 April 11, made at the Radcliffe Observatory, Oxford. Communicated by the Radcliffe Observer.

June 12. Note on the double star 31 *Leonis*. S. W. Burnham.

The companion to Σ 1594. S. W. Burnham.

On the verification of the Newtonian Law. E. W. Brown.

Note on the use of Peirce's criterion for the rejection of doubtful observations. S. A. Saunder.

On a probable relationship between the solar prominences and corona. W. J. S. Lockyer.

Note on the present condition of the lunar theory. E. Nevill.

On the relation existing between the light changes and the orbital elements of a close binary system, with special reference to the figure and density of the variable star *RR Centauri*. A. W. Roberts.

Recent observations of *Mars* and *Jupiter*. W. F. Denning.

The spectra of Sun-spots in the region B—D. Rev. A. L. Cortie.

Experiments as to the actuality of the "canals" observed on *Mars*. J. E. Evans and E. W. Maunder.

1903.

June 12. The great nebula in *Auriga*. Max Wolf.

Observations of the satellite of *Neptune* from photographs taken at the Royal Observatory, Greenwich, between 1902 November 12 and 1903 April 27. Communicated by the Astronomer Royal.

Examination of Mr. Whittaker's "Undulatory explanation of Gravity," from a physical standpoint. G. Johnstone Stoney.

Mean areas and heliographic latitudes of Sun-spots in the year 1902, deduced from photographs taken at the Royal Observatory, Greenwich, at Dehra Dûn (India), and in Mauritius. Communicated by the Astronomer Royal.

Observations of the new star in *Gemini* made at the Radcliffe Observatory, Oxford. Communicated by the Radcliffe Observer.

Further observations of the new star in *Perseus*, made at the Radcliffe Observatory, Oxford. Communicated by the Radcliffe Observer.

Further observations of the new star in *Auriga*, made at the Radcliffe Observatory, Oxford; with the mean magnitudes for the years 1902–1903. Communicated by the Radcliffe Observer.

On oscillating satellites. H. C. Plummer.

The National Argentine Observatory. J. M. Thome.

Positions of 166 stars around Nova *Geminorum*, with a discussion of the systematic differences between two exposures on the same plate. F. A. Bellamy.

On the possible identity of Nova *Geminorum* with a small star photographed before the outburst. H. H. Turner.

On the position of *X Geminorum*. F. A. Bellamy.

Nov. 13. On the construction of telescopes whose relative or absolute focal length shall be invariable at all temperatures. F. L. O. Wadsworth.

A derivation of Hill's equation by a direct substitution. R. A. Herman.

Ephemeris for physical observations of the Moon, 1904. A. C. D. Crommelin.

Results of double-star measures made with the 8-inch equatorial at Windsor, New South Wales, in 1902. John Tebbutt.

Observations of variable stars made under the direction of the late Sir C. E. Peek, Bart. Edited and discussed by H. H. Turner.

On the systematic proper motions of bright stars relatively to faint stars in the Oxford zones ($+25°$ to $+31°$). H. H. Turner.

Observations of *Mars* in 1903. Rev. T. E. R. Phillips.

1903.

Nov. 13. Measures of southern double stars made at Shanghai, 1902–1903. J. L. Scott.

Observations of Borrelly's Comet (*c* 1903) made at the Natal Observatory, Durban. Communicated by E. Nevill.

Remarks on a paper by Mr. W. E. Cooke on a new method of determining time, latitude and azimuth. E. B. H. Wade.

Preliminary note on the effect of the direction of gravity on lunar observations. E. B. H. Wade.

Observations of white spots on *Saturn* in 1903. A. Stanley Williams.

A spectrographic study of *β Lyræ*, Rev. W. Sidgreaves.

On the use of the stereo-comparator for plates on which a réseau has been impressed. Max Wolf (with introductory note by H. H. Turner).

Preliminary note on a method of photographing the Moon with surrounding stars. H. H. Turner.

Errors in the Moon's tabular longitude as affecting the comparison of the Greenwich meridian observations from 1750 with theory. P. H. Cowell.

On the large Sun-spots of 1903 October 4–18, and October 25–November 6, and the associated magnetic disturbances. Communicated by the Astronomer Royal.

Note on photographs of Comet *c* 1903 (Borrelly) taken with the 30-inch reflector at the Royal Observatory, Greenwich. Communicated by the Astronomer Royal.

Short method for the calculation of the orbits of celestial bodies. D. A. Pio.

Dec. 11. The rotation period of the planet *Saturn*. G. W. Hough.

The shower of *Leonids* in 1903. W. F. Denning.

On graphical methods of determining the local or Greenwich time of sunset at different places within a given region. H. H. Turner.

Observations of the *Leonid* meteors of 1903, made at the Royal Observatory, Greenwich. Communicated by the Astronomer Royal.

On oscillating satellites (second paper). H. C. Plummer.

An examination of the relative star-density in different parts of the plates forming the Harvard photographic sky-map. J. C. W. Herschel.

On the semi-diameter, parallactic inequality, and variation of the Moon from Greenwich meridian observations, 1847·0 to 1901–5. P. H. Cowell.

1903.

Dec. 11. Ephemeris for physical observations of *Saturn*, 1903–1904. A. C. D. Crommelin.

Two drawings of the Mare Serenitatis by John Russell, R.A., affording some hitherto unpublished evidence as to the appearance of Linné in the year 1788. A. A. Rambaut.

1904.

Jan. 8. Cape double-star results, 1903. R. T. A. Innes. Communicated by H.M. Astronomer.

Transformation of Hansen's Tables. P. H. Cowell.

Note on the use of long-focus mirrors for eclipse work. H. H. Turner.

New double stars detected with the $17\frac{1}{4}$-inch reflector during the year 1903. Rev. T. E. Espin.

Ephemeris for physical observations of *Jupiter*, 1904–1905. A. C. D. Crommelin.

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The "great" magnetic storms, 1875 to 1903, and their association with Sun-spots, as recorded at the Royal Observatory, Greenwich. E. W. Maunder. Communicated by the Astronomer Royal.

The aurora and magnetic disturbance. William Ellis.

Suggested connection between Sun-spot activity and the secular change in magnetic declination. Mrs. E. W. Maunder.

On the chromatic correction of object-glasses. A. E. Conrady.

An account of tables to facilitate the working of combined altitudes by Saint-Hilaire's method. Lieut. Simpson-Baikie.

Further note on the "great" magnetic storms, 1875–1903, and their association with Sun-spots. E. W. Maunder.

Note on Mr. Ritchey's photographs of the nebula in *Andromeda*. W. H. Wesley.

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ADDRESS

Delivered by the President, Professor H. H. Turner, on presenting the Gold Medal of the Society to Professor G. E. Hale.

The Council have awarded the Gold Medal to Professor George E. Hale "for his method of photographing the solar surface and other astronomical work," and it is now my pleasant duty to lay before you the grounds on which this award has been made.

"It cannot be too often repeated," remarks Professor Hale himself in a recent memoir, "that the Sun is the only star whose phenomena can be studied in detail." We all assent at once not only to the proposition itself, but to the obvious corollary that particular attention should be paid to solar phenomena; and yet it would appear from the history of astronomy that the statement, however often repeated, has fallen on deaf ears. If we turn, for instance, to the list of awards of our medal, which, unless the Society has been prejudiced, should reflect in some degree the relative interest taken in the various departments of astronomy, we find a score of recognitions for good work in lunar or planetary theory; another score for planetary and cometary discoveries; more than a score for catalogues of stars and nebulæ; nearly a dozen for the determination of important constants (such as that of aberration); and among awards which are more miscellaneous in character the spectroscopic study of the stars has three times been honoured. The study of the Sun, however, appears in the list but twice, and only once explicitly. In 1857 the medal was awarded to Schwabe "for his discovery of the periodicity of the solar spots," and in 1862 to Warren De la Rue "for his Astronomical researches, and especially for his application of photography," which included, though incidentally merely, the invention of the photo-heliograph. The poverty of this record is striking, and seems to show, unless the Society has failed in its duty to an extent which I am not prepared to admit, that far too little attention has been paid to the Sun hitherto. It is a happy augury for the new century that we desire so early in its course to do honour to one of those who are taking the lead in rectifying the omissions of the past.

May we, before passing to his work, dwell for a few moments on the two awards of half a century ago which I have just

mentioned, those of 1857 and of 1862? They afford a background of some historical interest for the events of to-day and yesterday.

Heinrich Schwabe, a gentleman of Dessau, began to observe the solar spots very zealously with a small telescope in 1826, and continued for seventeen years before he made his modest announcement of periodicity in 1843. The announcement was entirely new and unsuspected. In the Presidential Address, on awarding the medal in 1857, Mr. Manuel Johnson showed how little the discovery had been anticipated by quoting from various eminent writers explicit denials that there was law, or anything regular, in the appearance of Sun-spots. And even when the eleven-year period was announced by Schwabe the subject attracted little attention, until Humboldt reannounced it in 1851 in the third volume of his *Cosmos*. We could scarcely have a more characteristic illustration of the strange, almost obstinate, neglect of the Sun and all concerning him which appears instead of the particular attention we might have expected.

Dawning solar physics was represented by Schwabe alone—a modest, patient, untiring observer. "Twelve years," says Mr. Johnson, "he spent to satisfy himself; six more years to satisfy, and still thirteen more to convince, mankind. For thirty years never has the Sun exhibited his disc above the horizon of Dessau without being confronted by Schwabe's imperturbable telescope, and that appears to have happened on an average about 300 days a year."

And then follows a paragraph on which I need make no comment; it is sufficient to remember that it was spoken to this Society in 1857.

"Let us hope that the example will not be lost. Men are apt to speak of astronomy as an exhausted science, meaning that all that can be known is known. No doubt, being the most perfect, it is in one sense the most exhausted science. But the astronomer of Dessau has taught us that there are still mines rich in ore, though they lie deep buried, and must be worked with more assiduity and with more care. I can conceive few more unpromising subjects from which to extract a definite result than were the solar spots when Schwabe first attacked them."

In 1862 Dr. Lee, in an able exposition of the many good works of Mr. Warren De la Rue, devotes about a page to "a department in celestial photography where Mr. De la Rue stands almost alone," that of *Heliography*. Although one or two isolated photographs of the Sun had been obtained previously, no uniformly successful method had been devised until Mr. De la Rue took up the problem. By applying the stereoscope to the pictures taken he showed that the faculæ were elevated above the photosphere, and he obtained traces of what were then called Nasmyth's willow leaves. His instrument, the Kew photo-heliograph, is so familiar to us that I need not dwell on its work,

but the following passage from Dr. Lee's address is of interest to us to-day :

"I cannot refrain from expressing my belief that the success already achieved by our friend warrants us in entertaining the hope that before long he will be able, with the aid of stereoscopic pictures, to exhibit to us the rose-coloured prominences depicted on the sensitive plates as plainly as the faculæ have already been photographed."

Considering the date (1862) this is a somewhat remarkable expression of opinion. It must refer to the photography of the prominences in full sunlight, for Mr. De la Rue had obtained pictures of them at a total eclipse two years before, and this fact is duly recorded in the address. And of course Dr. Lee can have had no notion at the time of the particular manner in which his prophecy would be fulfilled ; he must have been speaking merely from a general but deep-rooted confidence in the steady advance of scientific discoveries and methods. The achievement was not to fall to the lot of Mr. De la Rue, nor was it accomplished in his lifetime ; but within two years of his death not merely the prominences themselves but their *spectra* were photographed for the first time* in full sunlight by our medallist of to-day.

The thirty years intervening between this last event and Dr. Lee's utterance, though it is not marked by the award of any medal by this Society for solar physics, was still far from being a barren period. But it was a time of steady general work rather than of conspicuous individual achievement. The honours of the chief step in advance—the discovery that the prominences could be seen with the spectroscope in full daylight —were shared by at least three people, being predicted by one and realised independently by two others. It does not, however, concern us to-day to follow the history of this period. The work of our medallist is essentially more recent. It began, indeed, so early as 1889 with some experiments conducted at Harvard College Observatory on the photography of prominences with an improvised spectro-heliograph ; but his most remarkable achievements are not yet many months old. It is at any rate only within the last few months that we have had the pleasure of seeing on the screen the beautiful pictures which open up a new and vast field of research on the Sun.

We must, however, by no means neglect the early work which laid the sure foundations for these later triumphs. The experiments of 1889, though not very successful in themselves, contained the germs of the spectro-heliograph, and therefore of all the recent successes. In such a brief historical review, however, as can be attempted within the limits of this address we may begin with the year 1891, when the prominence spectrum was first photographed without an eclipse, as above mentioned.

* *Astronomy and Astrophysics*, vol. xi. (1892), p. 51.

The announcement of success was made to the British Association at the Cardiff meeting; and the paper is reproduced in the first number of the new journal *Astronomy and Astrophysics*, afterwards the *Astrophysical Journal*, which the energy of Professor Hale has done so much to found and maintain at its high level of excellence. Alongside the paper are placed two others by Professor C. A. Young and M. H. Deslandres, both containing similar announcements of success in photographing the spectrum of the chromosphere during the year 1891. Professor Hale's priority was of a few weeks only; and we may note with satisfaction the increased interest in solar physics which this threefold achievement implies. But he had already made a new departure by which he established a clear lead. In 1892 April he announced the successful completion of the spectro-heliograph, which represented, in his own words at the time, "a method by which photographs are now made of all the prominences visible round the entire circumference of the Sun with a single exposure, and by which faculæ are clearly shown even in the brightest portions of the Sun's disc." It is the latter part of this sentence to which I would call special attention. In the photography of the spectra of the prominences Professor Hale was one of several workers who reached results about the same time; but he was the first to realise that the faculæ could be photographed all over the Sun's disc, and thereby he curiously inverted the expectations of Dr. Lee. His special achievement was not only "to exhibit to us the rose-coloured prominences . . . as plainly as the faculæ," but to photograph the faculæ as plainly as the prominences; and in this achievement of 1892 the key note is struck of the theme which he has recently developed so magnificently.

The early history of a new idea is of such great interest that I will reproduce here verbatim Professor Hale's own account * of the manner in which he was led to his discovery.

"In his *Catalogue of the Bright Lines in the Spectrum of the Chromosphere*, published in 1872, Professor Young remarks as follows in regard to the H and K lines: 'They were also found to be regularly reversed upon the body of the Sun itself, *in the penumbra and immediate neighbourhood of every important spot.*' The observations referred to were made under the exceptional atmospheric advantages enjoyed at the summit of Mount Sherman, but even with the less favourable conditions common at the sea-level the same observer has repeatedly made out similar reversals in many spots. I do not know that these observations were confirmed elsewhere until the photographs made at this (Kenwood) Observatory in 1891 April brought out the same thing with great clearness. A few months later Professor Young secured at Princeton some photographs of the reversals, but my own attention has until recently been so fully

* *Astronomy and Astrophysics*, vol. xi. (1892), p. 413.

occupied with work on the prominences that I have had but little opportunity to go on with my proposed photographic study of spot spectra. Late in December, however, I secured some photographs of the spectrum of a spot in which the lines were so sharply defined that they were given a very careful examination. Not only were the bright lines at H and K more prominent in the penumbra than in the umbra of the spot, but their extent in the surrounding region was so great as to arouse the suspicion that similar reversals might be found on the disc, at points remote from the spot regions. To test this idea a series of six photographs of the spectrum was taken, the slit in each case being placed parallel to the position it had occupied in the exposure just preceding, and about 3′ distant from it. My expectations were not only realised by these photographs but greatly surpassed. In each of the six positions of a slit not more than 0·002 inch wide the K line was reversed in from 3 to 10 places. H was, without doubt, similarly affected in all of these points, but in some cases it was too faint to be certainly seen. Most, if not all, of these reversals were double, *i.e.* a dark line ran through the centre of the bright line, as is frequently observed in the spectrum of the electric arc. I have since suspected in several cases a strengthening of the broad dark absorption bands of the solar spectrum for a short distance on both sides of the bright reversals.

"Having thus found the surface of the Sun to be dotted over with regions in which the H and K lines are bright, I at once concluded that the *forms* of the reversed regions might be photographed with the spectro-heliograph in exactly the same way that prominences around the circumference are obtained. The first attempt to do this was made on January 12, when the adjustments of the instrument were incomplete, and connection had not been established between the accumulator and the clepsydra. In lack of more suitable motive power the slits were moved by hand, and even in this way bright forms near a spot group were shown on the photograph, though they could not be seen with the helioscope. The completion of the apparatus a few days later enabled me to secure very good photographs of the bright regions, and on comparing them with drawings and photographs taken in the ordinary way of faculæ near the limb it was found that the forms were identical."

It is almost needless at this date to describe the now well-known principle of the spectro-heliograph, but Professor Hale has himself given such a concise and simple statement of it recently that I cannot forbear quoting his words. "Imagine a direct-vision spectroscope," he says, "in which the eyepiece ordinarily employed is replaced by a (second) slit. If an image of the Sun is formed on the first slit of this spectroscope the second slit will permit the passage of only a narrow region of the spectrum corresponding in width to this slit. If the slit is now moved until it coincides with the Hβ line, for example, only hydrogen

light will pass through the instrument. If, then, a photographic plate is placed behind and almost in contact with the second slit, and the spectroscope is moved at right angles to the optical axis, an image of the Sun, in monochromatic hydrogen light, will be built up on the plate from the successive images of the slit. If the exposure is suitable the chromosphere and prominences will be shown surrounding this image."

It should be added that, though this principle suggested itself independently to our medallist in 1889, he subsequently learnt, and at once acknowledged, with that cordial appreciation of the work of others which he manifests unfailingly in his scientific writings, that it was not new, but had been stated essentially by Janssen as early as 1869, while instruments had been designed, and even constructed, by Braun and Lohse, in 1872 and 1880 respectively. But their experiments were unsuccessful, as indeed were the early experiments of Professor Hale himself. He was, however, the first to make a *successful* instrument and obtain results with it in the Kenwood Observatory.

It was completed early in 1892 and set to vigorous, regular, and successful work. Photographs of both prominences and faculæ were taken daily, and the importance of this new method of watching the Sun's activity was soon demonstrated, for in July the connection of a violent magnetic storm with faculous changes was clearly illustrated from the Kenwood photographs.* But before the end of the year which had seen its birth the doom of the Kenwood instrument was sealed. In the last number of *Astronomy and Astrophysics* for 1892 occurs a note about the 40-inch telescope of the Yerkes Observatory, stating that the large discs of optical glass made by Mantois for the University of Southern California had been purchased for the University of Chicago, and that Mr. Alvan Clark had contracted to grind the objective within eighteen months; that the site of the observatory was still undecided, but would probably be several miles outside the city. We know enough to recognise a rare piece of self-denial on the part of Professor Hale. We know that this great project, now carried to so successful an issue, originated in a visit paid by him and President Harper to Mr. Yerkes, a Chicago millionaire; and we know that Professor Hale must have foreseen the possibility, soon to become stern reality, that on him would fall the main work of founding a great observatory, which would necessitate the neglect or temporary abandonment of the infant spectro-heliograph which had just been born to him with such promise; and we know that he never flinched for a moment in the choice between his own personal work, sure and important

* Professor Hale's recent work, showing the astonishingly different results which may be obtained by setting the slit on a slightly different position of the broad K line, have given a new importance to questions of adjustment. Seeing that the spectro-heliograph was dismounted during July, it is just possible that some of the conclusions from these early records may need modification.

though it was, and the laborious and hazardous enterprise which might, if successfully piloted, make it possible for others to do great work. We know too with what rare skill and devotion he did pilot this enterprise, so that the Yerkes Observatory, the very site of which had not been determined a dozen years ago, now ranks not merely in magnificence of equipment, but in solid and brilliant work done, among the foremost in the world. We are content that our medallists should do one thing well, though it often happens by good fortune that we can find more than one excellence to admire in them; but I venture to think that we have seldom had a better example of versatility than on the present occasion, when we find the same man initiating and conducting a new and important journal, initiating and establishing one of the foremost observatories, inventing a new instrument which has an obviously great future before it, and recently finding an entirely new field of research for that instrument.

To attempt to follow the activities of such a man during a dozen years in the space allotted to this address would be merely bewildering; to touch on a few of the salient points is all that is possible. I will ask you to pass rapidly over the events of about ten years during which the Yerkes Observatory was being built and equipped. During the first three or four the spectro-heliograph was in regular use on every clear day at the Kenwood Observatory, and a series of valuable records was thus obtained, which is still in existence and of which we hope to hear more. But in 1895 May this series was interrupted by the dismounting of the instrument for removal to the new Yerkes Observatory, where it was hoped soon to remount it. Unfortunately there came a series of unexpected delays. The 12-inch telescope was wanted for other purposes, and to use a spectro-heliograph on the 40-inch telescope meant making a new instrument, for which funds were at first not forthcoming at all, and then came but slowly. There were difficulties in finding suitable lenses; and, worst of all, an accidental fire destroyed a nearly completed instrument. Looking back on the enterprises in hand during these years, our surprise must, however, be not that such delays occurred, but that they were not more effective. Patiently and resolutely each difficulty was met and conquered, and within four years (in 1899) the new and improved spectro-heliograph was ready for trial; within four more it was successfully at work exploring an entirely new domain of solar physics. A full description of this beautiful instrument has just been published in the third volume of the "Yerkes Observatory Publications." "The design finally adopted was reached only after long and careful consideration of the special conditions of the problem," after numerous forms had been designed, and some of them actually tried. Professor Hale states clearly his opinion that "the ideal form of spectro-heliograph is that in which the instrument is moved as a whole, while the image of the Sun and the photographic plate are stationary." But this presupposes a fixed telescope with some form of heliostat. To

attempt it with a large equatorial would be to throw the telescope into vibration and render good images hopeless. The only feasible solution of the problem seemed to be to move the Sun's image across the first slit by a motion of the telescope, the photographic plate being moved sympathetically across the second slit: and accordingly the telescope is moved in declination by the slow-motion motor. As regards dimensions, the slits are eight inches long and properly curved: the collimator and camera lenses are 6¼-inch Voigtländer portrait lenses; and two prisms in conjunction with a mirror give a total deviation of 180°. These figures, however, though they suggest a fine instrument, give no idea of the care and trouble necessary to work it. The principle of the spectro-heliograph is exceedingly simple, but the working of the instrument is quite sufficiently complex. In the description of it Professor Hale gives, under the heading "Adjustments,"* a list of *thirteen* distinct operations to be carried out with great nicety before a satisfactory photograph can be taken. We can imagine a veteran transit-circle observer thanking Providence that he was not born in these days. An important feature in the design is the method of setting the second slit on any desired line in the spectrum; we may almost say on any desired *part* of any line, for it is by the partition of the broad K line into significant regions that Professor Hale has within the past year opened up a new field of research with this instrument, of which I proceed to speak. But I will first follow his example in uttering a word of caution against too literal an interpretation of what is only intended as a working hypothesis. "The hypothesis," he says,† "is used mainly as a guide to further research, for, while it seems to describe in a fairly satisfactory manner many of the phenomena photographed, it is, of course, open to modification or rejection in the light of future results."

With this caution in mind we remark in the first place that what are called the H and K lines in the solar spectrum are not narrow and simple lines, but wide and complex bands. Since a line may be widened by pressure of the vapour producing it, sources of light at different pressures will give bands of different widths; and if the results be superposed, then at the middle we shall get light from all the sources, though at the edges we shall only get that from the source where the pressure is greatest. If we put the slit of a spectro-heliograph on one of the edges we get light from this source of great pressure *only*. Now in looking at any point of the Sun's disc we get just this state of things: we receive light from the successive sections of his surroundings traversed by the rays; and the pressure, and therefore the width of the band, will be different for each section. By putting the slit of the spectro-heliograph on an edge of the wide H or K band, however, we exclude the light from all but the section at greatest pressure, and practically view this section only. We can cut it out of the

* *Yerkes Obs. Pub.* vol. iii. p. 12. † *Ibid.* p. 13.

Sun's atmosphere as cleanly as a biologist cuts out a section of a specimen for microscopic examination. But we cannot proceed, as he does, to cut other sections as cleanly ; for by moving the slit within the edge of the H or K line we get light not from the section of next greatest pressure only, but from it and the last combined ; and as we go further inwards the complexity increases. We must also remember, though the fact has been omitted in what precedes for the sake of simplicity, that the continuous spectrum is also superposed in all cases. But it is clear that by setting the slit to various positions on the broad band different results will be obtained, from which it is not hopeless, though it may be very difficult, to compile a complete account of the form of the calcium vapour at different levels in the Sun's surroundings.

One thing is to be specially noted. Professor Hale may be photographing faculæ, but he is certainly not photographing faculæ *alone*. A more general term is required for the whole class of phenomena to which faculæ are closely related ; and Professor Hale has suggested the name *flocculi*. The phenomena vary not only with the setting of the slit in different parts of the H or K band, but with a change to the line of a different element. On changing, for instance, to a line of hydrogen the remarkable fact is disclosed that, whereas the calcium flocculi are bright, those of hydrogen are *dark* ; and, following up this result, Professor Hale has since found that even calcium flocculi are sometimes dark. Indeed, new results are multiplying so fast that it is perhaps prudent not to attempt to summarise them here. A new degree of freedom has been imparted to solar research by the discovery of this method. The new development is still in its infancy ; but we have already had some most beautiful illustrations of its practical working exhibited to us, and from these alone we may confidently predict a great and important future for the study of the "flocculi."

I am tempted by the new name to make one more reference to the events of forty years ago. It fell to my lot on succeeding to the Savilian Chair at Oxford to collect for publication the astronomical correspondence of my predecessor, the Rev. Charles Pritchard. Among the letters were a number dealing with Nasmyth's discovery of "willow leaves" in the surface of the Sun, which created great excitement at the time, and led to a voluminous literature, now practically obsolete. A few of the letters were selected for publication,* and from the first of them it appears that the earliest confirmation of Nasmyth's views originated in an observation made by De la Rue in Pritchard's observatory at Freshwater ; and Pritchard attributed the detection of the "willow leaves" to the use of the new solar eyepiece, suggested by Sir John Herschel, to whom he was writing. An

* *Memoirs of Professor Pritchard* (London : Seeley & Co., 1897), p. 240.

eyepiece with one reflection from a piece of plate-glass and a neutral-tinted shade was the most elaborate addition made to the telescope for solar work in those days. The second letter, to the same person, opens with the following words, which recurred to my mind on hearing Professor Hale's new name for solar phenomena:

"We had a strong debate on 'willow leaves' last Friday evening. Mr. Dawes will have it that he sees nothing but a 'flocculent precipitate.' Our 'willow leaves' are a sort of 'crystalline precipitate.'"

The date of this letter is 1863 December 15. May we hope that the shade of that highly strung, keen-eyed observer, W. R. Dawes, will rest the more peacefully now that, just forty years after he did battle for the flocculent precipitate,* the term flocculi is re-introduced into solar physics by so competent an authority as our medallist of to-day!

I have already disclaimed any intention of giving a complete account of Professor Hale's work; but there are at least two important researches, independent of his work with the spectro-heliograph, which must be mentioned.

The first is his determined search for some method of observing the corona without an eclipse. Hitherto it has been unsuccessful, but that is owing to the difficulty of the problem, and not to any lack of skill or assiduity in the attack. A special expedition was made to Mount Etna in 1894 to try one of the experiments, and Professor Hale, after erecting the instrument and making the early observations himself, left the work for further trial in the able hands of Professor Riccò, who, however, had no better success. Several distinct methods have been tried at different times, utilising both the ultra-violet end of the spectrum and the heat radiation, the suggestion for exploring the corona with the bolometer being particularly happy, as it depends on a differential observation which ought theoretically to eliminate the sky glare; but even this ingenious attack has been hitherto fruitless, for the coronal heat radiation, as has been shown from experiments made by Langley at a total eclipse, is extremely small compared with its light radiation.

Secondly, I must make a brief reference to the work done at the Yerkes Observatory from 1898 onwards in photographing the spectra of the fourth-type stars and measuring the photographs. The results of this laborious and thorough investigation have just appeared as one of the Decennial Publications of the University of Chicago. It contains 135 quarto pages of closely printed text and tables, and eleven beautiful plates. Forty-three photographs of the spectra of eight stars were selected for measurement, preference being given to well-established results

* See also *Mem. Not.* xxix. p. 119, line 36, where the word "flocculi" is actually used.

in a few cases. The stars being all faint, the research was a specially appropriate use of the great telescope; but at the same time the difficulties introduced precluded any very great accuracy being obtained, as is frankly announced in the volume. Nevertheless the results for motion in the line of sight show a precision which is by no means to be despised. An average of 15 lines for each star were compared with known positions for terrestrial elements, and velocities ranging from $+4km$ to $-28km$ per second were deduced. A corrected table of 537 lines in these spectra is then formed. Such figures may give some idea of the amount of work represented and the thoroughness with which it has been done. There follows a most able and interesting discussion of the physical character of these stars, summarised in fifteen conclusions, the last of which is that "fourth-type stars probably develop from stars like the Sun through loss of heat by radiation"; and though the other fourteen which lead up to it are full of interest I can here select one only for mention. Professor Hale finds a noticeable agreement of the fourth-type spectra with those of Sun-spots. In the region which is not masked by the carbon flutings the forty-six lines which are most strongly and frequently widened in the spots, as recorded in the *Greenwich Results* for 1880, are found to be the most prominent dark lines in the star. Hence it is suggested that spots similar to those on the Sun may possibly be numerous in fourth-type stars.

This result is in itself a justification of the twofold attack on the problem of the Sun, and carries us back to the proposition which I quoted at the outset—"It cannot be too often repeated that the Sun is the only star whose phenomena can be studied in detail." The reasons for studying Sun and stars side by side are obvious: and there is still a third line of research, necessary for properly conducting both the others, to be carried on in the laboratory. That Professor Hale is fully alive to this third necessity we have evidence in his exhaustive investigations of the fluting of magnesium in 1890: and in his work on the spark spectra of metals in liquids and under pressure, recently completed but awaiting funds for publication. It is perhaps noteworthy, and to many will come as a surprise, that one of the chief difficulties encountered by our medallist throughout has been lack of funds; for it is liable to be forgotten that other things are wanted besides a large telescope itself, and the funds for publication are the most likely to be forgotten of all. Several times the deficiencies have been supplied by the father of our medallist, the late William E. Hale; and later from a fund established in memory of him by his children, including of course our medallist himself.

I have indicated the great variety of Professor Hale's activities. It remains to be added that they are connected by a notable unity of purpose. Throughout this address I have quoted freely from his writings as the simplest and most efficient way of explaining the value of his work; and I will quote once

more, in order that this purpose may be declared in his own words. He is, of course, not speaking consciously about himself, but of the "aims of the Yerkes Observatory." In his address * at the Dedication, in 1897, he said—

"If I mistake not the signs of the times the Yerkes Observatory can render no better service to both astronomy and physics than to contribute, in such degree as its resources may allow, towards strengthening the goodwill and common interest which are ever tending to draw astronomers and physicists into closer touch. During its three years of publication the *Astrophysical Journal* has had the same end in view. The annual meetings of its editors, of late devoted mainly to the informal discussion of astrophysical investigations, have invariably been of great interest and value."

These are no empty sentiments; they are the words of a man who organised under the name of a meeting of editors the first gatherings of astrophysicists in a land where, owing to the wide separation of workers, scientific gatherings are far more difficult than for us; of a man who took special care that at the Yerkes Observatory physical laboratories should exist side by side with telescopes and spectroscopes; and especially of a man who could cheerfully lay aside his own fascinating work for the general good in founding a great observatory. We recognise in our medallist one who has more thought for others than for himself; and I am sure it would be his own wish that before concluding this address I should say one word in recognition of the efficient help which he has received from others, as it is only to be expected he would. To select any one name among many may be invidious, but that of Mr. Ferdinand Ellerman, who has been associated in all Professor Hale's work, experimental and otherwise, for the past thirteen years, and whose able assistance is often acknowledged by him with gratitude, is certainly worthy of mention to-day.

It is, of course, a disappointment to us that Professor Hale cannot be here in person to receive the medal, but we have knowledge which mitigates the disappointment. If our medallist had pleaded that the journey was too serious we, who so seldom venture to cross the Atlantic from this side, could scarcely have objected to the plea, though we might have urged that a visit to Europe had on at least one occasion proved apparently beneficial to him; for we remember with gratification that he returned from the meeting of the British Association in 1891 to originate the spectro-heliograph and the new journal of *Astronomy and Astrophysics*. But Professor Hale enters no plea of the kind. That he is not on our front bench, where we should have been so delighted to see him, to-day is due simply to the fact that he is filling a place of even greater honour: he is initiating a new work of science.

* *Astrophysical Journal*, vol. vi. (1897), p. 310

He was led to pay a visit to California, to study the conditions for solar work, and found them so admirable that he felt compelled to commence operations at once on such a modest scale as is possible with his present very limited resources. We shall earnestly hope that these will soon be supplemented in such a manner as to put the new enterprise on a proper basis. Those who have the administration of funds for scientific research doubtless find the choice between various admirable proposals no easy matter. Indeed, we hear of more than one new project in which Professor Hale himself is interested. We hear not only of a possible new solar observatory, but of a somewhat novel type of observatory in the southern hemisphere; and no one who knows anything of the recent history of astronomy can fail to recognise the pressing importance of both these needs. The southern hemisphere has been unduly neglected from the first, and the leeway to be made up in studying it becomes greater every year. If half a dozen new observatories instead of one were established in the southern hemisphere to-morrow it would take them many years to atone for past neglect.

But there are equally strong reasons, having their origin in undue neglect in the past, for the establishment of a new solar observatory, as I endeavoured to show in the early part of this address. If Schwabe "taught us that there are still mines rich in ore" by merely counting the Sun-spots, what vast new gold-fields has not Professor Hale prospected by showing that every spot, which to Schwabe was a mere unit, is a region which must be explored from hour to hour, from section to section, and from one chemical element to another! He has pointed out the defective state of our knowledge as clearly as Flamsteed did two centuries ago; and, when the obvious need of a fine steady climate for the work is added, he has made out just as good a case for the foundation of a solar observatory as did Flamsteed for the foundation of the Royal Observatory at Greenwich. And if by good fortune it should be founded and the query should again be put, "Who is to take charge of it?" let us hope that the answer will be, in the words of King Charles II., "Why, he who told us the need of it!"

But it is unprofitable to venture on the quicksands of anticipation when we can tread on the solid ground of accomplished fact. I have surely said enough, however imperfectly, to justify to you fully the award of the Council, and it remains only to hand to the American Ambassador for transmission to Professor Hale the outward and visible symbol of our esteem.

The President then, addressing the American Ambassador, said:

"Mr. Choate, it is but three years since your Excellency did us the honour to attend our Annual Meeting to receive the

medal on behalf of Professor Pickering. It is, in every way, a great pleasure to us to see you here again so soon.

"It will, I am sure, not lessen the value of this medal which we ask you to send to Professor Hale as a token of the high value we set upon his work if I add that it is also, in some sort, a symbol of our cordial admiration of the great advances made in our science by the astronomers of the United States and those citizens who have so generously aided them."

The meeting then proceeded to the election of Officers and Council for the ensuing year, when the following Fellows were elected :

President.

H. H. Turner, Esq., D.Sc., F.R.S., Savilian Professor of Astronomy, Oxford.

Vice-Presidents.

W. H. M. Christie, Esq., C.B., M.A., D.Sc., F.R.S., Astronomer Royal.
J. W. L. Glaisher, Esq., M.A., Sc.D., F.R.S.
Major P. A. MacMahon, D.Sc., F.R.S.
E. J. Spitta, Esq.

Treasurer.

W. H. Maw, Esq.

Secretaries.

F. W. Dyson, Esq., M.A., F.R.S.
E. T. Whittaker, Esq., M.A.

Foreign Secretary.

Sir William Huggins, K.C.B., O.M., LL.D., D.C.L., F.R.S.

Council.

Sir William Abney, K.C.B., R.E., D.C.L., F.R.S.
Sir R. S. Ball, M.A., LL.D., F.R.S., Lowndean Professor of Astronomy and Geometry, Cambridge.
Rev. A. L. Cortie, S.J.
Alfred Fowler, Esq., Assistant Professor of Physics, Royal College of Science.
Major E. H. Hills, C.M.G., R.E.
A. R. Hinks, Esq., M.A.
E. B. Knobel, Esq.
Thomas Lewis, Esq.
Frank McClean, Esq., M.A., LL.D., F.R.S.
H. F. Newall, Esq., M.A., F.R.S.
Captain William Noble.
A. A. Rambaut, Esq., D.Sc., F.R.S., Radcliffe Observer.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

Percy Morris, Holmwood, Camborne Road, Sutton, Surrey, was balloted for and duly elected a Fellow of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Zia Uddin Ahmad, B.A., Isaac Newton Student in the University of Cambridge, Trinity College, Cambridge (proposed by E. T. Whittaker) ;
Daniel Buckney, 61 Strand, W.C. (proposed by W. H. M. Christie) ; and
Robert James Wallace, Yerkes Observatory, Williams Bay, Wis., U.S.A. (proposed by S. W. Burnham).

Eighty-five presents were announced as having been received since the last meeting, including, amongst others :—

Bonn Observatory, Veröffentlichungen : C. Mönnichmeyer, Beobachtungen der internationalen Polhöhensterne (presented by the Observatory) ; Cape Observatory Annals : Revision of the Cape Photographic Durchmusterung (presented by the Observatory) ; J. H. Schroeter, Beiträge zu den neuesten astronomischen Beobachtungen, 1788 (presented by Dr. Dreyer) ; Yerkes Observatory Publications, vol. 3 ; The Rumford Spectroheliograph

(presented by the Observatory); Portrait of Bishop Brinkley (presented by Sir R. Ball); Portrait of Manuel Johnson (presented by Dr. Rambaut);

And, in addition to the above, a series of photographs presented by the Director of the Yerkes Observatory which had been shown at the annual meeting—viz. 11 transparencies from Professor Hale's spectroheliographs of the solar surface, and 39 paper prints from photographs of stellar spectra, the Sun, instruments, &c.

Note on the Instrumental Errors affecting Observations of the Moon. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In the November number of the *Notices* I described a method of photographing the Moon among the stars as a way of obtaining its place which seemed to be freer than the methods now in use from certain systematic errors, and the following sentence occurs in the first paragraph :—

> It was to obtain observations of the Moon in the first and last quarters that Airy set up the altazimuth at Greenwich in 1847; and unfortunately the instrument did not satisfactorily solve the problem.

At the time of writing these words it occurred to me that the evidence for or against the value of the altazimuth observations had never, so far as I know, been definitely published. Disparaging criticisms of a general nature have been made more than once, and were at any rate so familiar to me that the above sentence suggested itself naturally. But at the same time, for my own satisfaction, it seemed desirable to make some investigation of the behaviour of the instrument; and as an involuntary pause in our work for the Astrographic Catalogue set a computer free, an inquiry was started.

2. Soon afterwards I learnt that Mr. Cowell was at work on the Greenwich lunar observations, and it seemed probable that we were merely duplicating his work. But in reply to a definite question I learnt that he was not discussing the altazimuth observations at all—perhaps another illustration of the general disrepute into which they have fallen. Accordingly the inquiry was not interrupted; and it now seems fortunate that it was started, for, unless I am mistaken, the results indicate that the instrumental errors of the transit circle observations may be much larger than Mr. Cowell appears to have allowed for, and are liable to modify some of the deductions which he has extracted with such skill from the *Greenwich Observations*. In view of the fact that he is contemplating further work of the same kind, it seems important to draw attention at once to the possibilities.

3. In the Appendix to vol. l. of the *Monthly Notices* the Greenwich observations, with both transit circle and altazimuth, from 1847 to 1861 are compared with Hansen's Tables. In discussing these observations from the point of view of theory, it is important to make certain corrections to the tables, as Mr. Cowell has done (see *Monthly Notices*, lxiv. p. 85); but my immediate concern was with the difference between transit circle and altazimuth observations—both made practically at the same time and compared with the same tables, so that any error of the tables does not matter, and was neglected. The only *instrumental* inequality of the Moon is that which runs its period with the lunation, though the season of the year must also be taken into account. Thus the observations were grouped according to the day of the lunation, counting full moon as 15; and all the lunations with full moon in January were taken together; all those with full moon in February, and so on. Blank days were filled in by simple interpolation between the dates which had observations. Sometimes this process had to be carried to an extent which might be condemned; but it does not seem likely that the general results shown below can be essentially incorrect. Then the means were taken for all the January lunations for each day, then for all the February lunations, and so on. Finally, taking the mean value for the five days 13, 14, 15, 16, 17 near full moon in January, this was subtracted from all the January days; the mean for five days near full moon in February from all the February days, and so on.

4. Assuming the tables correct, we should then have the *instrumental* error for each day compared with that at full moon, for both transit circle and altazimuth, the observations with the two instruments being of course kept separate. If the tables are not correct, the tabular errors will be common to both series; and the *differences* should give merely the differences of instrumental errors.

5. Now what should these differences be? Before full moon the first limb is observed with both instruments; after full moon the second limb.* The exceptions are confined entirely to the three days numbered 14, 15, 16, and are so few that they may be probably neglected for the purposes of the present inquiry. We should be prepared for a different apparent diameter for the two instruments, which would mean a constant difference between them, changing sign at day 15. This corresponds to the assumption made by Mr. Cowell for the transit circle observations. He says (*Monthly Notices*, lxiv. p. 96):—

> The hypothesis that the error of semi-diameter is a constant

* It might at first sight appear that a complication is introduced into the altazimuth observations, because the error in longitude is deduced from errors in azimuth and Z.D.; and in Z.D. sometimes the upper and sometimes the lower limb is observed. But on the hypothesis of a simple error in semi-diameter, the effect on the longitude will in all cases be reversed after full moon.

is the best that I can make. . . . The mean correction for different ages of the Moon is no doubt a function of the age of the Moon and not a constant. If there were any means of obtaining its numerical values they could be analysed into the form

$$f(D)+\alpha \sin D+\beta \sin 2D,$$

when α, β would be corrections applicable to the results of this paper. . . . I do not think that α can possibly be as large as $0''\cdot30$, and it is possibly a good deal smaller.

6. From the point of view of the paragraph just quoted the comparison of the transit circle and altazimuth observations throws light on the possible values of α and β, though it does not determine them. If α and β are small for the transit circle they should be smaller for the altazimuth, since the observations made with the latter are made far more often in a dark sky. If the differences mentioned in § 4 are *not* constant we cannot feel so sure that α and β are small.

7. These differences are shown in the following table, wherein the separate results for both altazimuth and transit circle are also given, though these are made up of instrumental errors combined with errors of theory in a manner which need not concern us at present. In the first instance the means of all the months have been taken.

TABLE I.

Mean Error in Longitude of the Moon's Centre as compared with that at full Moon for all seasons.

Day of Lunation.	Altaz. 1847–1850.	Altaz. 1847–1861.	T.C. 1847–1861.	T.C.–A. Uncorr.	T.C.–A. Corr. I.	T.C.–A. Corr. II.
	"	"	"			
2	...	+2·1	...	...	...	...
3	...	+1·2	...	...	...	...
4	...	−0·1	−0·2	−0·1	+0·9	+0·7
5	−0·1	−0·3	−1·2	−0·9	+0·1	−0·1
6	−0·5	−0·5	−2·2	−1·7	−0·7	−0·9
7	+0·1	−0·9	−2·3	−1·4	−0·4	−0·6
8	−1·2	−1·2	−2·3	−1·1	−0·1	−0·3
9	−1·7	−1·2	−2·0	−0·8	+0·2	0·0
10	−1·4	−1·1	−1·9	−0·8	+0·2	0·0
11	−1·4	−1·2	−1·8	−0·6	+0·4	+0·2
12	−0·5	−1·0	−1·7	−0·7	+0·3	+0·1
13	−0·3	−0·5	−1·4	−0·9	+0·1	−0·1
14	0·0	−0·1	−0·9	−0·8	+0·2	0·0
15	+0·5	+0·4	0·0	−0·4	...	...
16	−0·8	−0·2	+0·9	+1·1	−0·2	0·0

Day of Lunation.	Alt.-az. 1847-1859.	Alt.-az. 1847-1861.	T.C. 1847-1861.	T.C.−A. Uncorr.	T.C.−A. Corr. I.	T.C.−A. Corr. II.
	"	"	"	"	"	"
17	+0·5	+0·4	+1·4	+1·0	−0·3	−0·1
18	+0·3	+0·5	+1·8	+1·3	0·0	+0·2
19	+0·5	+0·5	+1·9	+1·4	+0·1	+0·3
20	+0·9	+0·7	+1·5	+0·8	−0·5	−0·3
21	+0·6	+0·9	+2·3	+1·4	+0·1	+0·3
22	+1·4	+1·1	+2·5	+1·4	+0·1	+0·3
23	+0·8	+1·0	+2·4	+1·4	+0·1	+0·3
24	+0·7	+0·8	+2·2	+1·4	+0·1	+0·3
25	−0·5	−0·2	+1·7	+1·9	+0·6	+0·8
26	...	−0·5	...	...	...	...
27	...	−0·2	...	...	...	...

8. In the first column is given the day of the lunation counting full moon in all cases as 15; to the second column reference will be made later in the paper; in the next two columns the mean errors with the different instruments. In the fifth column the simple difference is shown. Now, on the hypothesis that a different value of the semi-diameter is applicable to the two instruments, *and that only*, we may take the mean of the differences before full moon and the mean of those after to determine this difference. On the other hand, if we allow for a progressive change as well, we may establish continuity by taking the means of days close to full moon only—say three days before and three days after. The means on these two suppositions are as follows:

	Hypothesis I.		Hypothesis II.	
	10 days before.	10 days after.	3 days before.	3 days after.
TC−A	−1·0	+1·3	−0·8	+1·1

9. We notice that on either hypothesis there is a sensible divergence (0″·3) from symmetry. No probable explanation of this difference occurs to me at present, but it is worthy of note that we here have a curious and unexpected instrumental difference half as large as the value allowed as possible by Mr. Cowell for a, as quoted in § 5.

10. Applying these corrections for semi-diameter we get the columns shown as corr.$_{\text{I}}$ and corr.$_{\text{II}}$ Whichever column we take it seems improbable that the value of a can be so small as 0″·3; and we are here dealing with the *difference* between two instruments. The effect for either may be much larger. On the other hand it is possible that the whole error is in the alt-azimuth observations, and the transit circle observations are comparatively correct. But this should not be assumed without good reason being shown; for, as already remarked, the alt-

azimuth observations, rough though they may be, are made under more constant conditions than the meridian observations.

11. To check the above results it was determined to collect the information in an independent manner as follows. Only those days were included when there was both a T.C. and an altazimuth observation, and the direct difference of the errors in longitude was formed and tabulated under the day of the lunation, as before. Any error of the tables was thus eliminated by a more direct process, and the necessity of filling in blank days was avoided. Further, the observations made with the present transit circle, set up in 1851, were kept separate from those of the old one * used from 1847 to 1850. By an oversight this was not done in the previous work.

12. Let us first consider the comparisons on the hypothesis that there is simply a diameter peculiar to each instrument. Taking the means of all the comparisons before and after full, and taking groups corresponding to different seasons, we have

TABLE II. (Hypothesis I.)

	Old T.C.—Altaz. (1847-1850).		New T.C.—Altaz. (1851-1861).	
	Before Full.	After Full.	Before Full.	After Full.
	$''$	$''$	$''$	$''$
Nov., Dec., Jan.	$-1\cdot6_{38}$	$+1\cdot8_{48}$	$-0\cdot7$	$+2\cdot2$
Feb., Mar., Apr.	$-1\cdot9_{35}$	$+0\cdot7_{30}$	$-1\cdot7$	$+2\cdot6$
May, June, July	$-1\cdot9_{67}$	$+0\cdot1_{45}$	$-0\cdot4$	$+2\cdot3$
Aug., Sept., Oct.	$-1\cdot3_{53}$	$+0\cdot7_{59}$	$-1\cdot7$	$+2\cdot3$
Mean	$-1\cdot6$	$+0\cdot8$	$-1\cdot1$	$+2\cdot4$

13. We see that the curious asymmetry is much accentuated now that the comparison is made in this new way. Further, that it is quite different in character for the two periods 1847-1850 and 1851-1861. The mean error is negative for the former period ($-0''\cdot4$) and positive for the latter ($+0''\cdot6$); which seems to imply that, unless there was a corresponding change in the altazimuth observations themselves, the mean places of the Moon found by the new transit circle differ systematically by $+1''\cdot0$ in longitude from those found by the older instrument. It is of course quite possible that the change was in the alt-azimuth and due to

(*a*) *Instrumental causes*, such as the introduction of the chronographic method of registration. Even such changes as are mentioned in the volume for 1851, viz. a change in the method of illuminating the divisions or of mounting the object glass, may be responsible.

(*b*) *Personal.* A remarkable personality in the case of Mr. Hugh Breen was detected almost as soon as the altazimuth was erected. That observer had a lunar personality (applying to both limbs of the Moon) of $0^s\cdot31$. But this was detected in 1848 and the corrections made for existing observations, and Mr. Breen

* The term "old T.C." is used for brevity to denote the combination of transit instrument and mural-circle which was in use before 1851.

ceased forthwith to observe with the instrument. We may infer that such personalities were looked for with some care and probably eliminated; but some may have been overlooked.

But in either case, if the change was really in the altazimuth observations, it ought to be possible to prove this and free the transit circle observations from the suspicion of such a grave discontinuity.

14. To compare the results of Table II. with those previously found from Table I. we must combine them with the approximate weights 1 to 3; since the first series extended over three and a half years only and the second over eleven. We should then get $-1''\cdot2$ before full and $+2''\cdot0$ after, as compared with $-1''\cdot0$ before and $+1''\cdot3$ after (§ 8, Hypothesis I.) Our first results were therefore numerically too small as compared with the second. I feel tolerably sure that this is due to the way in which the first results are affected by the imperfection of the record, though I have not been able to establish this point completely. One way in which this imperfection tends to diminish these numbers is as follows:—Blank days were filled up by simple interpolation. Now, suppose we had observations on days 12 and 16. The former is subject to a correction $+1''\cdot2$, say, and the latter to a correction $-2''\cdot0$. On interpolation for day 13 we get a result subject to a correction $+1''\cdot2-\frac{1}{4}(3''\cdot2)$ instead of $+1''\cdot2$, as it should be; *i.e.* $0''\cdot8$ too small, and for day 14 $+1''\cdot2-\frac{1}{4}(3''\cdot2)$, or $1''\cdot1$ too small. Any days near full moon are thus liable to dilution in this way. In fact the process of §§ 7–10 is sensibly defective for our present purpose, and is to be regarded as superseded by the more direct method now being considered. The disadvantages of the present method, which prevented its use in the first instance, are of course (1) that a good many observations are lost; but this is not serious; (2) that we get no information as to the behaviour of the altazimuth in the part of the lunation where it is specially valuable, *i.e.* when the transit circle observations fail.

15. If now, instead of including *all* the results "before full," we limit ourselves to the three days, 12, 13, 14, as in Hypothesis II. of § 8, and similarly for "after full," we get

Table III. (Hypothesis II.)

	Old T.C.—Altaz. (1847–1850). Before Full.	Old T.C.—Altaz. (1847–1850). After Full.	New T.C.—Altaz. (1851–1862). Before Full.	New T.C.—Altaz. (1851–1862). After Full.
Nov., Dec., Jan....	$-1''\cdot7_{13}$	$+2''\cdot5_{13}$	$-0''\cdot5$	$+0''\cdot9$
Feb., Mar., Apr.	$-2''\cdot5_{14}$	$+1''\cdot5_{11}$	$-0''\cdot7$	$+2''\cdot1$
May, June, July	$-0''\cdot8_{16}$	$+0''\cdot5_{17}$	$-0''\cdot7$	$+1''\cdot3$
Aug., Sept., Oct.	$-0''\cdot8_{21}$	$+0''\cdot7_{23}$	$-1''\cdot6$	$+2''\cdot2$
Simple Mean	$-1''\cdot4$	$+1''\cdot3$	$-0''\cdot9$	$+1''\cdot6$

16. Inspection of Tables II. and III. suggests that the effect of the seasons is comparatively small. We will therefore, in the first instance, disregard it and take all the observations through-

out the year together, as was done in Table I. The results are given in Table IV., which corresponds with the arrangement of Table I., but is to be taken as superseding it for reasons already given.

TABLE IV.

Differences between Meridian and Altazimuth Observations of the Moon's Longitude.

Day of Lunation.	Old T.C.—Altaz. (1847–1850).			New T.C.—Altaz. (1851–1861).		
	Uncorr.	Corr. I.	Corr. II.	Uncorr.	Corr. I.	Corr. II.
	″	″	″	″	″	″
5	$-4\cdot28$	$-2\cdot6$	$-2\cdot8$	$-0\cdot4$	$+0\cdot7$	$+0\cdot5$
6	$-2\cdot6_{16}$	$-1\cdot0$	$-1\cdot2$	$-2\cdot0$	$-0\cdot9$	$-1\cdot1$
7	$-3\cdot2_{14}$	$-1\cdot6$	$-1\cdot8$	$-1\cdot6$	$-0\cdot5$	$-0\cdot7$
8	$-1\cdot9_{14}$	$-0\cdot3$	$-0\cdot5$	$-1\cdot8$	$-0\cdot7$	$-0\cdot9$
9	$-0\cdot4_{21}$	$+1\cdot2$	$+1\cdot0$	$-1\cdot5$	$-0\cdot4$	$-0\cdot6$
10	$-1\cdot1_{21}$	$+0\cdot5$	$+0\cdot3$	$-0\cdot9$	$+0\cdot2$	$0\cdot0$
11	$-1\cdot8_{24}$	$-0\cdot2$	$-0\cdot4$	$-0\cdot6$	$+0\cdot5$	$+0\cdot3$
12	$-2\cdot1_{22}$	$-0\cdot5$	$-0\cdot7$	$-0\cdot8$	$+0\cdot3$	$+0\cdot1$
13	$-0\cdot1_{27}$	$+1\cdot5$	$+1\cdot3$	$-0\cdot8$	$+0\cdot3$	$+0\cdot1$
14	$-1\cdot8_{25}$	$-0\cdot2$	$-0\cdot4$	$-1\cdot0$	$+0\cdot1$	$-0\cdot1$
15	$-1\cdot0_{25}$	...	...	$-0\cdot5$	...	...
16	$+0\cdot9_{21}$	$+0\cdot1$	$-0\cdot4$	$+1\cdot7$	$-0\cdot7$	$+0\cdot1$
17	$+0\cdot4_{26}$	$-0\cdot4$	$-0\cdot9$	$+1\cdot5$	$-0\cdot9$	$-0\cdot1$
18	$+2\cdot8_{18}$	$+2\cdot0$	$+1\cdot5$	$+1\cdot8$	$-0\cdot6$	$+0\cdot2$
19	$+0\cdot4_{19}$	$-0\cdot4$	$-0\cdot9$	$+2\cdot7$	$+0\cdot3$	$+1\cdot1$
20	$-0\cdot5_{18}$	$-1\cdot3$	$-1\cdot8$	$+2\cdot1$	$-0\cdot3$	$+0\cdot5$
21	$+0\cdot9_{26}$	$+0\cdot1$	$-0\cdot4$	$+2\cdot3$	$-0\cdot1$	$+0\cdot7$
22	$+0\cdot4_{22}$	$-0\cdot4$	$-0\cdot9$	$+2\cdot7$	$+0\cdot3$	$+1\cdot1$
23	$+0\cdot9_{15}$	$+0\cdot1$	$-0\cdot4$	$+2\cdot6$	$+0\cdot2$	$+1\cdot0$
24	$+0\cdot3_{10}$	$-0\cdot5$	$-1\cdot0$	$+2\cdot0$	$-0\cdot4$	$+0\cdot4$
25	$-2\cdot5_4$	$-3\cdot3$	$-3\cdot8$	$+4\cdot4$	$+2\cdot0$	$+2\cdot8$

17. The comparison with the old T.C. is affected with much larger accidental errors, owing to the smaller number of observations. To reduce the accidental errors a little let us assume that the errors are repeated with reversed sign after full moon. Hence we may reverse the sign of one set—say those before full moon—and take the mean. Further, we may smooth the results by taking means of every consecutive three. It will be sufficient to take the columns headed Corr. II.

TABLE V

Day.	Means of Before and After		Smoothed.		N−O.
	Old.	New.	Old.	New.	
	″	″	″	″	″
16	0·0	+0·1	...	...	...
17	−1·1	−0·1	0·0	0·0	0·0
18	+1·1	0·0	−0·1	+0·1	+0·2
19	−0·2	+0·4	−0·1	+0·2	+0·3
20	−1·1	+0·2	−0·7	+0·4	+1·1
21	−0·7	+0·6	−0·7	+0·6	+1·3
22	−0·2	+1·0	−0·1	+0·8	+0·9
23	+0·7	+0·8	+0·2	+0·9	+0·7
24	+0·1	+0·8	+0·1	+0·9	+0·8
25	−0·5	+1·2	...	...	...

18. These results are striking. They show, either (*a*) that the old transit circle and the new would give very different coefficients for the parallactic inequality; or (*b*) that the altazimuth was capable of behaving quite differently in the years 1847–1850, and in the years 1851–1861; that is to say, *the same* instrument was capable of assigning two very different values to this inequality.

In either case it seems to be necessary to make a careful study of the instrumental errors before deducing a value for the solar parallax entitled to any weight at all.

19. It throws some light on the question whether the altazimuth observations are at fault to recur to the results tabulated in Table I. and separate the group for the dates 1847–1850. The defects of these figures have already been noticed, but they chiefly concern the observations near full moon, and, further, they would be practically common to the whole series. Hence the second column of Table I. was prepared, and by comparing it with the next column it will be seen that there is a fairly strong presumption against any essential change in the altazimuth observations.

Summary.

(*a*) During the years 1847–61 observations of the Moon were made at Greenwich with three different instruments, viz. the altazimuth for the whole period, the present transit circle from 1851 to 1861, and the old transit and mural circle from 1847 to 1850. The altazimuth observations can thus be compared with those made with both the other instruments. The comparison is made through the medium of Hansen's tables, from the columns H—O in the Appendix to *Monthly Notices*, vol. I.

(*b*) The object of the comparison is to discuss *instrumental* errors, and not errors of theory; but it will show how far certain

terms in the lunar theory, especially the parallactic inequality, are affected by instrumental errors.

(*c*) The Moon's longitude determined by the old meridian instruments differs from that by the altazimuth, *after allowing for different apparent diameter*, by a *variable* quantity which rises to $-0''.7$ at about the quarters compared with full moon, and is represented in the fourth column of Table V.

(*d*) The new transit circle, on the other hand, shows an increasing positive difference, slowly increasing even past the quarters (see the fifth column of Table V.)

(*e*) The evidence is against this change being due to any change in the altazimuth.

(*f*) It therefore seems probable that meridian instruments may give very different values for the parallactic inequality ; and the results derived by Mr. Cowell on pp. 95–98 of *Monthly Notices*, vol. lxiv., especially that for the Sun's parallax, must, until these instrumental errors have been more fully discussed, be received with caution.

University Observatory, Oxford:
1904 *February* 20.

Methods of Analysis of Moon's Errors and some Results.

By P. H. Cowell, M.A.

I have had a very short glance at Mr. Nevill's paper * in MS. before it was sent to the printers, and in consequence I am now giving a full account of my methods, as my recollection is that Mr. Nevill misunderstands me in a few points. I gladly take the opportunity of acknowledging the assistance that Mr. Nevill's paper is sure to be to me, and I must also admit two or three errors in my paper that Mr. Nevill points out. Nothing in my investigations at present has turned upon these errors, but it is naturally of service to me to have them pointed out. I may add that as regards the term in cos ☊ I followed Airy too rashly. He expressly states that this term is not required by theory.

My method briefly is to analyse the observed errors. I naturally use the theory to suggest periods, and as far as possible to interpret the results, but my method is essentially empirical. I hope to obtain from the observations all inequalities in the errors of Hansen's tables exceeding $0''.5$, except, perhaps, in a few instances of terms of very long period, where such accuracy is probably unattainable. A term of less than $0''.5$ will probably not be detected, but coefficients of terms examined should in

* It has been found necessary to send the proofs of Mr. Nevill's paper to the author for correction. The paper cannot therefore appear in the present Number. Professor Brown's paper is also deferred for a similar reason.

general be obtained from fifty-four years' observations correctly to 0″·1 or less. There are two essential points in such an analysis, both of them well known. The first admits of no variation in its method of treatment. The second I treat in a somewhat novel fashion, which I shall endeavour to explain.

The first of these points is the near equality of many of the periods. Let us, for example, consider the inequalites with arguments nearly equal to the mean anomaly g. Let them be denoted by $\Sigma_s a_s \sin(g+\theta_s)$, where the quantities θ denote angles of very long period (differing from suffix to suffix). The observations are analysed into the form for each period of analysis—

$$\text{Error} = (\text{coefficient of } \sin g) \times \sin g + (\text{coefficient of } \cos g) \times \cos g$$

It is then attempted to equate

$$(\text{coefficient of } \sin g) \text{ to } \Sigma a_s \cos \theta_s$$

and

$$(\text{coefficient of } \cos g) \text{ to } \Sigma a_s \sin \theta_s$$

The quantity θ_s is obtained approximately from the observations, and then when possible identified with some argument suggested by theory. The coefficient a_s is obtained by solving the equations.

Investigations usually proceed on these lines. Certainly Professor Newcomb's (1876), Mr. Nevill's (1884), and Mr. Bakhuyzen's (1903) do. I have departed in one small point only from the practice established by these writers. My period of analysis is 400 lunar days instead of one year. My reasons for this are: (i.) 400 lunar days is fifteen periods of the mean anomaly, fourteen of the parallactic inequality, and thirteen of the evection; it is also a round number; (ii.) my plan is to remove all new inequalities from the tabular places, and to effect this speedily I have arranged the observations in columns of forty lines each. Each line of every column corresponds to a transit of the Moon, whether observed or not. This arrangement greatly facilitates the analysis as well as the application of corrections. It obviously requires the period of analysis to contain an exact number of columns.

The second point depends upon the want of symmetry in the distribution of the observations with reference to the age of the Moon. It is evident that, if the Moon was only observed when it was full, a real inequality $a \sin \phi$ could not be distinguished from apparent inequalities $-a \sin(\phi+D)$ and $-a \sin(\phi-D)$. In my paper in the December number of the *Monthly Notices* I showed that in 5647 observations the mean value of $\cos D$ is $-2721 \div 5647 = -0{\cdot}48$, and of $\sin D + 416 \div 5647 = +0{\cdot}07$. The latter quantity may be neglected, but from the former I infer the rough rule that a real inequality $a \sin \phi$ will give rise to two apparent inequalities $-\frac{1}{2}a \sin(\phi+D)$ and $-\frac{1}{2}a \sin(\phi-D)$. Conversely, given an apparent inequality $\beta \sin \phi$, it is not certain

that such a term is required by the tables. The term actually required may be $-2\beta \sin(\phi+D)$ or $-2\beta \sin(\phi-D)$. The terms $\phi \pm D$ must be examined before the nature of the apparent inequality can be determined. Below, when I come to the results, a few instances will be given where a second analysis (after removing some terms) has amply justified the method of reasoning.

n observations are considered to indicate an apparent inequality $\beta \sin \phi$, where

$$\beta = (\Sigma \varepsilon \sin \phi)/\tfrac{1}{2}n$$

ϕ being any argument, of period short compared with the period of analysis, and not either equal, or nearly equal, to D, the age of the Moon, and ε being the observed error.

It will be seen below that ϕ has been put equal to g, the mean anomaly, $2D-g$, the evection, and subsequently g a second time after the removal of some terms, the period of analysis being 400 lunar days, and 48 such periods being used. Also ϕ has been put equal to g', the Sun's mean anomaly, and $g-D$, an argument whose period is 400 lunar days, the whole 48×400 lunar days being in these cases taken as a single period of analysis.

In the December *Monthly Notices*, at the beginning of my paper on the semi-diameter &c., I mentioned that the tabular places from 1847 to 1901 had been corrected, when necessary, for Newcomb's corrections and for Hansen's term with wrong sign. Previous to the investigation of that paper they were also corrected for five other terms, making seven corrections in all. They have subsequently been corrected for

(8) semi-diameter (correction $0''{\cdot}4$ in accordance with the results of my paper, this being a reasonable approximation to $\pm 0''{\cdot}346 + 0''{\cdot}120 \sin D$).

(9) The long-period terms were taken out approximately by subtracting suitable constants from all the errors throughout a whole period of analysis.

This may be looked upon as equivalent to equating each error to $c+\beta \sin \phi$, instead of $\beta \sin \phi$ only. The normal equation for β will then contain c with a small coefficient; as an approximate value for c, the mean error of the period may clearly be taken. Since in some periods of analysis c exceeds $6''$, it must be more accurate to take account of it. The error of omitting it is $\frac{2c}{n} \Sigma \sin \phi$, and $\frac{1}{n} \Sigma \sin \phi$ is liable to be about 0·1. By this method the error is reduced to $\frac{2\Delta c}{n} \Sigma \sin \phi$, Δc being the quantity by which the estimate of c differs from the truth. When the observations happen to be well distributed as regards the argument ϕ, $\Sigma \sin \phi$ will be nearly zero.

(10) $\qquad -0''{\cdot}246 \sin(2D-g)$

a correction to the evection.

(11) $+0''.316 \sin (g+2=-3J+7^\circ)$

a term due to the action of *Jupiter*.

(12) $+0''.55 \sin g'$

a correction to the annual equation.

(13) $-0''.55 \sin (g-D)$,

a correction to Hansen's tables that I have discovered by analysis, but which is confirmed by the revised results of the *Darlegung*.

The following table gives the results of the different analyses.

Coefficients for 48 periods of analysis of 400 lunar days each of terms in the tabular minus observed errors of longitude in tenths of a second of arc.

	cos g.	sin g.	cos $(2D-g)$.	sin $(2D-g)$.	cos g.	sin g.
86	− 4	− 1	+ 4	+ 6	− 3	− 1
87	− 3	−10	+ 2	+10	− 1	− 7
88	−18	−13	0	+ 8	−15	− 4
89	−24	− 6	− 5	+ 9	−19	0
90	−14	+ 9	+ 2	+ 2	−12	+ 6
91	−18	+ 9	+ 1	+ 2	−13	+ 6
92	− 2	+ 7	− 1	+ 6	− 1	+12
93	+ 5	− 5	+ 1	+ 8	+ 5	− 1
94	+ 9	− 9	0	+11	+ 3	+ 2
95	−12	−11	− 1	+10	+ 1	− 6
96	−22	−20	− 6	+ 4	−13	− 9
97	−21	+ 2	−12	+ 6	− 9	− 2
98	−18	+ 6	− 6	− 1	−13	− 6
99	+ 4	+ 4	− 7	− 6	− 9	+ 7
100	+ 7	− 1	− 1	− 4	+ 6	+ 5
101	+ 8	0	− 3	0	+ 1	+ 2
102	− 3	− 7	+ 1	0	+ 1	− 2
103	0	− 6	+ 2	+ 1	− 1	− 6
104	− 5	−11	+ 4	+ 4	− 6	− 3
105	− 8	0	− 7	+ 6	−10	0
106	− 9	0	− 6	+ 6	− 8	+ 4
107	0	+10	− 1	+ 8	0	+ 7
108	+ 6	+13	− 2	+ 4	+ 4	+11
109	+ 5	+ 5	− 1	+ 6	+ 1	+ 5
110	+ 7	− 1	+ 2	+ 5	+ 4	− 1
111	+13	+ 1	− 7	+13	+ 9	+ 4
112	− 2	0	− 4	+12	0	0
113	− 1	+ 4	− 3	+10	− 1	− 1
114	− 3	+15	− 1	+ 2	+ 3	+ 9
115	+ 5	+17	− 4	+ 3	+ 7	+14

	cos g.	sin g.	cos (2D−g).	sin (2D−g).	cos g.	sin g.
116	+10	+ 4	− 8	+ 3	+10	+ 3
117	+ 6	+ 3	+ 3	0	+ 7	+ 1
118	+ 9	− 3	+ 1	+ 7	+11	0
119	− 9	−15	0	+10	− 7	− 9
120	−19	− 5	0	+	−13	− 2
121	−14	+ 3	− 2	+10	− 7	+ 5
122	−11	+ 9	− 2	+10	− 6	+11
123	− 7	+ 6	− 1	+ 9	− 3	+ 7
124	+ 3	+ 8	− 7	+ 4	+ 4	+11
125	+11	− 4	− 4	− 2	+14	+ 1
126	+10	−10	− 5	+ 9	+ 9	− 5
127	+ 3	−15	− 7	+14	+ 6	− 6
128	− 6	−13	− 6	+ 5	− 3	− 5
129	− 9	+ 1	− 3	+ 8	− 6	+ 6
130	− 2	− 4	− 6	+ 3	0	+ 3
131	+ 3	+ 4	− 4	− 3	+ 1	+ 7
132	+ 9	− 3	− 6	+ 2	+ 7	+ 6
133	+ 7	−14	− 3	+ 5	+ 1	− 3
Means	−0″·26	−0″·10	−0″·25	+0″·52	−0″·13	+0″·16

The first two columns ($\cos g$ and $\sin g$) in the table were derived from the errors after correction (8) and before correction (9). They are liable therefore to moderately large accidental errors in certain periods proportional to $\Sigma \cos g$ and $\Sigma \sin g$ respectively.

The third and fourth columns ($\cos (2D-g)$ and $\sin (2D-g)$) were derived after correction (9) and before correction (10).

The fifth and sixth columns ($\cos g$ and $\sin g$) were derived after correction (13).

In addition to the above, by including all the observations into one single period of analysis, after correction (9), I have obtained the following *apparent* corrections. The significance of the word *apparent* will be evident from the introductory remarks to this paper:

$$+0''\cdot59 \sin (g-D) + 0''\cdot34 \cos (g-D)$$
$$-0''\cdot83 \sin g' - 0''\cdot33 \cos g'$$
$$+0''\cdot40 \sin (\omega-\omega') - 0''\cdot35 \cos (\omega-\omega')$$

(The notation is Hansen's, with the additional symbol $D = g-g'+\omega-\omega'$.)

The coefficients of $\cos g$ and $\sin g$ (first set) have been analysed for planetary terms.

Since in the case of planetary terms there is no question of

arguments differing by D, the distinction between real and apparent corrections does not arise.

Coefficient of $\cos g$ = $-0''\cdot21 \sin(2\varpi - 3J + 7°)$
$+0''\cdot37 \sin(2\varpi - 2J) - 0''\cdot17 \cos(2\varpi - 2J)$

Coefficient of $\sin g$ = $-0''\cdot52 \cos(2\varpi - 3J + 7°)$
$+0''\cdot02 \sin(2\varpi - 2J) + 0''\cdot09 \cos(2\varpi - 2J)$

Combining the two terms and omitting small terms we obtain

(from periods 86–133) $-0''\cdot36 \sin(g + 2\varpi - 3J + 7°)$
$+0''\cdot23 \sin(g + 2\varpi - 2J)$

In a similar manner, using the values of a and b published in the November *Monthly Notices*, we obtain

(from periods 1–48) $+0''\cdot20 \sin(g + 2\varpi - 2J)$

(from periods 42–89) $+0''\cdot28 \sin(g + 2\varpi - 2J)$

It appears therefore that the tabular places require the correction

$$+0''\cdot36 \sin(g + 2\varpi - 3J + 7°) - 0''\cdot23 \sin(g + 2\varpi - 2J)$$

The first term agrees closely with theory, for Radau gives $+0''\cdot316$ as the coefficient.

The second term indicates a correction $-0''\cdot23$ to the $-0''\cdot893$ already applied. This term is of especial interest, for it will be remembered that it was discovered empirically by Professor Newcomb in 1876, who assigned it a coefficient $1''\cdot5$. Mr. Neison first pointed out that the term is due to the action of *Jupiter*. It appears from this paper that the coefficient given by the observations is $1''\cdot1$. Hill and Radau both obtain $0''\cdot9$ from theory.

Coming now to the solar terms, I obtain the following *apparent* terms in the tabular *minus* observed errors:

$$-0''\cdot52 \sin(g - 2D) - 0''\cdot25 \cos(g - 2D)$$
$$+0''\cdot59 \sin(g - D) + 0''\cdot34 \cos(g - D)$$
$$-0''\cdot10 \sin g - 0''\cdot26 \cos g$$

Some more terms are given below after the discussion of the above. The coefficients of $\sin g$ and $\cos g$ are from the first analysis.

The cosine terms, if real, denote an error of epoch. Clearly no change of the epoch of g will explain all three terms. The terms are, however, small. I hold over the discussion until the tabular places from 1750 to 1851 have been revised and the observations discussed.

The three sine terms suggest the real corrections

$$+0''\cdot25\sin(g-2D)-0''\cdot55\sin(g-D)-0''\cdot17\sin g$$

are required by the tabular places, for a real correction of this form will produce an apparent correction

$$\begin{aligned}&(+0''\cdot25+0''\cdot27)\sin(g-2D)\\&(-0''\cdot12-0''\cdot55+0''\cdot08)\sin(g-D)\\&(+0''\cdot27-0''\cdot17)\sin g\end{aligned}$$

on the principles explained above, thus annihilating the terms indicated by the errors with an accuracy that is almost suspicious.

It will be noticed that the terms $+0''\cdot25\sin(g-2D)-0''\cdot55\sin(g-D)$ have been applied, and the second analysis gives for the tabular minus observed errors $+0''\cdot17\sin g$. This second analysis bears out in a remarkable degree the substantial accuracy of the assumption that a real correction $\beta\sin\phi$ produces apparent corrections

$$\beta\sin\phi-\tfrac{1}{2}\beta\sin(\phi+D)-\tfrac{1}{2}\beta\sin(\phi-D)$$

The explanation of these corrections is interesting. The coefficient of the evection in Hansen's tables is $4586''\cdot690$ (see January *Monthly Notices*); the true coefficient is therefore $4586''\cdot44$. The theoretical coefficient for an elliptic coefficient $22639''\cdot5$ is about $4586''\cdot34$. The difference $0''\cdot10$ is either accidental or points to an ellipticity in the figure of the Moon only one-sixth of what Hansen assumed. It may be recalled that my paper in the December *Monthly Notices* on the evidence of the variation tended to support Hansen's hypothesis; but there can now be little doubt that the ellipticity of the Moon is insensible or nearly so, and that the apparent increase of the variation is due to the conditions of observation, as Professor Newcomb has pointed out.

I did not expect to find so large a correction as $-0''\cdot55\sin(g-D)$ in a term where Hansen's *Darlegung* and Delaunay are in such close accordance. The coefficients given by these two authorities are respectively $18''\cdot54$ and $18''60$ when reduced to a parallax $8''\cdot76$, and when Delaunay's series is completed by induction. The difficulty is explained, however, by the fact that Hansen's tables differ considerably from the *Darlegung* in this term. The coefficient in the tables is $19''\cdot15$, and the correction $-0''\cdot55$ might therefore have been anticipated.

The value of the principal elliptic coefficient now appears to be

$$22640''\cdot17-0''\cdot55-0''\cdot17=22639''\cdot45$$

on the assumption that the terms in $g\pm D$ are

$$-8''\cdot54\sin(g+D)+18''\cdot60\sin(g-D).$$

As, however, the former term apparently should be

$$-8''\cdot46 \sin (g+\mathrm{D})$$

the principal elliptic coefficient should be

$$22639''\cdot49$$

My preliminary discussion of Airy's period 1750–1851 if reduced to the same terms

$$-8''\cdot46 \sin (g+\mathrm{D}) + 18''\cdot60 \sin (g-\mathrm{D})$$

gives $22639''\cdot53$

a substantial agreement. The result, however, cannot be considered definite until Professor Brown has determined the two "allied" coefficients.

It will be noticed that all the coefficients given in the principal table of this paper show a periodicity in eight periods of analysis or nine years. This period is to be identified with $\varpi - \varpi'$ or $\omega - \omega'$. Solving on this assumption, I obtain for the tabular *minus* observed errors

$$+0''\cdot24 \sin (g-2\mathrm{D}+\omega-\omega')$$
$$-0''\cdot83 \sin (g-\mathrm{D}+\omega-\omega') - 0''\cdot33 \cos (g-\mathrm{D}+\omega-\omega')$$
$$+0''\cdot81 \sin (g+\omega-\omega') + 0''\cdot46 \cos (g+\omega-\omega')$$

the last line referring to the first analysis for g after correction (8). The argument of the second line is g', the Sun's mean anomaly.

The above apparent corrections seem to point to true corrections

$$+0''\cdot55 \sin g'$$
$$-0''\cdot57 \sin (g+\omega-\omega') - 0''\cdot46 \cos (g+\omega-\omega')$$

being required.

It will be seen that I have applied $+0''\cdot55 \sin g'$. As far as I can make out only half this term is not required by theory, but perhaps Professor Brown's theory when completed will verify the whole. This at present seems unlikely; but as to the reality of the term, I consider the evidence immensely strengthened by the corroboration derived from the "allied" term $g'-\mathrm{D}$.

I have not applied corrections

$$-0''\cdot57 \sin (g+\omega-\omega') - 0''\cdot46 \cos (g+\omega-\omega')$$

because of the extreme uncertainty of their real nature. No cosine term is required by theory, and the coefficient of the sine

term appears to require theoretically an alteration of only $0''\cdot27$. This suggests that for the outstanding part the period may not be $g+\omega-\omega'$, but some slightly different period. Two hypotheses have occurred to me: one is that the argument should be $g+\varpi$ the Moon's longitude, or that the term is due to systematic errors of observation (since ϖ' is practically a constant, analysis of the errors cannot possibly distinguish between $g+\varpi$ and $g+\varpi-\varpi'$); the other hypothesis is that the argument is $g+2\varpi+3\mathrm{V}-5\mathrm{E}$, and that there is something wrong with the theoretical coefficient of the *Venus* term. The argument $g+2\varpi+3\mathrm{V}-5\mathrm{E}$ moves more slowly than the argument $g+\omega-\omega'$ by $4^\circ\cdot22$ in 400 lunar days, or by one revolution in a century. Hence, fifty years' observations cannot distinguish between the two arguments, and the point must be held over until the Airy period 1750–1851 is re-discussed.

A comparison of my January paper with Professor Newcomb's transformation of Hansen's theory and comparison with Delaunay shows that the following corrections are still required in the tabular places:

$$
\begin{aligned}
&-0''\cdot27 \sin(g+\omega-\omega')\\
&+0\cdot40 \sin(g-g')\\
&-0\cdot45 \sin(g+2g')\\
&-0\cdot29 \sin(g'-2\omega+2\omega')\\
&+0\cdot27 \sin(g-g'+2\omega-2\omega')\\
&-0\cdot26 \sin(g+2\omega)\\
&-0\cdot72 \sin(\omega-\omega')\\
&-0\cdot22 \sin(g+g'-\omega+\omega')\\
&-0\cdot32 \sin(2g-3g'+4\omega-4\omega')
\end{aligned}
$$

The first term has just been alluded to; the coefficient of the seventh term seems too large, judging from the observations; as to the sixth term I have estimated that corrections

$$-0''\cdot14 \sin(g+2\omega)+0''\cdot12 \cos(g+2\omega)$$

are required by the observations. The agreement is not very close, but the analysis can only be approximate in view of the large movement of 2ω in one period of analysis. The fourth term is "allied" to the argument $g-\omega+\omega'$, and at present my analysis throws no light upon any of the other terms.

I conclude this paper with giving the necessary data for identifying my periods of analysis with the calendar, and for obtaining the values of the arguments at various epochs. The last two lines refer to Hansen's empirical *Venus* terms.

Values of various Arguments in the middle of the 44th period of analysis and their movements in 400 lunar days or one period of analysis.

Argument. Calendar	Value. 1600 Jan. 3^{d}6 G.M.T.	Movement. 414·02 Solar Days.
g	159°·7658	5409°·1681
g'	4·1375	408·0583
ϖ	192·7447	68·0478
ϖ'	246·4312	21·9435
Ω	33·0749	−21·9241
D	101·9418	5047·2141
$g-D$	57·8240	361·9540
$\varpi=\varpi+\Omega$	225·8196	46·1237
$\varpi'=\varpi'+\Omega$	279·5061	0·0194
$\varpi-\varpi'=\varpi-\varpi'$	306·3135	46·1043
Venus V	151·2198	663·3141
Earth E	103·6435	408·0619
Mars M	234·5551	216·9601
Jupiter J	82·1455	34·4014
$2\varpi-2J$	287·3482	23·4445
$2\varpi-3J+7°$	212·2027	−10·9570
$2\varpi+3V-5E$	27·0811	41·8802
$A+30°$	213·8946	1·4953
$8V-13E+274°\ 14'$	136·6262	1·7081

Comparisons of the Geocentric Places of the Sun and Major Planets calculated from the Tables of the American Ephemeris Office, with their Places calculated from Le Verrier's Tables, for the year 1906. By A. M. W. Downing, D.Sc., F.R.S.

This paper is supplementary to those published in *Monthly Notices*, vol. lix. No. 10, and vol. lxii. No. 1.

The American Tables of the Sun, *Mercury*, *Venus*, *Mars*, *Uranus*, and *Neptune* are due to Professor Newcomb; those of *Jupiter* and *Saturn* to Mr. G. W. Hill. These Tables are adopted in calculating the ephemerides given in the *Nautical Almanac* for 1906. The corresponding places in the *Connaissance des Temps* for the same year are from Le Verrier's Tables.

The comparisons have been made by reducing the quantities given in the *Nautical Almanac* from Greenwich noon to Paris noon at convenient intervals throughout the year, and then finding the differences between the reduced quantities and the

corresponding quantities in the *Connaissance des Temps*. For the right ascensions the discordances are given in arc of a great circle, as well as in time, to render them comparable with the discordances in declination.

The abnormal discordances in the case of *Venus*, near the time of inferior conjunction with the Sun in 1906, are noteworthy as appearing to show that the ordinary methods of computation are not sufficiently refined to ensure accuracy in the geocentric places corresponding to these critical positions of the planet in its orbit.

With regard to *Saturn* it will be remembered that M. Gaillot has succeeded in rectifying Le Verrier's theory of this planet, which has been found to be defective in certain particulars. The publication of the corresponding corrections to the Tables will be awaited with interest.

Sun, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	R.A. Time.	R.A. Arc.	Decl.	Day. 1906.	R.A. Time.	R.A. Arc.	Decl.
	s	″	″		s	″	″
Jan. 2	−·07	−1·0	−0·4	July 5	−·01	−0·1	+0·3
10	−·05	−0·8	−0·3	13	·00	0·0	+0·2
18	−·04	−0·6	−0·4	21	−·03	−0·4	+0·2
26	−·04	−0·6	−0·4	29	−·03	−0·4	+0·4
Feb. 3	−·04	−0·6	−0·4	Aug. 6	−·03	−0·4	+0·3
11	−·03	−0·4	−0·3	14	−·03	−0·4	+0·2
19	−·01	−0·2	−0·2	22	−·04	−0·6	+0·3
27	−·02	−0·3	−0·3	30	−·04	−0·6	+0·3
Mar. 7	−·01	−0·2	−0·2	Sept. 7	−·04	−0·6	+0·1
15	−·01	−0·2	−0·1	15	−·05	−0·8	+0·2
23	·00	0·0	−0·1	23	−·06	−0·9	+0·2
31	−·01	−0·2	0·0	Oct. 1	−·05	−0·9	+0·2
Apr. 8	+·01	+0·2	0·0	9	−·06	−0·9	+0·1
16	+·01	+0·2	+0·2	17	−·07	−1·0	+0·2
24	·00	0·0	+0·1	25	−·08	−1·2	+0·2
May 2	·00	0·0	+0·2	Nov. 2	−·07	−1·0	+0·2
10	−·01	−0·1	+0·1	10	−·06	−0·9	+0·1
18	·00	0·0	+0·3	18	−·09	−1·3	+0·1
26	−·01	−0·1	+0·1	26	−·08	−1·1	−0·1
June 3	−·01	−0·1	+0·2	Dec. 4	−·06	−0·8	−0·1
11	+·01	+0·1	+0·2	12	−·06	−0·8	−0·1
19	·00	0·0	+0·1	20	−·06	−0·8	−0·3
27	−·02	−0·3	+0·2	28	−·06	−0·8	−0·2

MERCURY, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	R.A. Time.	R.A. Arc.	Decl.	Day. 1906.	R.A. Time.	R.A. Arc.	Decl.
	s	"	"		s	"	"
Jan. 2	−·03	−0·4	−0·4	July 5	−·06	−0·9	+0·5
10	+·01	+0·1	−0·3	13	−·05	−0·7	+0·5
18	+·02	+0·3	0·0	21	−·07	−1·0	+0·6
26	+·03	+0·4	−0·1	29	−·12	−1·8	+0·7
Feb. 3	+·03	+0·4	0·0	Aug. 6	−·12	−1·8	+1·1
11	+·03	+0·4	+0·2	14	−·11	−1·6	+0·4
19	·00	0·0	0·0	22	−·05	−0·7	+0·3
27	−·02	−0·3	−0·4	30	−·05	−0·7	−0·1
Mar. 7	−·04	−0·6	−0·5	Sept. 7	−·07	−1·0	+0·1
15	−·01	−0·2	−0·5	15	−·04	−0·6	+0·1
23	·00	0·0	−0·4	23	−·05	−0·8	0·0
31	−·03	−0·4	−0·3	Oct. 1	−·01	−0·2	0·0
Apr. 8	−·04	−0·6	−0·2	9	·00	0·0	−0·1
16	−·05	−0·9	−0·1	17	·00	0·0	−0·1
24	−·06	−0·9	0·0	25	·00	0·0	−0·1
May 2	−·05	−0·8	+0·3	Nov. 2	·00	0·0	−0·3
10	−·02	−0·3	+0·2	10	−·01	−0·1	−0·3
18	−·03	−0·4	+0·1	18	·01	−0·1	−0·5
26	−·06	−0·9	−0·2	26	+·03	+0·4	−0·6
June 3	−·09	−1·3	−0·3	Dec. 4	+·08	+1·1	−1·1
11	−·05	−0·7	−0·2	12	+·03	+0·4	−0·3
19	−·07	−1·0	0·0	20	+·02	+0·3	−0·2
27	−·07	−1·0	+0·4	28	−·01	−0·1	−0·1

NOTE.—*Mercury* is in inferior conjunction with the Sun, 1906 November 29^d 18^h.

VENUS, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	R.A. Time.	R.A. Arc.	Decl.	Day. 1906.	R.A. Time.	R.A. Arc.	Decl.
	s	"	"		s	"	"
Jan. 2	−·01	−0·1	−1·2	Apr. 8	−·16	−2·4	+0·1
10	+·01	+0·1	−1·1	16	−·16	−2·3	+0·3
18	+·01	+0·1	−1·0	24	−·18	−2·6	+0·3
26	−·02	−0·3	−0·9	May 2	−·17	−2·4	+0·5
Feb. 3	−·03	−0·4	−0·8	10	−·17	−2·4	+0·5
11	−·05	−0·7	−0·9	18	−·14	−1·9	+0·7
19	−·05	−0·7	−0·6	26	−·15	−2·1	+0·8
27	−·08	−1·2	−0·6	June 3	−·14	−1·9	+0·6
Mar. 7	−·10	−1·5	−0·3	11	−·12	−1·6	+0·5
15	−·11	−1·7	−0·4	19	−·11	−1·5	+0·5
23	−·13	−2·0	−0·3	27	−·08	−1·1	+0·3
31	−·15	−2·2	−0·2	July 5	−·09	−1·3	−0·1

Day. 1906.	R.A. Time.	R.A. Arc.	Decl.	Day. 1906.	R.A. Time.	R.A. Arc.	Decl.
	s	"	"		s	"	"
July 13	−·07	− 1·0	−0·3	Oct. 9	+·10	+ 1·4	−0·6
21	−·07	− 1·0	−0·6	17	+·17	+ 2·3	−0·2
29	−·07	− 1·0	−0 8	25	+·27	+ 3·6	+0·7
Aug. 6	−·07	− 1·1	−1·3	Nov. 2	+·32	+ 4·2	+1·2
14	−·06	− 0·9	−1·3	10	+·48	+ 6·4	+2·4
22	−·06	− 0·9	−1·6	18	+·59	+ 8·0	+2·7
30	−·05	− 0·7	−1·5	26	+·69	+ 9·4	+2·8
Sept. 7	−·03	− 0·4	−1·7	Dec. 4	+·72	+10·0	+2 8
15	−·01	− 0·1	−1·6	12	+·62	+ 8·8	+2·6
23	−·01	− 0·1	−1·3	20	+·43	+ 6 2	+2·4
Oct. 1	+·04	+ 0·6	−1·0	28	+·29	+ 4·2	+2·3

NOTE.—*Venus* is in inferior conjunction with the Sun, 1906 November $29^d\ 17^h$.

MARS, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	R.A. Time.	R.A. Arc.	Decl.	Day. 1906.	R.A. Time.	R.A. Arc.	Decl.
	s	"	"		s	"	"
Jan. 2	−·03	−0·4	−0·3	July 5	−·11	−1·5	+0·6
10	−·04	−0·6	−0·4	13	−·10	−1·4	+0·6
18	−·03	−0·5	−0·2	21	−·11	−1·5	+0·7
26	−·04	−0·6	−0·3	29	−·10	−1·4	+0·7
Feb. 3	−·04	−0·6	−0·3	Aug. 6	−·09	−1·3	+0·9
11	−·05	−0·8	−0·2	14	−·09	−1·3	+0·8
19	−·04	−0·6	−0·3	22	−·09	−1·3	+0·8
27	−·06	−0·9	−0·3	30	−·08	−1·2	+0·7
Mar. 7	−·06	−0·9	−0·3	Sept. 7	−·08	−1·2	+0·6
15	−·07	−1·0	−0 2	15	−·06	−0·9	+0·7
23	−·07	−1·0	−0·1	23	−·07	−1·0	+0·7
31	−·08	−1·2	−0·2	Oct. 1	−·07	−1·0	+0·7
Apr. 8	−·09	−1·3	−0·1	9	−·07	−1·0	+0·4
16	−·07	−1·0	+0·1	17	−·08	−1·2	+0·6
24	−·08	−1·1	0·0	25	−·07	−1·1	+0·5
May 2	−·09	−1·3	+0·1	Nov. 2	−·06	−0·9	+0·4
10	−·10	−1·4	+0·2	10	−·06	−0·9	+0·5
18	−·09	−1·2	+0·2	18	−·07	−1·1	+0 5
26	−·10	−1·4	+0·3	26	−·06	−0·9	+0·3
June 3	−·10	−1·4	+0·3	Dec. 4	−·06	−0·9	+0·3
11	−·09	−1·2	+0·4	12	−·05	−0·7	+0·4
19	−·10	−1·4	+0·6	20	−·04	−0·6	+0·1
27	−·11	−1·5	+0·6	28	−·06	−0·9	+0·2

JUPITER, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	R.A. Time.	R.A. Arc.	Decl.	Day. 1906.	R.A. Time.	R.A. Arc.	Decl.
	s	″	″		s	″	″
Jan. 2	+·10	+1·4	+0·5	July 5	·00	0·0	+0·3
10	+·09	+1·3	+0·4	13	+·01	+0·1	+0·3
18	+·09	+1·3	+0·5	21	−·01	−0·1	+0·3
26	+·07	+1·0	+0·2	29	−·03	−0·4	+0·5
Feb. 3	+·07	+1·0	+0·3	Aug. 6	−·01	−0·1	+0·3
11	+·08	+1·1	+0·3	14	−·02	−0·3	+0·3
19	+·06	+0·9	+0·4	22	−·03	−0·4	+0·1
27	+·09	+1·3	+0·3	30	·00	0·0	+0·2
Mar. 7	+·08	+1·1	+0·4	Sept. 7	−·04	−0·6	+0·2
15	+·05	+0·7	+0·4	15	−·02	−0·3	+0·3
23	+·06	+0·9	+0·3	23	−·02	−0·3	+0·2
31	+·04	+0·6	+0·2	Oct. 1	·00	0·0	+0·2
Apr. 8	+·05	+0·7	+0·4	9	−·01	−0·1	+0·3
16	+·04	+0·6	+0·3	17	−·02	−0·3	+0·3
24	+·03	+0·4	+0·3	25	−·02	−0·3	+0·4
May 2	+·02	+0·3	+0·3	Nov. 2	−·02	−0·3	+0·4
10	+·01	+0·3	+0·4	10	−·03	−0·4	+0·6
18	+·01	+0·1	+0·3	18	−·02	−0·3	+0·5
26	+·02	+0·3	+0·3	26	−·01	−0·1	+0·4
June 3	·00	0·0	+0·3	Dec. 4	−·03	−0·4	+0·7
11	+·01	+0·1	+0·2	12	−·03	−0·4	+0·4
19	+·01	+0·1	+0·2	20	−·03	−0·4	+0·6
27	+·01	+0·1	+0·5	28	−·04	−0·6	+0·5

SATURN, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	R.A. Time.	R.A. Arc.	Decl.	Day. 1906.	R.A. Time.	R.A. Arc.	Decl.
	s	″	″		s	″	″
Jan. 2	−·42	−6·1	−2·3	Mar. 31	−·48	−7·1	−2·8
10	−·42	−6·2	−2·1	Apr. 8	−·48	−7·1	−2·9
18	−·42	−6·2	−2·3	16	−·49	−7·3	−3·0
26	−·42	−6·2	−2·4	24	−·52	−7·7	−3·0
Feb. 3	−·43	−6·3	−2·4	May 2	−·52	−7·7	−3·1
11	−·44	−6·5	−2·3	10	−·53	−7·9	−3·1
19	−·44	−6·5	−2·5	18	−·55	−8·2	−3·3
27	−·46	−6·8	−2·5	26	−·55	−8·2	−3·3
Mar. 7	−·47	−6·9	−2·7	June 3	−·57	−8·5	−3·5
15	−·46	−6·8	−2·7	11	−·58	−8·6	−3·5
23	−·48	−7·1	−2·8	19	−·60	−8·9	−3·6

Day. 1906.	R.A. Time.	R.A. Arc.	Decl.	Day. 1906.	R.A. Time.	R.A. Arc.	Decl.
	s	″	″		s	″	″
June 27	−·62	− 9·2	−3·8	Oct. 1	−·68	−10·0	−4·0
July 5	−·62	− 9·2	−3·9	9	−·68	−10·0	−4·1
13	−·63	− 9·4	−4·0	17	−·68	−10·0	−4·0
21	−·66	− 9·8	−4·0	25	−·67	− 9·9	−4·0
29	−·65	− 9·7	−3·9	Nov. 2	− 67	− 9·9	−3·9
Aug. 6	−·67	−10·0	−4·1	10	−·66	− 9·7	−3·9
14	−·67	−10·0	−4·0	18	− 65	− 9·6	−3·9
22	−·70	−10·4	−4·1	26	−·66	− 9·7	−3·9
30	−·68	−10·1	−4·0	Dec. 4	−·66	− 9·7	−3·9
Sept. 7	−·68	−10·1	−3·9	12	− 65	− 9·6	−3·8
15	−·68	−10·1	−3·9	20	− 64	− 9·5	−3·9
23	−·68	−10·1	−4·0	28	−·65	− 96	−3·9

URANUS, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	R.A. Time.	R.A. Arc.	Decl.	Day. 1906.	R.A. Time.	R.A. Arc.	Decl.
	s	″	″		s	″	″
Jan. 1	−·31	−4·3	−0·3	July 4	−·35	−4·8	−0·1
Feb. 2	−·32	−4·4	−0·3	Aug. 1	−·33	−4·6	−0·1
Mar. 2	−·33	−4·6	−0·2	Sept. 2	−·31	−4·3	−0·2
Apr. 3	−·34	−4·7	−0·2	Oct. 4	−·31	−4·3	−0·1
May 1	−·34	−4·7	−0·1	Nov. 1	−·29	−4·0	−0·1
June 2	−·35	−4·8	−0·2	Dec. 3	−·30	−4·1	−0·2

NEPTUNE, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	R.A. Time.	R.A. Arc.	Decl.	Day. 1906.	R.A. Time.	R.A. Arc.	Decl.
	s	″	″		s	″	″
Jan. 1	−·45	−6·3	−1·9	July 4	−·43	−6·0	−1·6
Feb. 2	−·45	−6·3	−1·9	Aug. 1	−·44	−6·1	−1·7
Mar. 2	−·44	−6·1	−1·9	Sept. 2	−·44	−6·1	−1·7
Apr. 3	−·44	−6·1	−2·1	Oct. 4	−·45	−6·3	−1·8
May 1	−·45	−6·3	−2·0	Nov. 1	−·46	−6·4	−1·6
June 2	−·44	−6·1	−1·9	Dec. 3	−·48	−6·7	−1·6

H.M. Nautical Almanac Office:
1904 *February* 29.

Note on the Drawings of the Mare Serenitatis by John Russell, R.A. By S. A. Saunder, M.A.

On pp. 156–159 of the present volume of the *Monthly Notices* Dr. Rambaut gives an account of two drawings of the region around Linné made by Mr. John Russell, R.A., and arrives at the conclusion that they render it probable that Linné then presented much the same appearance as at the present day when viewed with a low-power eyepiece attached to a small telescope.

The drawing reproduced in plate 3, fig. 2, has not much direct bearing on the question of change, for we are told by Mädler that near the time of full moon Linné was seen in his time as here represented, namely, as a round white spot almost as white in the middle as at the edges. It is to the drawing in fig. 1 that the real interest attaches. My own introduction to this drawing was through a photograph shown me by Dr. Rambaut, and from an inspection of this I came to the conclusion that it did offer distinct evidence of a change. As I was afterwards led to modify this opinion when, by the kindness of Dr. Rambaut, I was allowed to inspect the original, it may be not without interest that I should put on record both my first impression and the reasons which led me to modify it, as many will see the reproduction and but few can hope to have the opportunity of comparing it with the original.

Referring to plate 3, fig. 1, the only genuine marks on the floor besides Bessel and the bright streaks are :

(1) A small dark spot 0·45 in. from Bessel at position-angle 311°. This is perhaps a little more conspicuous in the original, but not much so. It almost certainly denotes Bessel A.

(2) A somewhat larger dark mark 0·61 in. from Bessel at position-angle 352°. This is much more conspicuous in the original, but careful inspection shows that it is due to a stain and is not part of Russell's drawing.

(3) The large dark mark 0·85 in. from Bessel at position-angle 70°. This is referred to by Dr. Rambaut in his note on p. 157, and, like the last, is a stain on the paper.

(4) Linné, 1·37 in. from Bessel, at position-angle 53°. This, as stated by Dr. Rambaut, is whiter and more conspicuous in the original, where also the dark curved shading to the west of the white spot is clearly seen and has every appearance of being intentional. The large dark mark trending to the south-east of the white spot has its importance exaggerated in the reproduction, where it looks decidedly like a shadow, and I was led to hope that some estimate might be formed of the height of the mountain to which it was due ; but from its faintness in the original, coupled with its southerly direc-

tion, I now agree with the opinion expressed by Dr. Rambaut, that it was not intended to represent a shadow. At the base of this, however, bordering the white spot on the east side, and extending a little to the south of it, is a much darker line in the original, perhaps half as thick again as the line on the west side. It is more conspicuous in the original than in the reproduction, and conveys to my mind the impression that it was really intended to represent a shadow.

We do not know the size of the refractor most frequently employed by Russell, and can only judge of its powers and of the definition on the occasion on which the drawing was made from the drawing itself. We note that Bessel A, a conspicuous crater under this illumination in a 3 or 4 inch telescope, and one which was clearly seen by Schröter, is only just indicated. The smaller craters north and north-west of Linné are not shown at all, yet at the present time these are very clearly seen with their interiors filled with shadow under similar illumination, when in my 7-inch refractor there is no trace of shadow in the white spot surrounding Linné. It should be noted, too, that at present, when Bessel is still full of shadow, the white spot round Linné has barely developed and is far from being a conspicuous object.

It must be remembered that the special purpose of the drawing was to show the real nature of Bessel, but the great care with which the detail south of the Mare is drawn shows that other parts were not neglected. We may therefore infer that the shadow in Bessel A was only just seen, that the other small craters were not seen, and yet (if my interpretation of these two dark lines is correct) that decided shadow was seen on both sides of the bright spot, marking the position of Linné, and which, if we accept the accounts of Lohrmann, Mädler, and Schmidt, we may take as denoting the floor of the crater.

If this is admitted the drawing does furnish clear evidence of a change. But, on the other hand, it was pointed out to me by Dr. Rambaut that in the drawing representing Linné under what must have been nearly vertical illumination there are similar dark borders, nearly as thick as those under consideration, to the north and south of the white spot by which Linné is there represented. These may be seen in plate 3, fig. 2; they could not possibly have been shadows, and their existence there renders doubtful the conclusion that similar marks in fig. 1 are to be taken as real shadows.

On the whole, then, the drawing, whilst, in my opinion, quite consistent with the theory of change, cannot fairly be held to prove it, and I can only regret that after looking through these drawings I have come to the conclusion that in spite of the care with which they were made there will be very great difficulty in obtaining satisfactory information from them with regard to such small changes as there is reason to think may have occurred

from time to time upon the Moon. I may further illustrate the difficulty from the drawing now published in fig. 2. Much of the detail is evidently drawn with the greatest care, and yet the large ring shown to the north of *Cassini* does not exist upon the Moon. Careful study will show how it was suggested ; but at the same time it will also show that it was only suggested, and that a part of what is there drawn has no real existence.

I cannot conclude this short note without endeavouring to express a sense of the debt which all who are interested in this question must feel that they owe to Dr. Rambaut and the staff of the Radcliffe Observatory for the care with which they have gone into the matter and the trouble they have taken, including as it did the learning of the system of shorthand in which Russell made the only notes that we have in explanation of the drawings. That the result has not been more definite is due to the inherent difficulty of the inquiry.

On the Relative Efficiency of Different Methods of Determining Longitudes on Jupiter. By A. Stanley Williams.

1. Two chief methods have been employed for the purpose of determining the longitudes or positions of markings on *Jupiter*, and therefrom the rotation period of the planet. One of these consists in recording the times when the markings appear to be exactly in mid-transit across the disc of the planet. This is termed *the method of transits*. In the other method the positions of the markings are measured with a micrometer from the preceding and following limbs of the planet, and the time of transit derived from these measures. This is termed *the micrometric method*.

2. As is well known, very great claims have been made as to the superiority of the micrometric method over the method of transits as regards the accuracy of the results obtained. Thus, the former has been described as being "infinitely preferable" to the latter ; and positions determined by the method of transits have even been said to bear the same relation to those derived from the micrometric method, as eye-estimates of the position angles and distance of double stars possess with respect to micrometer measures of the same. As the number of positions already determined by the method of transits can hardly amount to less than 40,000, and as these constitute the foundation of much of our knowledge of the rotation of *Jupiter* and of the various surface currents known to exist, it seems desirable to make some inquiry into the relative efficiency of the two methods.

3. Reference should be made in this connection to some statements and comparisons on this subject by Professor G. W. Hough in *Astronomy and Astro-physics*, vol. xi. p. 193, and in *Popular*

Astronomy, 1903, p. 297. A few words by Professor E. E. Barnard will also be found in *Publications of the Astronomical Society of the Pacific*, vol. i. 91, and in the *Monthly Notices*, vol. lii. 11. The writer may likewise, perhaps, be permitted to refer here to a few remarks on the subject by himself published in *Popular Astronomy*, 1903, p. 188. But, somewhat singularly, in view of the strong claims that have been made as to the superiority of the micrometric method, no adequate investigation or comparison as to the relative accuracy of the results obtained by the two methods seems to have been hitherto undertaken.

4. In the "Report of the Director of the Dearborn Observatory" for 1882 it is stated that the mean error on the concluded time of transit from a single pair of measures was $\pm 0^{s}\cdot 9$ minute, and for the "average mean probable error" for any day was $\pm 0^{s}\cdot 4$ minute. But these values were based on observations of the red spot, when at its greatest plainness, made on thirty-one nights only; and they seem to have been derived from the accordance of the separate measures of each night *inter se*. Observers of double stars are well aware that the separate measures of a star made on the same night may be very accordant, although the mean results of several different nights may differ considerably, and for this reason it is usually considered preferable to make a small number of measures of a star on each of several different nights rather than a large number on a single night. But there are other reasons for expecting a similar but much larger difference in the case of measures of planetary markings. Nevertheless, in whatever manner derived, it is from the comparison of the above small mean or probable errors with those derived by comparing positions obtained by the method of transits on *different* nights that the supposed superiority of the micrometric method would seem to have been founded.*

5. In order to make any proper comparison between the two methods it is obviously essential to choose some mode that shall be perfectly fair and do equal justice to both. Now since an observer can only observe a transit of a spot once in the course of the same passage of the marking across the planet's disc, it is clear that it is impossible to directly derive the mean or probable error of such transit on any particular night from the observations of that night. Hence the only simple mode of comparison that appears to be suitable and yet perfectly fair to *both* methods is to compare all the observations of a spot with an ephemeris representing its motion, and thence to deduce the mean or probable error of a night's work for that particular spot. The mean result derived in this manner from a number of spots will then give a good idea of the real degree of accuracy attained by either method.

* The actual mean error of a night's work on the red spot, according to the eighty-two observations published in the above cited report, is $\pm 1^{M}\cdot 5$, or nearly four times greater than the figure stated above for the average mean probable error.

6. It is obviously essential not to rest our conclusions on the results obtained from one or two spots only and a limited number of observations, but to discuss a considerable amount of work accomplished by both methods in order to satisfactorily compare the two. The present paper therefore includes in the first place a discussion of the micrometrical work published by Hough in the *Monthly Notices*, vol. lx. p. 546 *et seq.* This should give a very fair idea of the general accuracy of the micrometrical method, as a good many spots were observed, and the observations of each spot are usually fairly numerous. Moreover, the very clear manner in which the work is presented renders the discussion of the results easy and pleasant. In order to render the comparison more complete, and to avoid doing any apparent injustice to the micrometric method, the fine series of observations of the red spot, eighty-two in number, made in the years 1879–82 has been included in the discussion.

7. We have in the first place, besides the results of 1879–82, four years' observations of the red spot. The first column below gives the year or years in which the observations were made, the second column the *mean error* * of an observation or night's work, and the third the number of observations from which this has been derived. Then follow similar particulars for the other spots observed, in the order in which they come in the *Monthly Notices*, with an additional column descriptive of the spots. The work on the red spot has been considered separately from that done on the other spots.

The Red Spot.

Year.	Mean Error.	Observations.
	″	
1879–82	1·5	82
1895–6	1·7	23
1897	2·0	15
1897–8	1·7	23
1898–9	1·6	27
Average mean error for red spot = ±	1·7	170

Other Spots.

Black spot B_3	1895–6	1·6	7
,, ...	1897	2·2	9
,, ...	1898	0·8	7
Black spot *a*	1895–6	2·2	16
Black spot *b*	1895–6	3·6	22
Black spot *a*	1898	2·7	18
Black spot *b*	1898	2·1	16

* Airy's "Theory of Errors of Observations," articles 26 and 61.

	Year.	Mean Error. M	Observations.
Black spot	1897	0·8	4
Black spot	1897	0·6	4
Black spot	1897	1·4	4
Black spot	1897	3·0	6
Black spot C	1898	1·5	7
Long black spot	1899	3·9	5
White spot	1899	1·9	14
White spot	1899	2·7	7
Average mean error for other spots = ±2·1			146

8. It should be mentioned that three spots have not been included in the discussion, owing to their having only been observed on three nights each, and this number is too small to enable the mean error to be derived with any certainty, besides which there is necessarily usually some doubt as to identification, where there are only three observations of a spot. The first white equatorial spot referred to on p. 555 of the *Monthly Notices* has also been excluded, as the identification does not seem to the writer to be correct, the four observations given appearing to relate to three different spots. Moreover, the equatorial spots are seldom suitable for our present purpose on account of the great and rapid changes of position and appearance to which they are subject.

9. It will be seen from the foregoing table that the average mean error of an observation or night's work is $\pm 1^{M}\cdot 7$ in the concluded time of transit of the red spot, and $\pm 2^{M}\cdot 1$ in the case of other spots. With regard to the latter there are considerable variations for different spots, as might have been anticipated, for spots presenting the size and uniformity of shape and aspect of the red spot are rare. These figures differ very considerably from the $\pm 0^{M}\cdot 4$ given by Hough as the probable error of a night's work. But they are the only ones at all comparable with what has been hitherto published regarding the errors of the method of transits. Even when the red spot was at its maximum plainness (1879–82) the mean error of an observation by the micrometric method was only reduced to $\pm 1^{M}\cdot 5$; a figure which, rather curiously, is exactly the same as that derived from ninety-seven contemporary observations of Schmidt made by the method of transits, as will be seen later on. It may therefore be concluded that the times of transit of spots on *Jupiter* derived by the micrometric method are subject to an average mean error of $\pm 1^{M}\cdot 7$ in the case of the red spot and $\pm 2^{M}\cdot 1$ for other spots.

10. In order to fairly compare the results obtained by the two methods it will be necessary to determine the mean error of an observation, or night's work, in the same manner as has now been done for the micrometric method, for a number of spots observed by the method of transits. Only comparatively few of

the numerous observations made according to this method have been published in the very clear and detailed manner that has been employed by Hough, so that in the great majority of cases a considerable amount of labour and time would necessarily have to be devoted to the purpose in order to extract the necessary information. Nevertheless sufficient material is already to hand to enable the mean error to which the method of transits is subject to be determined in a manner comparable with that done for the micrometric method.

11. The results contained in the following table have been extracted from various sources, and are based on the work of several observers. The list is not by any means supposed to be exhaustive, but, as every instance found by the writer and published in sufficient detail has been made use of, it is probable that they are fairly representative of the method. No attempt has been made to select cases in which the observations are especially accordant or to reject any that are unusually discordant. The only cases excluded are two or three instances of spots of which only three or four observations are available, and where consequently a satisfactory value of the mean error cannot be determined. The table contains the same particulars as the previous one, with the addition in the last column of the name of the observer and a reference to the source from which the result has been derived.

The Red Spot.

Year.	Mean Error. M	Observations.	Observer.
1880-2	1·0‡	44	Barnard (*Pub. A.S.P.*, i. 91)
1891	0·8*	11	„ (*Monthly Notices*, lii. 12)
1898	2·2†	10	Denning (*ibid.*, lix. 82)
1899	1·3†	35	„ (*ibid.*, lx. 217)
1898	1·5†	14	Gledhill (*ibid.*, lix. 82)
1899	1·0†	23	„ (*ibid.*, lx. 217)
1880-1	0·5	11	Jedrzejewicz (*A.N.*, 2366)
1898	1·4†	8	Phillips (*Monthly Notices*, lix. 82)
1899	1·6†	26	„ (*ibid.*, lx. 217)

‡ Barnard states that "among the observations of the red spot I have forty-four complete and carefully estimated transits—that is, observations of the preceding end, middle, and following end of the spot. Twenty-one of these are from a single but careful estimate of each phase. These give the probable error of a transit of the centre from the mean of the three observations = ± 1ᵐ·0. In twenty-three of these transits three estimations were made of each phase: from these I get for the transit of the middle from the mean of the nine observations the error of the transit, = ± 0ᵐ·7" (*Pub. A.S.P.* vol. i. p. 91). The mean error of a transit for 1880-82, the period in which these 44 observations were made, has been assumed to be ± 1ᵐ·0 in the table. In all other cases the mean error has been derived directly by the writer.

† In these cases the "hollow" or bay in the S. equatorial belt was observed, and not the actual spot. Owing to the faintness of the latter all observers except the writer seem during the last few years to have relinquished observing the spot in favour of the "hollow."

Year.	Mean Error. M	Observations.	Observer.
1879–81	1·5	97	Schmidt (*A.N.*, 2410)
1892	2·0*	25	Williams (*Mem. B.A.A.*, ii. 143)
1893–4	2·6*	14	,, (*ibid.*, iii. 138)
1898	2·0	15	,, (*Monthly Notices*, lix. 82)
1899	3·1	9	,, (*A.N.*, 3596)
1900	0·8	4	,, (*ibid.*, 3675)
1901	2·3	14	,, (*ibid.*, 3786)
1902	2·2	22	,, (*ibid.*, 3875)
Average mean error	= ± 1 6	382	

Other Spots.

	Year.	Mean Error. M	Observations.	Observer.
Dark spot ...	1895–6	2·4	13	Brenner (*Monthly Notices*, lvi. 531)
White spot I...	1898	2·9	6	Denning (*ibid.*, lix. 85)
Dark ellipse ...	1898	2·7	5	,, (*ibid.*, lix. 87)
Dark spot II....	1898	(7·5)	6	,, (*ibid.*, lix. 88)
Dark spot ...	1895–6	2·0	30	Gledhill (*ibid.*, lvi. 531)
Dark spot ...	1895–6	2·4	5	Merfield (*ibid.*, lvi. 531)
Dark ellipse ...	1898	(5·8)	8	Phillips (*ibid.*, lix. 87)
Dark spot ...	1898	2·1	9	,, (*ibid.*, lix. 88)
Dark spot ...	1895–6	0·7†	7	Rambaut (*ibid.*, lvi. 531)
Dark spot ...	1862	0·6	12	Schmidt (*A.N.*, 2342)
Sharp elbow (Ecke)	1862	3 0	6	,, (*ibid.*, 1973)
White spot ...	1866	1·2	5	,, (*ibid.*, 1973)
White spot ...	1890	2·6*	20	Williams (*Monthly Notices*, lviii. 323)
End of belt ...	1881	1·2	9	,, (*ibid.*, lix. 29)
Dark spot ...	1888	2·6	10	,, (*ibid.*, lix. 30)
White spot II.	1898	1·0	5	,, (*ibid.*, lix. 84)
White spot I...	1898	2·7	5	,, (*ibid.*, lix. 85)
White spot III.	1898	0·6	5	,, (*ibid.*, lix. 85)
Dark spot ...	1892	1·7	16	,, (*A.N.*, 3528)
Dark spot ...	1890	2·5	8	,, (*ibid.*, 3528)
Average mean error	= ± 1·9		176	

12. There are some circumstances affecting the results contained in the foregoing table that do not favour the method of

† It is assumed that Rambaut's observations were made by the method of transits, but the writer has not seen the original paper from which they were taken.

transits. In the cases marked with an asterisk the motion of the spot was assumed to conform to Marth's ephemeris. This, however, was probably never exactly so, and consequently the stated values of the mean error are no doubt somewhat too great. Also in several cases the observations of a spot were compared with an ephemeris representing its mean motion as derived from the work of several observers, and a discussion based solely on the observations of the particular observer would probably result in reducing the residuals, and so the mean error. It may therefore be safely assumed that the average mean errors derived from the table are, if anything, too large. Schmidt's fine observations of the red spot are corrected for his "Correction C," representing a constant error varying according to the hour-angle of *Jupiter* at the time of observation. Schmidt seems to have connected this constant error with the inclination of the belts to the line joining the two eyes when the planet is far east or west of the meridian, and the head is held level. Probably he was right in this, as no similar error seems to be revealed by the work of other observers, most of whom, it is believed, take the precaution of keeping the eyes parallel to the belts when observing the transits. The two bracketed results in the list of "other spots" have been omitted in forming the average, as they are largely in excess of any others, and relate to two faint and difficult spots in a rather high S. latitude.

13. We are now in a position to make a proper comparison of the results obtained by the two methods. We have:—

Red Spot.

	Average Mean Error. M	No. of Series or Spots.	No. of Obs.
Micrometric Method ...	= ± 1·7	5	170
Method of Transits ...	= ± 1·6	17	382
Other Spots.			
Micrometric Method ...	= ± 2·1	15	146
Method of Transits ...	= ± 1·9	18	176

It will be seen that, so far from the micrometric method showing any superiority in point of accuracy over the method of transits, the advantage is rather in favour of the latter. The difference is, however, only slight, and it may therefore be concluded that the two methods yield results of about equal accuracy.

14. By largely increasing the number of measures of a spot theoretically* greater accuracy could be secured by the micrometric method than by the method of transits, though it would

* Practically it seems doubtful, however, whether any real increase in accuracy would result, on account of liability to bias in the later measures. Some years ago, when measuring some photographs of *Jupiter*, the writer found this liability to bias so marked as to render it expedient not to measure the different images of the planet in order of time; but this cannot be managed in the case of direct measures on the planet at the telescope.

seem that the beneficial effect of this would be slight. For the mean error is no doubt made up chiefly of three quantities: (*a*) the real errors of observation; (*b*) apparent errors due to the motion of the spot not being perfectly uniform, and (*c*) apparent errors produced by changes in the form and appearance of the spot. The apparent errors *b* and *c* will balance themselves when a large number of observations and many spots are discussed, so that their effect may be considered as the same for both methods. Now if $\pm 0^{M}\cdot 4$ be taken to represent the real error, then doubling the accuracy of the observations would only result in reducing the mean error from, say, $2^{M}\cdot 0$ to $1^{M}\cdot 8$. Considering the nature of the problems requiring to be investigated, it seems doubtful whether the great increase in the labour and time necessitated by the larger number of measures would be compensated for by the slight benefit obtained.

15. It has been said that an eye-estimate of a transit is, in a manner, equivalent to a single micrometer measure, and liable to the same error of bisection, &c.; but this, I think, is clearly not so. In a complete single measure, as described by Hough, one micrometer wire is placed tangent to the preceding limb of the planet, and the other made to bisect the spot. Both wires are then carried across the disc, and one placed tangent to the following limb and the other made to bisect the spot. It would seem, therefore, that in every complete measure there are four distinct sources of error. There is the liability to error in twice placing the wires tangent to the two limbs, and there is the liability to error in twice bisecting the spot. In the method of transits the comparisons are made direct with the true limbs, so that the liability to error in placing the wires tangent to the latter does not arise. Then the spot itself takes the place of the other wire, and there is consequently only a single bisection here—the bisection of the disc. Hence there are four distinct sources of error to which a single measure by the micrometric method is liable, compared with one in the method of transits. The effect produced by the rapid rotational motion of the spots must also not be lost sight of in making comparisons of this kind. If the rotation of *Jupiter* were performed in one hour, instead of nearly ten, the advantage would be all on the side of the method of transits.

16. It is not proposed to consider here the question of personal equation. It is only of importance where contemporary observations of the same marking by two or more observers are combined, but there is no reason for supposing that it would be any less in the case of the micrometrical method than it is known to be where the method of transits is used. It is no doubt chiefly due to different observers not observing markings, irregular both in shape and intensity, and often very highly so, in exactly the same way.

17. The conclusions derived from the foregoing investigation may be shortly stated as follows:

(1) The alleged great or indeed any superiority in accuracy of the micrometric method over the method of transits does not exist.
(2) The two methods give results of about equal accuracy.
(3) The times of transit derived from either method are subject to an average mean error of $\pm 2^{M}\cdot 0$, but in the case of very prominent and definite spots, such as the red spot in the years 1879–82, the mean error may be reduced to $\pm 1^{M}\cdot 5$ or even less. On the other hand, it often largely exceeds $2^{M}\cdot 0$, especially in the case of faint or irregular markings.

18. As the rotation of *Saturn* is performed in nearly the same time as that of *Jupiter*, it is evident that the first two conclusions apply with equal force to the former planet; so that the statement made in the *Monthly Notices*, vol. lxiv. 122, as to the great superiority of the micrometric method for determining the positions of spots on *Saturn* does not appear to be justified.

Hove: 1904 *February* 12.

Positions and Photographic Magnitudes of ninety Stars surrounding the Variable R Cygni. By Joel H. Metcalf, Ph.D.

(*Communicated by Professor H. H. Turner.*)

1. The suggestion that such work as the following might have value came from the recent experience of the Oxford Observatory in the work on the Rousdon variable stars. Existing positions for the comparison stars are usually given only roughly, and sometimes there is a doubt about identification in consequence. It was therefore felt that accurate places would be desirable. In this particular case it will be seen that the places good enough for a working chart at Rousdon were sometimes 5′ in error.

2. There would also seem to be good general reasons for accurate surveys of special regions, especially those that were previously known to contain objects of interest. On this account the region of variables is sure to be under close observation, and a detailed study of them will be given which it will not be possible to extend to the whole sky.

3. The photographic plate, which is the basis of the following catalogue, was taken at the University Observatory with the 13-inch astrographic telescope on the night of 1903 November 15. The plate was exposed from $1^{h}\ 8^{m}\ 50^{s}\cdot 0$ to $2^{h}\ 9^{m}$ local sidereal time. As a few seconds of time were lost in the middle of the exposure the time was approximately one hour. The plate is a good one and shows stars of about the 14th magnitude, as one

would judge by comparison with stars of known brightness. The variable itself has a diameter of 614, or 18″·42, which from the formulæ which were used would give a magnitude of M_1 7·4 and M_2 7·8 respectively.

The centre of the plate adopted was R.A. $19^h\ 32^m\ 56^s$, D.+49° 52′ 30″·0, which is approximately the position of the variable. It was measured in the following weeks by myself in the direct and reverse positions, as is the custom in the work on the astrographic chart, with the ordinary micrometer of the observatory, with the réseau and divided scale in the eye-piece.

4. To obtain the constants of the plate twelve stars were selected from the A.G. Catalogues of Bonn and Harvard, which were distributed in four groups of three each as near as possible to the edge of the plate. To Bonn 13117 the proper motion given in the catalogue was applied. To the others none was applied.

For these twelve stars standard coordinates were calculated in units of 5′ = 1 réseau interval for epoch 1855, referred to plate centre $19^h\ 32^m\ 56^s{\cdot}0$ +49° 52′ 30″·0.

TABLE I.

Star.	Standard Coords. x+13·0000.	y+13·0000.	Δx_1.	Δy_1.	Δx_2.	Δy_2.
East						
B 13316	24·1407	10·1577	−·3867	+1·1613	+·0016	−·0030
B 13323	24·4883	14·8213	−·4503	+1·1632	+·0009	−·0005
H 6152	22·9616	18·3597	−·4966	+1·1448	+·0008	+·0022
West						
B 13102	2·4846	14·5730	−·4276	+·8925	+·0019	+·0005
B 13101	2·4267	16·0430	−·4447	+·8870	+·0046	−·0030
B 13117	3·8283	12·9422	−·4118	+·9103	−·0027	+·0003
North						
H 6095	9·9691	25·2356	−·5826	+·9779	−·0037	+·0025
H 6116	15·8947	24·1169	−·5687	+1·0466	·0000	−·0033
H 6129	18·2566	24·1675	−·5716	+1·0820	−·0004	+·0026
South						
B 13230	14·9414	1·9205	−·2669	+1·0655	+·0035	+·0078
B 13228	14·6251	2·5349	−·2831	+1·0476	−·0048	−·0052
B 13244	17·1651	4·5080	−·3091	+1·0820	−·0023	−·0007

Under Δx_1, Δy_1 are given the differences directly obtained from the measures. From these the formulæ for correction were found to be

$$\Delta x_1 = -{\cdot}00066x - {\cdot}01343y - 0{\cdot}2208$$

$$\Delta y_1 = +{\cdot}01243x - {\cdot}00070y + 0{\cdot}8772$$

Applying these corrections to the differences, we obtain the residuals shown as Δx_2 and Δy_2.

5. Applying these constants of reduction to the stars measured, and afterwards reducing the standard coordinates to R.A. and Decl., we obtain the results shown in Table II.

Columns 2 and 3 refer to Father Hagen's Atlas of Variable Stars, and

Columns 4 and 5 to the *Memoirs R.A.S.* Vol. LV. p. xxxiii (now in the press). The magnitudes were formed by adding to the "Adopted Mag." the adopted correction given in the last column whenever available. The rough places there given are sometimes as much as 5′ in error, as already remarked in § 1; but it is hoped that the stars have been correctly identified.

In column 6 is given the diameter of the star disc on the photograph measured in units of 0″·03 (or 0·0001 réseau intervals).

In columns 7 and 8 magnitudes are deduced from these diameters from two formulæ hereafter to be explained.

TABLE II.

Stars near R Cygni.

Ref. No.	Hagen No.	Hagen Mag.	Rouslon No.	Rouslon Mag.	Photographic Diameter.	M_c	M_p	Coordinates, Epoch 1855·0. Standard X.	Standard Y.	R.A. 19h + m	R.A. 19h + s	N. Decl. 49° + ′	N. Decl. 49° + ″
1	...	...	8	9·8	347	10·5	10·3	−5·921	−0·07;	29	52·29	51	58·7
2	19	9·5	...	...	469	9·1	9·0	−4·633	+4·995	30	30·97	77	22·9
3	...	...	15	11·0	170	12·5	13·4	−4·554	+1·263	30	34·36	58	43·5
4	20	9·6	...	...	398	9·9	9·7	−4·297	−2·580	30	43·25	39	31·2
5	5	8·6	3	8·8	500	8·7	8·7	−4·137	+1·240	30	47·34	58	37·6
6	24	9·8	...	...	439	9·4	9·3	−3·688	−3·225	31	2·18	36	19·0
7	4	8·4	4	8·5	644	7·0	7·6	−3·233	+5·176	31	14·77	78	20·0
8	40	11·3	...	...	295	11·1	11·0	−2·747	+0·812	31	30·63	56	31·7
9	42	11·4	...	...	297	11·1	11·0	−2·734	+1·848	31	30·87	61	42·5
10	37	10·9	...	...	349	10·5	10·3	−2·601	−0·945	31	35·41	47	44·8
11	58	12·4	...	...	2,6	11·7	12·2	−2·357	+1·747	31	42·63	61	12·7
12	12	9·1	...	...	424	9·6	9·4	−2·363	−5·366	31	43·34	25	38·8
13	25	9·8	...	...	484	8·9	8·9	−2·332	−1·830	31	43·86	43	19·6
14	47	11·7	...	...	236	11·8	12·0	−2·258	+1·840	31	44·77	61	40·7
15	61	12·5	...	...	140	12·9	14·2	−2·177	+2·820	31	48·11	66	34·8
16	44	11·5	...	...	295	11·1	11·0	−2·177	−1·253	31	48·59	46	12·9
17	48	11·7	...	...	220	12·0	12·3	−2·029	−0·141	31	53·06	51	46·7
18	28	10·0	...	...	407	9·8	9·6	−1·988	−2·217	31	54·54	41	23·9
19	57	12·2	...	...	220	1·.0	12·3	−1·783	+2·597	32	0·42	65	28·3
20	...	...	14	11·5	204	12·1	12·6	−1·759	−0·135	32	1·44	51	48·7
21	18	9·5	...	...	408	9·8	9·7	−1·760	−3·560	32	1·71	34	41·2
22	53	11·9	...	...	181	12·4	13·1	−1·738	+2·209	32	1·86	63	31·9

Ref. No.	Hagen No.	Hagen Mag.	Rousion No.	Rousion Mag.	Photographic Diameter.	M_1.	M_2.	Coordinates, Epoch 1855·0. Standard X.	Standard Y.	R.A. 19^h+ m	R.A. s	N. Decl. 49°+ ′	N. Decl. ″
23	7	8·9	...	...	520	8·5	8·5	−1·740	−6·074	32	2·56	22	7·0
24	2	7·0	2	7·4	800	5·2	6·7	−1·711	+0·488	32	2·86	54	55·7
25	22	9·7	...	...	395	9·9	9·7	−1·530	+5·263	32	8·09	78	48·3
26	17	9·4	...	...	394	9·9	9·7	−1·465	−4·173	32	10·86	31	37·6
27	51	11·9	13	11·5	220	12·0	12·3	−1·443	−0·809	32	11·28	48	26·8
28	38	11·1	...	...	326	10·8	10·6	−1·419	+1·418	32	11·87	59	34·9
29	50	11·8	...	...	246	11·7	11·8	−1·369	−0·112	32	13·52	51	55·9
30	...	...	...	...	164	12·6	13·6	−1·287	−0·225	32	16·08	51	22·1
31	34	10·7	...	...	287	11·2	11·1	−1·248	+2·711	32	17·09	66	2·9
32	...	...	...	...	136	13·0	14·4	−1·141	−0·035	32	20·59	52	19·2
33	...	...	...	...	100	13·3	15·7	−1·126	+0·170	32	20·96	53	20·7
34	10	9·0	...	...	465	9·1	9·0	−1·096	−4·489	32	22·24	30	3·0
35	...	...	...	...	100	13·3	15·7	−1·076	−0·769	32	22·65	48	39·0
36	...	...	...	...	100	13·3	15·7	−1·043	+0·848	32	23·58	56	44·1
37	...	...	...	...	100	13·3	15·7	−0·983	+0·635	32	25·46	55	40·3
38	...	...	...	...	100	13·3	15·7	−0·858	−0·769	32	29·41	48	39·1
39	...	...	...	...	240	11·8	11·9	−0·744	+0·038	32	32·91	52	41·3
40	1	4·9	1	6·4	1440	− 2·2	4·1	−0·734	+0·182	32	33·22	53	24·5
41	...	...	...	...	150	12·8	13·9	−0·724	−0·189	32	33·55	51	33·2
42	55	12·1	...	...	248	11·7	11·8	−0·590	+1·251	32	37·65	58	45·2
43	60	12·5	...	...	141	12·9	14·2	−0·584	−0·544	32	37·89	49	46·7
44	...	...	17	12·5	215	12·0	12·4	−0·560	+0·408	32	38·61	54	32·3
45	36	10·8	11	10·7	352	10·5	10·2	−0·507	−1·473	32	40·30	45	8·0
46	...	...	...	...	120	13·1	15·0	−0·398	+0·179	32	43·65	53	23·7
47	56	12·2	...	...	284	11·2	11·2	−0·296	−0·460	32	46·82	50	12·0
48	...	...	...	...	100	13·3	15·7	−0·132	−0·182	32	51·91	51	35·4
49	...	...	16	13·3	210	12·1	12·5	−0·121	−0·424	32	52·25	50	22·8
50	...	...	...	...	100	13·3	15·7	−0·090	+0·098	32	53·21	52	59·4
51	59	12·4	12	11·8	218	12·0	12·3	−0·047	−0·587	32	54·54	49	33·9
52	R Cygni		...	...	614	7·4	7·8	−0·016	−0·003	32	55·52	52	29·1
53	14	9·3	10	9·6	488	8·9	8·8	+0·061	+0·294	32	57·89	53	58·2
54	...	...	...	...	100	13·3	15·7	+0·366	+0·866	33	7·37	56	49·8
55	...	...	...	...	160	12·6	13·7	+0·420	+0·537	33	9·05	55	11·1
56	...	...	...	...	100	13·3	15·7	+0·466	+0·875	33	10·48	56	52·5
57	49	11·8	...	...	193	12·3	12·8	+0·472	−1·773	33	10·60	43	38·1
58	23	9·8	...	...	420	9·7	9·5	+0·501	+3·658	33	11·64	70	47·3
59	...	...	...	...	100	13·3	15·7	+0·623	−0·311	33	15·32	50	56·6
60	...	...	...	...	100	13·3	15·7	+0·636	−1·030	33	15·70	47	20·9

Ref. No.	Hagen No.	Hagen Mag.	Rosedon No.	Rosedon Mag.	Photographic Diameter.	M_x.	M_y.	Coordinates, Epoch 1855·0. Standard X.	Standard Y.	R.A. 19^h+ m	s	N. Decl. $49^\circ+$ ′	″
61	...	...	...	...	100	13·3	15·7	+0·688	+0·913	33	17·38	57	3·8
62	...	...	...	...	110	13·2	15·3	+0·743	+0·134	33	19·06	53	10·1
63	27	10·0	...	...	335	9·9	9·7	+0·763	+2·579	33	19·78	65	23·6
64	...	...	...	...	100	13·3	15·7	+0·838	+0·962	33	22·06	57	18·4
65	35	10·8	...	...	327	10·8	10·6	+0·870	+1·356	33	23·07	59	16·6
66	43	11·4	...	...	269	11·4	11·4	+0·909	−1·301	33	24·16	45	59·5
67	...	...	...	...	200	12·2	12·7	+0·911	−0·135	33	24·27	51	49·3
68	54	11·9	...	...	226	11·9	12·2	+0·956	−2·162	33	25·56	41	41·2
69	...	...	...	...	100	13·3	15·7	+0·981	+0·449	33	26·47	54	44·5
70	39	11·2	...	...	294	11·1	11·0	+0·985	+1·303	33	26·64	59	0·7
71	...	...	...	...	110	13·2	15·3	+1·111	−0·191	33	30·47	51	32·4
72	46	11·7	...	...	207	12·1	12·5	+1·186	−2·283	33	32·66	41	4·8
73	52	11·9	...	...	203	12·2	12·7	+1·194	−2·579	33	32·89	39	35·9
74	41	11·4	...	...	269	11·4	11·4	+1·209	+1·268	33	33·60	58	50·0
75	...	...	...	...	100	13·3	15·7	+1·253	+0·705	33	34·93	56	1·1
76	8	8·9	...	...	504	8·7	8·7	+1·288	+4·759	33	36·30	76	17·3
77	31	10·2	...	...	413	9·7	9·5	+1·420	+0·563	33	40·11	55	18·4
78	45	11·6	...	...	293	11·1	11·0	+1·489	−0·369	33	42·18	50	38·7
79	32	10·3	...	...	323	10·8	10·6	+1·610	−0·214	33	45·94	51	25·1
80	29	10·0	...	...	333	10·7	10·5	+2·001	+1·230	33	58·23	58	38·0
81	30	10·1	...	...	355	10·4	10·2	+2·098	+1·908	34	1·32	62	1·3
82	33	10·5	...	...	349	10·5	10·3	+2·473	−0·570	34	12·67	49	37·4
83	11	9·1	5	9·2	424	9·6	9·4	+2·618	−1·383	34	17·05	45	33·4
84	16	9·4	...	...	415	9·7	8·8	+3·013	+3·226	34	30·03	68	35·4
85	6	8·8	6	8·8	418	9·7	9·5	+3·468	−0·002	34	43·62	52	26·3
86	15	9·3	...	...	388	10·0	9·8	+3·440	+5·672	34	43·81	80	48·5
87	13	9·1	7	9·1	407	9·8	9·6	+4·370	+1·510	35	11·97	59	58·0
88	26	9·9	...	...	418	9·7	9·5	+4·930	+3·931	35	30·04	72	2·9
89	21	9·6	...	...	352	10·4	10·2	+5·189	+5·351	35	38·54	79	8·2
90	9	9·0	...	...	405	9·8	9·6	+5·655	−2·352	35	50·78	40	36·2
91	3	8·2	...	...	648	7·0	7·6	+8·469	+2·034	37	19·71	62	21·5

6. To convert the diameters of the star images into photographic magnitudes they were first plotted against Hagen's magnitudes, and it was found that a straight line would very fairly represent the results from mag. 12·5 to 8·5, but for the brighter stars the diameters were too large.

The formula deduced was

$$\text{Magnitude} = 14{\cdot}5 - {\cdot}0116 \times \text{diameter}.$$

The magnitudes computed by using it are given under heading 7, designated M_1. It will be seen that the Hagen stars Nos. 1 to 4 cannot be represented by this even approximately.

Accordingly the log (diameter) was plotted against the Hagen magnitudes, and it was found that the following straight line represented the results very nearly :—

$$\text{Magnitude} = 35{\cdot}7 - 10 \log (\text{diameter}).$$

This formula is much better for the brighter stars, but fails for the fainter ones.

It may be noticed that on transforming it to the form

$$\text{Intensity of light} \propto 10^{-0{\cdot}4 \times \text{mag.}} \propto (\text{diameter})^4$$

this formula corresponds to the supposition that the diameter increases as the fourth root of the exposure time, if we suppose exposure time exactly equivalent to increased intensity of light. This law was obtained by Pritchard, *Proc. R.S.* vol. xl., though other laws have been found by other observers—the inverse square by Rutherfurd and the inverse cube by Prof. Turner at Greenwich in 1889 (see *Monthly Notices*, xlix. p. 292).

The results from this formula are given under head M_2.

7. Father Hagen gave the places of his comparison stars only approximately to the nearest second of time in R.A. and the nearest 0′·1 in D, and so no comparison is printed, though one was made and sent to him. The differences between his positions and those obtained by me are often quite large. This would seem to show the need of similar work being done for the whole list of variable stars.

Oxford University Observatory.

On the Absolute Proper Motions of Certain Double Stars showing Large Relative Motion. By H. Furner and J. Storey.

(*Communicated by the Astronomer Royal.*)

During Mr. Lewis' work on double stars he has found a number which show large relative motions, and it is of interest to examine the meridian observations in order to determine these absolute motions. A list of these stars was given to us, and a first instalment of our work upon them we now beg to lay before the Society.

Some 100 stars have been examined, but owing to the lack of sufficient material we have confined our present paper to such stars as show by micrometer measures a relative motion of at least 0″·1 per annum.

All available catalogue places have been made use of and reduced to epoch 1900·0, applying Professor Auwers' systematic corrections as given in *Ast. Nach.* 3195-6 and 3463. The stars from the Lalande Catalogue were reduced to the epoch 1800 by the help of Dr. E. von Asten's tables (*Vierteljahrschrift der Astronomische Gesellschaft*, Jahrgang III., Supplement). The graphical method has been employed in determining the proper motions, greater weight having been given to Struve's *Positiones Mediæ* 1830·0 for the earlier catalogues. It is of interest to note that in about half the stars here considered the larger proper motion belongs to the fainter component; a circumstance which may be explained by the fact that if the brighter star had the larger proper motion in all probability this would have already been detected.

Catalogue.	Epoch.	R.A. 1900·0.	P.M. applied.	N.P.D. 1900·0.	P.M. applied.
		Σ 63. Mag. 8·2, 11·2.			
		$0^h\ 44^m$	s	78° 42′	″
W. B.	1825	58·47	58·63	41·6	45·0
Pos. Med. ...	32·4	58·62	58·76	45·4	48·4
Camb. Obs. ...	43·9, 42·8	58·68	58·79	45·7	48·3
Göttingen ...	60	58·75	58·83	42·1	43·9
Leipzig	71·4	58·71	58·77	48·2	49·5
Washington ...	89·7	58·72	58·74	46·4	46·9
Cincinnati ...	89·8	58·72	58·74	48·9	49·4
Greenwich ...	89·8	58·75	58·77	48·4	48·9
Cape	89·8	58·76	58·78	47·9	48·4
Radcliffe ...	90·9	58·74	58·76	47·6	48·0
P.M. of A		+0″·031		+0″·045	

The relative motion of B to A from micrometer measures:

	−0″·159	0″·000
P.M. of B	−0″·128	+0″·045

Σ 175. Mag. 7·9, 8·4.

		$1^h\ 45^m$	s	69° 22′	″
Pos. Med. ...	1835·5	30·92	31·03	44·4	47·9
Berlin B.... ...	82·0	31·00	31·03	46·9	47·9
P.M. of A		+0″·024		+0″·054	

The relative motion of B to A from micrometer measures:

	−0″·026	−0″·098
P.M. of B	0″·000	−0″·044

Catalogue.	Epoch.	R.A. 1900·0.	P.M. applied.	N.P.D. 1900·0.	[P.M. applied.
		Σ 197. Mag. 7·2, 8·2.			
		1h 55m		55° 10′	
		s	s	″	″
Lalande	1794·9	10·27	10·74	59·0	50·1
W. B.	1828·8	10·56	·90	51·6	45·6
Pos. Med. ...	33·8	10·58	·90	54·3	48·7
Brussels	67·6	10·60	·75	50·8	48·1
Leiden	73·5	10·73	·85	50·7	48·4

P.M. of A +0″·056 −0″·085

The relative motion of B to A from micrometer measures :

−0″·082 +0″·064

Σ 436. Mag. 7·0, 8·2.

			3h 36m		2° 56′	
			s	s	″	″
W. B.	1825		7·94	8·19	22·6	20·4
Pos. Med. ...	36·8		7·96	·17	24·0	22·2
Schjellerup ...	65		7·88	·00	26·1	25·1
Radcliffe Obs. ...	67·9		7·96	·07	...	...
Radcliffe... ...	91·7		8·16	·19	22·4	22·2

P.M. of A +0″·050 −0″·029

The relative motion of B to A from micrometer measures :

−0″·185 +0″·040

P.M. of B −0″·135 +0″·011

Σ 499. Mag. 9·2, 9·3, 11·2.

			4h 3m		66° 11′	
			s	s	″	
Lalande	1798·4		44·17	44·36	22·1	...
W. B.	1825		44·16	·30	25·9	...
Pos. Med. ...	27·0		44·08	·22	25·7	...
Rumker	36		43·97	·09	24·7	...
Romberg	75·5		44·26	·31	25·5	...
Berlin B.... ...	80·7		44·20	·24	26·1	...

P.M. of A +0″·029 0″·000

The micrometer measures show that the stars A and B are relatively fixed ; hence A and B have the same P.M. The relative motion of C to A from micrometer measures :

−0″·210 −0″·037

P.M. of C −0″·181 −0″·037

Catalogue.	Epoch.	R.A. 1900·0.	P.M. applied.	N.P.D. 1900·0.	P.M. applied.
		Σ 853. Mags. 7·8, 8·3.			
		$6^h\ 3^m$		78° 19′	
		s	s	″	″
Pos. Med. ...	1833·7	35·59	35·11	34·5	...
Schjellerup ...	65	35·31	·06	32·5	...
Leipzig I. ...	70·6	35·26	·04	35·4	...
Washington ...	76·9	35·31	·14	34·1	...
P.M. of A		−0″·106		0″·000	

The relative motion of B to A from micrometer measures:

	+0″·071	−0″·084
P.M. of B	−0″·035	−0″·084

Σ 1142. Mag. 8·0, 10·4.

		$7^h\ 42^m$		76° 20′	
		s	s	″	″
Lalande	1796·2	46·96	46·74	3·9	2·5
W. B.	1825	47·23	47·07	2·7	1·7
Pos. Med. ...	37·2, 37·7	46·91	46·78	2·1	1·3
Leipzig I. ...	70·6	46·87	·81	2·4	2·0
Paris	74·1, 75·1	46·80	·75	1·7	1·4
P.M. of A		−0″·031		−0″·013	

The relative motion of B to A from micrometer measures:

	+0″·043	+0″·175
P.M. of B	+0″·012	+0″·162

Σ 1202. Mag. 7·7, 9·8.

		$8^h\ 8^m$		78° 30′	
		s	s	″	″
Lalande	1796·2	5·13	4·91	54·2	60·4
Piazzi	1800	5·23	5·03	55·9	61·9
W. B.	25	5·09	4·94	53·2	57·7
Pos. Med. ...	32·7	5·05	4·92	55·9	59·9
Madras	36·4	5·25	5·12	55·8	59·6
Paris, 1860 ...	60·6, 60·1	4·99	4·91	57·2	59·6
Leipzig I. ...	69·5	4·98	4·92	57·7	59·5
Romberg ...	75·4	5·00	4·95	58·5	60·0
Paris, 1875 ...	79·2	5·00	4·96	59·8	61·0
P.M. of A		−0″·029		+0″·060	

The relative motion of B to A from micrometer measures:

	−0″·105	+0″·060
P.M. of B	−0″·134	+0″·120

Catalogue.		Epoch.	R.A. 1900·0.	P.M. applied.	N.P.D. 1900·0.	P.M. applied.
		Σ 1329.	Mag. 8·3, 8·5.			
			$9^h\ 10^m$ s	s	90° 49′	″
Pos. Med.	...	1827·2	38·67	38·30	21·1	27·0
Munich ...	...	42·9	38·52	38·24	21·7	26·3
Nicolajew	...	88·5	38·33	38·27	26·2	27·1

P.M. of A $-0''\cdot075$ $+0''\cdot081$

The relative motion of B to A from micrometer measures:

$+0''\cdot065$ $-0''\cdot099$

Σ 1847. Mag. 8·5, 9·8.

			$14^h\ 23^m$ s	s	99° 45′ ″	″
W. B. ...	...	1825	18·09	17·90	22·5	29·3
Camb. Obs.	...	1843·2	18·40	18·26	22·0	27·1
Pos. Med.	...	43·3, 45·5	18·59	18·45	21·7	26·7
Munich...	...	61·4	18·40	18·30	24·2	27·7
Paris ...	...	63·4, 60·7	18·53	18·44	24·4	27·9
Romberg	...	75·0	18·50	18·44	24·5	26·7

P.M. of A $-0''\cdot035$ $+0''\cdot090$

The relative motion of B to A from micrometer measures:

$-0''\cdot105$ $-0''\cdot042$

P.M. of B $-0''\cdot140$ $+0''\cdot048$

Σ 1893. Mag. 8·4, 10·0.

			$14^h\ 51^m$ s	s	60° 7′ ″	″
Lalande ...	...	1795·5	1·22	0·91	18·0	22·7
Pos. Med.	...	1834·0	1·12	0·92	15·3	18·3
Paris ...	...	67·4	0·94	0·84	15·0	16·5
Leiden ...	...	71·2	1·00	0·91	17·3	18·6

P.M. of A $-0''\cdot036$ $+0''\cdot045$

The relative motion of B to A from micrometer measures:

$+0''\cdot062$ $+0''\cdot079$

P.M. of B $+0''\cdot026$ $+0''\cdot124$

Catalogue.	Epoch.	R.A. 1900·0.	P.M. applied.	N.P.D. 1900·0.	P.M. applied.
		Σ 2185. Mag. 7·0, 10·0, 7·7.			
		$17^h\ 27^m$ s	s	83° 55′	
W. B.	1822·5	55·23	52·52	74″·1	46″·2
Munich	61·6	54·18	·84	60·7	46·9
Brussels	71·1	53·65	·64	..	...
Glasgow	79·0, 77·4	53·50	·71	53·5	45·4
Romberg ...	80·4	53·47	·78	51·9	44·8
Leipzig II. ...	84·4	53·33	·78	51·0	45·4
Cincinnati ...	90·3	53·18	·84	50·0	46·5
P.M. of C		−0″·522		−0″·360	
		$17^h\ 29^m$ s	s	83° 55′	
Lalande	1794·6	55·62	55·36	22″·5	27″·6
W. B.	1825	55·38	·19	21·9	25·6
Pos. Med. ...	42·4	55·30	·16	24·3	27·1
Munich	61·6	55·30	·20	23·2	25·1
Brussels	70·0, 70·8	55·06	54·99	25·1	26·5
Paris	76·5	55·01	·95	25·1	26·2
Glasgow	79·3	55·13	55·08	26·0	26·9
Leipzig II. ...	84·4	55·12	·08	25·3	26·0
Cincinnati ...	90·3	55·20	·18	26·6	27·0
P.M. of A		−0″·037		+0″·049	

The micrometer measures give no appreciable motion between the stars A and B, so they probably have the same P.M. Now

P.M. of C	−0″·522	−0″·360
P.M. of A	−0″·037	+0″·049

Hence relative motion of C to A from computed P.M.'s is

−0″·485	−0″·409

The relative motion of C to A from micrometer measures :

−0″·580	−0″·488

		Σ 2514. Mag. 9·0, 11·3.			
		$19^h\ 16^m$ s		82° 29′	
Pos. Med. ...	1833·0	49·86	...	19″·1	22″·5
Christiania ...	75·5	49·68	...	23·1	24·4
Greenwich ...	99·6	49·83	...	22·5	22·5
P.M of A		0″·000		+0″·052	

The relative motion of B to A from micrometer measures :

+0″·024	−0″·122
P.M. of B +0″·024	−0″·070

Catalogue.	Epoch.	R.A. 1900·0.	P.M. applied.	N.P.D. 1900·0.	P.M. applied.
	Σ 2515.	Mag. 8·0, 9·0.			
		19h 20m		68° 40′	
		s	s	″	″
Lalande	1794·7	14·71	14·80	53·6	48·2
W. B.	1825·6	14·46	·53	49·2	45·4
Pos. Med. ...	28·6	14·87	·93	54·3	50·7
Berlin B. ...	81·6	14·93	·95	51·6	50·7

P.M. of A +0″·013 −0″·051

The relative motion of B to A from micrometer measures :

−0″·012 +0″·109

P.M. of B 0″·000 +0″·058

Σ 2658. Mag. 7·0, 9·1, 10·1.

		20h 11m		37° 11m	
		s	s	″	″
Lal. F.	1790	0·89	1·58	17·0	59·6
Groombridge ...	1811·7	0·66	·22	15·2	0·3
Pos. Med. ...	24·7	0·89	·37	14·4	1·8
Cambridge Obs. ...	44·7, 43·2	1·29	·65	10·4	0·8
Radcliffe ...	45·4, 45·7	1·05	·40	10·0	0·9
Brussels	71·4, 65·8	1·12	·30	6·3	0·6
Cambridge U.S....	77·3	1·24	·38	4·4	0·5
Cincinnati ...	95·2	1·17	·21	2·6	1·1
Greenwich ...	1900·5	1·36	·36	0·6	0·6

P.M. of A +0″·058 −0″·168

The relative motion of B to A from micrometer measures :

+0″·005 −0″·003

Hence B as the same P.M. as A.

The relative motion of C to A from micrometer measures :

−0″·032 +0″·136

Hence P.M. of C is small.

Σ 2734. Mag. 8·2, 8·7.

			20h 49m		77° 16′	
			s	s	″	″
Pos. Med. ...	1828·2		18·28	18·48	30·3	30·5
Leipzig I. ...	82·3		18·43	18·48	30·5	30·5

P.M. of A +0″·040 +0″·003

The relative motion of B to A from micrometer measures :

−0″·090 −0″·070

P.M. of A −0″·050 −0″·067

Catalogue.	Epoch.	R.A. 1900·0.	P.M. applied.	N.P.D. 1900·0.	P.M. applied.
		Z 2865. Mag. 8·5, 9·0.			
		$22^h\ 9^m$		20° 16′	
		s	s	″	″
Lalande F. ...	1789·7	10·52	11·90	31·3	29·1
Pos. Med. ...	1823·6	10·62	·57	29·6	28·1
Oeltz. Arg. (N.)...	41·6	10·50	·23	31·8	30·6
Brussels	67·3, 67·8	11·17	·58	28·7	28·1
Christiania ...	75·5	11·12	·43	27·6	27·1
Greenwich ...	99·2	11·52	·53	28·2	28·2

P.M. of A +0″·064 −0″·020

The relative motion of B to A from micrometer measures :

−0″·072 +0″·054

P.M. of B −0″·008 +0″·034

Note on the Determinations of Positions and Magnitude of Stars in the Greenwich Astrographic Catalogue.

(Communicated by the Astronomer-Royal.)

The Introduction to vol. i. of the Greenwich Section of the *Astrographic Catalogue*, which is now in the press and will shortly be published, contains short discussions of the personality of the measurers, the probable error of the measures and of resulting right ascensions and declinations, and of the relation between magnitude of stars and the diameters of their photographic images. It may be of interest to lay before the Society a brief summary of these investigations.

I. *Personality of Measurers.*

The duplicate measurement of the plates in reversed positions, with a view to an increase of accuracy, was undertaken as a result of the meeting of the Astrographic Committee at Paris in 1896 June. Zones 64°, 65°, 66°, 67°, had by this time been measured at Greenwich in the direct positions of the plates. They were accordingly re-measured, the plates and the glass diaphragm being reversed right for left. The direct and reversed measures were compared, the investigation involving many thousands of measures of about 300 plates by eight different measurers. In the direct measures the same measurer measured both the 6^m and 3^m images, but in the reversed measures there were separate measurers for the two images.

The means of the measures of the 6^m and 3^m images having been already formed, the differences of these means direct and reversed were tabulated for each combination of measurers, and for convenience each book of measures was dealt with separately. As an example the following table is given, showing the result of the collection of these differences for the second book of zone 66°, extending from R.A. $6^h\ 0^m$ to $11^h\ 24^m$. It should be stated that the "direct" measures of this book were made in 1896 March and the "reversed" measures in 1897 April.

Differences of Measures Direct–Reversed as made by different Observers for Zone 66° R.A. $6^h\ 0^m$–$11^h\ 24^m$.

Unit = ·0001 of a réseau interval = ″·03.

Measurers.				Centre +66′ (South halves).				Centre +67′ (North halves).			
				x		y		x		y	
Direct.		Reversed.		No. of Stars.	Sum of Diffs.	No. of Stars.	Sum of Diffs.	No. of Stars.	Sum of Diffs.	No. of Stars.	Sum of Diffs.
6^m	3^m	6^m	3^m								
J.,	J.	J.,	P. M.	36	+536	39	+288	59	+1214	59	+273
J.,	J.	W. S.,	P. M.	6	+ 35	6	+ 76	8	+ 96	8	+ 33
J.,	J.	C. D.,	P. M.	15	+142	15	+792	29	+210	29	− 49
W. S.,	W. S.	J.,	P. M.	32	+249	32	−219	57	−286	57	− 1
W. S.,	W. S.	J.,	E. S.	37	+192	37	−292	29	−145	29	− 35
W. S.,	W. S.	W. S.,	P. M.	67	+316	65	−672	67	−824	68	− 76
W. S.,	W. S.	W. S.,	E. S.	102	+692	102	−1087	104	−1503	107	+669
W. S.,	W. S.	P. M.,	E. S.	16	+ 62	16	− 96	17	−199	18	+ 72
W. S.,	W. S.	C. D.,	J.	18	+132	19	−221	19	+ 65	20	− 21
P. M.,	P. M.	J.,	W. S.	29	− 90	27	+ 21	38	−202	36	+ 27
P. M.,	P. M.	J.,	P. M.	18	−198	19	+ 32	55	−330	55	+261
P. M.,	P. M.	W. S.,	E. S.	50	−293	49	−525	47	−561	47	+290
P. M.,	P. M.	C. D.,	P. M.	23	−236	23	− 71	19	−264	19	+ 27
E. S.,	E. S.	W. S.,	E. S.	53	+ 53	52	−526	61	−680	62	−701
E. S.,	E. S.	C. D.,	P. M.	3	− 8	3	− 21	4	+ 47	4	− 38
E. S.,	E. S.	J.,	W. S.	7	+ 41	7	−103	10	− 57	10	− 86
E. S.,	E. S.	J.,	P. M.	31	+205	31	− 57	30	+ 5	29	−378
E. S.,	E. S.	J.,	E. S.	20	+ 89	20	− 69	25	+ 48	25	− 8
Total No. of Stars ...				563		562		678		682	

Denoting by the subscripts 1 and 2 measures made in the direct and reversed positions respectively, the above table gives equations of condition of the form

$$36\left(\mathrm{J.}_1-\frac{\mathrm{J.}_2+\mathrm{P.M.}_2}{2}\right)=+536$$

Normal equations were formed and solved in the usual way. The zero being arbitrary, it was assumed so that $\mathrm{J.}_1+\mathrm{J.}_2+\mathrm{W.S.}_1$

$+W.S._2=0$, as J. and W.S. had measured for a long period. The solution of these equations gave the following values for the personalities of the observers, in which a distinction is made according to the microscope of the duplex micrometer used :

	J.$_1$	J.$_2$	W.S.$_1$	W.S.$_2$	P.M.$_1$	P.M.$_2$	E.S.$_1$	E.S.$_2$
x Left-hand microscope	+ 9	− 7	+ 3	−4	−10	+1	− 2	0
x Right-hand microscope	+16	−11	−10	+6	− 9	+2	− 5	+3
y Left-hand microscope	+ 6	− 5	− 8	+5	− 4	−1	− 6	+4
y Right-hand microscope	+ 1	− 1	− 5	+4	+ 3	−7	−11	−7

Nineteen books of measures were treated in this manner, and the results, which are given in the Introduction to the Greenwich Astrographic Catalogue, show that these personalities were fairly constant. The mean results (with changed signs) are given in the following table of corrections. Different measures show clearly that the mean of the corrections, direct and reversed, is nearly zero for each measurer ; that is to say, the mean of a direct and reversed measure by the same measurer is almost free from personality.

Corrections for Personality.

Unit = $^{int.}\cdot0001 = 0''\cdot03$.

Measurer.	Measures of x Coordinate. Right-hand Microscope. Direct	Right-hand Microscope. Reversed.	Left-hand Microscope. Direct.	Left-hand Microscope. Reversed.	Measures of y Coordinate. Both Microscopes. Direct.	Both Microscopes. Reversed.
Miss Everett (A.E.)	+ 3	...	+1	...	+5	...
Mr. Davidson (C.D.)	+ 5	+2	+3	+ 1	+6	−3
Mr. Johns (J.) ...	−11	+8	−9	+ 7	+1	+1
Mr. Stevens (W.S.)	+ 6	−5	−2	+ 3	+6	−5
Mr. Melotte (P.M.)	+ 3	−2	+3	− 1	−4 or +2*	+4 or −2*
Mr. Skells (E.S.)...	+ 3	−7	+2	+ 2	0	+4
Mr. Evans (E.) ...	+ 1	−4	...	+ 9	0	0
Mr. Stiles (St.) ...	+13	−9	0	−12	−4	−5

As a result of this investigation it was arranged that the same measurer should measure the 6^m images in the direct and reversed positions of the plate, and another measurer the 3^m images also in both positions.

II. *Probable Error of the Measures.*

In 1896 July two investigations were made of the probable error of the measures, including the errors inherent in the photographic images of a star and the réseau lines to which it is referred.

* The values +2 and −2 were found for some measures made (direct and reversed) at a later date in 1897 June–August.

(i.) Measures of the same plate were made by five measurers, each measurer measuring both the 6^m and 3^m images on the plate in direct and reversed positions.

(ii.) Five plates of the same field taken at different times and under different conditions were measured by one measurer in direct and reversed positions, giving ten measures.

(i.) The values for the discordance of the mean of the measures of a 6^m and 3^m image by one observer in one position (direct or reversed), corrected for personality, from the mean of ten such measures of 113 stars was found to be

Measurer.	x Coordinate. Direct.	x Coordinate. Reversed.	y Coordinate. Direct.	y Coordinate. Reversed.
Mr. Davidson ...	″ ±·120	″ ±·135	″ ±·150	″ ±·195
Mr. Johns ...	±·219	±·186	±·219	±·174
Mr. Stevens ...	±·177	±·174	±·171	±·159
Mr. Melotte ...	±·231	±·222	±·219	±·219
Mr. Skells ...	±·213	±·186	±·186	±·183

The means of the above discordances are $\pm''\cdot186$ and $\pm''\cdot187$ in the two coordinates respectively, leading in each case (after multiplication by $\sqrt{\frac{10}{9}}\times\cdot845$) to a probable error of $\pm''\cdot166$. Thus the probable error of a complete measure (two images, direct and reversed) is $\pm''\cdot117$.

It is to be noted that a complete measure involves eight readings or estimations of the positions of the two réseau lines relatively to the star images, these being practically quite independent, as owing to irregular curvature of the plate the "run" for a réseau interval varies from point to point.

If, instead of applying a correction for personality, the means of the measures, direct and reversed, are taken for each observer, the discordances are found to be

Measurer.	x Coordinate.	y Coordinate.
Mr. Davidson	″ ±·108	″ ±·131
Mr. Johns	±·153	±·159
Mr. Stevens	±·114	±·126
Mr. Melotte	±·132	±·141
Mr Skells	±·144	±·132

The probable error of a complete measure (multiplying the mean discordance by $\sqrt{\frac{5}{4}}\times\cdot845$) is found to be $\pm''\cdot127$. This agrees satisfactorily with the result given above, showing that personality of the measurer is virtually eliminated in the mean of measures direct and reversed.

(ii.) Five plates of the same field were completely measured by Mr. Stevens, direct and reversed. In each case the means of the measures of the 6^m and 3^m images were taken. The plate

constants were obtained exactly as is done in practice by comparing the measured coordinates with the standard coordinates of the reference stars of which there were twenty-two on the plate. Corrections of the form $ax+by+c$, $dx+ey+f$ were applied to the measured coordinates, thus giving ten comparable values of the coordinates of 121 stars. The means of the ten measures, direct or reversed, were taken for each star, and the discordances from the mean formed. The mean discordances are given in the following table:

Plate No.	x Coordinate. Direct.	x Coordinate. Reversed.	y Coordinate. Direct.	y Coordinate. Reversed.
	″	″	″	″
419	±·42	±·35	±·34	±·31
3139	±·25	±·35	±·20	±·29
3150	±·34	±·26	±·19	±·24
3151	±·28	±·25	±·21	±·36
3089	±·27	±·29	±·26	±·24

The mean of these discordances is ±″·285, giving ±″·254 for the probable error of a single determination of position (6^m and 3^m image) and ±″·180 of a complete determination, direct and reversed.

If the discordances of the means of the direct and reversed measures are taken for the five plates, the results are:

Plate No.	x Coordinate.	y Coordinate.
	″	″
419	±·34	±·30
3139	±·18	±·17
3150	±·27	±·20
3151	±·20	±·23
3089	±·25	±·21

and the probable error of a complete measure deduced in this way is ±″·222. This increase in the probable error may be taken as due to the systematic error which is common to the plate, both for direct and reversed measures.

It should be noted that in this probable error are included (i.) the errors of the measures, (ii.) the errors inherent in the photographic images of stars and réseau lines, (iii.) the errors introduced through errors in the plate constants arising from these causes, but not the part arising from the assumed right ascensions and declinations of the reference stars.

A further discussion of probable errors of places of the stars deduced from the photographs has been made by comparing the separate results from pairs of plates for a limited number of stars. The right ascensions and declinations of 240 stars between 70° N.Dec. and 72° N.Dec. have been deduced, with the provisional plate constants given in the Introduction to the Catalogue, one

star being taken in each quarter of the overlapping plates in this zone. A comparison of the results from the two plates on which each star was shown was made, the mean differences being found to be $\pm''\cdot43$ in R.A. and $\pm''\cdot46$ in Dec. From this it results that the probable errors (in arc of a great circle) of the right ascension and declination of a star deduced from the measures on one plate are $\pm''\cdot26$ in R.A. and $\pm''\cdot28$ in Dec., and that the probable errors of a catalogue place from the mean of two plates may be taken as $\pm''\cdot19$ in both R.A. and declination. These results were obtained with provisional places of the reference stars, the accidental errors of which affect the plate constants to some extent. In order to obtain more accurate places of the reference stars for determination of plate constants, these stars are being re-observed with the transit-circle at Greenwich; and when these observations are completed in the course of a couple of years the accuracy of the plate constants will be materially increased. From a discussion of the observations of the reference stars from N.P.D. 0° to 5°, now completed, it appears that the probable error of a star's place in the new Greenwich Meridian Catalogue (five observations in each element) is $\pm0''\cdot23$ in R.A. or N.P.D.

III. *Determination of Photographic Magnitudes.*

In the publication of the Greenwich Measures the diameter of the 6^m image is given as a measure of the photographic magnitude. In each case the diameter is estimated by two measurers in divisions and tenths of a division of the scale and the sum taken. Allowance is made for images which are sensibly elongated by taking the mean of the diameters in the horizontal and vertical directions. Allowance is also made by the measurers for the want of density in the images and also for the shading off at the edge, and the diameter set down is that of an equivalent black image, *i.e.* one which would contain the same amount of silver deposit.

In order to obtain direct information as to the magnitude of the faint stars shown on the plates, comparison has been made with the determinations on a photometric scale given by Professor E. C. Pickering (*Harvard Annals*, vol. xxxvii. pp. 4–11) of the faint stars near certain variables which lie in the Greenwich section. The photometric magnitudes of faint stars near the pole (*ibid.*, vol. xviii. p. 134) have also been compared with the diameters of the images on a catalogue plate (exp. 6^m, 3^m, and 20^s) and on four chart plates (exp. 40^m). Some photographs of *Nova Persei* taken under the same conditions as the astrographic plates also admit of comparison with the magnitudes given by Father Hagen (Georgetown College Observatory, *Supplementary Notes to the Atlas Stellarum Variabilium*).

The following table, giving the stars near *R Draconis* R.A. $16^h\ 32^m\ 23^s$, Decl. $+66°\ 57'\cdot8$ (1900·0), shown on plates with

centres at R.A. $16^h\ 30^m$, Decl. $+67°$, may serve as a specimen of the comparisons made, which deal with 19 chart plates and 18 catalogue plates of seven different fields.

Diameters of Images of Stars near R Draconis.

Unit = 0″·15.

Reference Letter.	Photom. Mag.	Plate 2652. Exp. 20ᵐ.	Plate 2652. Exp. 6ᵐ.	Plate 3412. Exp. 40ᵐ.	Zone and No. in Greenwich Measures.
a	6·88	29	74	121	67, 5303
c	7·44	28	70	117	67, 5299
f	8·28	14	43	82	66, 5250
g	8·56	20	51	84	67, 5395
h	9·06	12	42	80	67, 5283
k	9·40	7	32	62	66, 5248
l	9·80	6	20	52	67, 5317
m	10·16	5	28	54	67, 5370
n	10·56	5	24	46	66, 5223
o	10·97	not shown	15	34	66, 5222
p	11·59	,,	9	26	66, 5251
q	12·01	,,	8	20	66, 5234
r	12·52	,,	6	20	66, 5254
s	12·85	,,	not shown	18	...
t	13·33	,,	,,	13	...
u	13·84	,,	,,	15	...
w	14·43	,,	,,	13	...
x	15·02	,,	,,	10	...

The faintest stars shown with an exposure of 6^m on all the plates where a comparison with photometric magnitudes could be made are shown in the following table :—

Plate.	Mag. m	Plate.	Mag. m	Plate.	Mag. m
4061	12·61	4027	12·92	6160	11·6
1678	11·08	2652	12·52	6168	12·7
4767	11·87	2660	12·01	6178	11·9
3409	12·02	2776	11·96	6200	11·8
2469	12·02	2771	11·96	6202	12·8
2708	12·92	5013	13·2	6204	12·7

The mean for the above plates is $12^m·25$, and this may be taken as an approximate value of the average limit of magnitude reached by the catalogue plates at Greenwich with an exposure of 6 minutes.

For the conversion of diameters into magnitudes the em-

pirical law connecting the measured diameter d and the magnitude m determined by the Astronomer Royal (*Monthly Notices*, vol. lii. pp. 125–146), viz.:—$m = C - n\sqrt{d}$, has been adopted and appears to give a satisfactory accordance with the photometric determinations. The following determinations of the values of the constants C and n are derived by direct comparison of the bright and faint stars on each of 19 chart plates (exp. 40^m) :—

Plate.	Field.	Range of magnitudes and No. of stars.				Deduced formula $m = C - n\sqrt{d}$.	Diameter given by formula			
		m		m			$9^m{\cdot}0$	$11^m{\cdot}0$	$13^m{\cdot}0$	$14^m{\cdot}0$
1517	S. Cassiop.	8·76	1	13·93	3	$15{\cdot}7 - 0{\cdot}68\sqrt{d}$	97	48	16	6
5800	,,	8·76	1	13·93	3	$15{\cdot}7 - 0{\cdot}68\sqrt{d}$	97	48	16	6
3719	,,	9·92	2	14·12	2	$16{\cdot}0 - 0{\cdot}75\sqrt{d}$	87	44	16	7
3878	R. Urs. Maj.	8·49	2	14·01	3	$16{\cdot}7 - 0{\cdot}74\sqrt{d}$	109	59	25	13
3436	,,	7·94	2	14·01	3	$15{\cdot}8 - 0{\cdot}85\sqrt{d}$	64	32	11	5
4019	R. Urs. Min.	8·76	4	12·55	3	$16{\cdot}6 - 0{\cdot}90\sqrt{d}$	71	40	16	8
3515	,,	8·94	1	12·55	3	$16{\cdot}1 - 0{\cdot}87\sqrt{d}$	67	35	13	6
3469	R. Draconis	8·91	3	14·43	3	$17{\cdot}2 - 1{\cdot}00\sqrt{d}$	67	38	18	10
3482	,,	7·16	2	14·43	3	$17{\cdot}6 - 1{\cdot}00\sqrt{d}$	74	44	21	13
4587	T Cephei	8·46	2	12·55	2	$16{\cdot}4 - 0{\cdot}87\sqrt{d}$	74	40	16	8
3223	,,	8·12	1	12·55	2	$16{\cdot}3 - 0{\cdot}81\sqrt{d}$	82	43	17	8
6058	Pole	10·00	4	14·40	4	$16{\cdot}6 - 0{\cdot}95\sqrt{d}$	64	35	15	7
6050	,,	10·00	4	14·40	4	$16{\cdot}2 - 0{\cdot}78\sqrt{d}$	85	45	17	8
5955	,,	10·00	4	14·40	4	$16{\cdot}1 - 0{\cdot}95\sqrt{d}$	56	29	11	5
5964	,,	10·00	4	14·40	4	$16{\cdot}4 - 0{\cdot}98\sqrt{d}$	57	31	12	6
6200	Nova Persei	7·70	5	12·50	9	$14{\cdot}6 - 0{\cdot}77\sqrt{d}$	53	22	4	1
6202	,,	7·70	5	12·50	9	$16{\cdot}0 - 0{\cdot}79\sqrt{d}$	79	40	14	6
6204	,,	7·70	5	12·50	9	$15{\cdot}2 - 0{\cdot}84\sqrt{d}$	54	25	7	2
6161	,,	7·70	5	12·50	9	$14{\cdot}7 - 0{\cdot}78\sqrt{d}$	53	22	5	1

The mean formula derived from the chart plates (exposure 40^m) is $m = 16{\cdot}1 - 0{\cdot}84\sqrt{d}$.

Formulæ were derived in a similar way for the exposures of 6^m, 3^m, and 20^s of the catalogue plates, the mean results being as follows :—

$$\left.\begin{array}{ll} m = 16{\cdot}1 - 0{\cdot}84\sqrt{d} & \text{for exposure } 40^m \text{ (19 plates)} \\ m = 13{\cdot}7 - 0{\cdot}77\sqrt{d} & \text{,, ,, } 6^m \text{ (18 plates)} \\ m = 12{\cdot}9 - 0{\cdot}76\sqrt{d} & \text{,, ,, } 3^m \text{ (6 plates)} \\ m = 11{\cdot}5 - 0{\cdot}79\sqrt{d} & \text{,, ,, } 20^s \text{ (9 plates)} \end{array}\right\}$$

Putting $d=4$ as practically the diameter of a star which is shown quite clearly (though there are a number of stars measured with diameters 3 or 2), we find

m =	14·4	for the average limiting magnitude with exposure			40^m
m =	12·3	"	"	"	6^m
m =	11·4	"	"	"	3^m
m =	9·9	"	"	"	20^s

The following table gives the observed difference of magnitude in passing from one exposure to another compared with the values of $2{\cdot}5 \log t_2/t_1$:

	Obs. Diff.	$2{\cdot}5 \log \frac{t_2}{t_1}$.
	m	m
20^s to 3^m	1·5	2·39
3^m to 6^m	0·9	0·75
6^m to 40^m	2·1	2·06
20^s to 40^m	4·5	5·20

It appears from this that in passing from 3^m to 6^m, and again from 6^m to 40^m, the law

Exposure × brightness = constant

holds almost exactly, while the exposure of 20^s gives fainter stars than would be expected in accordance with the law. As a possible explanation of this it is to be noted that the 20^s exposure was given after the exposures of 6^m and 3^m, and that this preliminary exposure to diffused light may have increased the sensitiveness of the plate for the 20^s exposure.

In a paper on the statistics of stars in a zone of 5° from +65° to +70° Dec. (*Monthly Notices*, 1903 January), the logarithms of the number of stars per square degree with different exposures are given as follows :—

I. *Shown on both plates.*

20^s	3^m	6^m	40^m
0·966	1·598	1·724	2·437

II. *Total number shown.*

20^s	3^m	6^m	40^m
1·186	1·818	1·830*	2·496*

Between the 20^s and 3^m exposure the log increase of number

* As stated in the above paper the number 1·830 is too small, as stars which show a 6^m image on one plate only are not counted. The number 2·496 is probably too small, though not to the same extent.

of stars per square degree is 0·632 or (the difference of magnitude being 1·5) at the rate of 0·42 per magnitude.

Between 20^s and 6^m the log increase of number of stars per square degree is 0·757, or (the difference of magnitude being 2·4) at the rate of 0·32 per magnitude.

Taking the mean of these results we find that between the magnitudes $10^m{\cdot}0$ and $12^m{\cdot}0$ the logarithmic increase per magnitude in the total number of stars is 0·37, and therefore the ratio of increase per magnitude is 2·34.

Between 3^m and 40^m there is a logarithmic increase of 0·839, corresponding to a difference of magnitude of $3^m{\cdot}0$ or 0·28 per magnitude, and between 6^m and 40^m of ·713, corresponding to a difference of magnitude of $2^m{\cdot}1$ or of 0·34 per magnitude.

Taking the mean of these results we have between $12^m{\cdot}0$ and $14^m{\cdot}4$ a logarithmic increase of number of stars per magnitude =0·31, corresponding to a ratio of increase of 2·04 per magnitude.

It is satisfactory to find that these results are in good accordance with those recently given by Professor Pickering in his valuable memoir on the "Distribution of Stars". (*Harvard Annals*, vol. xlviii. No. 5, p. 178). Taking the values given in his Table XXI., the logarithmic increase per magnitude in the total number of stars would be 0·39 (as compared with 0·37 found above) between 10 and 12 mag., and 0·31 (as compared with 0·31) between $12^m{\cdot}0$ and $14^m{\cdot}4$, the corresponding ratios of increase per magnitude in the total number of stars being 2·45 and 2·04 found by Professor Pickering, as compared with 2·34 and 2·04 found above.

Royal Observatory, Greenwich:
1904 *March* 8.

On the Chromatic Correction of Object-glasses. Second Paper.

By A. E. Conrady.

In my first paper * I limited myself to the consideration of a luminous point in the optical axis. Precisely the same reasoning may, however, be applied to any point in the field of an object glass; but as in the case of oblique pencils there is no such complete symmetry as in axial pencils, a greater number of rays have to be followed through the system of lenses and their paths in the successive media determined in order to compare the value of $\Sigma dn\mathrm{D}$ for the several rays, and thus to determine the state of the chromatic correction.

It will generally be sufficient to compute four rays through the system: three of these lie in the plane containing the optical axis and the luminous point; these should be computed

* *Monthly Notices*, January 1904.

by the trigonometrical formulæ given in my previous paper. They are those rays in that plane which pass through the centre and margins of the limiting aperture of the system. The fourth ray passes through the margin of the same aperture vertically above or below the central ray; it does not cut the optical axis and must be computed by formulæ adapted to this case; probably those given by von Seidel * are the most convenient. The paths of each of these four rays in the several constituent lenses of the system can then be computed by the formula given in my previous paper (*l.c.* page 187). In the case of the fourth ray, and adopting the nomenclature used in Steinheil and Voit's reprint of Seidel's classical paper, the angle at the centre of curvature, which I call α, is there given as η, and the angle between the ray and a parallel to the optical axis, which takes the place of my angle β', is also given as τ', so that my formula for D may be applied with the same facility to this ray as to those proceeding in planes containing the optical axis.

Then, if $\Sigma dn \,.\, \mathrm{D}$ has exactly the same value for all four rays, the system will be perfectly achromatic for the luminous point chosen and for the selected wave-length. But the values of the sum will generally, owing to the extreme difficulty of perfect correction for oblique pencils, differ more or less from each other. Let $\Sigma dn\mathrm{D}_0$ be the value of the sum for the central ray, $\Sigma dn\mathrm{D}_1$ and $\Sigma dn\mathrm{D}_2$ the respective values for the marginal rays in the plane containing the optical axis, and $\Sigma dn\mathrm{D}_3$ the value for the marginal rays farthest from that plane; then it is not difficult to see that:

1. If $\Sigma dn\mathrm{D}_1 \gtrless \Sigma dn\mathrm{D}_2$ the refracted coloured waves will form an angle with, and will therefore proceed in a direction differing from, that of the refracted principal waves; in other words, the object-glass will produce different-sized images for different colours, or the image of the luminous point will be drawn out into a spectrum.

2. If $\Sigma dn\mathrm{D}_1 = \Sigma dn\mathrm{D}_2$ but $\gtrless \Sigma dn\mathrm{D}_0$, the coloured and principal waves will have the same direction, but different curvature and therefore different foci, and the image of the luminous point will be affected with symmetrically arranged coloured fringes.

3. If $\Sigma dn\mathrm{D}_3$ differs from $\frac{1}{2}(\Sigma dn\mathrm{D}_1 + \Sigma dn\mathrm{D}_2)$ there will be an astigmatic difference for different colours, and similar considerations will lead to a correct interpretation of any other case that may be met with.

The reasoning employed in my first paper may also be applied to prisms, for there is nothing in the train of thoughts which leads to my expression for the chromatic aberration of a system of lenses which cannot be equally well applied to plane or curved waves proceeding through a train of prisms; hence if d

* "Sitzungsberichte der math.-phys. Classe der kgl. bayr. Akademie der Wissenschaften vom 10. Nov. 1866," reprinted in Steinheil and Voit, *Handbuch der angewandten Optik*, vol. i., Leipzig, 1891.

and D are taken as the respective paths of the two marginal rays transmitted, $\Sigma dn(d-D)$ expresses the differential dispersion of the train of prisms, measured as the distance between the refracted principal and coloured waves on one ray when they intersect on the other; and accepting the postulate that this distance must be equal to the wave-length in order to give sufficiently separated foci of the two waves, we obtain at once Lord Rayleigh's famous formula for the resolving power of spectroscopes,* at which the great physicist arrived by a totally different path.

Conversely, my formula as applied to lenses may be interpreted in accordance with the familiar treatment of lenses as prisms of variable angle as demanding that the total resolving power of the system shall be zero for the selected wave-length, and so stated it has an almost axiomatic appearance.

With regard to suitable formulæ for determining the refractive and dispersive powers of optical glass for any wave-length, I have repeated the necessary calculations, with the result that the statements in my first paper are amply confirmed; it would indeed seem that the formula there given as the best:

$$n_\lambda = n_0 + \nu'\lambda^{-\frac{2}{3}} + \nu_2\lambda^{-4}$$

is even more accurate than I stated, and will generally represent the indices of any ordinary silicate glass within the visible spectrum to less than one unit of the fifth decimal if well selected lines are chosen for determining the constants.

These calculations led to the discovery of yet another empirical formula which is slightly more convenient and at least equally accurate, and should therefore be preferable, viz.

$$n_\lambda = n_0 + \nu_1\lambda^{-1} + \nu_2\lambda^{-3\frac{1}{2}}$$

Finally, an application of the various formulæ to Langley's determination of the refractive indices for a light flint prism from $\lambda = 20{,}300$ to $\lambda = 3440$ (O line) had the interesting result of showing that the empirical formulæ retain their superiority throughout that range, for when the constants were determined from the indices for the A, F, and O lines, Cauchy's formula (with three terms) erred by twice as much for the longest wave-length as any of the others, and for other wave-lengths the superiority of the empirical formulæ proved even greater.

* *Encyc. Brit.*, 9th edition, vol. xvii. page 807. Also "Investigations in Optics," *Phil. Mag.* 1879, 1880.

Bedford Park, W.: 1904 *March* 10.

On the Determination of the Division Errors of a Graduated Circle. By S. S. Hough, M.A., F.R.S.

§ 1. *Introductory.*

The accurate determination of the division errors of a graduated circle can only be made at the expense of a considerable amount of labour, and it becomes a matter of the highest importance at the outset of such an undertaking to investigate how the available appliances may be utilised to the highest advantage with the least possible work. The considerations dealt with in the following paper have arisen in the course of the preparation of a programme for the determination of the errors of divisions of the new transit circle of the Cape Observatory now in progress, and it will serve to coordinate ideas if we deal with the problem from the outset in the form in which it is presented by that instrument. The subsidiary appliances will, of course, vary with different instruments, but those attached to the Cape instrument may at least be taken as sufficiently typical, while the principles involved may be readily adapted to other instruments and appliances.

The circles of the Cape instrument are ordinarily read by means of six microscopes symmetrically situated round the centres. For the determination of errors of graduation the instrument is also provided with two pairs of subsidiary microscopes, situated each at extremities of a diameter at angular distances of 20° and 25° respectively from one pair of the primary six.

The six primary microscopes enable us to determine the relative errors of six equi-spaced divisions with a high degree of accuracy, and the subsidiary microscopes then furnish the means of determining the errors of division—first, of division marks separated from the former by an interval of 20° ; and, secondly, of those separated by an interval of 25°—*i.e.* we can ascertain the relative spacing of division marks separated by intervals of 5′ throughout the circle. A further appliance, consisting of a microscope provided with a divided object glass similar to that used in the determination of the division errors of the old Cape transit circle,* enables us still further to subdivide the spaces of 5′ into spaces of 1′ interval.

We proceed first to consider the geometrical significance of a single pointing made by means of one of the micrometers on a division mark.

§ 2. *Derivation of the Fundamental Equation.*

Assume, in the first instance, that the circles are perfectly plane and perfectly rigid, and that the division marks consist of approximately radial straight lines engraved on them.

* Introduction to *Cape Observations*, 1879–81.

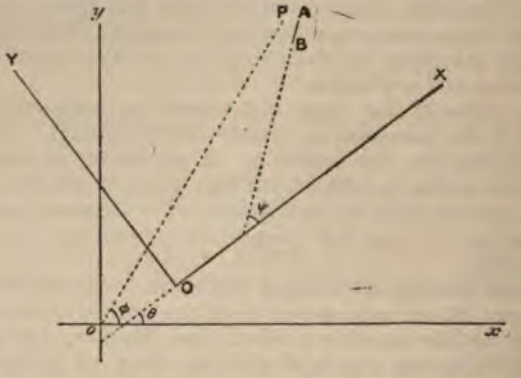
Take a pair of rectangular axes OX, OY *fixed relatively to a circle.* The origin O is supposed to be at the centre of the circle, and the direction OX directed towards the division mark 0°. Departures from these conditions of the order of magnitude of the instrumental errors may, however, be supposed to exist, and the precise definition of the point O and the direction OX with relation to these errors is deferred.

With reference to these axes the position of a division mark, assumed to be a straight line, may be represented by an equation of the form

$$X \sin \psi - Y \cos \psi = p,$$

where p will be a small quantity of the order of magnitude of the instrumental errors, and for the division mark marked $n°$ the the angle ψ will differ from $n°$ by a like small amount.

Now suppose that the circle under investigation is mounted in such a manner as to allow freedom of motion in its own plane



without variation of this plane in space, and that it can be viewed by a system of microscopes provided with micrometers, which are themselves regarded as *fixed in space.*

The operation of bisection consists in placing a pair of close parallel webs, movable by means of the micrometer screw, in the position symmetrical with respect to a division mark ; but, in case the division mark should not be strictly parallel to the moving webs, the bisection is always made between two fixed parallel webs at right angles to the moving webs and somewhat more widely separated than they.

If, then, a micrometer be set to its zero reading, the intersection of the pair of straight lines lying midway between the fixed and moving webs will define a point fixed in the focal plane of the microscope, which is conjugate to a fixed point in the plane of the circle. Thus the zero reading of the micrometer will define a point fixed in the plane of the circle, and the reading when the micrometer is pointed on any division mark

will measure in terms of the micrometer screw the perpendicular distance of this point from the division mark regarded as a straight line.

Now take a pair of fixed rectangular axes ox, oy in the plane of the circle, and let ξ, η denote the coordinates of the origin O, fixed relatively to the circle, with reference to the axes ox, oy fixed in space; further, let θ denote the angle between the two axes OX, Ox.

Let $R\cos\phi$, $R\sin\phi$ be the coordinates relatively to the fixed axes ox, oy of the fixed point associated with one of the micrometers, so that R, ϕ will denote two "constants of the microscope."

In the opposite figure this point is denoted by P and the division mark by the straight line AB.

On referring P to the moving axes OX, OY its coordinates will be

$$R\cos\phi\cos\theta+R\sin\phi\sin\theta-\xi\cos\theta-\eta\sin\theta$$
$$-R\cos\phi\sin\theta+R\sin\phi\cos\theta+\xi\sin\theta-\eta\cos\theta,$$

or

$$R\cos(\phi-\theta)-\xi\cos\theta-\eta\sin\theta,\quad R\sin(\phi-\theta)+\xi\sin\theta-\eta\cos\theta,$$

and the perpendicular distance of this point from the division mark AB, whose equation is

$$X\sin\psi-Y\cos\psi=p,$$

will be

$$p-R\sin(\theta+\psi-\phi)-\xi\sin(\theta+\psi)+\eta\cos(\theta+\psi).$$

If then m denotes the actual reading of a micrometer head, $m(1+r)+s$ the same corrected for "run" and screw error—*i.e.* referred to a unit and zero defined with reference to the plane of the circle instead of with reference to the screw—every micrometer reading will yield an equation of condition of the form

$$p-R\sin(\theta+\psi-\phi)-\xi\sin(\theta+\psi)+\eta\cos(\theta+\psi)-m-mr-s=0.$$

This equation involves three "constants of the microscope," viz. R, ϕ, r; two "constants for the division mark," viz. p, ψ; and three quantities, ξ, η, θ, which depend on the position of the circle beneath the microscopes, and which will be the same for all microscopes at any one setting of the circle.

Further, the equation expresses in its most general form the information which can be derived from any one pointing, provided that the geometrical conditions that we have presupposed have been fulfilled.

In practice the values of m which can occur will, of course, be limited by the field of view of the microscopes and the range of the micrometer screws; we may consequently regard $\theta+\psi-\phi$ as a small angle, and replace $\sin(\theta+\psi-\phi)$ by $\theta+\psi-\phi$.

Further, in operations for the determination of division errors

we may always suppose that the division mark is brought under the microscope in such a way as to be very approximately bisected when the micrometer is set to its zero reading. (The actual reading which may be regarded as zero can, of course, be arbitrarily selected.) Thus, if we suppose that θ_o, ϕ_o, ψ_o, R_o are the ideal values of θ, ϕ, ψ, R aimed at in the construction, adjustment, and setting of the instrument, we may suppose that

$$\theta = \theta_o + \delta\theta,\ \phi = \phi_o + \delta\phi,\ \psi = \psi_o + \delta\psi,\ R = R_o + \delta R,$$

where we have rigorously

$$\theta_o + \psi_o - \phi_o = o,$$

while R_o is the same for all microscopes, and $\delta\theta$, $\delta\phi$, $\delta\psi$, δR represent small errors whose squares we may regard as negligeable.

Our equation thus reduces to

$$p - R_o(\delta\theta + \delta\psi - \delta\phi) - \xi \sin\phi_o + \eta\cos\phi_o - m - s = o,$$

where in the small terms involving ξ, η we have replaced $\theta+\psi$ by its approximate value ϕ_o, and we have omitted the term mr, since m will now be a small quantity of the order of magnitude of the small errors of adjustment, &c. The runs of the microscopes may easily be reduced by mechanical adjustment so as to be practically insensible over the small range of the micrometer screw which need be called into play.

If, then, we put

$$p - R_o\delta\psi = x$$
$$R_o\delta\phi + s = y$$
$$-R_o\delta\theta = z$$

we obtain

$$x + y + z - \xi\sin\phi_o + \eta\cos\phi_o = m,$$

where x is a quantity which depends only on the division mark bisected, and which we may describe as the division error of the mark ξ.

y is a quantity which depends only on the micrometer used. If a sufficiently small range of the micrometer screw be involved the part s may be regarded as constant, and y will then denote a constant error in adjustment of the microscope. The screw errors, if considerable or liable to rapid variation, should, however, be independently determined and applied to the micrometer readings. There will then remain a residual correction y, which is constant for the microscope.

z is a quantity which depends only on the angular position in space of the divided circle, which we shall refer to in future as the error in setting of the circle.

If the pivots were truly cylindrical it would be possible to choose the points O, o at the axis of pivots which will be fixed

both with reference to the circle and with reference to the microscopes. In this case the quantities ξ, η would each be zero, and the equation would reduce to

$$x+y+z=m.$$

The same form of equation would result if the forms of the pivots were supposed known from previous investigations, since ξ, η could then be regarded as known functions of θ, their values being applied as a correction to the absolute term m.

§ 3. *Grouping of Observations.*

The problem involved in the determination of division errors consists in the arrangement of the observations in such a manner as to allow of the elimination of the quantities y, z, ξ, η and the evaluation of the quantities x with the greatest possible weight by means of the resulting equations.

The formation of our fundamental equation rests on the assumption that the microscopes are fixed in space. In practice the adjustment of the micrometers will be liable to incessant change, on account of changes of temperature, &c. These changes may, however, be assumed to take place slowly ; it will therefore be necessary that the observations should be grouped into short sets, during each of which the quantities y may be regarded as sensibly constant, and each of which will admit of the elimination of the y's, either of itself or in combination with other similar sets. The effective constancy of the y's during a set may be further ensured if a set consists of a series of operations immediately followed by an exactly similar series in reversed order.

The errors of the pivots will be completely eliminated from the mean of a pair of readings taken with opposite microscopes at any one setting of the circle, since the coefficients of ξ, η will have the same values with opposite signs for these two readings. Thus if we restrict our operations to the determination of the mean division errors of pairs of opposite division marks we need concern ourselves no further with the errors due to irregularity in the forms of the pivots. The restriction will be of no material consequence if in future use the circle is always read by pairs of opposite microscopes.

The only outstanding source of systematic error, at least of a geometrical character, which can affect our results would appear to be that involved in the assumption that the circle is perfectly rigid. Changes in the form of the circle due to change of temperature, &c., can, of course, only be guarded against by the process already proposed for the elimination of changes in the adjustment of the microscopes. There remain, however, to be considered the possible effects of a small degree of flexibility of the circle.

The only cause which could give rise to flexure of the Cape

circles in their own plane is the gravitational stress to which they are subject. The circles being supported by continuous discs, and not by spokes, unless there is some want of homogeneity in the construction of these discs we may anticipate that the displacement of all division marks from this cause when brought under a given microscope will be the same. Thus the errors of flexure will be inseparably involved with the errors of adjustment of the microscopes, and any process designed to eliminate the latter will also of necessity likewise eliminate the former.

Whatever be the construction of the circles we may at least assume that the strain due to gravitational stress will be reversed in sign with the stress which produces it, *i.e.* it will affect with opposite signs and by equal amounts the mean readings of a pair of opposite microscopes at two settings of the circle differing by 180°. Thus we may completely eliminate all errors due to a possible flexibility of the circle in its own plane by repeating all operations in positions of the circle differing by 180°.

The following "sets" of operations are suggested as being sufficiently short to allow of reasonable constancy in the instrumental conditions throughout a set. An observer can easily perform two or more such sets at a single sitting without undue strain on the eyesight.

I. The division x is brought in succession under each of the six primary microscopes, and all six microscopes are read at each setting.

The same series of operations is immediately repeated in reversed order.

Such a set involves seventy-two pointings.

II. The divisions x, $x+20°$, $x+40°$ are brought in succession first under one of the primary microscopes A, and secondly under the opposite microscope B, and at each setting the microscopes A, B and the pair of subsidiary microscopes a, b situated at an interval of 20° from a pair of primary microscopes are read.

The same operations are immediately repeated in reversed order.

Such a set involves 48 pointings.

III. The divisions x, $x+5°$, $x+10°$, $x+15°$ are brought in succession first under a microscope A and secondly under the opposite microscope B, and at each setting the microscopes A, B and the second pair of subsidiary microscopes α, β, situated at an interval of 25° from a pair of primary microscopes, are read.

The same operations are repeated in reversed order. Such a set will involve 64 pointings.

IV. The divisions x, $x+1°$; $x+1°$, $x+2°$; $x+2°$, $x+3$; $x+3°$, $x+4°$; $x+4°$, $x+5°$ are brought in succession under the divided object glass microscope, and the micrometer is pointed at the first setting on the images of the division marks x, $x+1°$; at the second on those of the division marks $x+1°$, $x+2°$, and so on ; *i.e.* the succession intervals by which the division marks x, $x+1°$,

$x+2°$, $x+3°$, $x+4°$, $x+5°$ are separated are compared in order with the fixed interval by which the two images of a single division mark are separated by the divided object glass.

The same operations are then repeated in reversed order.

Each set will involve 20 pointings.

§ 4. *Determination of Six Fundamental Intervals.*

Consider first a set of observations of Type I. We have in all 72 pointings, from which, if we combine into a single observation the means of the results obtained from the direct and reversed orders of measurement, we may derive 36 equations of condition.

Denote by x_0, x_{60}, x_{120}, x_{180}, x_{240}, x_{300} the division errors of the six division marks involved, and by y_A, y_B, y_C, y_D, y_E, y_F the errors of adjustment of the six microscopes; also let z_1, z_2, z_3, z_4, z_5, z_6 denote the errors in setting in the six positions of the circle.

Then, disregarding the errors of the pivots, the 36 equations of condition may be written in the form

$$\left.\begin{array}{llll}
x_0+y_A+z_1=m_{11}, & x_{60}+y_B+z_1=m_{12}, & x_{120}+y_C+z_1=m_{13}, \\
x_{180}+y_D+z_1=m_{14}, & x_{240}+y_E+z_1=m_{15}, & x_{300}+y_F+z_1=m_{16}, \\
x_0+y_B+z_2=m_{21}, & x_{60}+y_C+z_2=m_{22}, & x_{120}+y_D+z_2=m_{23}, \\
x_{180}+y_E+z_2=m_{24}, & x_{240}+y_F+z_2=m_{25}, & x_{300}+y_A+z_2=m_{26}, \\
x_0+y_C+z_3=m_{31}, & x_{60}+y_D+z_3=m_{32}, & x_{120}+y_E+z_3=m_{33}, \\
x_{180}+y_F+z_3=m_{34}, & x_{240}+y_A+z_3=m_{35}, & x_{300}+y_B+z_3=m_{36}, \\
x_0+y_D+z_4=m_{41}, & x_{60}+y_E+z_4=m_{42}, & x_{120}+y_F+z_4=m_{43}, \\
x_{180}+y_A+z_4=m_{44}, & x_{240}+y_B+z_4=m_{45}, & x_{300}+y_C+z_4=m_{46}, \\
x_0+y_E+z_5=m_{51}, & x_{60}+y_F+z_5=m_{52}, & x_{120}+y_A+z_5=m_{53}, \\
x_{180}+y_B+z_5=m_{54}, & x_{240}+y_C+z_5=m_{55}, & x_{300}+y_D+z_5=m_{56}, \\
x_0+y_F+z_6=m_{61}, & x_{60}+y_A+z_6=m_{62}, & x_{120}+y_B+z_6=m_{63}, \\
x_{180}+y_C+z_6=m_{64}, & x_{240}+y_D+z_6=m_{65}, & x_{300}+y_E+z_6=m_{66}.
\end{array}\right\} \quad (A)$$

Now on taking the mean of all the equations which involve x_0, i.e. those which appear in the first column, we obtain

$$x_0+\bar{y}+\bar{z}=\tfrac{1}{6}(m_{11}+m_{21}+m_{31}+m_{41}+m_{51}+m_{61}) \qquad (B)$$

where $\bar{y}$, $\bar{z}$ denote the means of all the quantities y, z involved.

By taking the mean of all the equations we derive

$$\bar{x}+\bar{y}+\bar{z}=\tfrac{1}{36}\Sigma m \quad \ldots \quad \ldots \quad \ldots \quad (C)$$

Taking the difference of the two equations (B) and (C) we can completely eliminate the quantities y, z and derive

$$x_0-\bar{x}=\tfrac{1}{6}(m_{11}+m_{21}+m_{31}+m_{41}+m_{51}+m_{61})-\tfrac{1}{36}\Sigma m$$

with similar expressions for x_{60}, x_{120} &c.

By combining the equations for x_0, x_{180} we obtain

$$\tfrac{1}{2}(x_0+x_{180})-\bar{x}=\tfrac{1}{12}(m_{11}+m_{21}+m_{31}+m_{41}+m_{51}+m_{61}+m_{14}+m_{24}+m_{34}+m_{44}+m_{54}+m_{64})-\tfrac{1}{36}\Sigma m.$$

We are thus enabled to eliminate the quantities y, z, and to determine the errors of division of a division mark or of a pair of opposite division marks, but only in relation to the mean of all the six division marks under investigation. Denote the division error of the mark 0° with relation to these six divisions by ξ_0, that of the pair of opposite division marks 0°, 180° by X_0, so that

$$\xi_0=x_0-\bar{x}=\tfrac{1}{6}(m_{11}+m_{21}+m_{31}+m_{41}+m_{51}+m_{61})-\tfrac{1}{36}\Sigma m,$$

$$X_0=\tfrac{1}{2}(x_0+x_{180})-\bar{x}$$
$$=\tfrac{1}{12}(m_{11}+m_{21}+m_{31}+m_{41}+m_{51}+m_{61}+m_{14}+m_{24}+m_{34}+m_{44}+m_{54}+m_{64})-\tfrac{1}{36}\Sigma m.$$

Since each of the quantities m depends on the mean of two pointings, if we suppose that the mean error of a single pointing is ϵ, the square of the mean error of each m is $\frac{1}{2}\epsilon^2$.

In the expression for ξ_0 all 36 m's appear, 6 of them with a coefficient $\frac{5}{36}$ and the remaining 30 with a coefficient $-\frac{1}{36}$.

Hence the square of the mean accidental error of ξ_0 is

$$\tfrac{1}{2}\epsilon^2[6(\tfrac{5}{36})^2+30(\tfrac{1}{36})^2]=\frac{5\epsilon^2}{72}.$$

In like manner the square of the mean accidental error of X_0 is

$$\tfrac{1}{2}\epsilon^2[12(\tfrac{2}{36})^2+24(\tfrac{1}{36})^2]=\tfrac{1}{36}\epsilon^2.$$

Thus the weight of the determination of a single division error, assumed unaffected by systematic error, is $\frac{72}{5}$, that of a pair of opposite divisions is 36, unit weight corresponding with that of a single micrometer pointing.

The difference of two of the quantities X—say X_0, X_{60}—will be expressible in the form

$$X_0-X_{60}=\tfrac{1}{12}[m_{11}+m_{21}+m_{31}+m_{41}+m_{51}+m_{61}+m_{14}+m_{24}+m_{34}+m_{44}+m_{54}+m_{64}-m_{12}-m_{22}-m_{32}-m_{42}-m_{52}-m_{62}-m_{15}-m_{25}-m_{35}-m_{45}-m_{55}-m_{65}]$$

whence the square of the mean error of X_0-X_{60} is

$$\tfrac{1}{144}\left[24\frac{\epsilon^2}{2}\right]=\frac{\epsilon^2}{12},$$

and this is the square of the mean amount by which the measurement of an angle which is approximately a multiple of 60°, but not of 180°, will be in error on account of imperfect determina-

tion of division errors, if the angle be measured by reading a pair of opposite microscopes on the division marks now under investigation.

An angle which is approximately a multiple of 180° will not be affected by division errors if determined by means of a pair of opposite microscopes. In like manner, if all six microscopes be read at any two settings of the circle which involve the six divisions under investigation the errors of division of the circle will be entirely eliminated from the angle through which the circle has been rotated between the two settings.

§ 5. *Derivation of Probable Error.*

It may readily be shown that the process of combination of the equation (A) which we have used is the exact equivalent of the "method of least squares."* We may thus derive from the residuals an estimate of the probable error of a single equation or that of a single micrometer reading, according to the usual rules laid down for the latter process, with, however, a slight modification.

We have already seen that the quantities x are not in themselves determinate from the equations (A), but only their differences from their mean value. In like manner the quantities y or z cannot be completely determined from equations (A). In fact, the normal equations derived from equations (A) will not be independent, and in order to obtain an unique solution from them it will be necessary to impose on the quantities x, y, z two linear relations which may to a large extent be arbitrarily chosen ; *e.g.* we might suppose

$$x_0 = 0, \qquad y_A = 0.$$

A more symmetrical pair of relations which will equally serve the purpose, and enable us to arrive at determinate residuals, is as follows :

$$\Sigma x = 0 \qquad \Sigma y = 0.$$

It is clear that the imposition of these conditions is equivalent to reducing the effective number of unknown quantities which enter into the left-hand members of the normal equations by 2, and hence the mean error of a single pointing must be derived by the formula

$$\epsilon^2 = \frac{\Sigma pvv}{m - n + 2},$$

where p denotes the weight of a single equation,
m the number of observation equations,
n the total number of quantities derived,
and v a "residual."

* P. H. Cowell, *Monthly Notices*, lxi. p. 527.

In the case of a set such as that under consideration we have

$$p = 2, \qquad m = 36, \qquad n = 18,$$

for we determine in all six quantities x, six quantities y, and six quantities z. Thus

$$\epsilon^2 = \tfrac{1}{15} \Sigma vv.$$

If, however, we combine into a single observation equation the pairs resulting from simultaneous pointings at opposite extremities of a diameter, we have

$$p = 4, \qquad m = 18, \qquad n = 12,$$

since we only determine three means of pairs of values of x and y, and six values of z.

Thus in this case

$$\epsilon^2 = \tfrac{1}{2} \Sigma vv,$$

and the derived value of the probable error will be independent of errors of the pivots.

If further we combine into a single observation equation the means of the four observations taken on a pair of opposite division marks in positions of the circle differing by 180°, so that the residuals will be free from errors of flexure, we have

$$p = 8, \qquad m = 9, \qquad n = 9,$$

and therefore

$$\epsilon^2 = 4 \Sigma vv.$$

§ 6. *Numerical Illustration.*

The following illustration is selected at random from among those already completed in connection with the Cape circles.

The division marks under investigation are the marks 0°, 60°, 120°, 180°, 240°, 300°. At the first setting of the circle the division mark 30° is brought under the pointer, and the marks under investigation are then respectively under the microscopes F, E, D, C, B, A. Care is taken to set the circle with such accuracy as to ensure that all measurements should be referred to the same parts of the micrometer screws. The results of a set (means of readings in direct and reversed directions of the circle) may then be expressed in the following tabular form, in which only the fractional parts of a revolution of the micrometer screw-head are quoted. One revolution of the screw-head is equivalent to a minute of arc, and the heads are divided into 100 parts.

Table of Micrometer Readings.

Pointer Reading.	Micr. A	Micr. B	Micr. C	Micr. D	Micr. E	Micr. F
°	′	′	′	′	′	′
30	·187	·142	·209	·270	·268	·196
90	·110	·104	·160	·218	·239	·185
150	·140	·106	·195	·242	·236	·191
210	·171	·130	·166	·243	·247	·188
270	·134	·102	·152	·199	·256	·192
330	·144	·116	·179	·223	·210	·193

The means of the quantities in the same horizontal line are respectively

′ ′ ′ ′ ′ ′
·212, ·169, ·185, ·191, ·172, ·177.

If we suppose that $\Sigma x = 0$, and $\Sigma y = 0$, these will represent the errors in setting, and on subtracting them from the separate entries we obtain the following revised table from which the errors in setting have been removed :—

Pointer reading.	Micr. A	Micr. B	Micr. C	Micr. D	Micr. E	Micr. F
°	′	′	′	′	′	′
30	−·025	−·070	−·003	+·058	+·056	−·016
90	−·059	−·065	−·009	+·049	+·070	+·016
150	−·045	−·079	+·010	+·057	+·051	+·006
210	−·020	−·061	−·025	+·252	+·056	−·003
270	−·038	−·070	−·020	+·026	+·083	+·019
330	−·033	−·061	+·001	+·045	+·032	+·015
Means	−·037	−·068	−·008	+·048	+·058	+·006

The means here quoted represent the relative errors of adjustment of the micrometers. Subtracting them from the separate entries we derive the following table of division errors free from errors in setting of the circle and errors in adjustment of the micrometers :—

Pointer Reading.	Micr. A	Micr. B	Micr. C	Micr. D	Micr. E	Micr. F
°	′	′	′	′	′	′
30	+·012	−·002	+·005	+·010	−·002	−·022
90	−·022	+·003	−·001	+·001	+·012	+·010
150	−·008	−·011	+·018	+·009	−·007	·000
210	+·017	+·007	−·017	+·004	−·002	−·009
270	−·002	−·003	−·013	−·022	+·025	+·013
330	+·003	+·006	+·009	−·002	−·026	+·009

The entries which correspond with a given division mark will appear in diagonals of this table. Collecting them into the same vertical columns, we obtain :

Division Mark / Pointer.	300°	240°	180°	120°	60°	0°
°	r	r	r	r	r	r
30	+·012	−·002	+·005	+·010	−·002	−·022
90	+·003	−·001	+·001	+·012	+·010	−·022
150	+·018	+·009	−·007	·000	−·008	−·011
210	+·004	−·002	−·009	+·017	+·007	−·017
270	+·025	+·013	−·002	−·003	−·013	−·022
330	+·009	+·003	+·006	−·002	−·002	−·026
Means	+·0118	+·0033	+·0010	−·0013	−·0013	−·0200

The residuals from the means are as follows :

Division Mark / Pointer.	300°	240°	180°	120°	60°	0°
°	r	r	r	r	r	r
30	+·0002	−·0053	+·0060	+·0025	−·0007	−·0020
90	−·0088	−·0043	+·0020	+·0045	+·0113	−·0020
150	+·0062	+·0057	−·0060	−·0075	−·0067	+·0090
210	−·0078	−·0053	−·0080	+·0095	+·0083	+·0030
270	+·0132	+·0097	−·0010	−·0105	−·0117	−·0020
330	−·0028	−·0003	+·0070	+·0015	−·0007	−·0060

and the sum of their squares expressed in units of the fourth decimal place is

156 302 ;

whence the square of the mean error of a single pointing is

156 30,

and the probable error of a single pointing $\pm 0^{r}\cdot 0083 = \pm 0''\cdot 50$.

The probable accidental error of a single pointing, as evidenced by the interagreement of the pointings made in direct and reversed measurements, can scarcely amount to as large a quantity as this, and it would thus appear that there is some considerable source of systematic error affecting the interagreement of our results. To ascertain whether the discordances may be attributed to pivot errors we combine, by taking means, the pairs of residuals derived from opposite microscopes at the same setting of the circle. The residuals now become :

Division Mark / Pointer.	300° and 120°	240° and 60°	0° and 0°
°	″	″	″
30	+·0014	−·0030	+·0020
90	−·0022	+·0035	·0000
150	−·0006	−·0004	+·0015
210	+·0009	+·0015	−·0025
270	+·0014	−·0010	−·0015
330	−·0007	−·0005	+·0005

The sum of the squares of the residuals expressed in units of the fourth decimal place now amounts to

$$5033,$$

and the square of the mean error of a single pointing as derived from the interagreement of the several determinations of the division errors of pairs of opposite division marks is

$$\tfrac{1}{2}\Sigma vv = 2517.$$

The probable accidental error of a single pointing as thus derived is therefore $\pm 0''{\cdot}0033$ or $\pm 0''{\cdot}20$.

This agreement is now satisfactory, the true value of the probable accidental error of a single pointing as derived by various tests involving a very large number of pointings being apparently about $\pm 0''{\cdot}18$.

To ascertain whether the agreement might be still further improved by attributing a small amount of flexure to the circle, we combine into a single residual the means of the four resulting from pointings on two opposite divisions under each of two opposite microscopes.

The resulting residuals are given in the following table :

Division Marks / Pointer Readings.	300° and 120°	240° and 60°	180° and 0°
°	″	″	″
30 and 210	+·0012	−·0008	−·0003
90 „ 270	−·0004	+·0013	−·0008
150 „ 330	−·0007	−·0005	+·0010

The sum of the squares of these residuals in units of the fourth decimal place is

$$640,$$

and the square of the mean error of a single pointing

$$= 4vv = 2560.$$

Thus no flexure exists of sufficient amount to materially influence the agreement of the observations of the present series.

The same conclusion is borne out from a far more extended series of observations.

§ 7. *Determination of* 20° *Spaces.*

Consider next the significance of a set of observations of type II. Such a set consists of forty-eight pointings, but we can combine into a single equation the means of the pointings made in direct and reversed orders of measurement, and we shall obtain twenty-four equations of condition, which we may write as follows :

$$\begin{aligned}
&x_0 + y_A + z_1 = m_{11},\ x_{20} + y_a + z_1 = m_{12},\ x_{180} + y_B + z_1 = m_{13},\ x_{200} + y_b + z_1 = \\
&x_{20} + y_A + z_2 = m_{21},\ x_{40} + y_a + z_2 = m_{22},\ x_{200} + y_B + z_2 = m_{23},\ x_{220} + y_b + z_2 = \\
&x_{40} + y_A + z_3 = m_{31},\ x_{60} + y_a + z_3 = m_{32},\ x_{220} + y_B + z_3 = m_{33},\ x_{240} + y_b + z_3 = \\
&\\
&x_{180} + y_A + z_4 = m_{41},\ x_{200} + y_a + z_4 = m_{42},\ x_0 + y_B + z_4 = m_{43},\ x_{20} + y_b + z_4 = \\
&x_{200} + y_A + z_5 = m_{51},\ x_{220} + y_a + z_5 = m_{52},\ x_{20} + y_B + z_5 = m_{53},\ x_{40} + y_b + z_5 = \\
&x_{220} + y_A + z_6 = m_{61},\ x_{240} - y_a + z_6 = m_{62},\ x_{40} + y_B + z_6 = m_{63},\ x_{60} + y_b + z_6 =
\end{aligned}$$

If we write for brevity

$$\begin{aligned}
X_0 &= \tfrac{1}{2}(x_0 + x_{180}),\ X_{20} = \tfrac{1}{2}(x_{20} + x_{200}),\ \&c., \\
Y_1 &= \tfrac{1}{2}(y_A + y_B) - \tfrac{1}{2}(y_a + y_b),
\end{aligned}$$

by combining the equations which appear in the same horizontal line we may completely eliminate all the quantities z and all the quantities y except as they appear in the combination Y_1.

Thus we obtain

$$\begin{aligned}
X_0 - X_{20} + Y_1 &= \tfrac{1}{2}(m_{11} + m_{13}) - \tfrac{1}{2}(m_{12} + m_{14}) \\
X_{20} - X_{40} + Y_1 &= \tfrac{1}{2}(m_{21} + m_{23}) - \tfrac{1}{2}(m_{22} + m_{24}) \\
X_{40} - X_{60} + Y_1 &= \tfrac{1}{2}(m_{31} + m_{33}) - \tfrac{1}{2}(m_{32} + m_{34}) \\
&\\
X_0 - X_{20} + Y_1 &= \tfrac{1}{2}(m_{41} + m_{43}) - \tfrac{1}{2}(m_{42} + m_{44}) \\
X_{20} - X_{40} + Y_1 &= \tfrac{1}{2}(m_{51} + m_{53}) - \tfrac{1}{2}(m_{52} + m_{54}) \\
X_{40} - X_{60} + Y_1 &= \tfrac{1}{2}(m_{61} + m_{63}) - \tfrac{1}{2}(m_{62} + m_{64}).
\end{aligned}$$

Since each m has weight 2, the weight of each absolute term of these equations will be 2. Taking the means of the first and fourth equations, the second and fifth, and the third and sixth, we derive three equations of the type

$$\begin{aligned}
X_0 - X_{20} + Y_1 &= n_1 \\
X_{20} - X_{40} + Y_1 &= n_2 \\
X_{40} - X_{60} + Y_1 &= n_3,
\end{aligned}$$

in which each absolute term will be of weight 4.

As these equations involve five unknown quantities, X_0, X_{20}, X_{40}, X_{60}, Y_1, they will not lead to a determination of any of the unknowns except in combination with other observations. If, however, we suppose X_0, X_{60} as already known from operations of Type I., we may derive X_{20}, X_{40}, Y_1 without ambiguity. We have, in fact,

$$Y_1 = \tfrac{1}{3}(n_1+n_2+n_3)-\tfrac{1}{3}(X_0-X_{60})$$
$$X_{20} = \tfrac{1}{3}(n_2+n_3-2n_1)+\tfrac{1}{3}(2X_0+X_{60})$$
$$X_{40} = \tfrac{1}{3}(2n_3-n_1-n_2)+\tfrac{1}{3}(X_0+2X_{60}).$$

The quantities n_1, n_2, n_3 each being of weight 4, if X_0, X_{60} be supposed absolutely known, the square of the mean errors of X_{20}, X_{40} will be

$$\tfrac{1}{4}\epsilon^2(\tfrac{1}{9}+\tfrac{1}{9}+\tfrac{4}{9}) = \tfrac{1}{3}\epsilon^2.$$

If, however, X_0, X_{60} have only been fallibly determined by means of sets of observations of Type I., concentrated on the set of division marks 0°, 60°, 120°, 180°, 240°, 300°, we may suppose that they have been subjected to the condition

$$x_0+x_{60}+x_{120}+\ \ldots\ = 0$$

or

$$X_0+X_{60}+X_{120} = 0;$$

we shall then have

$$2X_0+X_{60} = X_0-X_{120}$$
$$X_0+2X_{60} = X_{60}-X_{120}$$

and each of these quantities has been shown to be determined with weight 12 by means of a single set of Type I. Thus the square of the mean error of a single determination of

$$\tfrac{1}{3}(2X_0+X_{60}) \text{ or } \tfrac{1}{3}(X_0+2X_{60})$$

will be

$$\tfrac{1}{108}\epsilon^2.$$

Hence the square of the mean error of X_{20}, X_{40} supposed to depend on p sets of operations of Type I. and q sets of Type II. will be

$$\left(\frac{1}{108p}+\frac{1}{6q}\right)\epsilon^2.$$

It is evident from this formula that the operations of Type II. will have to be very frequently repeated if it is desired to determine the quantities X_{20}, X_{40} with anything approaching the precision with which the errors of the fundamental diameters are determined even from a single set of operations of Type I.

Since operations of Type I. are so much more effective than those of Type II., the question arises as to whether it would not be advantageous to subject the intermediate division marks x_{10}, x_{30} &c., directly to sets of operations of the first type.

§ 8. *Alternative Method of Combination of Observations.*

Suppose that three sets of observations of Type II. have been made, subdividing the intervals between the three fundamental diameters 0°, 60°, 120°. Let the resulting equations be expressed in the form

$$\begin{array}{lll} X_0 - X_{20} + Y_1 = n_1, & X_{60} - X_{80} + Y_2 = n_4, & X_{120} - X_{140} + Y_3 = n_7 \\ X_{20} - X_{40} + Y_1 = n_2, & X_{80} - X_{100} + Y_2 = n_5, & X_{140} - X_{160} + Y_3 = n_8 \\ X_{40} - X_{60} + Y_1 = n_3, & X_{100} - X_{120} + Y_2 = n_6, & X_{160} - X_{180} + Y_3 = n_9 \end{array}$$

Then if we put

$$\begin{aligned} \tfrac{1}{3}(X_0 + X_{60} + X_{120}) &= A_0, \\ \tfrac{1}{3}(X_{20} + X_{80} + X_{140}) &= A_{20}, \\ \tfrac{1}{3}(X_{40} + X_{100} + X_{160}) &= A_{40}, \end{aligned}$$

on taking means of the equations in the same horizontal line we obtain

$$\begin{aligned} A_0 - A_{20} + [Y] &= \tfrac{1}{3}(n_1 + n_4 + n_7) = N_1 \\ A_{20} - A_{40} + [Y] &= \tfrac{1}{3}(n_2 + n_5 + n_8) = N_2 \\ A_{40} - A_0 + [Y] &= \tfrac{1}{3}(n_3 + n_6 + n_9) = N_3, \end{aligned}$$

where $$[Y] = \tfrac{1}{3}(Y_1 + Y_2 + Y_3).$$

Since, if the observations depend only on a single set, each n will have weight 4, each N will have weight 12. As, however, these equations involve four unknown quantities, A_0, A_{20}, A_{40}, $[Y]$, they cannot be solved without being supplemented with some further condition. If we suppose that

$$A_0 + A_{20} + A_{40} = 0,$$

i.e. that the sum of the division errors of all the 18 division marks now under investigation is zero, we derive

$$A_0 = \tfrac{1}{3}(N_1 - N_3), \quad A_{20} = \tfrac{1}{3}(N_2 - N_1), \quad A_{40} = \tfrac{1}{3}(N_3 - N_2),$$
$$[Y] = \tfrac{1}{3}(N_1 + N_2 + N_3).$$

Since each N has weight 12, the square of the mean error of each A will be

$$\tfrac{2}{9}\left[\frac{\epsilon^2}{12}\right] = \tfrac{1}{54}\epsilon_2$$

and the weight of each of the A's will be 54.

Again, we have

$$A_0 - A_{20} = \tfrac{1}{3}(2N_1 - N_2 - N_3),$$

whence the square of the mean error of $A_0 - A_{20}$ is

$$\frac{\epsilon^2}{12}\left(\tfrac{4}{9} + \tfrac{1}{9} + \tfrac{1}{9}\right) = \frac{\epsilon^2}{18},$$

and its weight is 18.

Now if each of the quantities X_0, X_{20}, X_{40}, &c., have been previously derived from operations of Type I. subject to the conditions

$$X_0 + X_{60} + X_{120} = 0$$
$$X_{20} + X_{80} + X_{140} = 0$$
$$X_{40} + X_{100} + X_{160} = 0,$$

the quantities A_0, A_{20}, A_{40} will represent the corrections applicable to the different groups of division errors as thus derived, in order to refer the whole series of division marks to a system defined by

$$X_0 + X_{20} + X_{40} + X_{60} + X_{80} + X_{100} + X_{120} + X_{140} + X_{160} = 0.$$

Since the quantities X_0 are derived from a single set of operations of Type I. with weight 36, and the quantities A_0 from a single series of sets of Type II. with weight 54, the square of the mean error of the resulting division error with reference to the set of 18 division marks is

$$\epsilon^2 (\tfrac{1}{36} + \tfrac{1}{54}) = \frac{5\epsilon^2}{108}.$$

Thus the weight of the determination of each division error will be $21\frac{3}{5}$, and the repetition of the operations of Type I. on different series of fundamental divisions has increased the weight of the results more than threefold.

To determine the most economical distribution of the observations in order to derive the relative division errors of a set of 18 division marks with the highest precision, consider the following data :—

If the operations of Type I. be repeated p times and those of Type II. q times, the weight of the resulting determinations will be

$$\frac{1}{\frac{1}{36p} + \frac{1}{54q}} = \frac{108pq}{2p + 3q}.$$

Each set of Type I. will involve 72 pointings, the series of three sets will involve 216 pointings, and if repeated p times $216\,p$ pointings will be required.

Each set of Type II. will involve 48 pointings and a series of three sets each repeated q times will involve $144q$ pointings.

Hence the best distribution of the observations is that which with a given value of

$$216p + 144q \,[= 72(3p + 2q)]$$

gives the greatest value to

$$\frac{108pq}{2p + 3q}.$$

Now
$$\frac{pq}{2p+3q} = (3p+2q)\frac{pq}{6p^2+6q^2+13pq}$$
$$= (3p+2q)\frac{1}{\left(\sqrt{\frac{p}{q}}-\sqrt{\frac{q}{p}}\right)^2+25}.$$

Subject to a given value of $3p+2q$ this will evidently be greatest when
$$\sqrt{\frac{p}{q}}-\sqrt{\frac{q}{p}} = 0,$$

i.e. when
$$p = q.$$

Hence it is desirable that any repetition of the observations should as far as possible be symmetrically distributed over the sets of Type I. and Type II.

§ 9. *Determination of* 5° *Spaces.*

The subdivision of the 20° spaces into spaces of 5° interval may be effected by means of series of operations of Type III. Exactly as in the case of the last series, a set of observations may be supposed to lead to a set of equations of condition of the type

$$\begin{aligned} X_0 - X_{25} + Y_1 &= n_1 \\ X_5 - X_{30} + Y_1 &= n_2 \\ X_{10} - X_{35} + Y_1 &= n_3 \\ X_{15} - H_{40} + Y_1 &= n_4, \end{aligned}$$

in which each absolute term is of weight 4.

By means of nine such sets, starting at intervals of 20° round the circle, we can obtain thirty-six equations of the type

$$\begin{aligned} &X_0 - X_{25} + Y_1 = n_1, \quad X_5 - X_{30} + Y_1 = n_2, \quad X_{10} - X_{35} + Y_1 = n_3, \quad X_{15} - X_{40} + Y_1 = \\ &X_{20} - X_{45} + Y_2 = n_5, \quad X_{25} - X_{50} + Y_2 = n_6, \quad X_{30} - X_{55} + Y_2 = n_7, \quad X_{35} - X_{60} + Y_2 = \\ &X_{40} - X_{65} + Y_3 = n_9, \quad X_{45} - X_{70} + Y_3 = n_{10}, \quad X_{50} - X_{75} + Y_3 = n_{11}, \quad X_{55} - X_{80} + Y_3 = \\ &\qquad\qquad \&c., \qquad\qquad\qquad \&c., \end{aligned}$$

where the equations resulting from a single set are now written in the same horizontal line.

If we suppose that

$$\begin{aligned} \tfrac{1}{9}(X_0 + X_{20} + X_{40} + X_{60} + X_{80} + X_{100} + X_{120} + X_{140} + X_{160}) &= B_0, \\ \tfrac{1}{9}(X_5 + X_{25} + X_{45} + \ldots\ldots + X_{165}) &= B_5, \\ \tfrac{1}{9}(X_{10} + X_{30} + X_{50} + \ldots\ldots + X_{170}) &= B_{10}, \\ \tfrac{1}{9}(X_{15} + X_{35} + X_{55} + \ldots\ldots + X_{175}) &= B_{15}, \end{aligned}$$

on taking the means of all the equations in the same vertical column, we derive

$$\begin{aligned} B_0 - B_5 + [Y] &= N_1 \\ B_5 - B_{10} + [Y] &= N_2 \\ B_{10} - B_{15} + [Y] &= N_3 \\ B_{15} - B_0 + [Y] &= N_4 \end{aligned}$$

where

$$[Y] = \tfrac{1}{4}(Y_1 + Y_2 + Y_3 + \ldots + Y_0)$$

and each absolute term is of weight 36.

These equations must be supplemented by a further condition to render B_0, B_5, &c., determinate. Suppose we assume that

$$B_0 + B_5 + B_{10} + B_{15} = 0$$

i.e. that the sum of division errors of all seventy-two division marks now involved is zero. We then find

$$\begin{aligned} [Y] &= \tfrac{1}{4}(N_1 + N_2 + N_3 + N_4) \\ B_0 &= \tfrac{1}{8}(3N_1 + N_2 - N_3 - 3N_4) \\ B_5 &= \tfrac{1}{8}(3N_2 + N_3 - N_4 - 3N_1) \\ B_{10} &= \tfrac{1}{8}(3N_3 + N_4 - N_1 - 3N_2) \\ B_{15} &= \tfrac{1}{8}(3N_4 + N_1 - N_2 - 3N_3) \end{aligned}$$

Since each N has weight 36, the square of the mean error of each B will be

$$\frac{\epsilon^2}{36} \cdot \frac{9+1+1+9}{64} = \frac{5\epsilon^2}{576}$$

The difference of two of the quantities B, say B_0 and B_5, is

$$\tfrac{1}{4}(3N_1 - N_2 - N_3 - N_4)$$

and the square of its mean error is

$$\frac{\epsilon^2}{36} \cdot \frac{9+1+1+1}{16} = \frac{1}{48}\epsilon^2$$

To utilise these observations in combination with previous sets we may suppose that by means of the former sets we have derived values of

$$X_0, X_{20}, X_{40}, \ldots X_{160}$$

but only on the assumption that

$$A_0 + A_{20} + A_{40} = 0$$

or, what is equivalent,

$$X_0 + X_{20} + X_{40} + \ldots + X_{160} = 0$$

If the true value of this quantity be B_0, each X of the series as

previously derived will require a correction B_0. Suppose similar sets have been made to connect the division marks

$$X_5,\ X_{25},\ X_{45},\ \ldots$$
$$X_{10},\ X_{30},\ X_{40},\ \ldots$$
$$X_{15},\ X_{35},\ X_{55},\ \ldots$$

The derived values of the X's must be corrected by the appropriate amounts B common to each series as derived from observations of the present type.

Thus each division error will consist of three parts :

(1) A part derivable from a series of operations of Type I., which expresses the error of the division mark relatively to six symmetrically situated division marks of which it is one.

(2) A correction A, derived from observations of Type II., to refer it to a set of eighteen symmetrically situated divisions of which it is one.

(3) A correction B, derived from observations of Type III., to refer it to the set of seventy-two symmetrically situated divisions in which occurs.

Since these parts are independently derived, the square of the mean error of the result as thus obtained will be the sum of the squares of the mean errors of its component parts. Thus, if each division error is determined as the result of only one complete series of operations of each type, the square of its mean error will be

$$\varepsilon^2\left(\tfrac{1}{36}+\tfrac{1}{54}+\tfrac{5}{576}\right)$$

If the sets of observations of Type I. be repeated p times, those of Type II. q times, and those of Type III. r times, this expression will be replaced by

$$\varepsilon^2\left(\frac{1}{36p}+\frac{1}{54q}+\frac{5}{576r}\right)=\frac{\varepsilon^2}{1728}\left(\frac{48}{p}+\frac{32}{q}+\frac{15}{r}\right)$$

and such a series of observations will involve

$$864p+576q+576r \text{ or } 288(3p+2q+2r) \text{ pointings.}$$

Hence, in order to secure the highest precision with a given number of pointings, we must ascertain what values of p, q, r will make

$$\frac{48}{p}+\frac{32}{q}+\frac{15}{r}$$

a minimum, subject to the condition that

$$3p+2q+2r$$

should be constant.

By the ordinary rules of the calculus we find, on introducing an undetermined multiplier λ,

$$\frac{48}{p^2}-3\lambda=0,\ \frac{32}{q^2}-2\lambda=0,\ \frac{15}{r^2}-2\lambda=0$$

Whence

$$\frac{p^2}{32}=\frac{q^2}{32}=\frac{r^2}{15}$$

and

$$\frac{p}{r}=\sqrt{\frac{32}{15}}=1\cdot46$$

or roughly

$$p:q:r=3:3:2$$

This indicates how the work may best be distributed in order to secure the highest precision with the greatest economy of labour for the determination for the relative errors of a set of seventy-two equi-spaced division marks.

§ 10. *Determination of Single-degree Spaces.*

The operations previously discussed are restricted to a set of divisions symmetrically situated at intervals of 5° throughout the circle. If we suppose that these operations have been performed on each set of 5° marks, commencing with 0°, 1°, 2°, 3°, 4°, then exactly as we have combined operations on three sets of division marks spaced at intervals of 60° to determine the distribution of eighteen symmetrically situated division marks, and subsequently of four sets spaced at intervals of 20° to determine the distribution of seventy-two division marks, so we may combine the five series of operations on seventy-two division marks to obtain the distribution of the 360° division marks which correspond with the exact degrees on the circle by means of observations taken with the double object-glass micrometer.

A set of observations will lead to equations of condition of the type

$$\begin{aligned}
x_0-x_1+\mathrm{Y}&=n_1\\
x_1-x_2+\mathrm{Y}&=n_2\\
x_2-x_3+\mathrm{Y}&=n_3\\
x_3-x_4+\mathrm{Y}&=n_4\\
x_4-x_5+\mathrm{Y}&=n_5
\end{aligned}$$

where each absolute term is of weight unity, since it depends on the difference of two pointings each repeated twice in direct and reversed order of operations.

Seventy-two such sets will be required to complete the round of the circle, and if we put

$$\begin{aligned}
\tfrac{1}{72}(x_0+x_5+x_{10}+\ \ldots\ +x_{355})&=\mathrm{C}_0\\
\tfrac{1}{72}(x_1+x_6+x_{11}+\ \ldots\ +x_{356})&=\mathrm{C}_1 \text{ \&c.}
\end{aligned}$$

on taking the means of the first, second, third, fourth, and fifth equations resulting from each set, we obtain

$$\begin{aligned} C_0 - C_1 + [Y] &= N_1 \\ C_1 - C_2 + [Y] &= N_2 \\ C_2 - C_3 + [Y] &= N_3 \\ C_3 - C_4 + [Y] &= N_4 \\ C_4 - C_0 + [Y] &= N_5, \end{aligned}$$

which, if supplemented with the condition

$$C_0 + C_1 + C_2 + C_3 + C_4 = 0$$

(*i.e.* if we suppose that the zero of division error is referred to the mean of the 360 division marks), lead to

$$[Y] = \tfrac{1}{5}(N_1 + N_2 + N_3 + N_4 + N_5)$$

$$\begin{aligned} C_0 &= \tfrac{1}{5}(2N_1 + N_2 - N_4 - 2N_5) \\ C_1 &= \tfrac{1}{5}(2N_2 + N_3 - N_5 - 2N_1) \\ C_2 &= \tfrac{1}{5}(2N_3 + N_4 - N_1 - 2N_2) \\ C_3 &= \tfrac{1}{5}(2N_4 - N_5 - N_2 - 2N_3) \\ C_4 &= \tfrac{1}{5}(2N_5 + N_1 - N_3 - 2N_4) \end{aligned}$$

The difference of two of the quantities C will be expressed by formulæ of the nature

$$C_0 - C_1 = \tfrac{1}{5}(4N_1 - N_2 - N_3 - N_4 - N_5)$$

Hence, since each N is derived as the mean of seventy-two quantities of unit weight, the square of the mean error of each of the C's will be

$$\frac{\epsilon^2}{72}\left(\frac{4+1+1+4}{25}\right) = \frac{\epsilon^2}{180}$$

and that of the difference of two of them will be

$$\frac{\epsilon^2}{72}\left(\frac{16+1+1+1+1}{25}\right) = \frac{\epsilon^2}{90}$$

Thus the square of the mean error of the division errors of a pair of opposite division marks, supposed based on the minimum number of sets of the four types under investigation, which will lead to a complete determination of the 180 diameters involved will be

$$\epsilon^2\left(\tfrac{1}{36} + \tfrac{1}{54} + \tfrac{5}{576} + \tfrac{1}{180}\right) = \tfrac{523}{8640}\epsilon^2 = \frac{\epsilon^2}{16{\cdot}5}$$

i.e. each pair of division errors will be determined with weight 16·5.

The mean division error of a set of six symmetrically situated

division marks will be determined by operations of the Types II., III., IV., and thus we may omit the part of the error resulting from operations of Type I. The square of the mean error for a set of six divisions which appear simultaneously under the six primary microscopes will therefore be

$$\epsilon^2\left(\tfrac{1}{54}+\tfrac{1}{315}+\tfrac{1}{180}\right)=\tfrac{25}{840}\epsilon^2=\frac{\epsilon^2}{30}\text{ nearly.}$$

To summarise the results now obtained.

If a series of operations consisting of sixty sets of Type I., sixty of Type II., forty-five of Type III., and seventy-two of Type IV., involving in all 11,520 pointings, be made, we may derive the errors of division of pairs of opposite division marks in the form

$$X_0+A_0+B_0+C_0;\ X_1+A_1+B_1+C_1 \text{ \&c.}$$

where

X denotes the part derived from operations of Type I. and has weight 36;

A denotes the part derived from operations of Type II. with weight 54;

B denotes the part derived from operations of Type III. with weight $\frac{1}{2}\frac{7}{5}\frac{6}{0}$;

C denotes the part derived from operations of Type IV. with weight 180.

Further, we have seen that the difference of two of the quantities X, A, B, C which result from simultaneous determinations will have weights 12, 18, 48, 90 respectively.

Now the values of X, A, B, which refer to two diameters separated by an angle which is an exact multiple of 1°, but not an exact multiple of 5°, will be independently derived, whereas the values of C involved will have resulted from simultaneous determination. Hence the square of the mean error affecting the angle will be

$$\epsilon^2\left(\tfrac{2}{18}+\tfrac{2}{54}+\tfrac{5}{288}+\tfrac{1}{90}\right)=\frac{\epsilon^2}{8{\cdot}6}$$

If the diameters be inclined at an angle which is a multiple of 5°, but not a multiple of 20°, the square of the mean error becomes

$$\epsilon^2\left(\tfrac{2}{18}+\tfrac{2}{54}+\tfrac{1}{72}\right)=\frac{\epsilon^2}{8{\cdot}8}$$

since the quantities C involved will be the same for both diameters, and therefore will not enter into the angle expressed by their difference, while the B's will have been derived by a simultaneous determination.

In like manner the angle between two diameters, if a multiple of 20° but not a multiple of 60°, will be determined with an error whose mean square is

$$\epsilon^2\left(\tfrac{2}{36}+\tfrac{1}{18}\right)=\frac{\epsilon^2}{9}$$

since the quantities B, C will be the same for both diameters, while the A's will have been simultaneously determined.

Finally the angle between two diameters separated by 60° or any multiple thereof will be determined with weight 12, and the square of its mean error will be

$$\frac{\epsilon^2}{12}$$

These errors represent the amounts by which the measurement of an angle will be affected by imperfect determination of division error if the measurements are made with a pair of opposite microscopes. If three such pairs be used we may omit the parts of the errors resulting from imperfect determination of X, since the X's have been so determined that their mean value for six symmetrically placed division marks is zero.

Thus an angle which is a multiple of 1°, but not a multiple of 5°, will be affected by an amount whose mean square is

$$\epsilon^2\left(\tfrac{2}{54}+\tfrac{5}{288}+\tfrac{1}{100}\right)=\frac{\epsilon^2}{15}\text{ nearly};$$

an angle which is a multiple of 5°, but not a multiple of 20°, will be affected by an amount whose mean square is

$$\epsilon^2\left(\tfrac{2}{54}+\tfrac{1}{48}\right)=\frac{\epsilon^2}{17}\text{ nearly};$$

an angle which is a multiple of 20°, but not a multiple of 60°, will be affected by an amount whose mean square is

$$\frac{\epsilon^2}{18};$$

while an angle of 60°, or any multiple thereof, will be entirely free from the effects of division errors.

If the probable error of a single pointing be ±0″·18, which appears to about represent the precision attainable with the Cape micrometers, in the most unfavourable case the probable outstanding effect of division error on the determination of an angle measured with the aid of six microscopes will be

±0″·047

§ 11. *Alternative Method of Reduction.*

The process of subdivision above described might be continued with suitable appliances until the relative division errors of every division mark on the circle had been ascertained. As, however, the circles are divided into arcs of 5′ this will involve the determination of no fewer than 4320 unknown quantities, and the labour might well be regarded as prohibitive. We proceed to consider how it may best be further reduced.

Suppose, in the first instance, that the operations of Types I., II., III. had been restricted to the set of division marks 0°, 5°, 10°, . . . Then each set of Type IV. will, as before, lead to a set of equations of the form

$$\begin{aligned} x_0 - x_1 + Y &= n_1 \\ x_1 - x_2 + Y &= n_2 \\ x_2 - x_3 + Y &= n_3 \\ x_3 - x_4 + Y &= n_4 \\ x_4 - x_5 + Y &= n_5 \end{aligned}$$

from which we may obtain

$$\tfrac{1}{5}(x_0 - x_5) + Y = \tfrac{1}{5}(n_1 + n_2 + n_3 + n_4 + n_5)$$

and

$$x_1 = \frac{4x_0 + x_5}{5} + \tfrac{1}{5}(-4n_1 + n_2 + n_3 + n_4 + n_5)$$

$$x_2 = \frac{3x_0 + 2x_5}{5} + \tfrac{1}{5}(-3n_1 - 3n_2 + 2n_3 + 2n_4 + 2n_5)$$

$$x_3 = \frac{2x_0 + 3x_5}{5} + \tfrac{1}{5}(-2n_1 - 2n_2 - 2n_3 + 3n_4 + 3n_5)$$

$$x_4 = \frac{x_0 + 4x_5}{5} + \tfrac{1}{5}(-n_1 - n_2 - n_3 - n_4 + 4n_5)$$

On combining the results of two sets taken at opposite extremities of diameters * we obtain

$$X_1 = \frac{4X_0 + X_5}{5} + \tfrac{1}{5}(-4N_1 + N_2 + N_3 + N_4 + N_5) \text{ &c.}$$

where each N is the mean of two of the quantities n and therefore has weight 2.

If then we regard X_0, X_5 as determined from previous operations, the square of the mean accidental error of X_1, X_4

* The labour of observation will be somewhat reduced, while the errors of the pivots will be more effectively eliminated, if these two sets be taken simultaneously with a pair of microscopes provided with divided object-glasses. A second microscope of this character has accordingly been ordered from Messrs. Troughton and Simms.

resulting from the errors of observation in the present series will be

$$\tfrac{1}{2}\epsilon^2\left(\frac{16+1+1+1+1}{25}\right)=\frac{2\epsilon^2}{5}$$

and that of X_2, X_3 will be

$$\tfrac{1}{2}\epsilon^2\left(\frac{9+9+4+4+4}{25}\right)=\frac{3\epsilon^2}{5}$$

Thus in order to derive the quantities X_2, X_3 with the same precision as they may be derived by the previous process, it will be necessary to repeat the series of operations of Type IV. at least nine times, and this apart from the outstanding sources of error due to the previous imperfect determination of the quantities X_0, X_5. The complete series of observations will involve no less than

$$72\times 20\times 9(=12960)$$

pointings, or about half as many again as would be required to repeat the operations of Types I., II., III. on different sets of division marks starting at 1°, 2°, 3°, 4°.

Thus so long as the circle as a whole is under investigation it appears that it is highly advantageous to distribute as widely as possible the early observations; but the process now under discussion will be available for the more accurate determination of special divisions, *e.g.* the division marks called into play in the determinations of nadir readings, and the positions of close circumpolar stars may be regarded as worthy of special attention on account of the greater frequency with which they are put into subsequent use.

It has been shown that the complete set of operations requisite to determine the relative spacing of 360 division marks by means of sets of Type I., II., III., IV. will involve sixty sets of Type I., sixty of Type II., forty-five of Type III., and seventy-two of Type IV. This will require

$$60\times 72+60\times 48+45\times 64+72\times 20(=11520)$$

pointings.

In order to escape the difficulties resulting from irregularities of the pivots we have, however, confined our attention in the course of the discussion of such a set to the means of the errors of pairs of opposite division marks, which will be unaffected by pivot errors. We might have gone still further and restricted the discussion only to such errors as will affect the mean reading of the set of six primary microscopes. For any error which only affects the six readings of the microscopes differentially without affecting their mean will be automatically eliminated in the further use of the instrument, provided that all determinations are made with the aid of the six primary microscopes. Thus we

may omit all the operations of Type I. and the necessary number of pointings may be reduced by 4320.

Further, if the circles be free from flexure, the operations of Types II. and III. each involve a duplicate determination of the quantities A, B, since each division mark is brought under a pair of opposite microscopes, whereas the desired end could be attained without this reversal. This duplication of the observations, besides leading to the elimination of any flexure effects from the determination of the division errors, furnishes a valuable control of the observations, in addition to the primary control furnished throughout by the interagreement of the pointings in direct and reversed direction of motion of the circle, and it seems therefore undesirable to dispense with it; if, however, we did so, the number of pointings necessary might be still further reduced by 2880.

There will remain 4320 pointings as the minimum number necessary for the complete investigation of the single-degree spaces.

§ 12. *General Conclusions.*

The process in general use for the determination of the division errors of a graduated circle consists in the determination of the intervals into which the complete circle is divided by a set of fundamental division marks, and of the successive subdivision of these intervals by subsidiary appliances, such as those attached to the Cape Transit circle, which are designed to admit of the subdivision of the circle to the extent of a single degree.

It is usual to concentrate a large amount of labour on the determination of the fundamental intervals, but the precision attained, at least so far as it depends on the repetition of observations, is necessarily reduced from step to step as the number of spaces to be divided increases. The precision finally obtained will, however, be limited by that of the final step, and it is shown that the labour involved in obtaining results from such a step, with precision at all commensurate with that which may be obtained by the present process from the sets which precede it, may always be more economically bestowed on the sets which precede.

The essential feature of the process now indicated consists in the distribution as widely as possible of the early operations rather than of their concentration on a set of particularly selected division marks. Apart from the high accuracy resulting therefrom, the method is characterised by extreme simplicity in the reductions and uniformity in the weights of the results.

Royal Observatory, Cape of Good Hope:
1903 *December* 22.

Note on the Date of the Passage of the Vernal Equinox from Taurus into Aries. By E. Walter Maunder and A. S. D. Maunder.

In vol. iii. of the *Transactions of the Society of Biblical Archæology*, p. 237, Professor Sayce, after giving translations of a number of tablets, writes :—

> "The reports quoted above which refer to the vernal equinox establish the fact that that period of the year corresponded with *Aries*. The Accadian calendar was arranged so as to suit the order of the zodiacal signs; and Nisan the first month answered to the first zodiacal sign. Now the Sun still entered the first point of *Aries* at the vernal equinox in the time of Hipparchus, and would have done so since 2540 B.C. From that epoch backwards to 4698 B.C. *Taurus*, the second sign of the Accadian zodiac and the second month of the Accadian year, would have introduced the spring. The precession of the equinoxes thus enables us to fix the extreme limit of the antiquity of the ancient Babylonian calendar."

The paper from which this quotation is made was written in 1874, and the course of Assyriological discovery has modified some of the conclusions which Professor Sayce then drew. But the two dates given were always evidently erroneous, and yet they are even to this day quoted as if they were of undoubted authority (*e.g.* R. Brown's *Primitive Constellations*, vol. i. p. 54; and E. M. Plunket's *Ancient Calendars and Constellations*, preface, p. viii). It seems worth while therefore to examine into the epochs when the vernal equinox lay within the several constellations, and the connection, if any, between the constellations of the zodiac and the months of the year.

The first point is to ascertain, as far as possible, what were the limits of the primitive zodiacal constellations. The oldest record of use in this respect is Ptolemy's Catalogue, the poem of Aratus giving but little help. It must be borne in mind that the form of that part of a constellation which is at a distance from the ecliptic is here of no interest. Thus, though the figure of *Aquarius* sprawls over nearly the entire length of *Capricornus* and for a good way under *Pisces*, the Sun can only have been considered as in *Aquarius* whilst it was passing along that portion of the ecliptic which lay wholly within the constellation. In the following table the latitudes and longitudes are given of a number of bright stars near the ecliptic and near the boundaries of their respective constellations, stars which there is every reason to believe still remain attached to the same figures as those to which they were originally assigned. From this table the second table has been prepared, giving the approximate longitude of the beginning of each zodiacal constellation, and of

the dates when the spring equinox coincided with it in the past, or will do so in the future. In the first table several extra-zodiacal stars have been included either on account of their brightness or as having been polar stars at some epoch.

TABLE I.

Table of Latitudes and Longitudes for 1900.

Star.	Magnitude.	Latitude.	Longitude.
γ Pegasi	2·87	+ 12° 36′	7° 46′
α Piscium	3·94	− 9 4	27 59
γ Arietis	4·04	+ 7 10	31 47
α Arietis	2·23	+ 9 57	36 15
β Arietis	4·53	+ 1 49	49 27
ν Tauri	2·96	+ 4 3	58 35
Aldebaran	1·06	− 5 32	68 23
Rigel	0·34	− 31 8	75 25
Capella	0·21	+ 22 52	80 27
β Tauri	1·78	+ 5 22	81 11
ζ Tauri	3·00	− 2 12	83 24
Polaris	2·12	+ 66 5	87 10
Betelgeux	var.	− 16 3	87 22
ν Geminorum	1·93	− 0 54	92 2
Sirius	− 1·58	− 39 35	102 41
Castor	1·58	+ 10 5	108 51
Pollux	1·21	+ 6 40	111 50
Procyon	0·48	− 16 0	114 25
ζ Cancri	4·71	− 2 16	119 57
β Ursæ Minoris	2·24	+ 72 59	131 54
α Cancri	4·27	− 5 5	132 14
λ Leonis	4·48	+ 7 54	135 50
α Hydræ	2·16	− 22 26	145 47
Regulus	1·34	+ 0 27	148 26
ψ Draconis	3·64	+ 66 21	156 1
β Leonis	2·23	+ 12 17	170 14
β Virginis	3·80	+ 0 42	175 45
ε Virginis	2·95	+ 16 13	188 33
Spica	1·21	− 2 3	202 26
Arcturus	0·24	+ 30 48	202 50
λ Virginis	4·60	+ 0 32	215 34
α Libræ	2·90	+ 0 16	223 41
θ Libræ	4·34	+ 3 28	238 28

Star.	Magnitude.	Latitude.	Longitude.
		° ′	° ′
δ Scorpii	2·54	− 1 58	241 10
Antares	1·22	− 4 34	248 22
α Herculis	var.	+37 18	254 45
α Ophiuchi	2·14	+35 52	261 3
ι Scorpii	3·14	−16 44	266 8
γ Sagittarii	3·07	− 6 59	269 52
Vega	0·14	+61 44	283 56
Altair	0·89	+29 19	300 22
β Capricorni	3·25	+ 4 35	302 39
δ Capricorni	2·98	− 2 35	322 8
ε Aquarii	3·83	+ 8 5	324 44
α Aquarii	3·19	+10 41	331 57
Fomalhaut	1·29	−21 7	332 27
λ Aquarii	3·84	− 0 23	340 11
β Piscium	4·58	+ 9 3	347 12
α Pegasi	2·57	+19 26	352 4

TABLE II.

Constellation.	Longitude of First point. 1900 A.D.	Length of Constellation on Ecliptic.	Day of Sun's entry into Constellation (present time).	Dates of Coincidence of First Point with	
				Equinoxes. *Vernal.*	Solstices.
	°	°			
Aries	28	22	Apr. 18	110 B.C.	
Taurus ...	50	38	May 11	1680	*Summer.*
Gemini ...	88	27	June 20	4410	2040 A.D.
Cancer ...	115	20	July 18	6340	110
Leo	135	38	Aug. 8	*Autumnal.*	1330 B.C.
Virgo	173	45	Sept. 16	2400 A.D.	4050
Libra	218	22	Nov. 1	820 B.C.	7270
Scorpio ...	240	27	Nov. 23	2400	*Winter.*
Sagittarius ...	267	33	Dec. 19	4330	2120 A.D.
Capricornus ...	300	24	Jan. 20	6700	250 B.C.
Aquarius ...	324	22	Feb. 13	*Vernal.*	1970
Pisces	346	42	Mar. 6	2900 A.D.	3550
Aries	28	22	Apr. 18	110 B.C.	6560

It will be seen that the year 2540 B.C. corresponds to a time when the vernal equinox fell far within the limits of *Taurus*, since that date implies a precession of 62° before the present time, whilst the longitude of *Alcyone*, now the *lucida* of the *Pleiades*, is 58° 35′, and the longitude of *Aldebaran*, the chief

star of *Taurus*, only 68° 23′. The actual date when the equinox passed from *Taurus* into *Aries* was much more nearly 1700 B.C.

But it is one thing to say that the equinoctial point passed from *Taurus* to *Aries* about 1700 B.C. and quite a different thing to say that at this date *Aries* was recognised as having superseded *Taurus* as the constellation that led the year. As a matter of fact, *Aries* did not lead it in 1700 B.C., for, whilst the equinoctial point moves amongst the stars in the direction of diminishing longitude, the Sun in its apparent course through the year moves in the reverse direction. Consequently in 1700 B.C. the Sun was moving, not through *Aries*, but through *Taurus* during the whole of the month succeeding the equinox. This was the epoch when the equinox was at the first point of *Taurus*, just precisely as in the days of Hipparchus it was at the first point of *Aries*. Further, since the bright and distinctive stars of the *Ram* are all at the western end of the constellation, it could not be until the colure had receded as far as Hamal, the *lucida* of the constellation, that there would have been any obvious or pressing necessity for the ancient astronomers to look upon *Aries* as the constellation that opened the year. Now the colure passed through Hamal about 700 B.C. This date then—700 B.C., not 2540 B.C.—is the earliest date when the first month of the year can have been regarded as corresponding with the zodiacal constellation *Aries*, if we accept the assumptions, ordinarily made, that the year began with the vernal equinox, and that the successive constellations of the zodiac were connected with the successive months of the year by the position of the Sun amongst the stars.

The question therefore arises, whether these assumptions are justified, and we have to inquire: (1) Were the successive months connected with the successive constellations either in Accadian or early Semitic calendars? (2) How did the ancient astronomers ascertain the position of the Sun with respect to the stars? (3) Did the ancient astronomers connect the beginning of the year with the equinox, and if so, in what did the connexion consist? With regard to the first inquiry it is curious to note that some popular writers on antiquities systematically write as if it were an established fact both that the constellation figures owed their origin to the climatic conditions of the successive months of the calendar, and that those months owed their names to the constellation figures—two mutually exclusive propositions. Clearly there are only three possible hypotheses: (*a*) The constellations originated first, and the months and their names were derived from them. (*b*) The months came first, and from their characteristics the zodiacal constellations were designed afterwards. (*c*) The two were independent in origin. Any connexion therefore that has been established between them must have arisen in later times.

Of the first of these hypotheses we can dispose at once. It is not a thinkable proposition to assume that any nation devoted

itself to such a systematic study of the starry heavens as to effect the designing of the constellations when as yet it had not progressed sufficiently far to establish for itself a working calendar. Further, the constellations themselves show clearly that they are not more than 5000 years old. It is impossible to conceive that the months of the year and their characteristics had not been recognised prior to that date.

The second hypothesis is not, like the first, impossible upon its very face, but it has serious difficulties. The first is the great irregularity in the length of the zodiacal constellations on the ecliptic. If they were mapped out with a deliberate design to make them correspond to the several months of the year, it seems quite impossible that such a constellation as that of *Virgo* should have been planned to extend over 45° of longitude; still more that it should have been made to follow immediately upon another constellation almost as extended—viz. *Leo*—with 38° of longitude. This objection would have weight if the ancient year had been a solar tropical year, composed of precisely twelve months approximately equal in length. But as the primitive months were lunar months, and the year sometimes contained twelve and sometimes thirteen, the objection gains additional force. That the months of the Assyrian calendar were lunar months, and the year a luni-solar year, is apparent from a great number of tablets. About this there is no doubt. Thus the late Mr. George Smith refers us to *C.I.*, vol. iii. p. 51, lines 7–13, for the translation of a tablet:

> "On the 27th day the Moon was visible; on the 28th day, the 29th day, and 30th day observations for an eclipse were made; but it passed, and an eclipse did not take place; the first day when the Moon was seen, the day of the month Tammuz fixed."

The observations of eclipses, those of the Moon falling on the 14th or 15th day of the month, those of the Sun on the 29th or 30th, show that the months were lunations, commencing with the first actual observation of the new Moon. Thus Mr. George Smith comments on the above tablet:—

> "The Assyrian year consisted of twelve lunar months, each month commencing on the evening when the new Moon was first observed; or, in case the Moon was not observed, the new Moon started 30 days after the last month. The months were all thus either of 29 or 30 days, and these two numbers generally but not always alternated."

Similarly Mr. R. C. Thompson (*Astrological Reports*, p. xix) writes:

> "We know from several sources that the Assyrian year consisted of twelve months, each containing usually thirty days, and an intercalary month called the 'Second Adar' (the Ve'Adar

of the Hebrews). The Babylonian year consisted of the same number of months, but two intercalary months, Elul and Adar, were added. The examination of the texts, edited in this volume, indicates that the Assyrians, like the Babylonians, had a year composed of lunar months, and it seems that the object of the astrological reports, which relate to the appearance of the Moon and Sun, was to help to determine and foretell the length of the lunar month. If this be so the year in common use throughout Assyria must have been lunar. The calendar assigns to each month thirty full days; the lunar month is, however, little more than twenty-nine and a half days; therefore some of the calendar months must consist of twenty-nine days only. In the report of Balasî we read: 'When the Moon does not wait for the Sun' such and such things will happen. The prediction is followed by the words, which are clearly those of the astrologer, 'It appeared on the fifteenth day with the Sun.' 'When the Moon is not seen with the Sun on the fourteenth day of Adar' such and such things will happen. The prediction is followed by the words, which are again clearly those of the astrologer, 'The day will complete Nisan.' Since the Moon appeared without the Sun on the fourteenth, and with the Sun on the fifteenth, the Moon and the Sun will not be in conjunction before the afternoon of the twenty-ninth day; in this case the Moon would not be visible until the first day of the next month. It must be noted that when the astrologer uses the words 'this night' he refers to the eve of the day—*i.e.* 'last night.' The words 'The day will complete Nisan' refer to the thirtieth day of the month, and we know from other texts that they indicate that the month will contain thirty full days."*

Since the Assyrian year was thus luni-solar, and the months actual lunations, it is very clear that if a particular month corresponded with the passage of the Sun through a given constellation in one year, it would not do so a couple of years later, inasmuch as the year of twelve lunar months is eleven days short of a solar year. Bearing in mind the great irregularity of size of the zodiacal constellations, and that, as shown by Table II., whilst the Sun passes through *Cancer* in three weeks, it takes six and a half weeks to pass through *Virgo* and six to pass through *Pisces*, there can never have been any correspondence except of the very roughest nature between the constellations and the actual lunar months in any year, and even this rough correspondence would be seriously disturbed in the following year, and quite thrown out in the third year. The point will be made quite clear by the following table for the years 1901–3, showing the position of the Sun in the various

* It will be seen that both these observations are observations at sunrise. When "the Moon did not wait for the Sun" it means that she set before the Sun arose. When Sun and Moon were seen together, it means that the Sun had risen before the Moon had set.

zodiacal constellations on the evenings when the new moons were first observable.

TABLE III.

Probable Date of Observed New Moon.	Longitude of Sun.	Constellation.	Longitude in Constellation.	Month.	Length of Months.
	°		°		Days.
1901 Mar. 21	1	Pisces	15	Nisan	30
Apr. 20	29	Aries	1	Iyar	29
May 19	58	Taurus	8	Sivan	29
June 17	86	Taurus	36	Tammuz	30
July 17	114	Gemini	26	Ab	29
Aug. 15	142	Leo	7	Elul	30
Sept. 14	171	Leo	36	Tasrit	29
Oct. 13	200	Virgo	27	Marchesvan	30
Nov. 12	230	Libra	12	Cisleu	30
Dec. 12	260	Scorpio	20	Tebet	30
1902 Jan. 11	291	Sagittarius	24	Sebat	29
Feb. 9	320	Capricornus	20	Adar	30
Mar. 11	352	Pisces	6	Nisan	29
Apr. 9	19	Pisces	33	Iyar	30
May 9	48	Aries	20	Sivan	29
June 7	76	Taurus	26	Tammuz	29
July 6	104	Gemini	16	Ab	30
Aug. 5	132	Cancer	17	Elul	29
Sept. 3	160	Leo	25	Tasrit	29
Oct. 2	189	Virgo	16	Marchesvan	30
Nov. 1	218	Libra	0	Cisleu	30
Dec. 1	249	Scorpio	9	Tebet	30
Dec. 31	279	Sagittarius	12	Sebat	29
1903 Jan. 29	309	Capricornus	9	Adar	30
Feb. 28	339	Aquarius	15	Ve-Adar	30
Mar. 30	9	Pisces	23	Nisan	29
Apr. 28	37	Aries	9	Iyar	30
May 28	66	Taurus	16	Sivan	29
June 26	94	Gemini	6	Tammuz	29
July 25	122	Cancer	7	Ab	30
Aug. 24	150	Leo	15	Elul	29
Sept. 22	179	Virgo	6	Tasrit	29
Oct. 21	207	Virgo	34	Marchesvan	30
Nov. 20	237	Libra	19	Cisleu	30
Dec. 20	268	Sagittarius	1	Tebet	29
1904 Jan. 18	297	Sagittarius	30	Sebat	30
Feb. 17	328	Aquarius	4	Adar	30

The chief ground upon which the connexion between the months and the constellation figures has been alleged, has been the meaning of the names of the months in the Accadian and Assyrian calendars. The following table gives the usual interpretations :—

TABLE IV.

No. of Month.	Assyrian.	Accadian.
1	Month of beginning	Altar of righteousness
2	Month of light	The protecting bull
3	Month of bricks	Month of bricks or of twins
4	Month of Tammuz	Fulness of seed
5	—	Fire makes fire
6	Month of the spirit	Errand of Istar
7	Month of commencement	Month of the holy mound
8	The eighth month	The month opposite the foundation
9	Month of the giant	Month of clouds
10	Month of rain	The crossing of the sea
11	Month of destruction	The curse of rain
12	Month of darkness	Sowing of seed

It will be observed that in the Assyrian calendar there is not one single instance in which the name of the month has in it anything which could have served as a basis for the zodiacal design. The Accadian months give two or three which have been believed to have a zodiacal significance. These are : the second month, "the protecting or propitious bull"; the third month, "the month of twins"; and the sixth, "the month of the errand or message of Istar." If these be correct we should be forced to conclude that these months no longer retained their original names, but had been renamed with special reference to the three constellations, *Taurus*, *Gemini*, and *Virgo*, since these three meanings carry no meteorological or climatic significance, and have plainly been derived for the months from the constellations, and not for the constellations from the months. In particular the meaning "errand of Istar," if the constellation of *Virgo* is really intended, would be proof that the constellation had been formed, and was known, before the month received this name.

As a matter of fact there is considerable room for difference of opinion as to the meaning of the Accadian words in each of these three cases. Colonel C. R. Conder, R.E., has kindly given us what he considers to be the most probable interpretation for the names of the Accadian months. He would render the name of the second month as "herd becoming full." The third month is the month "of bricks," corresponding to the third Assyrian month. For the sixth month Colonel Conder writes : "I think the rendering 'message of Istar' is far-fetched," and he suggests

the meaning "parched earth" or "heated vegetation." His scheme for the substantial meaning of the Accadian month names would be as follows :—

1. The lambing month.
2. The calving month.
3. The brick month.
4. The harvest month.
5. The very hot month.
6. The dried-up month.
7. The thunder month.
8. Irrigation month.
9. Very cloudy weather.
10. Flood month.
11. Very rainy month.
12. Ploughing month.

It will be seen that this scheme has no zodiacal reference, but it is entirely climatic.

There is therefore no sufficient evidence that the zodiacal constellations owed their origin to the characteristics of the successive months, or, indeed, had any original connexion with them, whatever connexion may have been set up later.

2. How did the ancient astronomers ascertain the position of the Sun with respect to the stars? There are four obvious methods, any one of which might have been employed. The one which we should ourselves make use of nowadays, and therefore the one which is usually supposed to have been employed, is that of the conjunction with certain stars. This relation assumes that men had previously acquainted themselves very thoroughly with the positions of the chief stars near the ecliptic, and had defined the boundaries of the constellations with some precision. By noting what stars rose heliacally on the morning of a given day, and what set heliacally on the evening of the same day, it would be easy for the ancients, supposing they had already provided themselves with fairly correct maps of the stars, to have inferred what were the stars with which the Sun was then in conjunction.

But the employment of this method involves the previous making of the star maps and the actual observation both of certain heliacal risings and heliacal settings. It is simpler to suppose that one or other of these two relations was employed by itself as connecting the Sun with the stars. They required no instruments nor refinements of methods, no star maps, and so may be considered as likely to have been primitive. But the mere existence of the zodiac is sufficient to show that neither the method of heliacal rising nor that of heliacal setting was the one that was actually used. The relation given by either would be a relation to certain particular stars, and the brightness of the stars chosen would be of far more consequence than the fact that they lay within the zodiacal zone. Stars like *Sirius*, *Procyon*, *Betelgeux*, *Capella*, *Arcturus*, would inevitably be associated with the months rather than the zodiacal figures. Such a constellation as *Cancer*, which at best only claims twenty days out of the year, instead of its due proportion of thirty,

would probably have been eliminated altogether, its area being so small, and its stars so inconspicuous.

The fourth method which may have been adopted is the method of opposition—only this would have demanded some mechanical means for determining the passage of time in order that the moment of midnight might be fixed—and the construction and use of such instruments must have been late rather than early.

There is, however, a simple method by which the passage of the Sun through the several constellations might have been watched; a method which demands no instruments, and which does not assume that the early astronomers had furnished themselves with star maps. If month by month, when the new Moon was first seen in the evening sky, the constellation in which the Moon was then seen to be was considered as the one in which the Sun was, the method would be as simple and effective as could be desired.

This method would be substantially that of heliacal setting, but with a difference. If the first new Moon of the year was observed to set at the same time as a bright star, say *Aldebaran*, then twelve lunations later, when the new Moon was again observed to be setting, *Aldebaran* would still be several degrees high, and in the ordinary way Moon and star would set together, not on the first, but on the second evening of the month, since the sidereal year is longer by eleven days than twelve lunations, and the Moon in twenty-four hours passes over a slightly greater arc than the Sun does in eleven days. When twelve more lunations had passed the Moon and *Aldebaran* would usually set together on the third evening of the month. Twelve lunations later again the two bodies would set together on the fourth evening of the month, which would imply that they would also set together on the first evening of the thirteenth lunation. In other words, speaking generally, when Moon and star set together on the third evening of a given month they would set together on the first evening thirteen lunations later, *i.e.* that year would have thirteen months. This method of fixing the calendar is not one that has been handed down to us by tradition, nor does it seem to have been used in later or Western astronomy, but it does appear to have been an Accadian method. For Professor Sayce and Mr. Bosanquet give in the *Monthly Notices* of this Society, vol. xxxix., the following translation of an Accadian inscription:—

> "When, on the first day of the month Nisan, the star of stars (or *Dilgan*) and the Moon are parallel, that year is normal. When, on the third day of the month Nisan, the star of stars and the Moon are parallel, that year is full."

Professor Sayce and Mr. Bosanquet give no reference for the tablet, and we have not been able to trace it. But in all probability their translation is practically correct, for the method

indicated is just one of those eminently simple and practical ones which experience will dictate, but which is not likely to be thought out theoretically. It is clear that the tablet itself afforded to Professor Sayce and Mr. Bosanquet the first suggestion they had ever had of the method, the suitability of which they only ascertained after the tablet had pointed it out to them. They identified *Dilgan*, "the star of stars," with *Capella* by means of a tablet in the Semitic language, which reads :—

> "The appearance at the beginning of the year of the star *Icu* . . . one observe."

And again :

> "The star *Icu* in the month Nisan was seen."

These they take, and no doubt correctly, as being observations of the heliacal rising of *Capella*, and they point out that it rose heliacally at the time of the spring equinox about 2000 B.C., and, further, that its heliacal rising took place before its heliacal setting. These observations they join with the foregoing as together furnishing the determination of the beginning of the year. But the heliacal rising of a star necessarily gives a purely solar year, whilst the setting together of new moon and star gives a luni-solar year, so that the two methods are really incompatible. In both cases the year would be sidereal and not tropical ; and yet not precisely a sidereal year, since precession would slowly alter the declination of the star and change its position for rising and setting.

The stars which would be suitable for such an observation are the following : *Castor*, *Pollux*, and *Procyon* about 4000 B.C. ; *γ Geminorum* and *Sirius* about 2500 B.C. ; *Capella* and *β Tauri* about 2000 B.C. ; *Betelgeux* about 1600 B.C. ; *Aldebaran* about 700 B.C. ; and the *Pleiades* about 200 B.C. In all probability *Capella* was the star in use : it is much the brightest of the stars named, except *Sirius*, which is about 40° distant from the ecliptic. The result of determining the beginning of the year by this method, but using stars other than *Capella*, may be seen from Table V., which gives the mean date relative to the spring equinox of the first day of the first month for different stars, and at different epochs.

TABLE V.

Approximate number of days before or after the spring equinox for the mean beginning of years determined by the seleniacal setting of stars.

Star.	B.C. 0. Days.	B.C. 1000. Days.	B.C. 2000. Days.	B.C. 3000. Days.	B.C. 4000. Days.
α Arietis	−19	...	...	...	...
η Tauri	+ 2	−12	−26	...	...
β Orionis	+ 7	− 6	−19	...	...
Aldebaran	+ 9	− 4	−18	...	...

Star.	B.C. 0. Days.	B.C. 1000. Days.	B.C. 2000. Days.	B.C. 3000. Days.	B.C. 4000. Days.
Betelgeux	+23	+11	− 3	−16	−30
β Tauri	+27	+12	− 3	−17	−31
Capella	...	+18	+ 1	−14	−28
Sirius	+25	+14	+ 3	− 8	−20
Procyon	...	...	+21	+ 9	− 3
Castor	...	...	+30	+14	0
Pollux	...	...	+30	+14	0

Whenever the method originated, if *Capella* was *Dilgan*, the Sun must have been passing through the constellation *Taurus* during the whole of the first month of the year, so that if any connexion between the signs of the zodiac and the months of the year was then recognised, the *Bull* must have been regarded as the first sign, and the *Ram* as the last; and the connexion between *Taurus* and the first month of the year would have been the more natural, since the second brightest star of the constellation, *β Tauri—Alnath*, "*The Horn-Push*"—would have set together with the new moon and *Capella* at the opening of the first month of the year. And so long as this method remained in force the *Bull* must have continued to lead the zodiac, for the beginning of the year necessarily remained always attached to the same star, not to the equinoctial point. The effect of precession therefore would not be to disturb the relation between the beginning of the year and the constellation, but between the beginning of the year and the equinoctial point, so that it would slowly fall later and later, moving from the spring equinox towards the summer solstice. But though we can now see clearly that this forms a serious objection to the method, the early astronomers cannot possibly have known that there was a difference between the two kinds of years until the effect of precession, becoming in the course of centuries too great to be overlooked, forced itself upon their notice. Sooner or later the farmers, even if the astronomers did not, must have become aware that "the times were out of joint," and this method must have fallen into discredit and disuse.

If *Capella* was the "*Dilgan*"* of this method a curious result follows. One month later the new Moon at setting would find itself in almost the same position with regard to two bright stars, namely, *Castor* and *Pollux*, which it had held at the beginning of the first month with *Capella*. If it set with *Capella* the first day of the first month, it would as a rule set with *Castor* and *Pollux* on the thirtieth day afterwards, and so on. Thus *Gemini* would be marked out as connected with the

* As bearing upon this method Colonel Conder's rendering of DIL = iddu, or "times," and GAN = khasisu, or "determine," so that DIL-GAN = "determining times," is very appropriate.

second month, just as *Taurus* was associated with the first. We are not aware that any tablet has yet been found in which it is recorded that the beginning of the second month of the year was observed in connexion with the seleniacal setting of *Castor* and *Pollux*, as the first month was with the seleniacal setting of *Capella*. But there are two circumstances which strongly suggest that this relationship had been fully noted. First the different months had their tutelary deities; the second month had two, *Istar* and *Tammuz*, the Heavenly Twins. Next on the *Kuduru* stones, set up as landmarks in Babylonian fields, many curious figures are found, some obviously zodiacal, as, for instance, *Sagittarius*, *Capricornus*, and *Scorpio*. These vary in number and order on different stones, but the two first are almost always the same. The first is a crescent on its back, the second a pair of stars; and these are assigned to the patron deities of the first two months—the Moon-god and the Heavenly Twins. The appropriateness of the first sign to the first month of the year as determined by the method we have described is evident, since at the spring equinox the new Moon when setting is more completely "on its back" than at any other time of the year; and the appropriateness of the second symbol to the twin stars *Castor* and *Pollux*, setting together at the beginning of the second month with the new Moon, is not less clear. The same three symbols are shown on a very interesting tablet now in the British Museum, which is sculptured with a scene representing the worship of the Sun-god in the temple of Sippar, and inscribed with a record of the restoration of the temple by Nabu-pal-idinna, King of Babylonia about 870 B.C. It is thought that the sculpture, though thus made in the ninth century B.C., was probably reproduced from a much earlier one, as the temple had undergone restoration about two centuries earlier still. The inscription over the symbols is rendered by Colonel Conder :—

> "The Moon-god, the Sun-god, and Istar, dwellers in the abyss,
> Announce to the years what they are to expect";

an oracular statement, no doubt having an astrological significance, but none the less very appropriate to the symbols of the first two months, which by their relative positions indicated in advance whether the year was to have an intercalary month or not.

3. The above method had no necessary connection with the equinox. It may have originated at a considerably earlier date than the year 2000 B.C., given above, but in this case the mean date for the beginning of the year must have fallen considerably before the actual equinox. It may have continued long after in use, perhaps even as much as a thousand years. In this case the mean date for the beginning of the first month of the year will have been a fortnight after the equinox. But sooner or later the divergence of the first month from its true position must have

been too great to be tolerated, and there certainly came a time when in one way or another the equinox was observed directly, and the lunar year was corrected to fit the tropical year, and not to fit the sidereal. Thus we read in the tablet numbered in the British Museum as K 15 :

> "On the sixth day of the month Nisan the day and the night were equal. The day was six *kasbu*, and the night was six *kasbu*. May Nabu and Marduk be propitious unto the King my lord."

A similar report, translated by Professor Sayce in vol. iii. of the *Transactions of the Society of Biblical Archæology*, gives for the equinox the 15th day of Nisan. These two tablets, from the mound of Kouyunjik, are probably of the seventh century B.C., but cannot be earlier than the eighth. They are sufficient to show that at this date the beginning of the year was directly connected with the equinox, but they leave several questions unanswered. The form of the expressions used suggests that some form of time-measurer, probably a sand-glass or water-clock, was employed. We do not know what phase of sunrise and sunset were taken as defining the beginning and end of the day respectively. If the interval from the appearance of the upper limb of the Sun at sunrise to the disappearance of the upper limb at sunset was taken as defining the day, whilst from that disappearance till the following sunrise was taken as defining the night, it would throw the apparent spring equinox two days early in the year, and the apparent autumnal equinox two days late, reducing the winter half of the year to 177 days and increasing the summer half to 188. On the other hand, the irregularities on the actual horizon would tend to shorten the day and to lengthen the night, and so to bring the observed equinox nearer to the true.

A much more important point which is left undetermined is the question as to what was the connexion between the month Nisan and the equinox ; whether Nisan was so taken that the equinox always fell within it, or whether the mean date of the 1st of Nisan was made roughly coincident with the equinox. The difference is important, since the former method would imply that the mean date for Nisan 1 fell about a fortnight before the equinox, and this again would mean that the month Nisan might have been taken to correspond with *Aries* a thousand years earlier than it would have done if the mean date of Nisan 1 had corresponded with the true equinox, or 1200 years earlier, if we suppose that the Assyrians made the three days' error in their determination of the equinox suggested above.

Mr. George Smith states explicitly that "the Assyrian year commenced at the vernal equinox, the new Moon next before the equinox marking the commencement of the new year, the equinox thus falling some time during the first month, Nisan." But

apparently he bases this conclusion simply on the two equinox tablets cited above, neither of which are conclusive on the matter, as they would equally be consistent with Nisan 1 being taken as the *nearest* new Moon to the equinox, as in the Hebrew calendar. Further on Mr. Smith expresses his opinion that the intercalary months, rendered necessary by the defect of eleven days which a year of twelve lunations shows in each tropical year, were applied in every eighth year, when three months were intercalated. It need scarcely be pointed out that this arrangement, in itself very improbable, would render it impossible for the equinox to fall in Nisan in four years out of every seven.

We do not know of any early evidence bearing on the question of the exact connexion of Nisan with the equinox. There may be other tablets which would enable this point to be settled inferentially. For example, the dates of the Eponym Canon have been fixed by the eclipse of the Sun, which took place in the eponymy of Sagali in the month Sivan. This has been taken as the total eclipse of the Sun which took place on 763 B.C. June 15, and which was a very large partial eclipse at Nineveh. If this identification is accepted, the equinox of that year must have fallen about the ninth day of the month Nisan. This date, like the two others, is therefore inconclusive. It should, however, be pointed out that though the eclipse of 763 B.C. June 15 was very nearly total at Nineveh, and therefore may well have been the eclipse of Sagali, it is not absolutely certain that it was so.* The annular eclipse of 809 B.C. June 13 and the total eclipses of 744 B.C. June 15 and of 716 B.C. June 6 were all of them visible at Nineveh as considerable partial eclipses. It must be remembered that the Assyrians recorded this eclipse, not because they had computed that it was slightly the largest visible at Nineveh during any term of years, but simply because they had actually seen it. There is no reason why the eclipse of 809 B.C. should have been more easily overlooked than that of 763 B.C.†

Whichever of these eclipses we adopt it leaves the question indeterminate as to the precise relation of Nisan 1 to the equinox, and so also does an observation cited by Mr. George Smith of the occurrence of the longest night on the 15th day of the 10th month. The equinox in this case must have fallen about the fifth day of Nisan, and in order to settle the question in

* The reasoning by which Dr. Oppert has been led to fix on B.C. 809, and Mr. George Smith on B.C. 763, as the date of the eclipse of Sagali, implies that the seleniacal setting of *Capella* was then no longer used to determine the beginning of the month Nisan. If that method had been still in use, the eclipse of B.C. 820 July 15, or that of B.C. 803 August 6, would have to be substituted for the foregoing; since in that case Sivan 29 might fall in July or the first week of August, but could not fall in June.

† Or the reason of this eclipse being alone recorded of all those visible at Nineveh during the years covered by the Eponym Canon may have been that it coincided with a serious "revolt in the city of Assur." Eclipses that did not synchronise with any great national disaster may well have gone unnoted.

dispute it would be necessary to find a year in which the equinox had fallen either late in Adar or Ve-Adar, or in the last fortnight of Nisan. The point is one of first importance, for, as already indicated, it means a difference of a thousand years in the possible adoption of the Ram Zodiac. In the one case this may have taken place as early as 1700 B.C., or, allowing for the suggested error in the determination of the equinox, 1900 B.C.; whilst in the other case the earliest possible dates would be 700 B.C. or 900 B.C., according as the true or apparent equinox was observed.

But though there is this ambiguity about these earlier records, it seems quite clear that in later times the first day of Nisan could fall after the spring equinox. Fathers Epping and Strassmaier have translated and discussed three Babylonian calendars, and from the eclipses mentioned in them have been able to deduce the Julian date for Nisan 1 in five years.

Year of the Seleucidean Era.	Year B.C.	Julian date of Nisan 1.
188	124	April 4
189	123	March 25
190	122	April 12
201	111	April 10
202	110	March 30*

It will be seen that in each of these five instances Nisan 1 was later than the true spring equinox, which fell at that time on March 24, so that in the second century before our era the true equinox must have been observed and the day of the nearest new moon, whether before or after it, must have been taken in general as the first day of Nisan.

Even if we did not possess this direct evidence, we should be still strongly inclined to adopt the latest date, for it is quite contrary to experience to suppose that a method of determining the beginning of the year, so simple and effective as that of the seleniacal setting of *Capella*, would be abandoned in favour of a method on an entirely different principle before any inconvenience had arisen from the inherent defects of the first method. Men are far too conservative to make a change of this sweeping character until the pressure of necessity is brought to bear upon them.

Whenever the change took place, and however it was effected, it was nothing less than a complete revolution. The old "star of stars" was abandoned; the primacy of the zodiac was transferred from *Taurus* to the constellation which had previously been regarded as the last of all, *Aries*; the mean date of the beginning of the year was shifted earlier by more than three weeks; and the use of instrumental means of observation of one

* *Astronomisches aus Babylon*, p. 39.

kind or another was introduced in place of the simple scrutiny of the heavens. In the earlier age of astronomy, the age of *Taurus* and *Capella*, we have as yet no direct evidence of observation of the planets. In the second age, the age of *Aries*, the labours of Fathers Epping and Strassmaier have shown us that under the Seleucidæ and Arsacidæ not only were the places of the Moon and five planets carefully observed, but they were also predicted. But there is reason to think that much earlier than this, four or five centuries earlier, the movements of the planets had been carefully watched, and certain relations recognised—at least in the case of the planet *Jupiter*.

If the path of an exterior planet be watched it will be seen to come to a stationary point and then to retrograde for a considerable time, coming to a second stationary point, and afterwards resuming its forward movement. Midway between the two stationary points the planet is in opposition. Consequently, when once the apparent movements of the planets had become familiar to the observers and the period of retrogression for each was known it was only necessary to watch for the time when a planet became stationary as a morning star, and then it would be known that half the number of days of retrogression later it would be in opposition. There are three exterior planets known to the ancients, *Mars*, *Jupiter*, and *Saturn*: of these three *Mars* comes to opposition but once in somewhat over two years, and the time during which it is favourably placed for observation is comparatively short. *Saturn*, on the other hand, moves very slowly and is not conspicuous. *Jupiter*, the remaining planet and the brightest of the three, seems exactly fitted to draw attention. Its average rate of motion is 1° in four days—half the diameter of the Moon in twenty-four hours. This is an amount that the eye can readily detect, and with a little practice the ancient astronomers would no doubt soon have been able to fix the time of the stationary point of *Jupiter* with a fair degree of accuracy. The interval from this date to the time of opposition would be almost precisely two lunar months, and two lunar months again would bring it to its second stationary point. It would then be noted that from one opposition to the next *Jupiter* would pass through almost exactly one-twelfth part of the ecliptic—that is to say, the mean length of a constellation of the zodiac. Further, the arc traversed during the four months of retrogression would be, roughly, the third part of the mean length of a zodiacal constellation. In other words, it covered the arc which in later times was known as a "decan"—an arc, that is to say, of 10°. The march of *Jupiter* in a year corresponded therefore, in the mean, to the march of the Sun in a month; and hence no doubt it was that Marduk, the supreme god of the Babylonian pantheon, was intimately connected both with the Sun and with the planet *Jupiter*.

In the fifth tablet of the Babylonian Story of the Creation, translated by Mr. L. W. King, there seems a clear reference to

this action of the planet *Jupiter* as the divider of the ecliptic. The tablet runs:

"He" (*i.e.* Marduk) "made the stations for the great gods;
The stars, their images, as the stars of the zodiac he fixed.
He ordained the year and into sections he divided it;
For the twelve months he fixed three stars.
After he had . . . The days of the year . . . images.
He founded the station of Nibur to determine their bounds;
That none might err or go astray.
He set the station of Bel and Ea along with him."

The division of the year by Marduk into twelve parts and the allotment of three "stars" to each part corresponds to the division of the ecliptic by the planet *Jupiter* into twelve sections, each of which it subdivides again into three "decans." But such a division implies that already the old irregular partition of the ecliptic between the zodiacal constellations was being superseded by a division into twelve equal parts, each composed of three portions of ten degrees in length. The works of Autolycus the Æolian in the fourth century B.C. are the earliest extant in which this division into the "dodecatomoria" is explicitly mentioned.

The significance of the line "He founded the station of Nibur" lies in the fact that different names were applied to the planet *Jupiter* under different circumstances. Thus a different name was given to the planet according to the month of the year in which it was observed, and it became *Nibiru* in the month Tisri, that is to say, at the autumnal equinox. The name *Nibiru* was also given to it when it culminated, "When it stands in the meridian it becomes Nibiru" (*Astrological Report*, No. 94, in R. C. Thompson's translation). Thus the founding of the station of Nibur by Marduk is the equivalent of the planet *Jupiter* marking out the meridian and the ascending node. It is in complete accord with this meaning that Colonel C. R. Conder explains *Nibiru* as the "crossing over"; for the meridian is the crossing place for a celestial body from its ascent to its descent, and the ascending node is the crossing place from the southern hemisphere to the northern. The distinct recognition of the meridian, of the ecliptic and of the equator, of the nodes of these two great circles, and of the position of *Jupiter* as in opposition to the Sun, shows a great advance in the science of astronomy upon the times when the seleniacal setting of stars formed the chief fundamental observation. This knowledge does not appear to have reached the Ionian Greeks until about 600 B.C., when the recognition of the obliquity of the ecliptic and the division of the celestial sphere into five zones are ascribed to Thales of Miletus, or to his pupil Anaximander.

We can dimly discern from the foregoing considerations that primitive astronomy had passed through two very distinct and

well-defined epochs before the time of Hipparchus. To the first age, the age in which the *Bull* led the zodiac, we must assign the formation of the constellations, which cannot have taken place earlier than 3000 B.C., nor much later than 2500 B.C. In this age, amongst the Accadians at least, the beginning of the year was determined by the setting together of *Capella* and the new Moon. The second age was that in which the *Ram* was recognised as the leader of the celestial host, and it probably began not earlier than 700 B.C., possibly even later. It was the age in which the equinox was determined by instrumental observation, *i.e.* by the use of a mechanical time-measurer. It was the age also of the recognition of the apparent motions of the planets—at any rate of the planet *Jupiter*—and before it closed the regular observation of the planets had reached a remarkably high state of development. Its rise was probably connected very intimately with the great outburst of literary activity which characterised the reign of Assur-bani-pal. The first age, the age of the Bull Zodiac, cannot

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Ephemeris for Physical Observations of

Greenwich Mean Noon.	P.	L—O.	B.	A—L.	B.	Q.	d.
1904.	°	°	°	°	°	°	°
Nov. 7	23·45	267·92	+25·26	−30·35	+21·67	294·46	28·06
9	24·09	269·23	25·24	30·75	21·87	294·48	28·37
11	24·72	270·54	25·21	31·14	22·06	294·49	28·68
13	25·33	271·85	25·17	31·53	22·25	294·49	28·99
15	25·94	273·15	25·12	31·91	22·43	294·48	29·29
17	26·53	274·45	25·05	32·29	22·61	294·47	29·60
19	27·11	275·74	24·98	32·65	22·78	294·45	29·89
21	27·68	277·03	24·90	33·00	22·95	294·42	30·17
23	28·23	278·31	24·80	33·34	23·11	294·38	30·45
25	28·77	279·58	24·68	33·68	23·26	294·32	30·74
27	29·30	280·84	24·56	34·01	23·41	294·26	31·02
29	29·81	282·10	24·43	34·33	23·55	294·19	31·29
Dec. 1	30·31	283·35	24·28	34·64	23·69	294·12	31·56
3	30·80	284·59	24·12	34·94	23·82	294·04	31·83
5	31·27	285·83	23·96	35·24	23·95	293·95	32·11
7	31·72	287·06	23·79	35·52	24·07	293·85	32·37
9	32·16	288·28	23·61	35·80	24·18	293·74	32·64
11	32·59	289·49	23·42	36·06	24·29	293·63	32·89
13	33·00	290·69	+23·22	−36·30	+24·39	293·51	33·12

have passed gradually by insensible degrees into the second age, the age of the Ram Zodiac. The threefold change,—of the initial constellation, of the method of determining the beginning of the year, and of the relation of the year to the actual seasons by an interval of three weeks or more,—must have entailed a complete astronomical revolution.

The third age we may consider as beginning with Hipparchus It brought the recognition of precession, and in consequence the actual "constellations" of the zodiac were superseded by the "signs" of the zodiac; that is to say, for the actual groups of stars were substituted purely arbitrary, imaginary, and equal divisions of the ecliptic. In this age the apparent motions of the planets were not only noted, but under the Ptolemaic hypothesis were reduced to a definite mathematical system. For the fourth age men had to wait until the days of Galileo brought the acceptance of the Copernican doctrine and the invention of the telescope.

Mars, 1904-5-6. By A. C. D. Crommelin.

Greenwich Mean Noon. 1904.		⊙	Light-time.	Appar. Diam.	Defect of Illumination.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24^h+.
		°	m	″	″	°	h m	m
Nov.	7	60·10	17·249	4·48	0·26	312·23	3 16·40	40·10
	9		17·121	4·51	·27	292·72	4 36·59	
	11		16·991	4·55	·28	273·23	5 56·71	
	13		16·859	4·58	·29	253·74	7 16·83	
	15		16·725	4·62	·30	234·26	8 36·91	
	17		16·589	4·66	·30	214·78	9 56·99	
	19	65·34	16·452	4·70	·31	195·31	11 17·02	40·02
	21		16·312	4·74	·32	175·84	12 37·05	
	23		16·171	4·78	·33	156·38	13 57·04	
	25		16·028	4·82	·34	136·93	15 16·99	
	27		15·884	4·87	·35	117·48	16 36·94	
	29		15·739	4·91	36	98·04	17 56·85	
Dec.	1	70·58	15·592	4·96	·37	78·61	19 16·72	39·92
	3		15·444	5·01	·38	59·19	20 36·54	
	5		15·294	5·06	·39	39·77	21 56·37	
	7		15·143	5·11	·40	20·36	23 16·15	
	9		14·990	5·16	·41	0·96	24 35·89	
	11		14·836	5·21	·42	341·58	1 15·71*	
	13	75·82	14·681	5·27	0·43	322·20	2 35·37	39·82

Greenwich Mean Noon.	P.	L−O.	B.	Δ−L.	B.	Q.	d.
1904.	°	°	°	°	°	°	°
Dec. 15	33·40	291·88	+23·01	−36·54	+24·49	293·38	33·37
17	33·78	293·06	22·79	36·76	24·58	293·23	33·60
19	34·14	294·23	22·57	36·97	24·67	293·08	33·83
21	34·49	295·39	22·34	37·17	24·75	292·93	34·06
23	34·82	296·54	22·10	37·36	24·82	292·77	34·28
25	35·14	297·68	21·85	37·53	24·89	292·60	34·49
27	35·44	298·81	21·60	37·70	24·95	292·43	34·70
29	35·73	299·93	21·34	37·85	25·00	292·26	34·90
31	36·01	301·03	21·07	37·99	25·05	292·08	35·09
1905.							
Jan. 2	36·27	302·12	20·80	38·11	25·09	291·90	35·27
4	36·51	303·20	20·52	38·22	25·12	291·71	35·44
6	36·74	304·27	20·24	38·32	25·15	291·51	35·60
8	36·95	305·33	19·95	38·40	25·17	291·30	35·75
10	37·15	306·37	19·66	38·47	25·19	291·08	35·89
12	37·34	307·40	19·37	38·52	25·20	290·86	36·02
14	37·51	308·41	19·07	38·56	25·21	290·64	36·14
16	37·67	309·41	18·77	38·58	25·20	290·41	36·24
18	37·82	310·40	18·47	38·59	25·19	290·17	36·33
20	37·95	311·37	18·17	38·58	25·18	289·92	36·41
22	38·07	312·33	17·87	38·56	25·16	289·67	36·48
24	38·17	313·28	17·56	38·53	25·13	289·42	36·54
26	38·26	314·21	17·25	38·48	25·10	289·17	36·59
28	38·34	315·12	16·94	38·41	25·06	288·92	36·62
30	38·42	316·02	16·63	38·32	25·01	288·67	36·64
Feb. 1	38·48	316·90	16·32	38·22	24·96	288·42	36·64
3	38·53	317·77	16·02	38·09	24·90	288·17	36·62
5	38·56	318·62	15·72	37·95	24·83	287·92	36·60
7	38·59	319·45	15·42	37·79	24·75	287·67	36·57
9	38·61	320·26	15·12	37·62	24·67	287·42	36·52
11	38·62	321·05	14·83	37·43	24·58	287·17	36·44
13	38·62	321·82	14·54	37·21	24·49	286·92	36·34
15	38·61	322·57	14·25	36·98	24·39	286·66	36·22
17	38·60	323·31	13·97	36·73	24·28	286·40	36·09
19	38·58	324·02	13·70	36·46	24·17	286·15	35·93
21	38·55	324·71	13·43	36·17	24·05	285·89	35·74
23	38·52	325·38	13·17	35·85	23·93	285·63	35·53
25	38·49	326·02	12·92	35·49	23·80	285·38	35·29
27	38·45	326·64	+12·68	−35·10	+23·66	285·14	35·02

Greenwich Mean Noon.	☉	Light-time.	Appar. Diam.	Defect of Illumination.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24^h+.
1904.	°	m	″	″	°	h m	m
Dec. 15		14·525	5·32	0·44	302·83	3 54·97	
17		14·367	5·38	·45	283·47	5 14·53	
19		14·208	5·44	·46	264·12	6 34·06	
21		14·048	5·50	·47	244·78	7 53·54	
23		13·887	5·56	·48	225·45	9 12·98	
25	81·07	13·725	5·63	·50	206·14	10 32·33	39·67
27		13·562	5·70	·51	186·84	11 51·64	
29		13·399	5·77	·52	167·55	13 10·91	
31		13·234	5·84	·53	148·27	14 30·14	
1905.							
Jan. 2		13·069	5·92	·54	129·00	15 49·32	
4		12·903	6·00	·56	109·74	17 8·46	
6	86·34	12·736	6·08	·57	90·50	18 27·52	39·52
8		12·568	6·16	·58	71·27	19 46·54	
10		12·400	6·24	·59	52·06	21 5·50	
12		12·231	6·33	·60	32·86	22 24·39	
14		12·062	6·42	·62	13·67	23 43·24	
16		11·893	6·51	·63	354·49	0 22·64*	
18	91·64	11·723	6·60	·64	335·33	1 41·36	39·35
20		11·553	6·70	·65	316·18	3 0·03	
22		11·383	6·80	·67	297·04	4 18·67	
24		11·213	6·90	·68	277·92	5 37·21	
26		11·042	7·00	·69	258·81	6 55·72	
28		10·871	7·11	·70	239·72	8 14·14	
30	96·97	10·700	7·22	·71	220·65	9 32·48	39·17
Feb. 1		10·529	7·34	·73	201·59	10 50·77	
3		10·359	7·46	·74	182·55	12 8·99	
5		10·189	7·59	·75	163·53	13 27·13	
7		10·019	7·72	·76	144·53	14 45·17	
9		9·850	7·85	·77	125·54	16 3·17	
11	102·36	9·682	7·99	·78	106·57	17 21·08	38·93
13		9·514	8·13	·79	87·63	18 38·87	
15		9·346	8·28	·80	68·71	19 56·57	
17		9·179	8·43	·81	49·80	21 14·23	
19		9·013	8·58	·82	30·91	22 31·80	
21		8·848	8·74	·82	12·04	23 49·28	
23	107·79	8·684	8·91	·83	353·20	0 27·95*	38·67
25		8·521	9·08	·83	334·38	1 45·22	
27		8·360	9·25	0·84	315·58	3 2·41	

Greenwich Mean Noon.	P.	L−O.	B.	A−L.	B.	Q.	d.
1905.	°	°	°	°	°	°	°
Mar. 1	38·41	327·24	+12·44	−34·69	+23·52	284·90	34·73
3	38·37	327·81	12·22	34·27	23·37	284·67	34·41
5	38·32	328·35	12·00	33·83	23·21	284·44	34·06
7	38·27	328·87	11·79	33·38	23·04	284·21	33·68
9	38·22	329·36	11·60	32·90	22·87	284·00	33·27
11	38·17	329·81	11·42	32·39	22·69	283·81	32·83
13	38·13	330·23	11·26	31·84	22·51	283·63	32·36
15	38·09	330·62	11·11	31·25	22·32	283·46	31·85
17	38·05	330·98	10·98	30·62	22·12	283·29	31·30
19	38·01	331·31	10·87	29·96	21·91	283·13	30·71
21	37·98	331·60	10·77	29·26	21·70	282·98	30·07
23	37·95	331·85	10·69	28·53	21·48	282·85	29·39
25	37·92	332·06	10·63	27·77	21·25	282·73	28·66
27	37·90	332·23	10·59	26·96	21·02	282·62	27·89
29	37·88	332·36	10·57	26·11	20·78	282·52	27·07
31	37·88	332·46	10·56	25·21	20·54	282·44	26·21
Apr. 2	37·88	332·51	10·59	24·27	20·29	282·38	25·29
4	37·89	332·50	10·64	23·29	20·04	282·34	24·32
6	37·91	332·45	10·70	22·27	19·78	282·31	23·30
8	37·94	332·36	10·79	21·21	19·51	282·30	22·23
10	37·97	332·23	10·90	20·10	19·24	282·29	21·10
12	38·00	332·06	11·03	18·94	18·96	282·30	19·92
14	38·04	331·83	11·19	17·73	18·68	282·32	18·67
16	38·10	331·55	11·38	16·47	18·39	282·36	17·36
18	38·15	331·23	11·59	15·16	18·09	282·40	16·00
20	38·21	330·87	11·82	13·82	17·79	282·46	14·61
22	38·27	330·47	12·07	12·45	17·48	282·55	13·18
24	38·34	330·03	12·33	11·05	17·17	282·65	11·72
26	38·41	329·54	12·62	9·57	16·85	282·70	10·18
28	38·48	329·02	12·93	8·05	16·53	282·74	8·60
30	38·55	328·45	13·26	6·51	16·20	282·72	6·98
May 2	38·62	327·87	13·60	4·96	15·87	282·50	5·32
4	38·69	327·26	13·94	3·40	15·53	282·15	3·66
6	38·75	326·63	14·29	1·82	15·18	281·62	1·98
8	38·81	325·99	14·65	−0·21	14·83	...	0·28
10	38·86	325·33	15·01	+1·45	14·47	...	1·51
12	38·90	324·67	15·37	3·10	14·10	106·36	3·23
14	38·94	324·01	+15·73	+ 4·73	+13·73	105·85	5·01

Greenwich Mean Noon.	⊙	Light-time.	Appar. Diam.	Defect of Illumination.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24^h+.
1905-	°	m	″	″	°	h m	m
Mar. 1		8·199	9·43	0·84	296·80	4 19·51	
3		8·040	9·62	·84	278·05	5 36·48	
5		7·883	9·81	·84	259·33	6 53·31	
7	113·29	7·727	10·01	·84	240·64	8 10·02	38·34
9		7·573	10·21	·84	221·98	9 26·60	
11		7·421	10·42	·83	203·34	10 43·09	
13		7·270	10·64	·83	184·74	11 59·42	
15		7·121	10·86	·82	166·17	13 15·62	
17		6·975	11·09	·81	147·64	14 31·66	
19	118·87	6·832	11·32	·79	129·14	15 47·57	37·95
21		6·692	11·56	·78	110·66	17 3·39	
23		6·554	11·80	·76	92·22	18 19·05	
25		6·419	12·05	·74	73·83	19 34·50	
27		6·286	12·31	·72	55·48	20 49·78	
29		6·156	12·57	·69	37·17	22 4·89	
31	124·52	6·030	12·83	·66	18·90	23 19·82	37·44
Apr. 2		5·907	13·09	·63	0·67	24 34·60	
4		5·789	13·36	·59	342·48	1 11·89*	
6		5·674	13·63	·56	324·33	2 26·34	
8		5·562	13·90	·52	306·23	3 40·58	
10		5·456	14·17	·47	288·18	4 54·60	
12	130·27	5·354	14·44	·43	270·17	6 8·46	36·90
14		5·257	14·71	·38	252·20	7 22·15	
16		5·165	14·97	·34	234·27	8 35·68	
18		5·077	15·22	·30	216·39	9 49·00	
20		4·995	15·47	·25	198·55	11 2·14	
22		4·918	15·72	·21	180·76	12 15·08	
24	136·12	4·847	15·96	·16	163·01	13 27·85	36·36
26		4·781	16·18	·13	145·29	14 40·49	
28		4·721	16·38	·10	127·60	15 53·00	
30		4·667	16·57	·06	109·96	17 5·31	
May 2		4·620	16·74	·04	92·36	18 17·44	
4		4·578	16·89	·02	74·77	19 29·54	
6	142·07	4·543	17·02	·01	57·18	20 41·64	36·04
8		4·514	17·13	...	39·61	21 53·65	
10		4·492	17·22	...	22·05	23 5·57	
12		4·476	17·28	·01	4·52	24 17·46	
14		4·466	17·32	0·03	346·97	0 53·42*	

Greenwich Mean Noon.	P.	L−O.	B.	A−L.	B.	Q.	d.
1905.							
May 16	38°97	323°36	+16°08	+ 6°34	+13°36	105°84	6°71
18	39·00	322·72	16·43	7·94	12·98	105·85	8·42
20	39·02	322·10	16·77	9·53	12·60	106·00	10·12
22	39·03	321·51	17·09	11·11	12·21	106·14	11·82
24	39·04	320·94	17·40	12·67	11·82	106·28	13·48
26	39·05	320·40	17·69	14·19	11·43	105·42	15·10
28	39·05	319·90	17·97	15·66	11·03	106·56	16·65
30	39·05	319·43	18·23	17·08	10·63	106·70	18·18
June 1	39·05	319·01	18·47	18·46	10·22	106·84	19·67
3	39·05	318·65	18·68	19·80	9·81	106·97	21·12
5	39·04	318·34	18·87	21·10	9·39	107·10	22·53
7	39·04	318·07	19·04	22·35	8·97	107·21	23·89
9	39·04	317·85	19·19	23·56	8·54	107·31	25·20
11	39·04	317·67	19·32	24·72	8·11	107·39	26·45
13	39·05	317·57	19·43	25·82	7·68	107·46	27·64
15	39·06	317·52	19·51	26·87	7·24	107·52	28·78
17	39·07	317·52	19·57	27·85	6·80	107·57	29·87
19	39·08	317·57	19·61	28·77	6·35	107·60	30·91
21	39·09	317·67	19·63	29·64	5·90	107·62	31·91
23	39·11	317·82	19·64	30·48	5·45	107·62	32·87
25	39·14	318·02	19·62	31·27	5·00	107·62	33·77
27	39·16	318·28	19·58	32·02	4·54	107·61	34·62
29	39·19	318·58	19·52	32·72	4·08	107·59	35·43
July 1	39·21	318·93	19·44	33·38	3·62	107·56	36·20
3	39·24	319·32	19·34	33·99	3·15	107·52	36·93
5	39·26	319·74	19·22	34·56	2·68	107·46	37·62
7	39·28	320·21	19·08	35·09	2·21	107·39	38·27
9	39·31	320·72	18·93	35·59	1·74	107·30	38·87
11	39·33	321·28	18·76	36·06	1·26	107·21	39·43
13	39·35	321·87	18·57	36·49	0·78	107·10	39·96
15	39·36	322·49	18·37	36·88	+0·30	106·98	40·46
17	39·36	323·14	18·15	37·24	−0·18	106·84	40·93
19	39·36	323·82	17·91	37·58	0·66	106·69	41·37
21	39·35	324·53	17·66	37·90	1·14	106·53	41·79
23	39·34	325·27	17·39	38·19	1·63	106·36	42·17
25	39·31	326·03	17·11	38·46	2·11	106·18	42·52
27	39·27	326·82	16·81	38·71	2·60	105·98	42·85
29	39·22	327·64	+16·50	+38·95	−3·09	105·77	43·16

Greenwich Mean Noon. 1905.	☉	Light-time.	Appar. Diam.	Defect of Illumination.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages $24^h 4^m$.
	°	m	″	″	°	h m	m
May 16		4·462	17·33	0·06	329·39	2 5·53	
18	148·15	4·464	17·33	·10	311·80	3 17·68	36·07
20		4·472	17·29	·14	294·20	4 29·86	
22		4·486	17·24	·19	276·59	5 42·09	
24		4·505	17·17	·24	258·96	6 54·41	
26		4·530	17·07	·29	241·26	8 7·02	
28		4·560	16·96	·35	223·52	9 19·79	
30	154·34	4·595	16·83	·42	205·75	10 32·69	36·47
June 1		4·635	16·68	·49	187·95	11 45·71	
3		4·679	16·53	·56	170·10	12 58·94	
5		4·728	16·36	·62	152·19	14 12·48	
7		4·781	16·17	·69	134·22	15 26·27	
9		4·837	15·99	·76	116·20	16 40·26	
11	160·66	4·897	15·79	·83	98·14	17 54·44	37·10
13		4·961	15·59	·89	80·03	19 8·82	
15		5·028	15·38	0·95	61·86	20 23·47	
17		5·097	15·17	1·01	43·63	21 38·37	
19		5·169	14·96	1·06	25·34	22 53·52	
21		5·244	14·75	1·11	7·00	24 8·88	
23	167·11	5·320	14·53	1·16	348·62	0 46·71*	37·77
25		5·399	14·32	1·21	330·19	2 2·38	
27		5·480	14·11	1·25	311·70	3 18·30	
29		5·562	13·90	1·29	293·16	4 34·44	
July 1		5·647	13·69	1·32	274·57	5 50·79	
3		5·733	13·49	1·35	255·94	7 7·31	
5	173·70	5·820	13·29	1·38	237·28	8 23·96	38·36
7		5·908	13·09	1·41	218·57	9 40·81	
9		5·997	12·90	1·43	199·81	10 57·90	
11		6·088	12·70	1·45	181·02	12 15·12	
13		6·179	12·52	1·46	162·19	13 32·51	
15		6·271	12·33	1·48	143·34	14 49·98	
17	180·42	6·363	12·16	1·49	124·45	16 7·62	38·83
19		6·457	11·98	1·50	105·53	17 25·38	
21		6·550	11·81	1·50	86·58	18 43·27	
23		6·643	11·64	1·51	67·61	20 1·24	
25		6·738	11·48	1·51	48·61	21 19·34	
27		6·833	11·32	1·51	29·58	22 37·57	
29	187·28	6·928	11·16	1·51	10·52	23 55·92	39·18

Greenwich Mean Noon.	P.	L−O.	B.	A−L.	B.	Q.	d.
1905.	°	°	°	°	°	°	°
July 31	39·16	328·48	+16·17	+39·16	− 3·58	105·55	43·45
Aug. 2	39·09	329·34	15·83	39·35	4·07	105·32	43·72
4	39·01	330·22	15·47	39·52	4·56	105·07	43·97
6	38·91	331·12	15·10	39·68	5·05	104·81	44·19
8	38·80	332·04	14·72	39·84	5·54	104·54	44·40
10	38·67	332·98	14·32	39·99	6·03	104·25	44·59
12	38·52	333·94	13·91	40·12	6·52	103·94	44·76
14	38·36	334·91	13·48	40·23	7·01	103·62	44·90
16	38·18	335·89	13·05	40·33	7·49	103·29	45·02
18	37·98	336·89	12·61	40·43	7·98	102·95	45·12
20	37·77	337·90	12·15	40·52	8·46	102·59	45·23
22	37·54	338·92	11·68	40·60	8·94	102·22	45·32
24	37·29	339·96	11·20	40·67	9·42	101·84	45·39
26	37·01	341·01	10·70	40·74	9·90	101·45	45·45
28	36·72	342·07	10·20	40·80	10·38	101·04	45·50
30	36·42	343·14	9·69	40·86	10·85	100·62	45·54
Sept. 1	36·09	344·22	9·17	40·91	11·32	100·19	45·56
3	35·74	345·31	8·64	40·97	11·78	99·75	45·57
5	35·37	346·41	8·10	41·02	12·24	99·30	45·57
7	34·97	347·52	7·55	41·07	12·70	98·83	45·56
9	34·56	348·65	6·99	41·12	13·15	98·35	45·54
11	34·13	349·78	6·42	41·16	13·60	97·86	45·50
13	33·67	350·92	5·85	41·20	14·05	97·36	45·46
15	33·20	352·06	5·27	41·24	14·49	96·85	45·41
17	32·70	353·21	4·68	41·28	14·93	96·32	45·35
19	32·19	354·37	4·08	41·32	15·36	95·79	45·28
21	31·65	355·55	3·48	41·36	15·79	95·25	45·21
23	31·09	356·73	2·87	41·39	16·21	94·70	45·12
25	30·51	357·92	2·26	41·42	16·63	94·14	45·03
27	29·92	359·11	1·65	41·46	17·03	93·58	44·93
29	29·30	0·31	1·03	41·49	17·43	93·01	44·83
Oct. 1	28·67	1·52	+0·41	41·53	17·82	92·44	44·72
3	28·02	2·74	−0·22	41·56	18·20	91·86	44·60
5	27·35	3·97	0·85	41·60	18·58	91·28	44·47
7	26·66	5·20	1·49	41·63	18·95	90·69	44·34
9	25·96	6·44	2·13	41·66	19·31	90·09	44·20
11	25·24	7·69	2·77	41·69	19·66	89·49	44·06
13	24·50	8·95	− 3·41	+41·72	−20·00	88·88	43·91

Greenwich Mean Noon. 1905.	☉	Light-time.	Appar. Diam.	Defect of Illumination.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24^h+.
	°	m	″	″	°	h m	m
July 31		7·025	11·01	1·51	351·44	0 35·17*	
Aug. 2		7·120	10·86	1·51	332·34	1 53·66	
4		7·216	10·72	1·50	313·22	3 12·23	
6		7·312	10·58	1·50	294·09	4 30·89	
8		7·408	10·44	1·49	274·92	5 49·63	
10	194·28	7·504	10·31	1·48	255·74	7 8·46	39·42
12		7·600	10·18	1·47	236·55	8 27·33	
14		7·696	10·05	1·47	217·34	9 46·28	
16		7·792	9·93	1·46	198·12	11 5·27	
18		7·887	9·80	1·45	178·87	12 24·40	
20		7·984	9·69	1·43	159·62	13 43·53	
22	201·40	8·079	9·57	1·42	140·36	15 2·70	39·59
24		8·175	9·46	1·41	121·09	16 21·91	
26		8·271	9·35	1·39	101·80	17 41·21	
28		8·367	9·24	1·38	82·50	19 0·55	
30		8·462	9·14	1·37	63·19	20 19·93	
Sept. 1		8·557	9·04	1·36	43·87	21 39·36	
3	208·63	8·653	8·94	1·34	24·54	22 58·84	39·73
5		8·748	8·84	1·33	5·20	24 18·36	
7		8·843	8·74	1·31	345·85	0 58·16*	
9		8·939	8·65	1·29	326·49	2 17·74	
11		9·033	8·56	1·28	307·12	3 37·36	
13		9·128	8·47	1·26	287·74	4 57·03	
15	215·98	9·223	8·39	1·25	268·36	6 16·71	39·83
17		9·317	8·30	1·23	248·96	7 36·46	
19		9·414	8·22	1·22	229·55	8 56·26	
21		9·506	8·13	1·20	210·14	10 16·06	
23		9·600	8·05	1·19	190·73	11 35·86	
25		9·695	7·98	1·17	171·31	12 55·70	
27	223·42	9·790	7·90	1·15	151·88	14 15·59	39·94
29		9·885	7·82	1·14	132·44	15 35·52	
Oct. 1		9·979	7·75	1·12	112·99	16 55·49	
3		10·074	7·68	1·11	93·53	18 15·50	
5		10·169	7·61	1·09	74·07	19 35·51	
7		10·263	7·54	1·07	54·59	20 55·61	
9	230·93	10·357	7·47	1·06	35·11	22 15·71	40·05
11		10·451	7·40	1·04	15·62	23 35·85	
13		10·546	7·33	1·02	356·12	0 15·95*	

Greenwich Mean Noon.	P.	L−O.	B.	A−L.	B.	Q.	d.
1905.	°	°	°	°	°	°	°
Oct. 15	23·74	10·22	− 4·05	+41·75	−20·34	88·27	43·75
17	22·96	11·50	4·69	41·78	20·67	87·66	43·59
19	22·18	12·78	5·33	41·80	21·00	87·05	43·43
21	21·39	14·07	5·97	41·82	21·30	86·44	43·26
23	20·58	15·37	6·61	41·84	21·59	85·83	43·08
25	19·75	16·68	7·25	41·85	21·87	85·22	42·89
27	18·92	18·00	7·89	41·86	22·15	84·61	42·70
29	18·07	19·33	8·52	41·87	22·42	84·01	42·50
31	17·20	20·67	9·15	41·88	22·69	83·41	42·30
Nov. 2	16·33	22·02	9·78	41·88	22·93	82·81	42·10
4	15·45	23·38	10·40	41·88	23·16	82·22	41·89
6	14·56	24·75	11·02	41·87	23·38	81·65	41·68
8	13·65	26·13	11·63	41·86	23·59	81·08	41·46
10	12·74	27·52	12·24	41·84	23·78	80·52	41·24
12	11·82	28·92	12·84	41·81	23·96	79·96	41·02
14	10·90	30·32	13·44	41·78	24·13	79·41	40·80
16	9·97	31·74	14·03	41·74	24·29	78·87	40·57
18	9·03	33·17	14·61	41·69	24·44	78·33	40·34
20	8·08	34·61	15·18	41·63	24·57	77·79	40·11
22	7·13	36·06	15·74	41·57	24·69	77·26	39·87
24	6·17	37·52	16·29	41·50	24·80	76·75	39·63
26	5·21	39·00	16·83	41·41	24·90	76·25	39·38
28	4·24	40·49	17·36	41·31	24·98	75·76	39·13
30	3·27	41·99	17·88	41·21	25·05	75·29	38·88
Dec. 2	2·30	43·50	18·40	41·10	25·11	74·83	38·62
4	1·32	45·02	18·90	40·97	25·16	74·38	38·36
6	0·34	46·55	19·38	40·83	25·19	73·94	38·10
8	359·36	48·08	19·85	40·68	25·20	73·51	37·83
10	358·38	49·63	20·31	40·52	25·21	73·09	37·56
12	357·40	51·19	20·75	40·35	25·20	72·68	37·29
14	356·41	52·76	21·18	40·17	25·18	72·29	37·01
16	355·43	54·34	21·59	39·97	25·14	71·91	36·74
18	354·45	55·93	21·99	39·76	25·10	71·54	36·46
20	353·47	57·53	22·37	39·54	25·04	71·19	36·18
22	352·49	59·14	22·73	39·30	24·97	70·85	35·90
24	351·51	60·75	23·08	39·05	24·88	70·53	35·61
26	350·54	62·37	23·42	38·78	24·78	70·22	35·32
28	349·57	64·00	−23·74	+38·50	−24·68	69·93	35·03

Greenwich Mean Noon.	☉	Light-time.	Appar. Diam.	Defect of Illumination.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24^h+.
1905.	°	m	″	″	°	h m	m
Oct. 15		10·641	7·27	1·01	336·61	1 36·16	
17		10·736	7·20	0·99	317·10	2 56·38	
19		10·830	7·14	·98	297·58	4 16·64	
21	238·50	10·926	7·08	·96	278·05	5 36·95	40·15
23		11·021	7·02	·95	258·51	6 57·31	
25		11·116	6·96	·93	238·96	8 17·70	
27		11·212	6·90	·92	219·40	9 38·13	
29		11·308	6·84	·90	199·83	10 58·60	
31		11·404	6·78	·88	180·25	12 19·13	
Nov. 2	246·10	11·500	6·72	·87	160·66	13 39·69	40·28
4		11·596	6·67	·85	141·06	15 0·30	
6		11·692	6·62	·84	121·45	16 20·94	
8		11·788	6·57	·82	101·83	17 41·63	
10		11·884	6·51	·80	82·20	19 2·36	
12		11·981	6·46	·79	62·56	20 23·14	
14	253·71	12·078	6·41	·78	42·91	21 43·96	40·39
16		12·175	6·35	·76	23·26	23 4·78	
18		12·272	6·30	·75	3·59	24 25·68	
20		12·370	6·25	·74	343·91	1 6·17*	
22		12·468	6·20	·72	324·22	2 27·15	
24		12·566	6·15	·71	304·52	3 48·17	
26	261·31	12·664	6·10	·69	284·80	5 9·27	40·55
28		12·763	6·06	·68	265·07	6 30·42	
30		12·862	6·01	·67	245·33	7 51·61	
Dec. 2		12·962	5·97	·65	225·58	9 12·84	
4		13·061	5·92	·64	205·82	10 34·11	
6		13·161	5·88	·63	186·06	11 55·38	
8	268·88	13·260	5·83	·61	166·28	13 16·74	40·68
10		13·360	5·79	·60	146·49	14 38·14	
12		13·460	5·74	·59	126·69	15 59·59	
14		13·561	5·70	·57	106·88	17 21·17	
16		13·662	5·65	·56	87·05	18 42·74	
18		13·763	5·61	·55	67·22	20 4·31	
20	276·40	13·864	5·57	·54	47·38	21 25·92	40·79
22		13·966	5·53	·52	27·54	22 47·53	
24		14·067	5·49	·51	7·69	24 9·20	
26		14·169	5·45	·50	347·82	0 50·10*	
28		14·271	5·41	0·49	327·94	2 11·88	

Greenwich Mean Noon.	P.	L−O.	B.	Λ−L.	*B.*	Q.	*d.*
1905.	°	°	°	°	°	°	°
Dec. 30	348·61	65·65	−24·04	+38·21	−24·56	69·65	34·72
1906.							
Jan. 1	347·65	67·31	24·31	37·91	24·43	69·38	34·43
3	346·70	68·97	24·56	37·60	24·29	69·13	34·14
5	345·75	70·63	24·80	37·28	24·14	68·89	33·84
7	344·81	72·30	25·03	36·95	23·98	68·67	33·54
9	343·88	73·97	25·24	36·60	23·80	68·47	33·23
11	342·96	75·65	25 42	36·24	23·61	68·27	32·92
13	342·04	77·33	25·58	35·86	23·41	68·08	32·61
15	341·13	79·01	25·72	35·47	23·20	67·91	32·30
17	340·24	80·70	25·84	35·08	22·98	67·76	31·99
19	339·37	82·39	−25·94	+34·69	−22·75	67·62	31·68
Dec. 11	36·83	304·43	+20·77	−30·24	+25·16	292·06	28·13
13	37·08	305·70	20·44	30·50	25·12	291·82	28·45
15	37·32	306·96	20·10	30·76	25·08	291·57	28·77
17	37·54	308·22	19·75	31·02	25·03	291·31	29·10
19	37·75	309·46	19·39	31·27	24·98	291·05	29·42
21	37·93	310·70	19·03	31·52	24·93	290·77	29·74
23	38·10	311·93	18·66	31·76	24·86	290·49	30·06
25	38·25	313·16	18·28	32·00	24·79	290·20	30·39
27	38·38	314·38	17·89	32·23	24·71	289·90	30·71
29	38·48	315·59	17·49	32·46	24·63	289·59	31·04
31	38·57	316·79	+17·09	−32·69	+24·54	289·27	31·36

The position of *Mars'* north pole is assumed to be R.A. 317° 17′·28, N.P.D. 37° 21′·12 at the beginning of 1905, and R.A. 317° 18′·20, N.P.D. 37° 20′·65 at the end of 1906, these being the values deduced by Dr. Hermann Struve from observations of the satellites.

P denotes the position-angle of the axis of *Mars*; L−O+180° the right ascension of the Earth referred to the plane of *Mars'* equator, and reckoned from O, the point of the vernal equinox of *Mars'* northern hemisphere; Λ−O+180° the corresponding quantity for the Sun; B *B* the declinations of the Earth and Sun reckoned from the plane of *Mars'* equator; Q the position-angle of the greatest defect of illumination; *q* the amount of the same; *d* the areocentric angle between the Earth and the Sun.

Thus Λ−L is the angle between the meridians of *Mars*, which are central to the Earth and to the Sun; the point where an image of the Sun should be looked for is distant $\frac{1}{2}$(Λ−L) in longitude from the centre of the disc, on the side towards the fully illuminated limb, and is in areocentric latitude $\frac{1}{2}$(B+*B*);

Greenwich Mean Noon.	⊙	Light-time.	Appar. Diam.	Defect of Illumination.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24^h+.
1905.	°	m	″	″	°	h m	m
Dec. 30		14·374	5·38	0·48	308·06	3 33·67	
1906.							
Jan. 1	283·83	14·476	5·34	·47	288·17	4 55·49	40·91
3		14·579	5·30	·46	268·28	6 17·32	
5		14·681	5·26	·45	248·37	7 39·22	
7		14·784	5·23	·44	228·46	9 1·13	
9		14·887	5·19	·42	208·54	10 23·07	
11		14·990	5·16	·41	188·62	11 45·02	
13	291·18	15·093	5·12	·40	168·69	13 7·01	40·99
15		15·196	5·09	·39	148·76	14 29·01	
17		15·299	5·05	·38	128·82	15 51·05	
19	294·82	15·403	5·02	0·37	108·88	17 13·09	41·01
Dec. 11	93·86	17·122	4·52	0·27	157·26	13 53·47	39·97
13		16·983	4·55	·28	137·81	15 13·42	
15		16·843	4·59	·28	118·37	16 33·33	
17		16·700	4·63	·29	98·93	17 53·23	
19		16·557	4·67	·30	79·50	19 13·10	
21	98·32	16·412	4·71	·31	60·08	20 32·92	39·90
23		16·267	4·75	·32	40·67	21 52·70	
25		16·120	4·79	·33	21·26	23 12·47	
27		15·972	4·84	·34	1·86	24 32·21	
29		15·822	4·88	·35	342·47	1 12·06*	
31	102·80	15·672	4·93	0·36	323·09	2 31·72	39·83

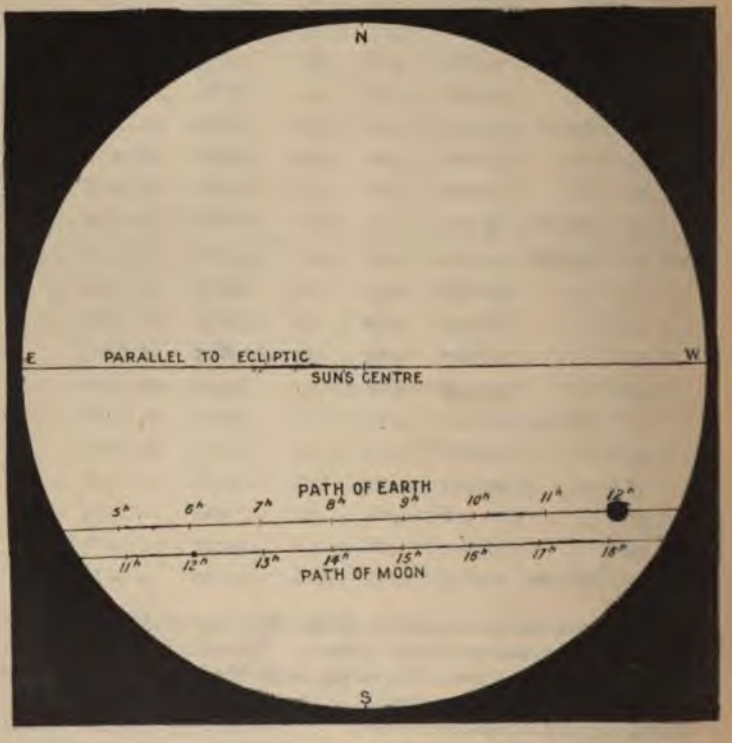
⊙ is the areocentric longitude of the Sun referred to the plane of *Mars*' orbit and reckoned from O. This column indicates the season on *Mars*; ⊙ = 0° corresponds to March 20 on the Earth, ⊙ = 30° to April 20, and so on. The diameter of the planet at distance unity is assumed to be 9″·30, the *Nautical Almanac* value being 9″·36. The assumed time for light to travel over the unit distance is $8^m·315$.

The zero meridian and period of rotation have been carried on unchanged from Mr. Marth's last ephemeris, the assumed sidereal rotation in 24^h of mean time being 350°·89202; the equivalent period of sidereal rotation is $24^h\ 37^m\ 22^s·654$.

The passage of zero meridian is given for alternate rotations except those marked with an asterisk, which are separated from those preceding them by one rotation only. Where 24^h occurs in this column it is to be read as 0^h on the following day; *e.g.* 1904 Dec. $9^d\ 24^h$ is to be read as Dec. $10^d\ 0^h$. The last column contains the interval between successive passages, and enables the intermediate passages to be readily deduced.

According to Mr. Lowell's observations the longitude of the central meridian should be increased by about 3°, and the times of transit of zero meridian diminished by about 13^m.

The ephemeris for 1906 December begins the following apparition of *Mars*, which is continued in the *Nautical Almanac* for 1907.



It is of interest to note that there will be a transit of the Earth and Moon across the Sun's disc as seen from *Mars* on 1905 May 8. The last two transits occurred 1879 November 12 and 1800 November 8 at the other node. The next will be in 1984 May. The following are approximate elements of the coming transit:

Semi-diameter of Sun seen from Mars	=	10′	18″·3
,, ,, Earth ,, ,,	=		16·2
,, ,, Moon ,, ,,	=		4·4
Least distance of centres of ⊙⊕	=	4	39

		d	h	m
Ingress of Earth's centre	= May	8	4	10
Egress " "	=	8	12	52
Ingress of Moon's centre	=	8	10	17
Egress " "	=	8	18	48

The transits generally recur in a period of 284 years, there being four transits, two at each node, in this period; this is analogous to the 243-year period of recurrence of transits of *Venus* (which likewise generally contains four transits), and the 217-year period for transits of *Mercury*.

Thus we obtain the following years for May transits of the Earth: 1621, 1700, 1905, 1984, 2189, 2268, &c. And the following for November transits: 1516, 1595, 1800, 1879, 2084, 2163.

The transit of 1879 was discussed by Mr. Marth (*Monthly Notices*, vol. xxxix. p. 513). I think, however, that through some slip he has made the tracks of both Earth and Moon 1′ too low on the Sun's disc.

Owing to the absolute symmetry of the illumination round the limb of the planet during the transit it will be a favourable opportunity for measuring the diameter and the polar compression.

The following are the dates of the commencements of the Martian seasons:

			d
Summer Solstice of N. Hemisphere	1905	Jan.	14·3
Autumnal Equinox " "	"	July	16·3
Winter Solstice " "	"	Dec.	9·8
Summer Solstice " "	1906	Dec.	2·3

Benevenue, 55 Ulundi Road, Blackheath, S.E.:
1904 March 9.

Errata in Cape Double Star Results

Pages 131, 132, 133, column 5, *for* 1902 *read* 1903.

Page 134, line 3, *for* distance decreasing *read* distance increasing.

Erratum in Professor Turner's Paper on finding time of Sunset.

Page 194, line 11, *for* $l + \theta$ *read* $l + \theta +$ const.

Erratum in Plate 14 *to Mr. Ellis's paper on Aurora and Magnetic Disturbance.*

On right-hand side of plate, Regions of the "Catalog," *for* 1861–1877 *read* 1761–1877.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXIV. APRIL 8, 1904. No. 6

Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

John Borthwick Dale, M.A., Assistant Professor of Mathematics, King's College, London; "Myosotis," New Malden, Surrey;
Rev. William Charles Eppstein, Reading School, Berks;
Walter Nuttall, B.A., Hallfold School, Whitworth, Rochdale, and
William Edward Rolston, Solar Physics Observatory, South Kensington, S.W.;

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:

Edward Barlow, Harewood, Andover, Hants (proposed by Major P. A. MacMahon);
Otto J. Klotz, Astronomer, Department of the Interior, Ottawa, Canada (proposed by W. H. M. Christie);
Major C. Leigh-Lye, 15 Mayfield Avenue, Chiswick, W. (proposed by A. C. D. Crommelin).

The following were proposed by the Council as Associates of the Society:

Henri Deslandres, D.-ès-Sc., F.R.A.S., Observatoire d'Astronomie Physique, Meudon, S.-et-O., France;
C. D. Perrine, Lick Observatory, Mount Hamilton, California, U.S.A.;
George W. Ritchey, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.

Seventy presents were announced as having been received since the last meeting, including, amongst others :

Annales de l'Observatoire de Meudon—Atlas de Photographies Solaires, presented by M. Janssen ; Observatoire d'Alger, Catalogue Photographique du Ciel, 3 fascicules ; and Introduction by Ch. Trépied, presented by the Observatory ; G. E. Hale and F. Ellerman (&c.), The Rumford Spectroheliograph of the Yerkes Observatory, and Spectra of Stars of Secchi's fourth type, presented by Professor Hale ; E. B. Frost and W. S. Adams, Radial Velocities of Stars of the *Orion* type, presented by Professor Frost ; S. W. Burnham, Measures of Double Stars, presented by the Author ; Tokyo Observatory, Annales, tomes 2, 3 (with report of Solar Eclipse of 1901), presented by the Observatory.

On the Degree of Accuracy of the new Lunar Theory and on the Final Values of the Mean Motions of the Perigee and Node. By Ernest W. Brown, Sc.D., F.R.S.

In the *Monthly Notices* for 1903 May I gave a general statement of the degree of precision existing between the calculated values of the mean motions of the perigee and node and their observed values. It was added that the final verifications of the higher terms due to solar action were not complete, but an upper limit was given for possible errors. Since that date the calculations for these higher terms have been tested and found to be correct, so that the portions due to solar action are completely finished, and the portion arising from each characteristic is calculated to $0''{\cdot}01$ in the annual motions.

In order to make the comparison complete to this degree of accuracy it was found necessary to undertake an examination into the numerical values of the constants used and into the effects produced by sources other than the direct solar action. This inquiry revealed several differences which had a perceptible effect. It was first necessary to develop a general method of dealing with the effects of planetary and other perturbations on these mean motions which would permit of their being found easily and accurately. Two methods for the solution of this problem were obtained ; they will be published elsewhere,* the numerical results only being given here. It was found necessary to make several changes in the values of these non-solar perturbations as collected in my papers in the *Monthly Notices* for 1897 March and June : four of these caused alterations of about half a second each in the annual motions. First, in multiplying the ratios of the motions to n—ratios which were found correctly from theory—by n (the mean motion), it was found that a

* *Trans. Amer. Math. Soc.* July 1904.

slightly inaccurate value of n had been used. Second, Hansen's value for the planetary action on the perigee was found to be wrong by a quarter of its whole amount. Third, Hill's values for the effects produced by the figure of the earth correspond to an ellipticity rather greater than would seem to be admissible. Fourth, Hansen's and Newcomb's values, deduced from observation, correspond to the epoch 1800, while the values of the constants used in my theory, especially that of the solar eccentricity, correspond to the epoch 1850. After these changes had been made it was found that the agreement between theory and observation was not impaired; in fact, the differences are still within the limits of error arising from doubts as to the correct values to be assigned to certain constants. As far as the lunar theory is concerned, every theoretical result set down here has been calculated or tested afresh by one, or in some cases two methods; the only exception is in a case where little doubt can be entertained and where in any case the effect is very small—namely, the perturbations of the solar radius vector, and these have been taken from Newcomb's *Tables of the Sun*.

There is only one constant whose observed value is so far doubtful as to affect the results by so much as $0''{\cdot}1$—the ellipticity of the earth. For this constant there seem to be really two competing values, one about $1/293$ and the other about $1/297$. As it appears difficult to make a choice between them I give two calculated results corresponding to two values of the ellipticity, the final determinations are as follows:

Annual Mean Motions, Epoch 1850·0.

	Perigee.			Node.		
Calculated (α)	+146	435″·27	±0″·10,	−69	679″·37	∓0″·05;
Calculated (β)	+146	435·11	±0·10,	−69	679·22	∓0·05;
Observed	+146	435·23	... ,	−69	679·45	... ;
C−O (α)	...	+0·04	... ,	...	+0·08	... ;
C−O (β)	...	−0·12	... ,	...	+0·23	... .

(α) corresponds to an ellipticity $1/292{\cdot}9$;

(β) " " " $1/296{\cdot}3$;

the quantities $\pm 0''{\cdot}10$, $\mp 0''{\cdot}05$ being extreme possible errors due to neglect of terms whose orders are higher than those calculated and to the doubt as to the correct value of the mass of *Venus*. The direct and indirect effects due to *Mercury* are nearly equal and opposite, so that the uncertainty as to the mass of this planet does not affect the results. No possible errors due to the use of a limited number of places of decimals have been set down, since nearly every number has been computed to at least one more place than that actually given; the errors from this cause are probably less than $0''{\cdot}05$.

The portions which various parts of the complete disturbing function produce will now be considered in detail. So far as I know, every purely gravitational cause which can give a sensible effect has been considered; the list in No. 7 below will, in any case, permit of any changes or additions to be easily made.

The references are collected at the end of the paper and are referred to by the letters (*a*), (*b*), ...

The adopted values of the constants are for the epoch 1850·0:

Moon.	Sun.
$n = 17\ 325\ 594''$,	$n' = 1\ 295\ 977''4$;
$e = \cdot 054\ 900\ 3$,	$e' = \cdot 016\ 772$;
$\gamma = \cdot 044\ 887\ 2$,	$\gamma' = 0$;
Parallax = 3422″·7 ,	Parallax = 8″·80 ;
E/M = 81·5 ,	$m'/(E+M) = 328\ 243$;

where n, n' are the annual mean motions; E, M, m' the masses of the Earth, Moon, Sun; e, the lunar eccentricity corresponding to Newcomb's coefficient 22659″·58 for the principal elliptic term (*a*); e', the solar eccentricity (*b*); γ, the sine of half the inclination corresponding to Newcomb's coefficient 18461″·48 for the principal term in latitude (*a*); the parallaxes are those generally accepted at the present time. No probable change in any of these constants will materially alter the values for the mean motions.

1. *The solar portions.*—These are obtained by using the disturbing function (*a*)

$$(E+M+m')\left[\frac{1}{\sqrt{(x-x')^2+(y-y')^2+z^2}}-\frac{xx'+yy'}{r'^3}\right]$$
$$=\frac{n'^2a'^3}{r'^3}\{r^2(\tfrac{3}{2}S^2-\tfrac{1}{2})+\frac{r^3}{a'}(\tfrac{5}{2}S^3-\tfrac{3}{2}S)+\frac{r^4}{a'^2}(\tfrac{35}{8}S^4-\tfrac{15}{4}S^2+\tfrac{3}{8})\}$$

where the solar coordinates x', y', z', r' refer to the elliptic motion of the Sun about the Earth with the correction

$$\frac{E-M}{E+M}\cdot\frac{a}{a'}=a \text{ for } \frac{a}{a'}$$

in the final results—a correction which accounts for the greater part of the motion of E about the centre of mass of E and M. Here S is the cosine of the angle between the radii to the Sun and Moon.

The portions calculated are those with characteristics 1, e^2, γ^2, e'^2, $(a/a')^2$, e^4, $e^2\gamma^2$, γ^4, $e^2e'^2$, $\gamma^2e'^2$. A consideration of the numerical values of other characteristics shows that the sum of the terms which arise from them will be less than 0″·04 in the case of the perigee and less than 0″·02 in the case of the node.

The terms of characteristic unity form respectively 71/70, 174/175 of the complete values.

In an earlier paper (*d*) I have given results (extensions of Adams' theorems) which directly connected the parts in the motion of the perigee having characteristics $e^2\gamma^2$, γ^4, $\gamma^2 e'^2$ with those in the motion of the node having characteristics e^4, $e^2\gamma^2$, $e^2e'^2$ respectively. The former set are determined in the theory with the terms in x, y (longitude and parallax), and the latter with the terms in z (latitude), which are of the fifth order with respect to their characteristics. The two sets are calculated in totally different ways, the former with the "homogeneous" equations and the latter with the non-homogeneous equation. There were thus three separate tests, and in all cases the results agreed within two units of the last place of decimals used. These searching tests, involving nearly all the results of the third order, a large proportion of those of the fourth order, and the greater part of the forms of computation for the terms of the fifth order, would seem to warrant a feeling of security in the general accuracy of the whole theory, so far as it has gone.

But these three tests, though they are exact and searching, are on the whole less valuable than an indirect test for general accuracy which arises automatically in every set of terms with a given characteristic. Each such set in the x, y coordinates and every other set in the z coordinate has one (long-period) or two (monthly) terms whose coefficients possess small divisors, and the process of division by these divisors is practically the last step in the determination of the coefficients. Now, the possibility of error in the *details* of numerical calculation has been practically eliminated ; the chief danger arises from the possible omission of a whole set of terms, or the use of a wrong set. But it is precisely the latter forms of error which the test guards against, for such mistakes would almost certainly cause the coefficients to be many times greater than their actual values, and a rough comparison with the results of Delaunay would show the existence of error at once. In fact, the numbers before division are composed of quantities which are large in comparison with their algebraical sum. Thus, the chief cause of the enormous extent of the computations furnishes the most valuable test of their accuracy.

2. *Correction due to using* $\mathrm{E}+\mathrm{M}+m'$ *instead of* m' *as the factor of the disturbing function.*—The additional sensible portion is (*e*)

$$-\frac{\mathrm{E}+\mathrm{M}}{\mathrm{E}+\mathrm{M}+m'}\, n'^2 r'^2 \left(\tfrac{3}{2}\mathrm{S}^2-\tfrac{1}{2}\right).$$

The terms of characteristic unity, due to this function, have been calculated by the method of variation of arbitraries as well as by the method of the theory, and the results agreed in giving $-0''{\cdot}71$ for the perigee and $+0''{\cdot}20$ for the node. Multiplying these by 70/71, 175/174 respectively, which accounts sufficiently

for the terms in higher characteristics, we obtain $-0''\cdot70$, $+0''\cdot20$, agreeing with the estimate which I gave in an earlier note (*e*). No possible alteration in the value attached to the ratio $(E+M)/(E+M+m')$ will change these by so much as $0''\cdot01$.

3. *Correction for the remaining effect of the mass of the Moon.*—The addition to the disturbing function (*c*) is a term

$$\frac{EM}{(E+M)^2}\, n'^2\, \frac{r^4}{a'^2}\left(\tfrac{35}{8}S^4-\tfrac{15}{4}S^2+\tfrac{3}{8}\right).$$

It gives $+0''\cdot02$ for the perigee, $-0''\cdot01$ for the node. The indirect action produced by a corresponding term in the Sun's disturbing function gives $-0''\cdot01$ and $0''\cdot00$ in the respective cases.

4. *Direct planetary action.*—This has been calculated by the method of the theory (for its principal part) and also completely by the variation of arbitrary constants. Using Hansen's values for the masses of the planets, and his expansions of the disturbing functions, only in so far as they depend on the positions of the Earth and planets, it appears that Hansen's result for the perigee is much too small (*f*). Adopting modern values for the masses and Newcomb's expansion for *Venus* (*h*), which has the greatest effect, I find $+2''\cdot66$ for the perigee and $-1''\cdot42$ for the node. Hansen (*f*) gave $+2''\cdot10$ and $-1''\cdot50$. My results include squares of the eccentricities and inclinations of the Moon and planets, and all powers of m. The use of these more accurate values for the parts depending on the planets would subtract about $0''\cdot1$ from Hansen's result for the perigee, so that it is $0''\cdot7$ in error. The node is practically unaffected. I have not been able to trace the source of this error in Hansen very far; it depends, however, on the terms factored by K in Art. 172 of the *Darlegung*, the value of K being correctly found with the adopted values of the masses.

The following table gives the values of the masses adopted here (*b*) with the corresponding motions and the extreme amounts by which the latter can be in error owing to the uncertainty in the mass of *Mercury*, which is doubtful to 50 per cent. of the given mass, and in the mass of *Venus*, which is doubtful to 5 per cent.:

Direct Action of Planets.

	Recip. of Mass.	Perigee.	Node.
		"	"
Mercury	6,000,000	$+0\cdot049 \pm \cdot025$	$-0\cdot016 \mp \cdot008$
Venus	408,000	$+1\cdot966 \pm \cdot098$	$-\cdot819 \mp \cdot041$
Mars	3,093,000	$+\cdot034$	$-\cdot025$
Jupiter	1,047·35	$+\cdot582$	$-\cdot536$
Saturn	3.501· 6	$+\cdot026$	$-\cdot025$
Sum	...	$+2\cdot657 \pm 123$	$-1\cdot421 \mp \cdot049$

The extreme errors, when taken in connection with those produced by the indirect action, are diminished to one-half of these values, as appears in No. 6 below.

5. We have put, in the solar disturbing function after the correction of No. 2 has been made,

$$E+M'+m' = n'^2 a'^3, \quad r' = a' + \text{elliptic terms}.$$

The latter value of a' requires a correction, the constant part of which will be denoted by $\partial a'$, on account of the perturbations produced in the Sun's motion by the planets. The addition to the disturbing function for the Moon is

$$-3\frac{\partial a'}{a'} n'^2 r^2 (\tfrac{3}{2}S^2 - \tfrac{1}{2}).$$

It is therefore similar to that of No. 2, and the corrections to the perigee and node are respectively

$$-\frac{3\delta a'}{a'} \cdot \frac{0''{\cdot}70}{328243'} \quad +\frac{3\partial a'}{a'} \cdot \frac{0''{\cdot}20}{328243}.$$

The value of $\partial a'/a'$ for *Mercury* has been obtained from Newcomb (*i*) and for the other planets and for the adopted value of the gravitation constant from his tables of the Sun (*b*). The corrections, with the extreme errors due to uncertainty in the adopted masses of *Mercury* and *Venus*, are given in the following table:

Indirect Planetary Action.

	$\log_{10}(1+\partial a'/a')$.	Perigee.	Node.
Mercury ...	+ ·000 000 028	$-0''{\cdot}041 \mp {\cdot}020$	$+0''{\cdot}012 \pm {\cdot}006$
Venus ...	+ 634	$-\ {\cdot}934 \mp {\cdot}047$	$+\ {\cdot}267 \pm {\cdot}013$
Mars	− 12	$+\ {\cdot}018$	$-\ {\cdot}005$
Jupiter ...	− 512	$+\ {\cdot}754$	$-\ {\cdot}215$
Saturn ...	− 24	$+\ {\cdot}036$	$-\ {\cdot}010$
Grav. const. ...	+ 13	$-\ {\cdot}019$	$+\ {\cdot}005$
Sum	+ ·000 000 127	$-0''{\cdot}186 \mp {\cdot}067$	$+0''{\cdot}054 \pm {\cdot}019$

The action of *Venus* produces a periodic inequality with a coefficient of about 15″. When squares of the action are included, this inequality will give rise to small changes in the mean motions of the perigee and node. These I have examined and found to be absolutely insensible.

6. *Total planetary action.*—It will be noticed that the direct and indirect actions of *Mercury* are opposite in effect and nearly equal, so that any uncertainty as to its mass does not affect the results by so much as 0″·01. The indirect action of *Venus* is about half the direct action with the opposite sign.

Hence, combining the results in Nos. 4, 5, we can subtract the two sets of extreme errors and obtain for the

Total Planetary Action.

Perigee.	Node.
$+2''{\cdot}48 \pm 0''{\cdot}06$	$-1''{\cdot}37 \mp 0''{\cdot}03$,

the extreme errors arising entirely from doubt as to the mass of *Venus*.

7. *Figure of the Earth effects.*—The part of the disturbing function due to the figure of the earth, which alone can give rise to sensible terms in the mean motions of the perigee and node, is

$$\lambda\left(\tfrac{1}{3}\frac{a^3}{r^3}-\frac{z^2 a^3}{r^5}\right).$$

Hill (*k*) has made an extended literal investigation by Delaunay's method of the terms arising from this function, but it is open to the objection that the results for the perigee and node are only carried as far as the orders λm^3, λe^2, $\lambda\gamma^2$. It appears from the expression for the perigee, which begins with a term of order λm^0, that slow convergence along powers of m might make a difference of as much as $0''{\cdot}3$ from Hill's result. I therefore made a new and different computation, using the numerical value of m and carrying the approximations so far as to include all sensible terms of the forms

$$\lambda f(m),\ \lambda e^2 f(m),\ \lambda\gamma^2 f(m),\ \lambda e'^2 f(m).$$

The terms of the form $\lambda f(m)$ were also computed by the method of my theory. It appeared that the apparent slow convergence had very little effect. Hill's results for the perigee and node were $+6''{\cdot}82$, $-6''{\cdot}41$ respectively, while mine gave $+6''{\cdot}86$, $-6''{\cdot}42$, using Hill's value for λ. The only possible source of error is therefore in the value to be substituted for λ.

The usual method, and probably the best one for the purposes of the lunar theory, is to determine λ by means of the coefficient of the principal term, due to the figure of the Earth, in latitude. This, according to Hill, has a coefficient $-8''{\cdot}726$, and, excluding the doubt as to λ, it is probably accurate within $0''{\cdot}003$. Hansen (*f*) finds $-8''{\cdot}382$ for the observational value of the coefficient. There is a term due to the motion of the ecliptic with the same argument, and with a coefficient, according to Hill's theory (*l*), $-0''{\cdot}226$ (according to Hansen $-0''{\cdot}240$) ; hence the part of the observed coefficient due to the figure of the Earth is $-8''{\cdot}156$. We must therefore multiply the numbers $6''{\cdot}86$, $-6''{\cdot}42$, obtained with Hill's value of λ, by $8{\cdot}156 \div 8{\cdot}726$ to reduce them to the value deduced from Hansen's observed coefficient in latitude. They respectively become $+6''{\cdot}41$, $-6''{\cdot}00$.

Faye (*m*), however, gives $-8''{\cdot}59$ for the observed coefficient in latitude, so that the part due to the Earth's action is $-8''{\cdot}36$. The corresponding motions are $+6''{\cdot}57$, $-6''{\cdot}15$. Helmert (*m*) has a still different value.

The coefficient λ, according to the theories of the figure of the Earth, is approximately equal to $k\,(e_1 - \frac{1}{2}\phi)$, where k is a constant determinable with all the accuracy required here, ϕ is the ratio of centrifugal force to gravity at the equator and is equal to ·003468, and e_1 is the ellipticity of the Earth's surface.* As is well known, this equation is obtained on certain suppositions concerning the constitution and distribution of the Earth's mass. Hill (k) finds λ from a collection of pendulum observations, and deduces $1/e_1 = 286{\cdot}7$. Helmert (m) has deduced $1/e_1 = 298{\cdot}3 \pm 1{\cdot}0$, also from pendulum observations. Clarke (m), from geodetical measurements, obtains $1/e_1 = 293{\cdot}5 \pm 1{\cdot}0$. The precessional constant used by Newcomb (b) is connected by the theory of the figure of the Earth with the value 297·3. As the value of the ellipticity deduced from Hill is greater than most of the other determinations, it seems worth while to give a list of the principal determinations, together with the corresponding motions of the perigee and node:

Author.	$\frac{1}{e_1}$	Annual Motions. Perigee.	Node.
Hill, pendulum obs.	286·7	$+6''.86$	$-6''.42$
(α) Faye, coef. in lat.	292·9	$+6''.57$	$-6''.15$
Clarke, geodesy	293·5	...	...
(β) Hansen, coef. in lat.	296·3	$+6''.41$	$-6''.00$
Precessional const.	297·3	...	...
Helmert, Moon pert.	297·8	...	...
Helmert, pendulum obs.	298·3	$+6''.33$	$-5''.94$

Hill's theoretical coefficient in latitude is compared with Faye's observed coefficient in order to find e_1 in (α) of the table, and Hansen's observed coefficient is similarly treated, so that Hansen's theoretical coefficient does not enter into this discussion.

According to the evidence Hill's value is too large. Faye's value puts the differences between the observed and theoretical motions of the perigee and node within the limits of errors arising from other known sources. Hansen's value is close to that obtained by Helmert, and also to that deduced from the precessional constant. I shall therefore in the final results, awaiting a new discussion of the observed value of the coefficient in latitude, give the two values (α), (β), these being practically the values round which a choice seems probable; they also make the theory consistent as far as the adopted values

* The ellipticity is defined by $e_1 = (a-c)/a$, where a, c are the equatorial and polar axes of the Earth. The complete relation between λ, e_1 is given by Professor Darwin in the *Monthly Notices* for 1899 December. I am much indebted to Professor Darwin for assistance in the discussion of this paragraph, and to Professor Helmert for communicating a calculation of the values of ϕ, k, from the relation just mentioned, and of the latest result, 298·3 ± 1·0 for $1/e_1$ (*Berl. Sitzungsber.*, 1901, pp. 328–336).

of the constants are concerned. A change of 1 per cent. in the ellipticity causes changes of 2 per cent. in the corresponding perturbations of the Moon's motion.

8. *Other perturbations.*—Hansen (*f*) considers the figure of the Moon as responsible for the differences he finds between the observed and theoretical motions—namely, +1″·50, −2″·56. As will be seen, no such differences exist, and in any case modern measurements of the Moon seem to put the large values Hansen assigned for the ellipticity of the Moon entirely out of the question.* In fact, until the question of the ellipticity of the Earth is settled it is unnecessary to invoke any other sources of perturbation beyond those which can be definitely calculated. In particular the law of gravitation at the distance of the Moon appears to be exact within the limits of accuracy at present attainable.

9. *The final results.*—Gathering together the results in the foregoing sections, we obtain the following :

Annual Mean Motions, Epoch 1850·0.

		Perigee.	Node.
Solar Action	char^{c}. 1	+148 524″·92	−69 287″·90
	,, e^2	− 519·31	− 616·09
	,, γ^2	− 1 739·85	+ 260·59
	,, e'^2	+ 156·27	− 25·46
	,, a^2	+ 2·24	− 1·11
	,, e^4	+ ·04	+ ·07
	,, $e^2\gamma^2$	+ 6·72	− 1·70
	,, γ^4	− 1·51	+ ·05
	,, $e^2e'^2$	− ·99	− ·57
	,, $\gamma^2e'^2$	− 1·61	+ ·08
	,, rem^{s}.	·00 ± ·04	·00 ∓ ·02
Terms in No. 2		− ·70	+ ·20
Terms in No. 3		+ ·01	− ·01
Plan. direct		+ 2·66 ± ·06	− 1·42 ∓ ·03
Plan. indirect		− ·19	+ ·05
Fig. of earth (α)		+ 6·57	− 6·15
Fig. of earth (β)		+ 6·41	− 6·00
Calcul. sum (α)		+146 435·27 ± ·10	−69 679·37 ∓ ·05
Calcul. sum (β)		+146 435·11 ± ·10	−69 679·22 ∓ ·05
Observed		+146 435·23	−69 679·45
C−O (α)		+ 0·04 ± ·10	+ 0·08 ∓ ·05
C−O (β)		− 0·12 ± ·10	+ 0·23 ∓ ·05

* It is to be noticed that if we add 1/12 of these differences to the result (β) in No. 9, the differences C−O (β) practically disappear. But both sets are too doubtful to build any argument on their possible existence.

Of the calculated results (α), (β) in the above table, the mer is obtained by using Faye's value of the observed fficient in latitude to determine the constant on which the ms due to the figure of the Earth depend, the latter by using ansen's observed coefficient. It is probably true that Hansen's efficient refers to his special axes, but the difference between and the ecliptic system will not materially alter it. The ors ±·10, ∓·05 are the extreme errors previously explained.

Nothing has been said concerning the probable errors of the served values of the mean motions. In view of Newcomb's confirmation of Hansen's (*f*) results (a change of 0″·1 in the tion of the node alone being made), it would seem that there little doubt of their accuracy as far as Hansen's theory with known corrections is concerned. But of late years the amination of planetary inequalities has revealed several ose periods are very close to that of the principal elliptic m. Three of them have coefficients approaching 1″ in magnide, according to Radau (*n*), and it is not impossible that a tematic search may reveal others whose inclusion in the ory may cause slight alterations in the observed values of the an motions of the perigee and node. The values adopted ove are those of Newcomb (*a*).

10. *Bibliography.*—The references above are to the following pers:

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(*m*) These references are extracted from F. Tisserand, *Méc. Cél.* . ii. chap. 21, and W. Harkness, "Solar Parallax and Related nstants," *Washington Observations for* 1885, app. 3.

(*n*) R. Radau, "Recherches concernant les Inégalités plané-

taires du mouvement de la Lune," *Par. Obs. Ann.* (*Mém.*) vol. xxi. 1892.

The work of Harkness (*m*) contains a collection of all the constants of astronomy and a full bibliography.

Haverford College: 1904 *January* 4.

The Parallactic Inequality and the Solar Parallax.

By Ernest W. Brown, Sc.D., F.R.S.

Of the principal solar terms in the Moon's motion the parallactic inequality is the only one whose coefficient can be determined correctly within $0''{\cdot}1$ when characteristics of order higher than the third are omitted. Like all the terms whose coefficients contain a/a' as a factor, the convergence of its coefficient along powers of m is slow, and consequently its value deduced from Delaunay's literal expression, even when allowance is made for higher powers of m, is not certain within half a second. The transformation of my theory to polar coordinates up to and inclusive of the terms whose characteristics are of order 3 was completed some time ago (*Monthly Notices*, 1899 December). As the coefficient of the parallactic inequality can be found, with a given set of constants, correctly to $\frac{1}{5000}$ of the whole from these terms it may perhaps be useful to give the result.

The following table gives the values of the various portions of the coefficient obtained by myself and Delaunay. It is expressed exactly in the form and with the constants of Delaunay, who uses a solar parallax of $8''{\cdot}75$. The column headed l.p. m. gives the value of the last power of m calculated by Delaunay. The result has not been multiplied by $(E-M)/(E+M)$, the factor necessary to obtain the final value.

Coefficient of Parallactic Inequality.

Characteristic.	Brown.	Delaunay.	l.p. m.
$a/a' = \alpha$	$-128''{\cdot}069$	$-127''{\cdot}621$	$-0''{\cdot}381$
$e^2\alpha$	$-$ 2·485	$-$ 2·378	$-$ ·119
$e'^2\alpha$	$-$ ·022	$-$ ·024	+ ·001
$p^2\alpha$	+ 3·105	+ 2·772	+ ·354
α^3	$-$ ·001	·000	$-$ ·0004
Higher	...	+ ·009	+ ·009
Sum	$-127''{\cdot}472$	$-127''{\cdot}242$	...

My result is thus $-127''{\cdot}47$ + higher terms. The latter, judging from Delaunay's calculation and the experience gained as to the

general rate of convergence of particular classes of terms, will give $+0''\cdot02\pm0''\cdot02$. Thus the coefficient is $-127''\cdot45\pm0''\cdot02$. Multiplying this by $(E-M)/(E+M) = 80\cdot15/82\cdot15$, and reducing to the parallax $8''\cdot790$, I find

$$\text{parallactic inequality} = -124''\cdot92\,\frac{\text{solar parallax}}{8''\cdot790}\sin D$$

with a possible error of $0''\cdot02$.

In the *Monthly Notices* for 1903 December Mr. Cowell finds $-124''\cdot75$ for the observed value of this coefficient. Comparing we obtain

$$\text{solar parallax} = 8''\cdot778.$$

Haverford College: 1904 *April* 1.

Some further Analyses of the Moon's Errors of Longitude, 1847–1901. By P. H. Cowell.

The columns 2–7 of the annexed table gives the "apparent" coefficient of $\sin 2\theta$, $\cos 2\theta$, $\sin(4D-2g)$, $\cos(4D-2g)$, $\sin D$ and the apparent error of semi-diameter as deduced from each of the forty-eight periods of analysis, consisting of 400 lunar days each, into which the period 1847–1901 has been divided.

Last month I explained how an apparent coefficient of $\sin\phi$ might in reality be due to a term $\sin(\phi\pm D)$. Periodicity in the apparent coefficient of $\sin\phi$ indicates a term different slightly in argument from ϕ or from $\phi\pm D$.

The angle θ is an auxiliary angle whose movement is $13^\circ\cdot5$ in a lunar day. θ therefore recurs in eighty days and 2θ in forty days; θ differs very slightly in movement from g, the mean anomaly, for the movement of g in 400 lunar days $= 13\cdot5\times400 + 9^\circ\cdot1681$. $4D-2g$ is double the evection.

The first four coefficients tabulated appear to be nearly, if not entirely, accidental. I cannot find any marked periodicity in any one of the four series. The conclusion from this is that the tables of Hansen contain either no error (with coefficient exceeding $0''\cdot3$ say) and argument $2g$, $4D-2g$, $2g\pm D$, $4D-2g\pm D$ or angles differing from any of the above by an argument of long period, such as $\omega-\omega'$ or 2ω, or that if an error in one term does exist, its apparent effect is cancelled (within a limit of about $0''\cdot3$) by another error in an allied term or errors in both the allied terms—an improbable supposition. Hansen's tables are therefore probably correct as regards terms of the above form. Subsequently I intend to give analyses of the allied terms $g+g'$ and $g-3g'$.

Apart from accidental error, therefore, I infer that all these

In Tenths of a Second of Arc.

Period.	Apparent Coefficients of					Apparent Corr. to Semi-diameter.	Resolved Coefficient of sin D.	Resolved Corr. to Semi-diameter.
	sin 2θ.	cos 2θ.	sin (4D−2g).	(cos 4D−2g).	sin D.			
86	−2	+3	+2	−2	+ 2	+ 1	+ 5	− 2
87	−4	+6	−7	−9	−11	− 8	− 7	− 3
88	0	+2	0	+2	− 1	− 2	+ 8	− 8
89	+4	+3	0	−4	0	0	0	0
90	−6	−2	−2	+3	− 5	− 4	− 1	− 4
91	+5	−1	+3	+3	+ 5	+ 3	+ 8	− 2
92	+4	−5	−4	−1	− 8	− 5	−10	+ 2
93	−2	−1	0	+1	0	− 2	+14	−12
94	+3	−1	+5	−2	− 5	− 3	− 8	+ 2
95	+8	−8	0	0	− 9	− 6	− 9	0
96	−3	−1	−1	−2	−12	− 8	−12	0
97	−9	+2	+1	+2	−13	−11	+ 3	−14
98	+3	+4	+5	+3	− 9	− 6	− 9	0
99	−3	−3	+3	0	− 1	0	− 6	+ 4
100	−1	+5	−2	−1	+ 4	+ 3	+ 2	+ 2
101	+8	−4	+2	0	− 1	0	− 6	+ 4
102	−5	+1	−2	−3	+ 4	+ 3	+ 2	+ 2
103	−6	0	−1	0	+ 4	+ 3	+ 2	+ 2
104	+1	+7	0	−2	− 6	− 3	−14	+ 6
105	+9	−1	+1	−3	+ 1	+ 1	− 1	+ 2
106	−2	−3	−2	0	+ 3	+ 1	+10	− 6
107	0	+4	0	+1	+ 2	+ 1	+ 5	− 2
108	−5	−6	+2	−4	+ 6	+ 4	+ 7	0
109	+1	+4	0	0	+13	+ 8	+18	− 4
110	−3	0	−7	+3	+ 2	+ 3	− 9	+ 9
111	+5	+4	−2	−4	− 3	− 2	− 3	0
112	−1	−4	0	+8	− 1	0	− 6	+ 4
113	−7	+1	0	+2	+ 2	+ 1	+ 5	− 2
114	−1	+5	−1	−3	+ 3	+ 2	+ 3	0
115	−2	−3	+6	+3	+ 3	+ 3	− 4	+ 5
116	0	+5	0	−1	+ 3	+ 3	− 4	+ 5
117	+5	+1	−6	+1	+ 4	+ 4	− 5	+ 8
118	+1	−6	+3	+1	+ 4	+ 3	+ 2	+ 2
119	−1	−2	−6	0	+ 2	+ 1	+ 5	− 2
120	−5	+4	+4	+3	0	0	0	0
121	−4	+1	+2	−1	+ 5	+ 3	+ 8	− 2
122	−5	+1	+2	−1	+12	+ 8	+12	0
123	−5	+5	+1	0	+ 5	+ 3	+ 8	2
124	+5	+4	−2	−4	+ 5	+ 3	+ 8	2
125	−2	+1	+1	+1	+ 1	+ 1	− 1	+ 2
126	−1	+4	+3	−4	− 6	− 4	− 7	0
127	+5	+2	+4	−1	+ 2	+ 2	2	+ 4
128	+2	−4	−1	+3	− 1	0	− 6	+ 4
129	−5	−3	+1	+1	− 4	− 4	+ 5	− 8
130	0	+3	+2	+2	− 5	− 3	− 8	+ 2
131	+2	−2	0	−3	− 6	− 5	+ 1	− 6
132	−6	+1	0	0	− 2	− 1	− 5	+ 2
133	+7	+4	−2	−6	− 2	− 1	− 5	+ 2

quantities should be zero. The probable error of the apparent coefficient of any periodic term can therefore be deduced. Adding the four columns without regard to sign and dividing by 48, and then multiplying by 0·85 to reduce "Mean of the Errors" to "Probable Error," I obtain from the apparent coefficient of

$\sin 2\theta$,	174	$\div 48 \times 0{\cdot}85$	$= \pm 0''{\cdot}31$
$\cos 2\theta$,	147	" "	$= \pm 0''{\cdot}26$
$\sin(4D-2g)$,	101	" "	$= \pm 0''{\cdot}18$
$\cos(4D-2g)$,	104	" "	$= \pm 0''{\cdot}18$

and from the mean of these four separate determinations the probable error of any entry in the first four columns of the table seems to be about $\pm 0''{\cdot}23$.

With this last number as the basis, the probable error of a single "tabular — observed" error of the Moon appears to be about $\pm 0''{\cdot}23 \times \sqrt{\frac{1}{2} \cdot \frac{5647}{48}} = \pm 1''{\cdot}7$. In my December paper I gave $1''{\cdot}5$ as an estimate of this quantity.

Again, the probable error of a coefficient deduced from all forty-eight periods is

$$\pm 0''{\cdot}23 \div \sqrt{48} = \pm 0''{\cdot}03$$

When the Airy period 1750–1851 is fully worked up there will be plenty of material for checking this estimate. At present I can only refer to the values of the principal elliptic coefficient and the coefficient of the Jovian evection in my paper of last month, from which it will be seen that $0''{\cdot}03$ is a fair estimate of the probable error.

As regards the tables of the Moon the results of these two analyses are of a negative character only, indicating the absence and not the presence of periodic errors. It is interesting to note, however, that had Airy analysed for the argument $4D-2g$ he would have found a periodic error of $4''$ in his tabular places, and this would probably have led to a more thorough analysis.

Coming now to the last four columns of the table the sixth column exhibits the apparent coefficient of sin D, that is to say, the quantity obtained by multiplying each error by sin D and dividing by half the number of observations. It is evident that an error of semi-diameter, or a term sin 2D, will each contribute to the "apparent coefficient of sin D." The seventh column exhibits the apparent error of semi-diameter, that is to say, the quantity obtained by reversing the errors of the second limb, adding on the errors of the first limb and dividing by the whole number of observations. It is evident that terms in sin D and sin 2D will contribute to the apparent error of semi-diameter.

I copy here the left-hand sides of the normal equations

obtained in my December paper, where I attempted to separate the semi-diameter, parallactic inequality, and variation.

$$+5647\mu+3788\delta_1-2327\delta_2 \ (=5647 \text{ apparent } \mu)$$
$$+3788\mu+3074\delta_1-1366\delta_2 \ (=3074 \text{ apparent } \delta_1)$$
$$-2327\mu-1366\delta_1+2645\delta_2 \ (=2645 \text{ apparent } \delta_2)$$

On analysing the apparent coefficient of sin D and the apparent correction to semi-diameter for a period $\omega-\omega'$ I obtain

(in the coefficient of sin D)	$+0''\cdot29\cos(\omega-\omega')$
(in the correction to semi-diameter)	$+0''\cdot19\cos(\omega-\omega')$

The first of the normal equations shows that a real inequality $+0''\cdot29\cos(\omega-\omega')$ in the coefficient of sin D will produce an apparent inequality $+0''\cdot29\times3788\div5647 = +0''\cdot19\cos(\omega-\omega')$ in the apparent correction to semi-diameter. Hence the conclusion is that there is no periodic inequality in the true correction to semi-diameter, and that the correction $-0''\cdot29\cos(\omega-\omega')\sin D$ is really required by the tabular places.

No attempt has yet been made to analyse the observations for a term cos D. On account of the grouping of the observations round full Moon the analysis will clearly be subject to an increased accidental error. Hence it is impossible to say at present in what proportions the term $-0''\cdot29\cos(\omega-\omega')\sin D$ must be distributed between arguments $g-g'$ and $g-g'+2\omega-2\omega'$.

The following are the coefficients of sin $(g-g')$ and sin $(g-g'+2\omega-2\omega')$ according to various authorities :

	$\sin(g-g')$.	$\sin(g-g'+2\omega-2\omega')$.
Hansen's tables	$+148''\cdot03$	$-28''\cdot60$
„ theory	$+148''\cdot02$	$-28''\cdot56$
Delaunay (1)	$+147''\cdot46$	$-29''\cdot50$
„ (2)	$+148''\cdot43$	$-28''\cdot32$

The last three lines are taken from Newcomb's paper (*Astron. Papers Amer. Eph.* vol. i.). Delaunay (1) and Delaunay (2) are defined in that paper. The difference is the result of estimating for the higher powers not calculated by Delaunay. It is well known that in these terms (that is, terms with characteristic ee') the convergence of Delaunay's series is extremely slow. The present paper shows that in respect of these terms Hansen's theory is considerably nearer the observations than either Delaunay (1) or Delaunay (2).

Lastly I have solved for "true" (in contradistinction to

"apparent") corrections to the coefficient of sin D and the semi-diameter by the formulæ

$$\mu = 5{\cdot}77 \text{ apparent } \mu - 3{\cdot}87 \text{ apparent } \delta_1$$
$$\delta_1 = 5{\cdot}77 \text{ apparent } \delta_1 - 7{\cdot}11 \text{ apparent } \mu$$

and the results are given in the last two columns.

From the normal equations above quoted it will be seen that the left-hand sides are strictly $\mu - 0{\cdot}7\delta_2$ and $\delta_1 + 0{\cdot}3\delta_2$; but as $\delta_2 = +0''{\cdot}03$ the difference is unimportant.

Since the December paper was written I have corrected the observations individually for an error of semi-diameter $0''{\cdot}4$. Judging from that paper, therefore, the solution to be expected is

$$\mu = -0''{\cdot}054 \qquad \delta_1 = -0''{\cdot}120$$

From the means of the last two columns in the present paper I get

$$\delta_1 + 0''{\cdot}01 = -0''{\cdot}021$$
$$\mu - 0''{\cdot}02 = -0''{\cdot}015$$

differing very slightly from the previous values.

Adding the last four columns without regard to sign I obtain for the probable error of the results of the separate periods

for δ_1 (apparent)	...	...	$211 \div 48 \times 0{\cdot}85 =$	$\pm 0''{\cdot}38$
„ μ „	...	...	$148 \div 48 \times 0{\cdot}85 =$	$\pm 0''{\cdot}26$
„ δ_1	...	...	$292 \div 48 \times 0{\cdot}85 =$	$\pm 0''{\cdot}52$
„ μ	...	...	$157 \div 48 \times 0{\cdot}85 =$	$\pm 0''{\cdot}28$

From the method of formation of the first of these quantities ($\pm 0''{\cdot}38$) there is no reason why it should be larger than the mean ($\pm 0''{\cdot}23$) of the four similarly formed quantities previously obtained. That it is larger is perhaps evidence that the errors are not entirely accidental with reference to the argument D, or that there are time changes from one period of analysis to another, such as might be introduced by personality in semi-diameter and a change of observers. The second quantity is also large, considering that it has double weight. $0{\cdot}26 \times \sqrt{2} = 0{\cdot}36$, and this quantity is therefore comparable with the preceding. The third quantity is determined with a weight diminished in the ratio 1030 : 5647 (see December *Monthly Notices*, footnote); $\pm 0''{\cdot}52$ is in close agreement therefore with $\pm 0''{\cdot}23$ obtained previously found. The fourth quantity is determined with a weight diminished in the ratio 1562 : 5647; $\pm 0''{\cdot}28$ is therefore comparable with a probable error $\pm 0''{\cdot}15$ in the coefficients of periodic inequalities. This additional evidence is therefore against the hypothesis that there are time-changes, for it appears

that the values of μ are more accordant than might have been expected, having regard to accidental error only.

On analysing the last column but one for argument ☊ I find

(tabular *minus* observed) $(+0''\cdot40 \sin ☊ - 0''\cdot12 \cos ☊) \sin D$

The former term is far too large to be accidental; but as sin ☊ sin D splits into cosines, and not sines, it is probable that ☊ is not the actual argument of the periodicity, but only a near approximation to it. The point is accordingly held over till the Airy period is reduced. As far as I can foresee, this reduction should be complete in about two months.

There is a point to which attention should be called, although I am at present unable to offer an explanation. The *apparent* coefficient of sin D and the *apparent* correction to semi-diameter are naturally of the same sign as a rule; in fact, they are never of unlike sign. One test of the validity of the solution into true corrections is that there shall be no periodicity in the correction to semi-diameter, but that all the periodicity should belong to the correction to parallactic inequality. The results stand this test. There is, however, another test, viz. that the signs should be independent. Now in the forty-eight periods we have thirty-two cases of unlike sign, six of like sign, and ten cases where one at least of the quantities is zero. This test is therefore not satisfied. As a similarity of sign has been replaced by, on the whole, a dissimilarity, I conclude that the process of separation has been overdone.

Note on Elliptic Motion. By Asaph Hall.

The difference of the ratio of the radius vector and the mean distance from unity and the difference of the true and mean anomalies, or the equation of centre, are two important quantities in the theory of this motion. If these quantities are wanted in explicit terms of the eccentricity and the mean anomaly, perhaps the easiest method is to apply the theorem of Lagrange and to use the equation

$$dv = \sqrt{1-e^2} \cdot \frac{a^2}{r^2} \cdot dnt.$$

The ratio $\frac{a^2}{r^2}$ can be found by the above theorem, and a substitution and integration will give the equation of centre, or $v - nt$. Bessel objected to this method as not being the simple and natural solution of the question.

In his *Mécanique* Poisson has indicated a method of obtaining these quantities by definite integrals, and this is the method

followed by Bessel. This problem is old, and the coefficients are accurately known. The aim of the writers is to find general expressions for the coefficients, which are complicated for the equation of centre, and I wish to point out that the coefficients can be found directly and easily from the series. Assume with Poisson,

$$r = A_0 + A_1 \cos nt + A_2 \cos 2nt + \ldots + A_i \cos int + \ldots$$
$$v - nt = \quad B_1 \sin nt + B_2 \sin 2nt + \ldots + B_i \sin int + \ldots$$

If i and i' are positive whole numbers, and different, the integrals of the differentials $\cos int \cos i'ntd.nt$ and $\sin int \sin i'ntd.nt$ are zero between the limits v and π. If $i = i'$, the value is $\frac{\pi}{2}$. Hence

$$A_i = \frac{2}{\pi}\int_0^{\pi} r \cos int \,.\, dnt$$
$$B_i = \frac{2}{\pi}\int_0^{\pi} (v - nt) \sin int \,.\, dnt$$

For $i = 0$, we have an exception, and

$$A_0 = \frac{1}{\pi}\int_0^{\pi} r \,.\, dnt = \frac{a}{\pi}\int_0^{\pi} (1 - e\cos u)^2 \,.\, du = a\left(1 + \frac{e^2}{2}\right)$$

where u is the eccentric anomaly. Since the anomalies have the values 0 and π at the same time, by means of the known values of the differentials of v and nt we change the variable to u, integrate by parts, and omit the second part of B_i which is a complete differential, and its integral is zero at the limits. Thus we have

$$A_i = -\frac{2ae}{i\pi} \,.\, \int_0^{\pi} \sin (iu - ie \sin u) \sin u du$$
$$B_i = \frac{2\sqrt{1-e^2}}{i\pi} \,.\, \int_0^{\pi} \frac{\cos (iu - ie \sin u)\, du}{1 - e\cos u}$$

By expansion

$$\sin (iu - ie \sin u) \sin u du$$
$$= + \sin iu \left(\sin u - \frac{i^2 e^2 \sin^3 u}{\lfloor 2} + \frac{i^4 e^4 \sin^5 u}{\lfloor 4} - \frac{i^6 e^6 \sin^7 u}{\lfloor 6} + \ldots\right) du$$
$$- \cos iu \left(ie \sin^2 u - \frac{i^3 e^3 \sin^4 u}{\lfloor 3} + \frac{i^5 e^5 \sin^6 u}{\lfloor 5} - \frac{i^7 e^7 \sin^8 u}{\lfloor 7} + \ldots\right) du$$

In order to integrate and find the coefficients A_i we need only a table for converting powers of $\sin u$ into sines and cosines of the multiple arcs. Then by making i equal to 1, 2, 3, 4 . . . successively we pick out the term that gives the value $\frac{\pi}{2}$, multi-

plied by a power of e, and a numerical coefficient. The rest of the terms vanish. When i is even, the first line is zero; and when i is odd, the second line is zero. As a trial I have computed in this way the twenty-five terms of the A_i, the coefficients including the ninth power of e. Comparing these, without revision, with Le Verrier's I made only three errors. The work is very simple.

Expanding the differential part of B_i, we have

$$+\left(1-\frac{i^2e^2\sin^2 u}{\lfloor 2}+\frac{i^4e^4\sin^4 u}{\lfloor 4}-\frac{i^6e^6\sin^6 u}{\lfloor 6}+\ldots\right)\cos iu\,.\,du$$

$$+\left(ie\sin u-\frac{i^3e^3\sin^3 u}{\lfloor 3}+\frac{i^5e^5\sin^5 u}{\lfloor 5}-\ldots\right)\sin iu\,.\,du$$

$$+\left(e-\frac{i^2e^3\sin^2 u}{\lfloor 2}+\frac{i^4e^5\sin^4 u}{\lfloor 4}-\frac{i^6e^7\sin^6 u}{\lfloor 6}+\ldots\right)\cos iu\cos u\,.\,du$$

$$+\left(ie^2\sin u-\frac{i^3e^4\sin^3 u}{\lfloor 3}+\frac{i^5e^6\sin^5 u}{\lfloor 5}-\ldots\right)\sin iu\cos u\,.\,du$$

&c. &c. &c.

The first two lines can be integrated as before. For the third line an integration by parts gives the partial value

$$B'_i=\frac{2}{\pi}\cdot\frac{\sqrt{1-e^2}}{n+1}\cdot\int_0^{\pi}\sin^{n+1}u\sin iu\,.\,du$$

and the fourth line

$$B''_i=-\frac{2}{\pi}\cdot\frac{\sqrt{1-e^2}}{n+1}\cdot\int_0^{\pi}\sin^{n+1}u\cos iu\,.\,du$$

The lines that follow in the expansion will return to the above forms. n is any exponent of $\sin u$, and for any value of n we make i, 1, 2, 3, 4 . . . &c. Here the work is longer, and we have to multiply by the expanded form of $(1-e^2)^{\frac{1}{2}}$.

Hansen has given some elegant forms for the equation of centre. He puts $e=\sin\phi$, and expands in powers of $\beta=\tan\frac{1}{2}\phi$. His β is the same as the λ of Laplace. The objection of Bessel to the method of Lagrange does not appear to be well founded. It should be a function of the higher analysis to furnish better methods for practical work.

Goshen, Conn.:
1904 March 21.

The Rousdon Variable Star Observations.

By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The observations of long-period variables made during the years 1886–1900 at the Rousdon Observatory, Lyme Regis, Devon, under the direction of the late Sir C. E. Peek, Bart., were put into my hands for discussion and publication after Sir C. E. Peek's death, and will appear as Vol. LV. of the *Memoirs*. A brief account of the discussion may be convenient for other observers.

2. Comparisons were made whenever possible between the variable and *five* stars within a field approximately 1° square. It was of course impossible always to find five stars in this area within a few tenths of a magnitude of the variable; and the differences estimated often exceed a magnitude, sometimes two or three. The accidental errors of such comparisons are naturally large; but there are, nevertheless, distinct advantages in the method which appear on discussion. As regards systematic error it appears that a certain quantity (about 0·2 mag.) must be *numerically subtracted* from all estimated differences greater than about ±0·4 mag. But the value of this quantity changes sensibly with the brightness of the stars compared, and also at two epochs during the work. Still it is not difficult to correct with tolerable accuracy for this arbitrary scale.

3. There is some evidence of systematic error depending on position-angle, but it is conflicting; and, after several vain attempts to establish laws for it from the material available, the quest was abandoned.

4. There seems to be no doubt that the magnitudes for the comparison stars determined at Harvard will not in all cases suit the Rousdon observations. Mr. Grover sees some of them systematically brighter or fainter, doubtless owing to peculiarities of colour. Some trouble was taken to deduce individual magnitudes which would suit his observations, while yet conforming generally to the Harvard scale. But the consequent corrections have been exhibited in a separate column (as also those for the scale value) so that they can be omitted if this is thought desirable.

5. A comparison with the Harvard observations (*Harvard Annals*, vol. xxxvii., Part I.) is given. Two important deductions are made from this comparison.

(*a*) *Systematic Errors.*—The stars observed were divided into two classes, A and B, according to their "redness" as given in Chandler's Third Catalogue. Six stars in Class A (viz. *R*, *S*, and *T Cassiopeiæ*, *S* and *T Cephei*, and *R Aurigæ*) have a mean redness 7·1; and five stars in Class B (viz. *S* and *T Ursæ Majoris*, *S Boötis*, *R Camelopardi*, and *R Draconis*) have a mean redness 2·4. The differences, Rousdon—Harvard, were

arranged according to the mean Rousdon magnitude, and gave the following mean results:

Rousdon Mag.	... Redness.	6·5	7·5	8·5	9·5	10·5	11·5	12·5
Class A ...	7·1	+·6	+·3	−·1	−·3	−·6	−·7	−1·1
Class B ...	2·4	...	0	0	0	−·1	0	+ ·2

There is thus a progressive difference *for red stars* amounting to 0·3 per magnitude! This personal equation far exceeds any other systematic error discussed in the *Memoir*, and seems to demand the most careful investigation if series of observations by different observers are to be combined. It throws some light on the special results for comparison stars mentioned in the last paragraph. It does not affect discussions of observations by the same observer unless he is found to change his habit; but it seems quite possible that there may be such changes of habit, and a series of comparisons between observers at different times would give valuable information.

T Cephei. Assumed

Residuals for each

Group. Julian Day.	1 0	2 32	3 64	4 96	5 128
1871	...	...	...	...	...
2255	− ·3	− ·2	− ·5	+ ·2	+ ·2
2639	− ·3	− ·6	− ·5	+ ·3	+ ·3
3023	+ ·4	− ·3	+ ·1	− ·1	+ ·5
3407	+ ·4	+ ·7	+ ·5	− ·1	+ ·5
3791	+ ·6	+ ·2	+ ·7	+ ·6	+ ·2
4175	+ ·5	+ 1·0	+ ·1	+ ·1	(− ·2)
4559	− ·5	− ·1	− ·3	− ·7	− ·8
4943	− ·2	+ ·6	·0	...	...
Mean	+0·08	+ 0·16	+0·01	+0·04	+0·10
Mag. from Curve ...	7·50	6·80	6·38	6·83	7·56
2255+(2)	− ·2	− 1·1	− ·9	+ ·4	+1·0
3791+(2)	+ ·6	+ 1·1	+ ·5	·0	− ·8
Difference	+ ·8	+ 2·2	+1·4	− ·4	−1·8
Factor 5·09	+4·1	+11·2	+7·1	−2·0	−9·2
$p-f$	+0·6	+ 1·2	+0·1	−1·3	−1·4
Corr. Period... ...	+6·8	+ 9·3	...	+1·5	+6·6

(*b*) *Accidental Errors.*—There are occasions when an observer records a sensible deviation from a mean curve, amounting to more than half a magnitude. It is very important to know whether these deviations are real or accidental. Observers themselves sometimes feel convinced of their reality. It is interesting to find that, in the case of *T Cassiopeiæ*, at any rate, which is discussed in detail in the *Memoir*, when a sensible deviation of this kind is observed at Rousdon it is not confirmed at Harvard, and *vice versâ*. Systematic difference was allowed for before making the comparison. From this and other evidence it would seem that the variation of light in these stars is possibly a strictly periodic phenomenon, though several waves may be superposed.

6. Besides tables giving the observations for each night separately, compendious tables have been formed summarising the results. The following table for *T Cephei* will sufficiently illustrate the method adopted :

period, 384 days.

Group of 32 *Days.*

6 160	7 192	8 224	9 256	10 288	11 320	12 352
...	+ 1·1	+ ·1	·1	− ·3	− ·5	− ·1
+ ·3	+ ·5	+ ·2	+ ·6	− ·1	·0	+ ·2
·0	(− ·1)	− ·2	− ·5	− ·6	·0	− ·4
·0	− ·2	+ ·3	+ ·7	− ·2	− ·1	+ ·3
+ ·1	− ·4	+ ·4	− ·1	− ·1	+ ·1	+ ·5
+ ·1	− ·2	(·0)	+ ·2	+ ·4	+ ·4	+ ·6
− ·5	− ·8	− 1·1	− ·3	− ·1	− ·1	− ·6
− ·6	− 1·0	− ·7	− ·4	− ·1	+ ·2	− ·7
...	...	...	...	...	...	...
−0·09	− 0·14	− 0·12	+0·03	−0·14	0·00	−0·03
8·32	9·07	9·62	9·53	8·93	8·10	7·60
+ ·3	+ ·2	+ ·3	+ ·8	− ·9	− ·1	+ ·1
−1·0	− 2·0	− 1·8	− ·5	+ ·2	+ ·5	− ·7
−1·3	− 2·2	− 2·1	−1·3	+1·1	+ ·6	− ·8
−6·6	−11·2	−10·7	−6·6	+6·0	+3·1	−4·1
−1·3	− 1·2	− 0·6	+0·7	+1·5	+1·2	+0·6
+5·1	+ 9·3	+17·8	−9·4	+4·0	+2·6	−6·8

7. Starting from an arbitrary date with an assumed period the observations were divided into groups of one-twelfth period and a mean light curve formed as shown in the line "Mag. from Curve." The individual observations were compared with this curve and the residuals for each group combined. The mean date to which any residual in the table corresponds is formed by adding the numbers at the top of the column and the end of the row and the omitted constant 2410000 to get the Julian date. Thus the first residual +1·1 corresponds to Julian date 2410000 +1871+192 = 2412063. From the means of the columns it is seen that the assumed light curve is not quite correct, but the corrections indicated, or any portion of them, can readily be applied.

8. In the lower part of the table it is shown how a correction to the assumed period may readily be deduced. Sums of residuals for the period 2255 and two following are denoted by 2255+(2). Comparing these with a later series, the differences are negative from maximum to minimum, and positive from minimum to maximum; indicating that the assumed period is too short. Applying the "factor 5·09" and the divisor "$p-f$" we get a correction to the period from each column, as fully explained in the memoir. The weighted mean of these corrections for *T Cephei* is +4·9 days, so that the true period deduced from the Rousdon observations is 388·9 days.

9. In this way an accurate value for the period has been deduced for each star from the Rousdon observations alone; and compared with that given in Chandler's Third Catalogue (*A. J.* No. 379). His revised "Elements" (*A. J.* No. 553) have since been received and compared with the Rousdon determinations in a postscript. The effect of the revision is so marked that the following summary of it may be repeated here. For the majority of the stars the period is variable according to a definite law, that for *S Cassiopeiæ*, for instance, being denoted by

$$610{\cdot}5\ \mathrm{E}+37\ \sin\ (15^\circ\ \mathrm{E}+59^\circ)$$

the mean period of 610·5 days being liable to regular variations of thirty-seven days each way. For comparison with Rousdon, the periods have been obtained from the formulæ by substituting the values of E appropriate to the Rousdon observations.

No.	Star's Name.	Chandler's Period. Old. d	Chandler's Period. New. d	Rousdon Period. d	R—Old. d	R—New. d
107	T Cassiopeiæ	445·0	443·5	444·4	− 0·6	+ 0·9
432	S Cassiopeiæ	606·2	619·4	630·7	+24·5	+11·3
1855	R Aurigæ	460·2	455·0	454·2	− 6·0	− 0·8
2100	U Orionis	375	375	374·4	− 0·6	− 0·6
2478	R Lyncis	380·0	377·4	377·8	− 2·2	+ 0·4
3825	R Ursæ Majoris	304·2	302·4	300·4	− 3·8	− 2·0

No.	Star's Name.	Chandler's Period. Old. d	New. d	Rousdon Period. d	R—Old. d	R—New. d
4511	T Ursæ Majoris	254·8	254·8	255·5	+ 0·7	+ 0·7
4557	S Ursæ Majoris	223·0	224·4	224·5	+ 1·5	+ 0·1
5157	S Boötis	259·8	266·5	265·9	+ 6·1	− 0·6
5190	R Camelopardi	269·5	273·9	273·3	+ 3·8	− 0·6
5504	S Coronæ	360·8	362·6	363·3	+ 2·5	+ 0·7
5955	R Draconis	245·6	245·6	245·8	+ 0·2	+ 0·2
6044	S Herculis	308·1	305·7	309·0	+ 0·9	+ 3·3
6449	T Draconis	(569)	426	428·3	...	+ 2·3
7045	R Cygni	425·7	426·6	427·0	+ 1·3	+ 0·4
7120	χ Cygni	406·2	406·2	407·0	+ 0·8	+ 0·8
7220	S Cygni	320·8	324·1	327·0	+ 6·2	+ 2·9
7609	T Cephei	387	387	388·9	+ 1·9	+ 1·9
7779	S Cephei	484	487·9	489·8	+ 5·8	+ 1·9
8600	R Cassiopeiæ	424·7	436·5	438·7	+ 14·0	+ 2·2

10. It will be seen that the new formulæ suit the Rousdon results much better than the old in every case but that of *S Herculis*. For *S Cassiopeiæ* the period is still capable of improvement; and perhaps also for *R Ursæ Majoris* and *R Cassiopeiæ*. But the general agreement is such as to warrant considerable confidence in Chandler's formulæ.

11. He gives no definite periods for *S Persei* or *R Ursæ Minoris*. For *S Persei* a main period of 840 days, and a subsidiary one of 1120 days are indicated by the Rousdon observations; though there are also traces of a much longer period (3360 days?). For *R Ursæ Minoris* the Rousdon observations indicate 320 days.

12. Some attempt is made in the Memoir to consider the possible origin of Chandler's periodic terms. The various light curves were analysed harmonically, and they seemed to fall into a series, the coefficients of the separate harmonics being related to one another. Thus dividing the stars into five groups, according to the value of A in the formula

$$A \sin \theta + B \cos \theta + C \sin 2\theta + D \cos 2\theta + E \sin 3\theta + F \cos 3\theta$$

where θ is reckoned from maximum, and the whole range of variation is represented by 6, we get the following values for the coefficients:

Group.	A	B	C	D	E	F
I.	+ ·83	−2·50	−·49	−·14	+·16	−·19
II.	+ ·40	−2·78	−·22	+·27	−·09	−·20
III.	− ·05	−2·86	+·01	+·04	−·02	−·13
IV.	− ·77	−2·70	+·30	−·23	−·03	−·04
V.	−1·19	−2·56	+·33	−·36	+·09	−·04

13. It is only natural that B should vary with A, since without the other terms, which are small, the range would be $2(A^2+B^2)^{\frac{1}{2}}$, and we have made the range constant. But there is less reason for the obvious run in the other terms. As regards D, the apparent anomaly in the value for Group I. is removed when we put the expression into the alternative form

$$H_1 \cos(\theta+G_1)+H_2 \cos(2\theta+G_2)+H_3 \cos(3\theta+G_3)$$

when the negative sign for D is seen to be merely a consequence of the continuous change in G_2. The following approximate formulæ are merely illustrative, to show roughly the dependence of all these quantities on a single variable, and have obviously no physical significance :

$$H_2 = 0{\cdot}5\,A^{\frac{3}{4}} \qquad H_3 = -0{\cdot}14 - 0{\cdot}20\,A \qquad G_2 = 130^\circ\,A$$

$$G_3 = 80^\circ\,A - 40^\circ$$

and the success with which these formulæ represent the observed quantities is shown in the following table :

				C		D				E		F	
Group.	A	H_2	G_2	Calc.	Obs.	Calc.	Obs.	H_3	G_3	Calc.	Obs.	Calc.	Ob
I.	+·83	+·44	+108°	−·42	−·49	−·14	−·14	−·31	+26°	+·13	+·16	−·27	−·
II.	+·40	+·27	+52	−·21	−·22	+·17	+·27	−·22	−8	−·03	−·09	−·22	−·
III.	−·05	+·06	−6	+·01	+·01	+·06	+·04	−·13	−44	−·09	−·02	−·09	−·
IV.	−·77	+·42	−100	+·41	+·30	−·07	−·23	+·01	−102	+·01	−·03	·00	−·
V.	−1·19	+·56	−155	+·24	+·33	−·51	−·36	+·10	−135	+·07	+·09	−·07	−·

14. If relations of this kind can be established among the harmonic coefficients, we may be able to separate out two classes of periodic terms, one set following laws similar to those sketched above, and another with smaller amplitude and slightly different period which, by interference with the main set, cause the oscillation in epoch of maximum which Chandler represents by such expressions as

$$610{\cdot}5\,E + 37 \sin(15^\circ E + 59^\circ)$$

At any rate a promising line of investigation seems to open here. It was my intention to collect more material so as to improve the formulæ ; and I made a preliminary search for well-established light curves published by other observers in order to submit them to harmonic analysis. But I had little success in finding them—perhaps through lack of acquaintance with the literature of the subject.

15. *I should be particularly obliged if observers who have formed well-determined light curves would kindly let me have copies of them or a reference to the publication in which they are given.*

16. While writing this note it occurred to me that possibly the sun-spot curve might exhibit features either resembling those

of the variable stars or obviously differing from them. Seen as a star, the variation in total light of the Sun, owing to its variations in spottedness, is no doubt extremely small, perhaps quite insensible; but the features of the light curves above studied seem to have little or no relation to the total range of variation, and may persist even when this has become so small as in the case of our Sun. To test the point, Wolf's sun-spot numbers for the forty-four years 1849–1892 were tabulated in four periods of eleven years,* and the sums taken as follows:

1	2	3	4	5	6	7	8	9	10	11
362	309	253	195	128	77	44	24	61	196	354

It was not difficult to draw a regular curve through these points and to read off the ordinates at twelve epochs for harmonic analysis.

17. But now came the question, Is the Sun to be regarded as (A) slightly brighter at a sunspot maximum, or (B) slightly fainter? If spots mean loss of light (B) is the proper supposition; but since they also mean increase of activity, it would be more natural to take (A). Both hypotheses were therefore tried, with the results given below.

Hypothesis A. Sun brighter when spotted.

18. Twelve equidistant ordinates in fractions of the greatest as unity, counting from sun-spot maximum as zero.

Max.	30°.	60°.	90°.	120°.	150°.	180°.	210°.	240°.	270°.	300°.	330°.
·00	·09	·23	·38	·54	·71	·84	·94	·99	·93	·65	·22

Analysing these by the same process as that used for the variables, we get:

	A.	B.	C.	D.	E.	F.	H_φ	G_φ	H_μ	G_μ
Observed	−1·49	−2·32	+·11	−·70	+·19	−·19	+·71	−171°	+·27	−135°
Formulæ	(−1·49)	−2·29	−·16	−·64	+·05	−·15	+·65	−194	+·16	−160

Below the "observed" values are given the values which would be deduced from the variable star formulæ given above by subtracting the value −1·49 for A. The agreement is good, the chief discrepancy being that in C, which would be removed by a comparatively slight change in the epoch G_μ.

A slight change either in A or in the formula $G_\varphi = 130° A$ would make a satisfactory adjustment. Considering that the value of A for the Sun is much larger than in any of the series from which the formulæ were deduced, so that we are extrapolating for the Sun, the accordance may be considered satisfactory.

* For our present purpose a slight error in the period does not matter.

Hypothesis B. Sun fainter when spotted.

19. Twelve equidistant ordinates as before, but counting from sun-spot minimum as zero.

Max.	30°.	60°.	90°.	120°.	150°.	180°.	210°.	240°.	270°.	300°.	330°.
·00	·10	·42	·85	1·00	·90	·74	·60	·43	·26	·13	·05

Analysing which we get :

	A.	B.	C.	D.	E.	F.	H_2.	G_2.	H_3.	G_3.
Observed	+1·51	−2·35	−·46	−·54	−·23	+·14	+·72	+138	−·27	+236
Formulæ (+1·51)		−1·67	+·17	−·63	+·43	−·08	+·65	+195	−·44	+ 80

The difficulties of reconciling the observed quantities with the formulæ are now more serious. The H*s* agree as well as before; but the discrepancy in G_2 is doubled, that in G_3 is as large as it can well be, and there is a serious discrepancy in B as calculated from the formula $B^2 = 8{\cdot}20 - 1{\cdot}8\,(A + 0{\cdot}20)^2$, deduced from the variable star results. Without a considerable modification of the formulæ, the sun-spot curve will not fit in with this end of the series at all.

20. We must not lay too much stress on the evidence, but so far as it goes it supports the conclusions :

(1) That the sun-spot curve falls into the series of variable-star curves, but outside the series presented by the Rousdon variables.

(2) That a maximum of sun-spot activity corresponds to a maximum of a variable star.

The Rotation Period of the Planet Saturn. By G. W. Hough.

In the *Monthly Notices* for January 1904, Mr. W. F. Denning suggests a wrong identification of the spots on *Saturn* as given in *Monthly Notices* for December, 1903. As so little is known regarding the time of rotation of the planet, the question is one of considerable importance. Of the spots observed on June 23–27 and July 13–29 there can be no question whatever. The latitudes agree very closely, and the spot was then conspicuous. And it was the only well-defined and conspicuous spot seen on the planet during the opposition. The spot observed by Burnham on August 19 was faint, and appeared to be out in latitude, and for these reasons its identity with the original spot may be questioned.

I think it highly probable, however, that previous to the middle of August the Barnard spot had lost its outline and ceased to be a well-defined object. The small spot observed on July 14, according to Barnard's micrometer measure, followed

thirty-nine minutes. The observation of August 19 was made when the planet was only thirty minutes from the meridian, and if the spot observed was not the Barnard spot, the other ought to have been visible on the disc. If, however, the observation of August 19 is rejected as doubtful, my results for rotation period remain substantially the same.

Since the rotation period was increasing the mean period should be slightly less than when the observation of August 19 was included. A mean period $R=10^h\ 38^m\ 24^s$ yields the following:

Date 1903.	Days.	Obs.	O—E. m
June 23	−4	Bar	+1·4
27	0	Ho	+0·0
July 13	16	Ho	−3·0
29	32	*B*	+1·3

The observations are all satisfied with a mean error $\pm 1^m\cdot 4$.

According to Mr. Denning's identification for spot (C) $R=10^h\ 38^m$ we have the following as given in his table:

Date 1903.	Obs.	Days.	O—E. m
July 6	Ho	0	+10·1
18	Ho	12	− 5·6
29	*B*	23	−14·4
Aug. 14	*B*	43	+ 9·9

We have here a range of $24^m\cdot 5$ in the residuals. Such discordant observations would not be good for eye estimates, and certainly have no connection whatever with micrometer work. These observations obviously do not refer to the same object. Mr. Denning seems to place implicit confidence in Barnard's four observations covering a period of 21 days. On the contrary, I have great faith in his micrometer measure of June 23, which is in harmony with Burnham and myself, although it differs 6·8 minutes from his estimate.

Using Barnard's micrometer measure for June 23, from his observations we get the following, $R=10^h\ 38^m\ 16^s$:

Date.	Days.	O—E. m	Date.	Days.	O—E. m
June 23	0	+0·0	July 13	20	+1·1
24	1	+6·6	14	21	+0·0

which is in confirmation of my value for the rotation period.

I am somewhat surprised that such an experienced observer as Mr. Denning, who has done so much good work on the planet *Jupiter*, should raise the question of the rotation period of the planet *Saturn* with the kind of material at his disposal.

The Barnard spot was observed on five nights by two or more observers.

The following is the range in the time of passage over the central meridian :

Date 1903.	Range. m	Date 1903.	Range. m
June 26	19	July 24	17
July 11	25	Aug. 5	20?
12	20?		

These figures imply an uncertainty of about ten minutes in the time of passage over the central meridian. During this period the spot was a conspicuous object.

In 1878, before the rotation period of the red spot on *Jupiter* was ascertained we had very much the same kind of uncertain observations.

Mr. Denning's suggestion that discordant observations may be due to oscillation in position of the spots on *Jupiter* and *Saturn* is not sound. There is no proof that any spot on *Jupiter* has changed the direction of its motion in a short interval of time. The smallest spot observed has an area of more than one million square miles, and those usually observed many times that area. Presumably there is mass, and according to our ordinary conception of matter in motion we should not expect any sudden stoppage or reversal of motion.

As between micrometer measures on a luminous disc and eye estimates, at this late day, there ought to be no difference of opinion.

The micrometer method for ascertaining the passage of a spot over the central meridian of *Jupiter* was used by Airy in 1835. At that time the chronograph was unknown and the parallel wire micrometer little used. Airy located the spot by transits of the limbs of the planet and spot by the eye and ear method as in meridian observations.

In 1879 I made a few observations of the red spot in the same way, using the chronograph.

I found, however, that much time was saved and greater precision secured by making the measures directly with the micrometer.

It is to be hoped that if any definite spots are seen on *Saturn* at the coming opposition, astronomers who have micrometers will use them, and determine both longitude and latitude at each observation. Two sets of measures of three bisections of the spot and the limbs of the planet, with the corresponding times when the spot is near the central meridian, will give the time of passage over the central meridian by simple proportion.

During the fifteen minutes employed by the observer in watching a spot in transit one may make from twenty to forty micrometer readings.

When observers on the same night differ in the estimated time of passage fifteen or twenty minutes, it is evident that the method is not a good one. Observers who have no driving clock or micrometer must necessarily do the best they can, but it ought to be evident to every working astronomer that the quality of the work cannot be the same for all instruments.

Dearborn Observatory of North-Western University:
1904 *April* 2.

Corrected Continuation of Brünnow's "Tafeln der Flora."

By A. M. W. Downing, D.Sc., F.R.S.

The materials collected in this paper are intended for the convenience of astronomers who may wish to use the corrected elements of *Flora*, published in *Monthly Notices*, vol. lii. p. 590, in combination with Brünnow's Tables, for the purpose of computing an ephemeris. By means of the continuations or modifications of certain of the tables, or of parts of them, here given, the necessary computations can be made with facility for the period 1904–1911.

From an observation of the planet made by Mr. Tebbutt at Windsor, N.S.W., on 1902 July 27, and published in *Astron. Nachrichten*, No. 3856, I find the correction to the place of *Flora*, computed in the manner indicated above, to be

in R.A. $-3^{s}\cdot85$, and in Decl. $-13''\cdot3$.

Continuation of Brünnow's Tafel I., "Argumente für die Jahre."

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.
B. 1904	100·83	13·06	113·90	214·74	188·63	87·8	126·9	289·4	315·6	285·3	298·4	140·0	276·4	311·3
1905	180·65	342·73	163·39	344·05	18·59	197·9	146·1	199·2	164·7	144 8	127·6	128·9	216·5	110·2
1906	260·47	312·40	212·88	113·36	208·55	165·2	308·1	108·9	13·8	4·4	316·8	117·7	156·6	269·1
1907	340·29	282·08	262·38	242·67	38·51	58·2	184·4	18·7	223·0	223·9	146·0	106·6	96·7	68·0
B. 1908	60·33	251·67	312·01	12·34	228·99	168·7	203·6	289·3	72·6	83·1	334·8	95·4	37·6	226·3
1909	140·15	221·34	1·50	141·65	58·95	278·8	222·8	199·1	281·7	302·6	164·0	84·3	337·8	25·2
1910	219·97	191·02	50·99	270·96	248·91	28·9	242·0	108·9	130·9	162·1	353·2	73·1	277·9	184·1
1911	299·79	160·70	100·49	40·28	78·88	139·1	261·2	18·7	340·1	21·6	182·3	61·9	218·0	343·0

	XV.	XVI.	XVII.	XVIII.	XIX.	XX.	XXI.	XXII.	XXIII.	XXIV.	XXV.	XXVI.	XXVII.	XXVIII.
B. 1904	30·4	131·1	56·5	328·6	4·2	118·2	224·0	136·2	0·3	136·6	224·4	311·8	48·4	175·9
1905	20·0	200·5	345·5	147·4	54·4	217·9	321·9	124·0	86·0	210·1	48·0	159·9	286·0	121·9
1906	9·6	270·0	274·4	326·2	104·7	317·7	59·9	111·8	171·7	283·6	231·7	8·0	163·6	67·9
1907	359·2	339·4	203·4	145·0	154·9	57·4	157·8	99·6	257·4	357·1	55·3	216·1	41·2	13·9
B. 1908	349·8	49·9	133·2	324·3	206·2	158·5	255·9	87·3	343·5	70·8	239·5	64·6	278·5	320·8
1909	339·3	119·3	62·1	143·1	256·5	258·2	353·8	75·1	69·2	144·3	63·1	272·7	156·2	266·8
1910	328·9	188·8	351·0	321·9	306·8	357·9	91·8	62·9	154·9	217·8	246·8	120·8	33·9	212·7
1911	318·5	258·2	279·9	140·7	357·1	97·5	189·8	50·7	240·5	291·3	70·4	328·9	271·6	158·7

In the formation of these arguments the mean anomalies of *Flora* have been computed from the corrected elements. The mean anomalies of *Jupiter* and *Saturn* have been taken from Le Verrier's Tables, and are corrected for the great inequality.

Table to be substituted for the First Part (p. 6) of Brünnow's Tafel IV., "Für die mittlere Anomalie."

	M.	t.
B. 1904	87° 47′ 23″·68	+55·998
1905	197 55 57·11	+56·997
1906	308 4 30·54	+57·997
1907	58 13 3·97	+58·996
B. 1908	168 39 43·74	+59·998
1909	278 48 17·17	+60·997
1910	28 56 50·60	+61·997
1911	139 5 24·03	+62·996

	M	t.
January ...	0° 0′ 0″·00	0·000
February ...	9 21 16·48	+0·085
March ...	17 48 13·95	+0·162
April	27 9 30·43	+0·246
May	36 12 40·58	+0·329
June	45 33 57·06	+0·413
July	54 37 7·21	+0·496
August ...	63 58 23·69	+0·580
September ...	73 19 40·17	+0·665
October ...	82 22 50·32	+0·747
November ...	91 44 6·80	+0·832
December ...	100 47 16·95	+0·915

Days.	M.	t.
1	0° 18′ 6″·34	+0·003
2	0 36 12·68	+0·005
3	0 54 19·01	+0·008
4	1 12 25·35	+0·011
5	1 30 31·69	+0·014
6	1 48 38·03	+0·016
7	2 6 44·37	+0·019
8	2 24 50·71	+0·022
9	2 42 57·04	+0·025
10	3 1 3·38	+0·027
20	6 2 6·76	+0·055
30	9 3 10·14	+0·082

In bissextile years (marked B above) subtract one day from the date during January and February.

Corrections to Brünnow's Tafel V., "Für die Mittelpunkts Gleichung," and Tafel VI., "Für den elliptischen Radius Vector."

M.	Δ(v−M).	Δ log r.	M.	Δ(v−M).	Δ log r
°	"		°	"	
0	0·00	+27	90	−3·50	−29
1	−0·06	+27	95	−3·42	−32
5	−0·49	+26	100	−3·33	−34
10	−0·97	+25	105	−3·17	−37
15	−1·42	+23	110	−2·97	−40
20	−1·85	+21	115	−2·77	−40
25	−2·24	+18	120	−2·57	−40
30	−2·59	+15	125	−2·37	−41
35	−2·89	+11	130	−2·17	−43
40	−3·15	+ 7	135	−1·96	−46
45	−3·38	+ 3	140	−1·75	−49
50	−3·60	− 2	145	−1·54	−49
55	−3·70	− 5	150	−1·33	−49
60	−3·73	− 8	155	−1·12	−49
65	−3·74	−11	160	−0·91	−50
70	−3·75	−15	165	−0·70	−50
75	−3·69	−20	170	−0·48	−50
80	−3·63	−24	175	−0·25	−51
85	−3·57	−26	180	0·00	−51

These corrections reduce the equation of the centre and the log radius vector, taken from Brünnow's Tables, to the corresponding values deduced from the corrected elements.

The corrections for intermediate values of M can be found with sufficient accuracy by interpolation.

When M is greater than 180° the argument is 360°−M. In this case the sign of $\Delta(v-M)$ must be changed, but the sign of $\Delta \log r$ remains unchanged.

The values of $\Delta \log r$ are expressed in units of the seventh decimal.

Continuation of Brünnow's Tafel XIV., "Für die Constanten in Bezug auf den Aequator."

	A.	B.	C.	log a.	log b.	log c.	log cos a.	log cos b.	log cos c.
1904	123° 38′ 45″49	35° 47′ 9″86	19° 53′ 6″24	9·9980036	9·9694658	9·5735743	8·98075	9·55892n	9·96717
1905	123 39 35·90	35 47 58·99	19 53 57·99	9·9980040	9·9694696	9·5735477	8·98071	9·55890n	9·96717
1906	123 40 26·30	35 48 45·12	19 54 49·74	9·9980044	9·9694734	9·5735211	8·98067	9·55887n	9·96717
1907	123 41 16·71	35 49 37·25	19 55 41·50	9·9980047	9·9694772	9·5754945	8·98063	9·55885n	9·96718
1908	123 42 7·25	35 50 26·52	19 56 33·40	9·9980051	9·9694811	9·5734678	8·98058	9·55882n	9·96718
1909	123 42 57·65	35 51 15·65	19 57 25·17	9·9980055	9·9694849	9·5734412	8·98054	9·55880n	9·96718
1910	123 43 48·06	35 52 4·79	19 58 16·94	9·9980059	9·9694887	9·5734145	8·98050	9·55878n	9·96719
1911	123 44 38·46	35 52 53·93	19 59 8·71	9·9980063	9·9694925	9·5733878	8·98046	9·55876n	9·96719

These quantities have been computed from the corrected elements, and the values of A, B, and C modified by the subtraction of a constant (9′ 46″·5) from each, so as to be applicable to the perturbed true anomalies when the perturbations are taken from Brünnow's Tables.

H.M. Nautical Almanac Office:
1904 *March* 31.

Observations of the Minor Planet (324) *Bamberga at Windsor, New South Wales.* By John Tebbutt.

The observations were made with the 8-inch equatorial and its filar micrometer in a bright field. The following notes were made with reference to the planet's magnitude:

August 12: Magnitude about 8·0. August 17: Apparently brighter than on previous occasions. August 20: Certainly did not exceed $8\frac{1}{2}$ magnitude. August 29: Planet and comparison star exactly equal. September 1: Much brighter than comparison star and of $8\frac{1}{2}$ magnitude. September 3: Planet slightly brighter than comparison star. September 4: Considerably less than star. September 5: Very much inferior to star. September 7: Planet becoming fainter. September 14: Slightly less than star. September 21: Equal to $8\frac{1}{2}$ magnitude.

(324) *Bamberga.*

Date.	Windsor Mean Time	Planet—Star, Δ R.A.	Δ N.P.D.	No. of Comps.	Planet's App. R.A.	Log. p_A.	Planet's App. N.P.D.	Log. p_A.	Reduction to App. Place. R.A.	N.P.D.	Comp. Star.
1903.	h m s	m s	′ ″		h m s	−	° ′ ″	+	s	″	
Aug. 12	10 51 56	+1 48·61	− 5 33·5	15	22 23 24·48	9·4355	101 37 19·2	0·5448	+3·34	−23·7	1
13	9 45 47	+0 58·36	−11 0·4	15	22 22 34·25	9·5725	101 31 52·3	0·5767	+3·36	−23·7	1
15	10 26 1	−4 51·32	...	5	22 20 42·88	9·4666	...	...	+3·37	...	2
15	10 26 1	−5 53·16	− 3 28·3	5	22 20 42·93	9·4666	101 20 17·7	0·5542	+3·36	−23·8	3
16	9 33 2	−5 45·97	+ 4 42·4	7	22 19 48·24	9·5690	101 14 47·0	0·5789	+3·38	−23·9	2
16	9 33 2	−6 47·82	...	7	22 19 48·29	9·5690	...	...	+3·38	...	3
17	9 21 59	−6 43·36	− 0 59·8	7	22 18 50·86	9·5791	101 9 4·8	0·5835	+3·39	−23·9	2
17	9 21 59	−7 45·29	....	7	22 18 50·83	9·5791	...	...	+3·39	...	3
20	9 32 58	−3 12·77	+10 48·0	12	22 15 50·48	9·5318	100 51 40·5	0·5736	+3·44	−24·0	4
22	9 15 26	−5 16·08	− 0 47·9	10	22 13 47·18	9·5464	100 40 4·5	0·5797	+3·45	−24·1	4
23	9 16 41	−6 19·02	− 6 40·5	10	22 12 44·25	9·5342	100 34 11·9	0·5781	+3·46	−24·1	4
29	9 13 24	+1 4·54	+ 6 53·5	15	22 6 20·84	9·4729	99 58 18·4	0·5744	+3·51	−24·2	5
Sept. 1	8 55 46	+0 59·78	+ 1 9·9	15	22 3 12·23	9·4794	99 40 5·2	0·5793	+3·52	−24·2	6
2	8 52 43	−0 2·40	− 5 1·4	7	22 2 10·06	9·4744	99 33 53·8	0·5800	+3·53	−24·3	6
3	8 26 24	−1 2·78	−11 8·7	15	22 1 9·68	9·5235	99 27 46·5	0·5895	+3·53	−24·3	6
4	8 33 19	+0 41·84	+10 43·5	15	22 0 8·61	9·4972	99 21 29·0	0·5863	+3·52	−24·3	7

Date. 1903.	Windsor Mean Time. h m s	Planet—Star. Δ R.A. m s	Δ N.P.D. ′ ″	No. of Comps.	Planet's App. R.A. h m s	Log. $p\Delta$. −	Planet's App. N.P.D. ° ′ ″	Log. $p\Delta$. +	Reduction to App. Place. R.A. s	N.P.D. ″	Comp. Star.
Sept. 5	8 30 35	−0 17·92	+ 4 25·8	15	21 59 8·85	9·4919	99 15 11·3	0·5868	+3·52	−24·3	7
7	8 23 35	−2 14·42	− 8 16·0	10	21 57 12·35	9·4851	99 2 29·5	0·5885	+3·52	−24·3	7
7	9 10 54	−2 16·36	...	5	21 57 10·41	9·3400	...	...	+3·52	...	7
7	9 10 54	+4 36·14	+ 1 5·0	5	21 57 10·34	9·3400	99 2 14·8	0·5740	+3·52	−24·2	8
8	7 44 48	−3 9·42	...	7	21 56 17·35	9·5560	...	...	+3·52	...	7
8	7 44 48	+3 43·07	− 4 55·9	7	21 56 17·27	9·5560	98 56 13·9	0·6022	+3·52	−24·2	8
9	8 6 35	−4 5·66	...	3	21 55 21·11	9·5019	...	...	+3·52	...	7
9	8 6 35	+2 46·73	−11 30·0	3	21 55 20·93	9·5019	98 49 39·8	0·5934	+3·52	−24·2	8
12	8 58 31	+6 43·60	+ 8 36·4	6	21 52 41·70	9·2911	98 29 54·6	0·5790	+3·50	−24·2	9
13	8 3 21	+5 56·32	+ 2 16·5	10	21 51 54·42	9·4624	98 23 34·6	0·5936	+3·50	−24·3	9
14	7 59 33	+5 8·88	− 4 19·9	10	21 51 6·97	9·4600	98 16 58·2	0·5950	+3·49	−24·3	9
20	8 3 33	−4 9·15	+10 28·7	8	21 47 1·26	9·3612	97 36 29·1	0·5957	+3·46	−24·5	10
21	7 58 38	−4 42·80	+ 3 37·4	7	21 46 27·60	9·3626	97 29 37·8	0·5972	+3·45	−24·5	10
23	7 48 59	−5 42·78	−10 11·3	7	21 45 27·61	9·3652	97 15 49·2	0·6005	+3·44	−24·4	10

Adopted Mean Places of the Comparison Star for 1903·0.

Star.	R.A. h m s	N.P.D. ° ′ ″	Authorities.
1	22 21 32·53	101 43 16·4	Argent, G. Cat. 1875, 30622; Radcliffe, 1890, 6026.
2	22 25 30·83	101 10 28·5	Greenw. Cat. 1880, 3769; Stone, 11769; Radcliffe, 1890, 6040.
3	22 26 32·73	101 24 9·8	Greenw. Cat. 1880, 3778; Stone, 11780; Radcliffe, 1890, 6043.
4	22 18 59·81	100 41 16·5	Argent, G. Cat. 1875, 30577; Radcliffe, 1890, 6012.
5	22 5 12·79	99 51 49·1	Lalande, 43217 and 8.
6	22 2 8·93	99 39 19·5	Yarnall$_2$, 9955.
7	21 59 23·25	99 11 9·8	Radcliffe, 1890, 5935.
8	21 52 30·68	99 1 34·0	Radcliffe, 1890, 5911.
9	21 45 54·60	98 21 42·4	Radcliffe, 1890, 5889.
10	21 51 6·95	97 26 24·9	Radcliffe, 1890, 5906.

Private Observatory, The Peninsula, Windsor, N.S. Wales:
1904 *February* 11.

MONTHLY NOTICES

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No. 2.

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"Sunspot Variation in Latitude, 1861—1902." By WILLIAM J. S. LOCKYER, M.A. (Camb.), Ph.D. (Gött.), F.R.A.S., Chief Assistant, Solar Physics Observatory. Communicated by Sir NORMAN LOCKYER, K.C.B., LL.D., F.R.S. Received January 16, —Read February 11, 1904.

In a previous communication,* Sir Norman Lockyer and I gave the results of a discussion of prominence observations, and pointed out the necessity of dealing individually with small zones on the solar surface.

In that paper brief reference was made to the law of spot zones, as discovered by Carrington, and corroborated by Spörer, and it was further stated that more modern observations had established these *general* deductions of spot distribution. The words "general deductions" were purposely used, as it was then noticed that there were many anomalies that required explanation.

The object of the present paper is to draw attention to these anomalies, and to give the results that have been deduced from a minute examination of the changes of heliographic latitudes of sunspots from year to year. The present evidence indicates that the law of Spörer, although of great importance, represents only a very general idea of a complicated sunspot circulation.

* "Solar Prominence and Spot Circulation, 1872—1901," 'Roy. Soc. Proc.,' *vol.* 71, p. 446.

From a valuable series of sunspot observations, made between the years 1853 and 1861, Richard Carrington* was the first to point out that spots had a general drift towards the Equator during a sunspot cycle, or, to use his own words, there was indicated "a great contraction of the limiting parallels between which spots were formed for two years previously to the minimum of 1856, and, soon after this epoch, the apparent commencement of two fresh belts of spots in high latitudes, north and south, which have in subsequent years shown a tendency to coalesce, and ultimately to contract, as before, to extinction." Spörer fortunately took up the work where Carrington left off, and his observations extended over the period 1861—1879. These were published in four different volumes,† and the conclusions at which he arrived practically corroborated those of Carrington. In the last of these publications, Spörer summed up all the observations for the period 1854—1879, and published curves, showing the relation between the sunspot frequency for these years and the variation of the mean heliographic latitude of the spots.

The law of zones, as definitely formulated by Spörer, is as follows‡ :—"Un peu avant le minimum, il n'y a de taches que près de l'equateur solaire, entre $+5°$ et $-5°$. A partir du minimum, les taches, qui avaient depuis longtemps déserté les hautes latitudes, s'y montrent brusquement vers $\pm 30°$. Puis elles se multiplient, un peu partout, à peu près entre ces limites, jusqu'au maximum, mais leur latitude moyenne diminue constamment jusqu'à l'époque du nouveau minimum."

As solar prominences appear on any part of the disc, it was sufficient, in order to trace their distribution, to divide the sun's surface into nine zones of 10 degrees each. Since, however, spots seldom occur above latitude 40°, the width of the zones had to be considerably diminished. For the present inquiry, it was finally decided to group the spots into belts 3 degrees wide, for even zones of 5 degrees in width were found to mask many important characteristics.

The necessity for such narrow zones will be seen from the accompanying figure (fig. 1), in which the yearly distribution of spots is shown for the years 1879—1883, taking zones of 10 degrees, 5 degrees, and 3 degrees in width respectively.

* 'Observations of the Spots on the Sun,' made at Redhill by R. C. Carrington, F.R.S., 1863, p. 17.

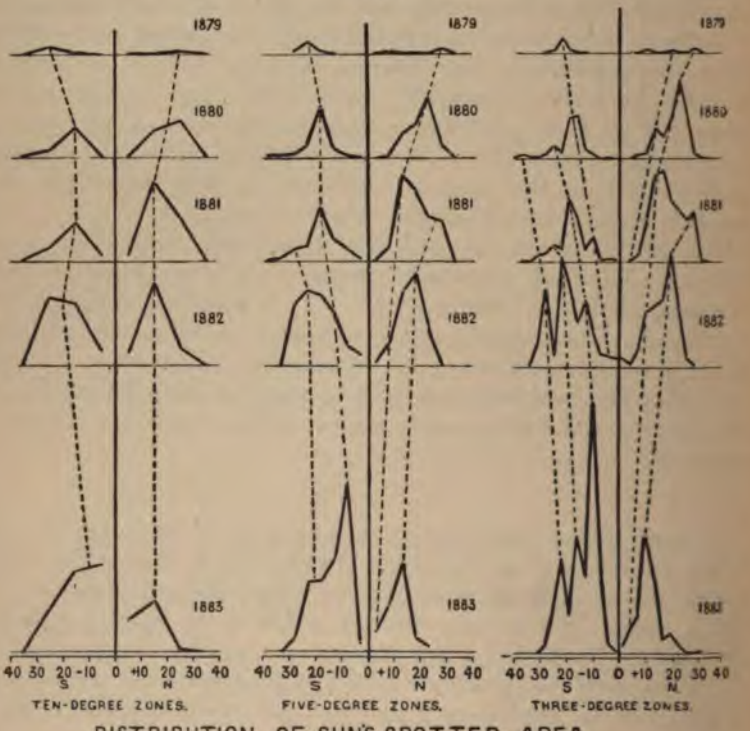
† 'Publication der Astronomischen Gesellschaft,' vol. 13 (Leipzig, 1874); 'Publication der Astronomischen Gesellschaft,' vol. 13. Fortsetzung (Leipzig, 1876); 'Publicationen des Astrophysikalischen Observatoriums zu Potsdam,' No. 1, vol. 1, Part I; No. 5, vol. 2, Part I.

‡ 'Comptes Rendus,' *vol. 108, p. 486.*

In these curves each broad vertical line corresponds to the solar equator, and the scales to the right and left of each represent the north and south latitudes respectively. The heights of the curves above each horizontal zero line indicates the different amounts of spotted area, and the scales of these are so arranged that the curves are all proportional to the spotted area.

The curves themselves are formed by determining the mean spotted

FIG. 1.



DISTRIBUTION OF SUN'S SPOTTED AREA.

FROM 1879–1883.

area for each zone, and plotting each value at the point representing the mean latitude of this zone; these points are then all joined together. Thus, in the case of the 0—10° zone, the mean spotted area is plotted at 5°, 10—20°, at 15°, etc. The other zone divisions are similarly treated, thus, 0—5° is plotted at 2·5°, 0—3° at 1·5°, etc.

In the 10° zone curves here shown there is only one maximum *in* each hemisphere for the years in question, and these, as indicated by the dotted curves which join them, do not progress gradually towards

the equator, as would probably be the case according to Spörer's law. With 5° zones it is possible to detect the presence of two maxima in one or other of the hemispheres, all of which have a trend towards the equator in succeeding years. Still more detail is displayed in the 3° zones, and here is apparent a spot distribution and movement which is practically masked in the two preceding sets of curves.

The advisability of adopting 3° zones for the present investigation being thus apparent, the whole series of observations from the year 1861—1902 was treated in the above manner, the points plotted and the curves drawn as shown in the figure previously referred to.

This reduction was rendered comparatively easy by the fact that the Astronomer Royal has quite recently published* the values of the amount of sunspot area for each degree of latitude for each hemisphere from the year 1874—1902. For information previous to that date use was made of the detailed observations of the positions and areas of sunspots collected by Spörer in the publications already referred to, and curves for each year were drawn. For this period the curves employed were of a less degree of accuracy than those drawn from the Greenwich reduction, as the number of days of observation throughout a year was not so great.

Advantage was taken of the fact that Spörer's observations and those reduced at Greenwich overlapped during the four years 1874—1877, and a comparison of the curves from each series was rendered possible. The close similarity of these in each case showed that the reduction of Spörer's observations exhibited the chief features of the movements of centres of spot-activity as indicated by the Greenwich curves.

By thus employing Spörer's observations, curves for each of the 42 years from 1861—1902 inclusive were drawn in the manner described above, and these were placed vertically one under the other, like those shown in fig. 1 for the years 1879—1883.

In this way it was possible to trace the varying positions, as regards changes of latitude, of the centres of action, or maxima points of the curves, from year to year, just as was previously attempted in the case of the prominences. These centres of action were then connected by lines passing from one yearly curve to the next. It is worthy of remark that very little difficulty was met with in deciding the maxima points to be joined. There was always, throughout the whole period, a most distinct march of these points individually towards the equator, and the method of placing the curves one beneath the other rendered such movement at once obvious to the eye. There was only one instance where it seemed necessary that a march from lower to higher latitudes ought to be considered. This was in the southern hemisphere, in the years 1889 and 1890 (see Plate [2], Curve A). There

* '*Monthly Notices R. A. S.*,' vol. 63, pp. 452—461.

was a small indication of the presence of a spot centre of action in latitude 19° in the former year, while next year the position of the centre of action was in latitude 24°. Since this case was unique, it was considered advisable not to connect these points together, but to leave the centre of action in the year 1889 as an isolated point.

The diagram (fig. 1) not only exhibits some of the types of curves met with, but shows how the various centres of maximum spot-activity were joined up with each other, year by year, for the period of time over which the curves extend, namely, from 1879, the year following a sunspot minimum, to about a sunspot maximum in 1883.

Considering the curves relating to the sun's northern hemisphere, it will be seen that in 1879, the year following a sunspot minimum, when the spots were ending a cycle near the equator, two new outbreaks occurred in latitudes about 20° and 30°.

These two centres of activity moved towards the equator next year, and by 1881 the former had disappeared, while the other rapidly grew in intensity and reached latitude 15°. During this year a new outbreak in latitude 30° made its appearance, and this in the two following years had an equatorial trend.

A somewhat similar occurrence took place in the southern hemisphere, each of the centres of action moving rapidly towards the equator.

It is interesting to note the rapid growth and decay of these centres of action, an example of which is shown commencing in 1879 in latitude 28° in the northern hemisphere.

Attention may particularly be drawn to the three prominent maxima of the curves for the southern hemisphere in the years 1882 and 1883, which indicate that at this period there were three definite centres of spot action in existence.

In order to bring within a small compass the results of the above analysis for the whole period of investigation (the above-mentioned forty-two curves, although drawn close together on a small scale, cover a strip of paper 5 feet in length), a method was adopted similar to that employed in the case of the prominence reduction.*

In the accompanying plates the two sets of curves marked A indicate for each hemisphere the changes in the positions of these centres of spot activity from year to year plotted at equal intervals of a year. The striped portion is deduced from Spörer's observations, and the remainder from the Greenwich reductions. These lines have been proportionally thickened to indicate approximately the relative amount of spotted area at these centres of action, or, in other words, the heights of the maxima points on the yearly curves. These curves thus indicate for each year the positions, as regards latitude, of the particular zones

* 'Roy Soc. Proc.,' vol. 71, pl. 6.

in which the centres of spot activity occur, and give an idea of the movements of these centres during each sunspot cycle.

In this paper these curves have been called "spot-activity tracks," but it is important to point out that this term is not necessarily applied to the proper motion of any individual spot, but simply to the changes of position of the regions in which they are most numerous. As, therefore, the term "spot-activity tracks" represents the different positions of the regions of greatest spot-activity, so "prominence-activity tracks" may be employed to indicate the equivalent variations as regards the prominences which were shown in a previous paper.*

These "spot-activity tracks" have possibly a terrestrial equivalent in the variations from year to year of the positions of the "Zugstrassen," or cyclone tracks of Köppen, it having been found that cyclones in general, which move in the direction of the great mass of air carried by primary currents, have a strong tendency to pursue somewhat the same tracks according to the place of origin.

For the sake of comparison, curves B, C, and D in each plate have been added. Curves B show the variations of the mean heliographic latitude of the total spotted area for each hemisphere as determined and described in a previous paper.†

Curves C illustrate the distribution and changes of position of the centres of prominence activity. These curves are somewhat different to those previously published,‡ being so arranged that they form a continuous series from the year 1870. The small circles in the years 1870—1871 represent Respighi's observations, the curves from 1872—1881 those of Tacchini, and the remainder, up to the year 1902, Ricco and Mascari's observations. The dotted curves previous to 1870 are intended only to give a rough idea of the prominence variations based on a repetition of the observations of 1872—1885.§ The last curves, namely, those marked D in the plates, represent the variation from year to year of the total spotted area on each hemisphere of the sun, and special attention was drawn in a previous publication‖ to the great differences between the two hemispheres at the times of sunspot maxima. The vertical broken and continuous lines indicate the epochs of sunspot minima and maxima as determined by combining the amount of spotted area on both hemispheres of the sun.

Reverting now to the curves marked A, which form the special subject of the present paper, the following *general* deductions may be made :—

1. From sunspot minimum to minimum there are three, but generally

* 'Roy. Soc. Proc.,' vol. 71, p. 446.
† 'Roy. Soc. Proc.,' vol. 71, p. 449.
‡ 'Roy. Soc. Proc.,' vol. 71, pl. 6 and 7.
§ 'Monthly Notices R. A. S.,' vol. 63, No. 8, p. 484.
‖ 'Roy. Soc. Proc.,' vol. 71, p. 246, footnote.

four, distinct "spot-activity tracks," or loci of movements of the centres of action of spot disturbance.

2. The first appearance of each of these "spot-activity tracks" occurs generally between a sunspot minimum and the following maximum. After about the epoch of maximum generally no new "spot-activity tracks" of large magnitude are commenced.

3. Their first appearance is mostly in higher latitudes than 20° in each hemisphere.

4. They are faintly indicated at first, become more prominent and distinct, and finally thin out and fade away.

5. They all fade away in regions close to the equator.

6. There seems to be a tendency for each successive "spot-activity track" to make its appearance in latitudes higher than the one preceding it.

7. At, or a little after, the time of sunspot maximum there is also a tendency for each "spot-activity track" to retain its latitude for a short time.

In the light of these curves it is interesting to analyse those formed by plotting the mean yearly heliographic latitude of spotted area for each hemisphere: these are given in the accompanying plates (Curves B). These latter curves represent the drift from year to year of the mean heliographic spot latitude, and illustrate Spörer's "Law of Zones." It will be noticed that each commences in high latitudes about the time of sunspot minimum, and gradually approaches the equator until the epoch of the following minimum, when a new cycle in high latitudes recommences.*

An important point to be noticed about this series of curves is that each is of a *wavy* nature. This peculiarity is clearly shown even in the plate illustrating Spörer's reduction† which covers the period 1854—1880. There the curves, representing the mean spotted area and the mean heliographic spot latitude for both hemispheres combined, were compared, and Spörer pointed out some anomalies between these mean curves and those formed by joining the actual points of observation. So conspicuous were these anomalies at one epoch (even after many of the peculiarities had been lost by combining the two hemispheres instead of treating them singly), that he drew attention to them in the following words‡: "Nun sieht man an den Beobachtungspuncten, welche bei der Breitencurve eingetragen sind, dass die aus specieller Rechnung hervorgehende Curve *ebenfalls eine*

* These curves should overlap, but the unit of time, namely, the year, here employed, masks this feature.

† 'Publicationen des Astrophysikalischen Observatoriums zu Potsdam,' vol. 2. Erstes Stück, No. 5, Tafel 32.

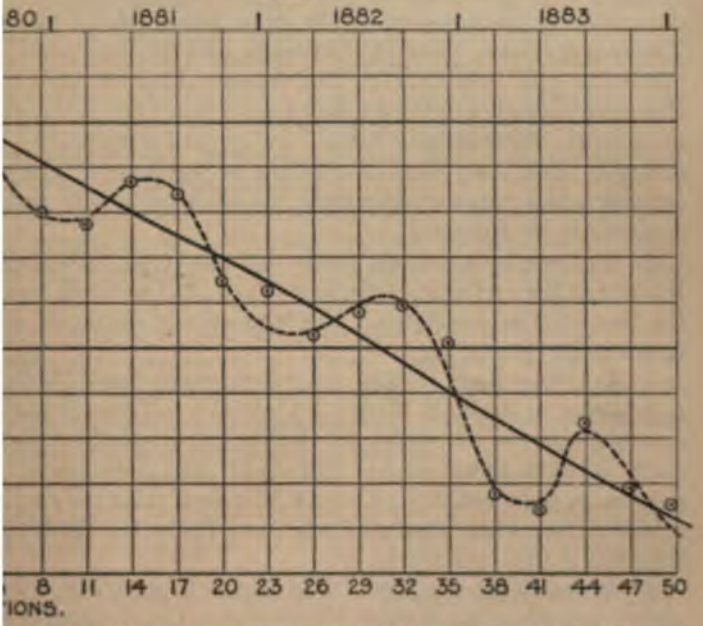
‡ 'Publicationen des Astrophysikalischen Observatoriums zu Potsdam,' vol. 2. Erstes Stück, No. 5, p. 81.

fern würde, und zwar hätte diese ihre grösste Erhebung gegen
s Jahres 1875." As will be seen from a further quotation
e same page, Spörer distinctly noticed subsidiary increases of
area and a reversion of spots to higher latitudes, and this drew
m the conclusion that the epoch of the following sunspot
n would be late.

rote (page 81), "die Ursache, welche eine Erhebung der
urve bewirkte, d. h. welche veranlasste, *dass in höheren Breiten
Flecke entstanden*, dadurch auch Veranlassung gewesen ist, dass
n Vermehrung der Flecke eintrat und damit auch das Flecken-
n für längere Zeit verzögert wurde."

, from the solar observations made at the Kalócsa Observatory
the years 1880—1884, both years inclusive, Dr. Braun*
in a graphic manner the progressive changes in the mean
phic latitude of the spots during this period, and drew attention
lifferences between the mean curve and that passing strictly
the points of observation (fig. 2). The mean curve he found

FIG. 2.



o illustrate for the years 1880—1883 the differences between the curve
ing the decrease of the mean heliographic latitude for the entire spotted
f both hemispheres taken together (continuous curve), and that obtained
less smoothed curve (dotted) passing through the points of the actual
vations.

ehte von dem Erzbischöflich Haynaldschen Observatorium zu Kalócsa in
von Carl Braun, S.J. Münster i. W., 1886.

to continually approach the equator, but superposed upon the line of uniform descent was a series of minor oscillations. In this connection he wrote:* "Die Spörer'sche Entdeckung der in jeder Periode auftretenden Annäherung der Flecken-Zone gegen den Aequator findet somit in unsern Beobachtungen eine unmisskennbare Bestätigung. Allerdings sind die Zeichnungen nicht durchaus von demselben Gehilfen ausgeführt worden, und dieser Umstand mag einigen Einfluss in Bezug auf diese Wahrnehmung haben . . . Sehr auffallend erscheint eine gewisse secundäre periodische Schwankung der mittleren Breite mit einer Periode von etwa 1 Jahr und einer Amplitude von reichlich 2 Graden . . . Diese mag wohl einem Zufall zuzuschrieben sein, welche nur während dieser vier Jahre obwaltete, vielleicht auch mit dem Wechsel der Zeichner in Zusammenhang steht. Doch wäre es immerhin von Interesse, dass diese Wahrnehmung durch die Discussion anderweitiger Beobachtungen geprüft und eventuell näher untersucht würde."

The present investigation seems to throw light on these peculiar changes of curvature shown by the mean heliographic latitude curves. These latter (Plates [1] and [2], Curves B) are individually really nothing more than the *integration* of the corresponding Curves A. Every change of curvature in Curves B is due to either the outburst of spots in another "spot-activity track" or by one "spot-activity track" becoming more intensified in relation to another, or lastly by the extinction of a "spot-activity track" as the equator has been reached as shown in the Curves A.

To illustrate this, let the curve for the mean heliographic spot latitude in the southern hemisphere (Plate [2], Curves B) beginning in the year 1879 be considered. This is practically the period referred to above by Dr. Braun.

At this time there is only one "spot-activity track" (latitude 22°) in existence, as shown in Plate [2], Curve A; so Curve B consequently commences in the same latitude. By the next year the "spot-activity track" (Curves A) has reached latitude 17°, and a new one has made its appearance in latitude 25°. Curve B, therefore, takes the mean position of about 20°, when allowance has been made for the difference of intensity of these two tracks.

In the following year 1881 both these "spot-activity tracks" have approached nearer the equator, but another has appeared in latitude 25°, so that the mean latitude for the whole hemisphere has only slightly changed.

By the year 1882 still another "spot-activity track" has come into existence in latitude 28°, while the first "spot-activity track" mentioned

* Ibid., p. 83.

above has vanished. The mean latitude for the whole hemisphere, as is indicated in Curve B for this epoch, is increased to latitude 20°. After this all three "spot-activity tracks" approach the equator and Curve B does the same, but owing to the relative changes in the amount of the spotted area in each of these "spot-activity tracks" as indicated by their thickness, the mean heliographic latitude curve suffers another change of curvature in 1885. In a similar way the various changes of curvature in all the other curves (Curve B) can be accounted for.

Particular attention has been drawn to the fact that about the times of sunspot maxima there is considerable spot activity in the highest spot latitudes, which according to Spörer's law would not be expected. The following extract expressing the impressions of Messrs. De La Rue, Stewart and Loewy on this point is therefore of interest, since it shows that such activity at the maximum of 1871 was even remarked as long ago as 1872.* "A striking feature of last year's observations seems to have been the occurrence of groups in comparatively high latitudes, especially in the southern solar hemisphere; a group observed between March 21 and 23 had the almost unprecedented high latitude of 43°, while latterly, towards the end of the year, several groups in almost as high a latitude have repeatedly made their appearance."

The Wilna observers also drew attention to the high latitudes of some spots about this maximum period, 1869—1871, as can be gathered from the following extract.†

"Generally speaking, during the last three years, about the last maximum, the spots were most distant from the equator; five spots were observed near latitude 38°; three about latitude 40—43°; one at 51½° latitude, . . ."

A word may finally be said as to the relationship between the curves representing the "spot-activity tracks" (Curves A), and those indicating the "prominence-activity tracks" (Curves C). As was pointed out in a previous communication‡ the general drift of the prominence activity is from low to high latitudes.

It is of interest here to note that from the time of a sunspot minimum when the "prominence-activity tracks" are approaching more rapidly high latitudes, up to about a sunspot maximum when they reach their highest positions, nearly all the "spot-activity tracks" come into existence. Further, the nearer the "prominence-activity tracks" approach the poles the higher in latitude do these "spot-activity tracks" also occur, and this is the case for each hemisphere of the sun separately.

* 'Monthly Notices R. A. S.,' 1872, vol. 32, p. 225.
† 'Report of the Committee on Solar Physics,' 1882, p. 155.
‡ 'Roy. Soc. Proc.,' *vol. 71, p. 452.*

What the actual connection between these two different sys
of currents is, it is not possible yet to say, but these facts su
a very close relationship.

In conclusion, I wish to express my thanks to Mr. T. F. Conr
computer in the Solar Physics Observatory, for his assistan
making the reductions and drawing the numerous curves.

Conclusions.

The result of the investigation leads to the following conclusion

1. Spörer's law of spot zones is only approximately true, and
only a very general idea of sunspot circulation.

2. Spörer's curves are the integrated result of two, three
sometimes four "spot-activity track" curves, each of the latter fa
nearly continuously in latitude.

3. Spörer's, and many other previous reductions have indi
the peculiar "wavy" nature of the integrated curve, which peculi
is here shown to be for the most part real and not due to erro
observation, etc.

4. Outbursts of spots in high latitudes are not restricted si
to the epochs at or about a sunspot minimum, but occur even
the time of sunspot maximum.

5. The successive commencement of the "spot-activity track
higher latitudes between a sunspot minimum and maximum s
to be closely related to the "prominence-activity tracks" at
periods.

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"The Spectra of Antarian Stars in Relation to the Fluted Spectrum of Titanium." By A. FOWLER, A.R.C.S., F.R.A.S., Assistant Professor of Physics at the Royal College of Science, South Kensington. Communicated by Professor H. L. CALLENDAR, F.R.S. Received February 18,—Read March 3, 1904.

The distinguishing feature of the spectra of the Antarian Stars* is the system of apparently dark flutings, sharp towards the violet and fading off towards the red end of the spectrum. The principal flutings are well seen in Antares, but they are more strongly developed in the spectra of α Herculis and o Ceti, in which stars additional details are also seen. These flutings have not hitherto received a definite chemical interpretation, and it has been uncertain, owing to the possibly misleading effects of contrast, whether the spectrum was to be regarded as one consisting wholly of absorption flutings fading towards the red, or as one partly consisting of emission flutings fading in the opposite direction.

The purpose of the present communication is to state the nature of the evidence which indicates that the spectrum is essentially an absorption spectrum, and that the chief substance concerned in the production of the flutings is titanium, or possibly a compound of that element with oxygen.

The first indication of this result was the striking general resemblance of the titanium flutings, as seen in photographs recently obtained, with the stellar flutings, both as to relative intensity and apparent position (Plate [3]). The interspaces between the flutings, as they appear on a negative, in some cases also strongly recall the corresponding bright spaces in the stellar spectra.

* Secchi's *Type III*; Vogel's Class IIIa.

The most extensive series of visual observations of the Antarian stars were made by Vogel* and Dunér† many years ago, and for the part of the spectrum extending from near D to the extreme red, no other measurements have yet been published. For the region more refrangible than D, however, wave-lengths derived from photographs are available, the most complete statements of these being due to Father Sidgreaves‡ and Mr. Stebbings.§ The individual results given by different observers vary considerably: visual observations are difficult, and, in the case of photographs taken with prismatic cameras, errors doubtless arise through the lack of suitable reference lines. There is also some difficulty in deciding where a fluting actually commences. The evidence in favour of a titanium origin for most of the flutings, however, depends on such a large number of coincidences that it is almost independent of a very precise knowledge of wave-lengths.

The flutings in question come out in the arc spectrum of titanium oxide, if the precaution be taken to provide a liberal supply of material and to use a very long arc, taking care also that the image of the "flame" is projected on the slit of the spectroscope. They are also seen in the arc spectrum of the chloride under similar conditions. Numerous lines accompany the flutings produced in this manner and some of the details are consequently masked or not recognized without careful study of the photographs.

So far the flutings have not been very successfully produced in the oxyhydrogen flame; they are visible in the flame spectrum of the fumes from the chloride, but their observation is difficult on account of the bright continuous spectrum.

The best representation of the flutings has been obtained by passing a spark, without jar, through the fumes of oxychloride which rise from the chloride of titanium on exposure to air. Under these circumstances the lines which appear are not numerous, and some of the secondary flutings which are masked by lines in the spectrum of the flame of the arc are readily detected, in spite of the continuous spectrum which is also present. The few lines which do appear in this spectrum are probably low temperature lines which may be found of special importance in the cooler stars.

Photographs have been taken over the region C to K, the instrument employed being one built up on the Littrow principle, having one prism of 60°,‖ and a 2-inch objective of 40 inches focal length, giving a linear dispersion from D to K of 5 inches. Wave-lengths were

* 'Beobachtungen zu Bothkamp,' vol. 1, p. 20, etc.

† 'Sur les Etoiles à Spectres de la Troisième Classe'; "K. Sven. Vet.-Akad. Hand.," vol. 21, No. 2, 1884.

‡ 'Monthly Notices, R.A.S.,' vol. 58, p. 344; vol. 59, p. 509.

§ 'Lick Observatory Circular,' No. 41, May, 1903.

‖ Lent by the Government Grant Committee.

determined in the usual manner by micrometric measurements of the photographs, using reference lines of titanium and iron, and calculating by the Cornu-Hartmann formula; though only provisional, they are probably not greatly in error.

It is instructive first to make a comparison between the more conspicuous flutings and those recorded visually in the stars by Vogel and Dunér. Details of the measurements are given in Table I, but reference should also be made to Plate 6, in which Dunér's drawing of the spectrum of α Herculis, as seen with a spectroscope of small dispersion, is compared with a negative of the titanium flutings, as they appear in the "arc" spectrum of titanium oxide.

Table I.—Comparison of Titanium Flutings with Visual Observations of the Spectra of Antarian Stars.

Titanium flutings.		Antarian flutings (more refrangible edge).	
Wave-length.	Visual intensity.	Wave-length.*	Dunér's number.
7055	10	..	Out of range.
..	..	6493	1
6162·5	10	6164	2
..	..	5862	3
5604·5	8	5596	4
5447·0	10	5453	5
5241·0	5	5243	6
5167·5	10	5169	7
4955·1	8	4960	8
4761·6	7	4769	9
4584·3	5	4608	10

It will be seen be seen that eight of the ten bands recorded by Vogel and Dunér agree within the possible limits of error with the flutings of titanium, and it is to be noted also that the only one of the principal titanium flutings which is not represented in the stellar spectrum is out of range in the extreme red. The origin of the two outstanding bands at 5862 and 6493 has not yet been ascertained. There are traces of titanium flutings near their positions, but they seem inadequate to account for two such distinct bands as those drawn

* The wave-lengths given are the means of Vogel's and Dunér's measurements, corrected to Rowland's scale (Scheiner's 'Astronomical Spectroscopy,' p. 301). For bands 5, 7, 8, 9, 10, the means of the wave-lengths derived from photographs by Lockyer, Pickering, Sidgreaves and Stebbings are respectively 5448, 5165, 4954, 4761, and 4584 (see *Table II*).

Table II.—Comparison of Titanium Flutings with Photographic Spectra of Antarian Stars.

Flutings of Ti (Fowler).*		Flutings in stars (Stebbings).		Flutings in *o* Ceti (Sidgreaves).		Remarks.
λ.	Intensity.	λ mean of five stars.†	*o* Ceti.	λ.	Intensity.	
			4313·4			
4352·9	3	..	4352·7	4352	7	
4395·2	3	..	4395·0	4395	5	Masked by lines in arc spectrum.
4422·0	2	4421·5	4421·5	4421	4	
4437·0	3	..	..	..	..	Masked in arc spectrum. Sidgreaves notes a "wide line" at 4436.
4461·9	4	..	4462·1	4460	6	Head well defined.
4505·0	4	..	4504·8	4504	6	Head not very well defined. Not recognised in arc spectrum.
4548·4	3	..	4547·9	4546	5	Head apparently strengthened by a line at 4548·9.
4584·3	6	4584·4	4584·1	4583	10	Beginning of Dunér's band 10, λ 4585 Lockyer,‡ 4586 Pickering.§ Head has the appearance of a double line.
4626·4	6	4626·3	4625·8	4625	8	Head well defined, and has the appearance of a double line.
4667·8	5	4667·6	4668·6	4669	9	
4714·5	3	4714·1	4713·2	4714	4	Head not sharply defined.
4738·0	2	4737·1	..	4736	3	Possibly only an ill-defined line which appears in "chloride" spark, but not in the arc.
4761·6	7	4761·0	4759·0	4758	10	Beginning of Dunér's band 9, λ 4763 Lockyer, 4762 Pickering. Head sharply defined, and has the appearance of a double line.
4805·6	7	4804·6	4803·2	4803	6	Head appears as a double line.
4842·0	1	..	4841·8	4842	8	A feeble brightening seen only in best photographs; it is not the Ti line λ 4841·0.
4848·6	5	4848·4	..	..	..	Head appears as a double line.
4894·0	2	..	..	4892	3	Head indefinite.

						sharply defined; a line at 4997·6 possibly forming part of a double head.
5002·8	4	..	..	4998	9	Head rather indefinite; not clearly distinguished in arc. Lines at 4981·9, 4991·2, 4999·7, 5007·4, 5014·4.
5050·5	2	..	..	5046	6	Head rather indefinite; not clearly distinguished in arc.
				5074	3	
				5098	4	
				5135	3	
5167·5	10	5165·8	5165·9	5162	10	Beginning of Dunér's band 7, λ 5165 Lockyer, 5168 Pickering. Head very sharply defined and perhaps double with 2nd head at 5169·8.
5241·0	5	..	..	5237	8	Beginning of Dunér's band 6. Begins with a line which is relatively stronger than in the arc spectrum.
5308·0	3	..	5307·2	5306	7	
5356·6	4	..	5357·7	5356	6	
5407·0	1	..	..	5406	2	
			5438·3			
5447·0	10	5446·8	5446·8	5447	10	Beginning of Dunér's band 5, λ 5455 Lockyer, 5445 Pickering. Head very sharply defined.
5497·5	5	5496·8	5496·9	5498	5	Head well defined.
5604·5	8	..	..	5597	8	Beginning of Dunér's band 4. There are strong lines near 5598, 5630, 5662, 5708, which, with small dispersion, might appear to be the heads of the adjacent flutings. Among other details within this group, Sidgreaves notes "bands" at 5603 and 5642, and a "wide line" at 5667.
5636·3	6					
5668·4	5	..	..	5660	7	
5713·9	3	..	..	5709	3	
5760·9	5	..	..	5756	8	Head well defined } Among the details in this region Sidgreaves notes "bands" at 5731, 5808, and 5845.
5811·0	5	..	..	5804	8	" "
5847·2	3	..	..	5840	8	" "

* As seen in spark without jar through fumes from titanium chloride; they are also seen in the arc unless otherwise stated.

† α Herculis, ρ Persei, β Pegasi, α Orionis, α Ceti.

‡ 'Phil. Trans.,' A, vol. 186, 1893, p. 702.

§ 'Annals Harv. Coll. Obs.,' vol. 28 Part 1, p. 8.

by Dunér. The association of vanadium with titanium in the spectra of sun spots suggested that they might be due to the former element, but this does not appear to be the case. The strongest fluting of vanadium is near 5472, and there is no certain evidence to show the presence of this fluting in the Antarian type of spectrum.

The evidence for titanium in the case of the remaining flutings, however, is enormously strengthened by a discussion of their structure and by extending the comparison further into the violet. Photographs of the stellar spectra, especially those of *o* Ceti and *α* Herculis, show that some of the principal flutings are composite, Dunér's band 10, for example, containing, according to Sidgreaves, four distinct flutings separated by intervals of about 44 tenth-metres, each of which is weaker than the one which precedes it on the more refrangible side. A precisely similar structure is found in the case of the titanium flutings, and a comparison of wave-lengths indicates that the various components occupy the same positions as those in the stars, so far as the available measurements permit the test to be applied. For this comparison (Table II) the wave-lengths derived from photographs by Father Sidgreaves and Mr. Stebbings are utilised. The relation may also be gathered by inspection of the reproductions of the photographs given in Plate [3], that of *o* Ceti having been very kindly placed at my disposal by Father Sidgreaves. Not all the details of the negatives, however, can be brought out in the reproductions, and the relative dispersions are not exactly the same.

It will be seen from the table that the details of the titanium flutings are reproduced with remarkable fidelity in the stellar spectra, and more especially in *o* Ceti. In the latter spectrum the number of flutings recorded is slightly greater than in the case of titanium, but it is by no means certain that every detail of the titanium spectrum has yet been photographed. It is possible also that some of the features described as flutings in the stellar spectrum may be groups of lines, and in at least one instance (4437) a fluting has been classed as a "wide line."

The points of difference are very slight, and are mostly in the less refrangible part of the spectrum, where the reductions of the stellar spectra present the greatest difficulty. There is a peculiar displacement of the fluting 4848 to λ 4842 in the spectrum of *o* Ceti, which may possibly be due to the superposition of a fluting or group of lines of undetermined origin; or, it may be that the feeble maximum observed at 4842 in titanium is strong enough in this spectrum to account for the apparent shift. There is also some uncertainty in connection with the complicated groups of flutings and lines extending from 5598 to D, which need further investigation in the stellar spectra with instruments of greater dispersion.

The general agreement is nevertheless such as to leave no reason-

(1) α Herculis (Dunér).
(3) ο Ceti (Stonyhurst).

(2) Titanium in flame of arc.
(4) Titanium in spark without jar.

able doubt that titanium is the main factor in the production of the dark flutings which characterise the Antarian group of stars.

This explanation of the dark flutings suggests that the appearance of bright flutings in the Antarian spectrum arises chiefly from effects of contrast. It does not, of course, exclude the possibility of the presence of bright flutings, such as might be indicated by local brightenings which are not exactly in coincidence with the edges of dark flutings.

Whether the absorption flutings are produced by the vapour of titanium or by that of the oxide has not yet been completely determined. As already pointed out, the flutings may be obtained either from the oxide or the chloride, but as the latter so readily unites with oxygen on exposure to air, it furnishes no evidence against the supposition that the flutings are due to the oxide.

The author has pleasure in acknowledging the very able assistance which has been rendered in the experimental work involved in this investigation by Mr. F. W. Jordan, B.Sc., Teacher in Training in the Department of Astronomical Physics, Royal College of Science.

DESCRIPTION OF PLATE [3].

1. Visible spectrum of α Herculis, as drawn by Dunér.
2. Flutings of titanium, as they appear in the spectrum of the "flame of the arc," when charged with titanium oxide.
3. Photographic spectrum of *o* Ceti, from a photograph taken at the Stonyhurst College Observatory, November 29, 1897.
4. Flutings of titanium, as they appear when a spark, without jar, is passed through the fumes which rise from titanium chloride on exposure to air.

(*Note.*—The coincidences cannot be very exactly shown in this manner, on account of the differences of dispersion of the three instruments with which the spectra were recorded.)

"Further Researches on the Temperature Classification of Stars." By Sir NORMAN LOCKYER, K.C.B., LL.D., F.R.S. Received January 30,—Read February 18, 1904.

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1. *Historical Review.*

In my first Bakerian Lecture in 1873 I dealt with the question of the spectra of stars, and pointed out that the facts accumulated up to that time by Rutherford and others led to the view that in the reversing layers of the sun and stars various degrees of dissociation are at work.

I also suggested that the stellar evidence indicated that one of the results of dissociation temperatures could be to "prevent the coming together of atoms which at the temperature of the earth, and at all artificial temperatures yet obtained here, compose the metals, the metalloids, and compounds."*

In a subsequent communication to the Paris Academy I wrote,† "Il semble que plus une étoile est chaude plus son spectre est simple, et que les éléments métalliques se font voir dans l'ordre de leurs poids atomiques."

This last generalisation rested upon the great preponderance of hydrogen in certain stars, which I classed as hottest on the ground that the blue end of the spectrum was open. Of the spectrum of helium,

* 'Phil. Trans.,' vol. 164, p. 479.

† 'Comptes Rendus,' vol. 77, 1873, p. 1357.

which I had first observed and named in 1869, we then knew nothing either on the earth or in the stars; the solar line D_3 being its only representative. The question of the relative temperatures of stars became of great importance in relation to the questions thus raised, but it was not till 1892 that I was able to approach it by means of photography. The interval was spent chiefly in researches bearing upon solar and terrestrial changes in spectra when differences of thermal and electric energies were obvious.

In a paper on stellar spectra in relation to temperature, communicated to the Royal Society in 1902,* I gave an account of an attempt at a temperature classification of stars, utilising the fact that an extension of spectra into the ultra-violet is produced by increased temperature, and further that a lower temperature in an atmosphere above a photosphere would increase the absorption in the blue end. The classification arrived at was based on photographs obtained with instruments having prisms and lenses made of glass which has a strong absorbing effect on the ultra-violet rays.

The general results of the discussion was the conclusion that the stars so far considered might be divided into two series, one of ascending, the other of descending temperature. Further, that the classification proposed was justified both by the relative extensions of the spectra into the ultra-violet and by the temperature sequence of the few typical lines then available for study.

By 1899, laboratory work on the spectra of different substances under different conditions, and the discovery of a terrestrial source of helium by Ramsay, which enabled me to investigate the complete spectrum, had so far facilitated the study of the typical lines in the various stellar spectra, that I felt myself justified in attempting to classify the stars in relation to the chemical sequence revealed by the presence of gaseous and metallic lines, using especially the lines of helium and hydrogen and the "enhanced" and arc lines of the metals. In this way I hoped to be able to test the classification of 1892 based on the relative lengths of spectra.

An account of this research was published in the Proceedings of the Royal Society (vol. 65, pp. 186—191, 1899), and ultimately the complete results obtained were included in a "Catalogue of 470 Brighter Stars Classified According to their Chemistry."†

In this catalogue the stars were arranged in sixteen groups along a temperature curve with its apex in the central portion. On the assumption that the chemical changes were due to temperature, including in that term the possible results of electrical energy, the general arrangement of the stellar groups in the order both of ascending and descending temperatures was indicated, the group

* 'Phil. Trans.,' A, vol. 184, p. 688.

† 'Publications of the *Committee* on Solar Physics,' London, 1902.

containing the hottest stars, on the dissociation hypothesis, being placed at the top of the curve. It will be convenient to reproduce this table here.

10		Argonian	
9		Alnitamian	
		Protohydrogen Stars	
8	Crucian	Cleveite-gas Stars	Achernian
7	Taurian		Algolian
6	Rigelian		Markabian
5	Cygnian	Proto-metallic Stars	—
4	—		Sirian
3	Polarian		Procyonian
2	Aldebarian	Metallic Stars	Arcturian
1	Antarian	Stars with fluted spectra	Piscian

There was abundant chemical evidence to show that the mean temperature of the stars occupying the same height on both branches of the curve was not very different, so that we have ten horizons, or stages, of mean temperature indicated in passing from the complete fluted spectra of Antarian-Piscian stars to the simplified line spectra of the γ Argus type.

So far as we could judge from the photographs then available, the chemical changes gave a sequence identical with that desired for the length of ultra-violet spectra in 1892, so that the latter classification was fully justified by the test which had been applied to it.

2. *Aims and Conditions of the Present Research.*

As before mentioned in the work of 1892, the relative lengths of the ultra-violet spectra were determined from photographs secured with instruments having glass optical parts. It was possible therefore to secure still another test by obtaining a new set of photographs using calcite and quartz in place of glass, to enable the far ultra-violet to be obtained and studied. Nor was this all, it seemed of the first importance to utilise not only the length of spectrum in the blue, but the relative brightness of the different parts. What happens regarding the relative brilliancy of the different parts of the spectrum, as well as the extension into the ultra-violet by increased temperature, was very

clearly stated by Sir George Stokes* in 1876, in the following words, "When a solid body such as a platinum wire, traversed by a voltaic current, is heated to incandescence, we know that as the temperature increases, not only does the radiation of each particular refrangibility absolutely increase, but the proportion of the radiations of the different refrangibilities is changed, the proportion of the higher to the lower increasing with the temperature."

This question was also investigated by Melloni and Crova; and in recent years exact determinations of the law of increase have been made by Lummer, Paschen, and others. Melloni showed† experimentally that the maximum radiation moved towards the more refrangible end of the spectrum as the temperature increased. Crova made use of this fact in determining the temperatures of various incandescent light sources, and was one of the first to suggest‡ that the method was applicable to the determination of the temperatures of the sun and stars.

3. *The Observational Conditions.*

The kind of spectroscope to be used and method of observation to be followed were indicated by the following considerations.

In order to utilise the effect of temperature changes to the full it was necessary to record the red end of the spectrum as well as the ultra-violet, as only in this way could the *relative* changes in intensity be recorded, *hence it was desirable to employ only a small dispersion.*

In addition to the natural differences photographed in the ultra-violet, artificial differences due to the absorbing effect of our atmosphere—which, even when clearest, is more or less opaque to the ultra-violet radiations—might be introduced, therefore it was considered advisable, in order to eliminate the effects of atmospheric absorption, to obtain the spectra of any two stars to be compared *whilst they were at approximately the same altitude.* Further, to avoid the many pit-falls to which those who compare photographs taken on plates of unequal sensitiveness, and differently exposed and developed, are liable, it was obviously important that any spectra to be compared should be obtained on the same plate in order to secure identical plate sensitiveness and development.

Again, in order to secure similar optical treatment it was arranged to photograph both spectra near to the optical axis of the camera; thus they are near together in the centre of the plate.

I am sorry to say this work has been considerably delayed by the long time taken in preparing a new camera and optical parts suitable for the research, as above defined, and latterly by a long spell of bad observing weather.

* 'Roy. Soc. Proc.,' vol. 24, p. 353, 1876.
† Taylor's 'Scientific Memoirs,' vol. 1, p. 56.
‡ '*Comptes Rendus*,' vol. 87, p. 981.

4. *Description of the Instrument used.*

With regard to the new apparatus, I may state that an objective prism camera (fig. 1), was devised having a 2-inch 30° calcite prism mounted in front of a 2½-inch quartz lens of 18 inches focal length.

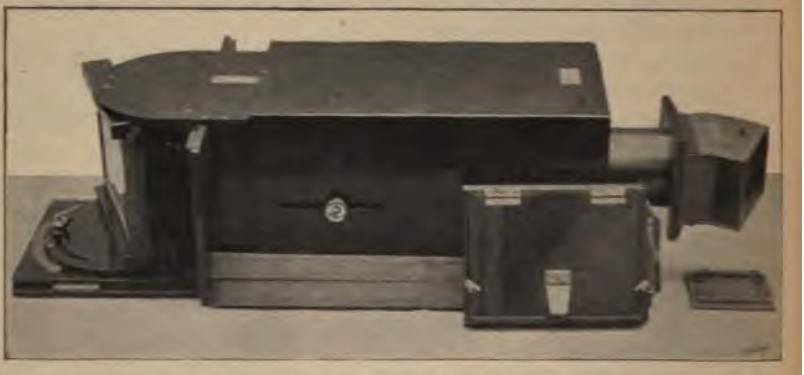

FIG. 1.—Quartz-Calcite Prismatic Camera.

The prism is so cut that its first face is perpendicular to the optic axis of the crystal, and it is arranged that the incident rays are normal to this face. All the rays, therefore, pass through the prism parallel to the optic axis and in this way there is no double refraction.

The whole length of the spectrum is brought into focus in a plane inclined at 42° to the optical axis. The apparatus is attached to the side of an equatorially mounted 6-inch Dallmeyer doublet camera with an angle in declination between the two optical axes equal to the angle of deviation of the calcite prism, so that the Dallmeyer is used as a finder.

Edward's snap-shot isochromatic plates have alone been used, and as they are not sufficiently sensitive for wave-lengths between λ 486 and λ 550 (approx.), there is a break in each spectrum about this region followed by a further portion of the spectrum having its centre about "D." The length of spectrum obtained is such that H_β to H_ϵ is 0·355 inch (9·0 mm.) and from H_ϵ to H_γ 0·165 inch (4·2 mm.).

5. *Method of Work.*

As shown in Table I, there are ten stages or steps of temperature to be investigated, and pairs of stars were selected in order to show the differences between adjacent stages or else the difference between stages widely apart.

Stages 1 and 10 are not available in consequence of the photographic faintness of the Antarian and Piscian stars, and the fact that the only

star as yet known representing the Argonian group is situated in the Southern Hemisphere. The stages already investigated are shown by brackets in the following table.

Table I.

10.		Argonian.	
9.		Alnitamian.	
8.	Crucian.	Achernian.	3.
7.	Taurian.	Algolian.	
6.	Rigelian.	Markabian.	5. 7.
5.	Cygnian.	———	2. 6.
4.	———	Sirian.	4.
3.	Polarian.	Procyonian.	1.
2.	Aldebarian.	Arcturian.	
1.	Antarian.	Piscian.	

Thus an Arcturian star (second stage), was compared with a Sirian (fourth stage), and so on. In the case of some stars, *e.g.*, Capella, this was repeated several times, each photograph showing a different comparison.

In securing the photographs an attempt was always made to obtain the pair of spectra with the region between H_β and H_γ of the same intensity in each. This condition is very difficult to fulfil in actual practice owing to the difference of magnitude of the two stars, their difference in declination, and hence in clock rate, and lastly the very important actual differences between the actinism of the two bodies in this region of the spectrum due to their different temperature conditions.

It was also arranged to obtain, whenever possible, the spectra of two stars near together, so that the chances of introducing atmospheric interference due to the different conditions possibly present in various parts of the sky might be reduced to a minimum. In every case where the observer had reason to suspect any change in the atmospheric conditions obtaining during the two exposures, which sometimes extended over a period of $1\frac{1}{2}$ hours, the result has not been included in the discussion.

Thus we have a series of comparison photographs from which all variable conditions except the natural variations in radiation have, as far as possible, been eliminated.

6. *Description of the Photographs.*

Many of the photographs taken are too faint to reproduce well enough to exhibit completely the striking differences shown between the pairs of spectra. I shall therefore content myself by giving a detailed description of each negative (Table II), only reproducing those photographs which are sufficiently dense to plainly show the phenomena under discussion.

Table II.

No. of negative.	Date.	Star.	Stage of temperature.	Type.	Altitude.	Description.
		From Stage 2 to Stage 4.				
19	7.5.03	α Lyræ	4	Sir.	56°	Faintly red, fairly bright to H_γ.
		α Boötis	2	Arct.	57	Intense red, scarcely extends beyond K.
4	29.12.02	β Ursæ Maj.	4	Sir.	59	Red scarcely visible, maximum about λ 400.
		α Ursæ Maj.	2	Arct.	58	Fairly bright red, maximum about λ 460.
14	17.2.03	α Geminorum	4	Sir.	42	Red scarcely visible, maximum about λ 416.
		α Aurigæ	2	Arct.	39	Fairly bright red, maximum about λ 455.
37	21.1.04	α Geminorum	4	Sir.	68	Weak red, maximum about λ 420.
		α Aurigæ	2	Arct.	68	Very dense red; blue maximum commences at F, centre at λ 460, weak beyond λ 388.
		From Stage 4 to Stage 6.				
11	28.1.03	β Orionis	6	Rig.	21	Red not so intense, and ultra-violet much more extended than in Sirius.
		α Canis Maj.	4	Sir.	22	Dense red, centre of maximum about λ 422.

Table II—*continued.*

of ive.	Date.	Star.	Stage of temperature.	Type.	Altitude.	Description.
		From Stage 6 to Stage 9.				
		κ Orionis	9	Alnit.	20	Although spectrum of Rigel is generally much stronger, that of Alnitam extends as far into the ultra-violet.
		β Orionis	6	Rig.	24	
		Various Intervals.				
	10.12.03	γ Ursæ Maj.	6	Mark.	78	Faint red, maximum about λ 416, well sustained up to Hε
		α Ursæ Maj.	2	Arct.	76	Fairly bright red, maximum at λ 460, rapidly falling off beyond λ 430.
	14.1.04	η Ursæ Maj.	8	Cruc.	51	Faint red, maximum at about λ 418, bright extension well beyond the end of the Capella spectrum
		α Aurigæ	2	Arct.	52	Very bright red, maximum at about λ 455, faint beyond K.
	14.11.03	ε Orionis	9	Alnit	38	Faint red, centre of maximum at about Hδ. Bright extension far beyond the hydrogen series.
		α Ursæ Min.	3	Proc.	41	Very bright red, maximum about Hγ, and falls quickly beyond Hε
		Extreme Interval.				
	10.12.03	ε Orionis	9	Alnit.	36	Very faint red, centre of maximum about Hδ, i.e., near the more refrangible end of the Aldebaran spectrum.
		α Tauri	2	Aldeb	36	Very strong red, centre of maximum about λ 465, end of spectrum about Hε

7. *Discussion of Photographs.*

The following is a detailed discussion of the photographs described in Table II.

Stage 2 *to Stage* 4.

No 19. This photograph of Vega (fourth stage), and Arcturus (second stage), is very striking in the relative intensities of the two spectra. The red portion of the Arcturian spectrum is considerably more intense and forms the one end of a maximum which—except in the green region where the plate is not very sensitive—extends from D to about λ 454. The part of the spectrum more refrangible than that rapidly becomes less intense, until at and beyond K it is very faint. Matters are very different in the spectrum of Vega. Here the maximum radiation occurs about a third of the distance from H_δ to H_γ (λ 422) and the spectrum extends without any great falling off in intensity to H_ν, beyond that it is weaker but continues without any great decrease to twice the distance on the more refrangible side of H_ν than the latter is from K. From that point to the end the spectrum gradually declines. Whilst the maximum radiation in this spectrum has moved wholesale towards the ultra-violet, the red is relatively only about half the density of the red in the Arcturian spectrum.

No. 4. The general appearance of these two spectra leads to the conclusion that that of β Ursæ Majoris (fourth stage) is much stronger than that of α Ursæ Majoris (second stage).

A more careful examination, however, shows that whilst the detached red part of the former is only seen with difficulty, the same portion of the latter is comparatively prominent.

The maximum intensity in α is situated at about λ 450 and does not vary a great deal between there and K. After the latter point is reached, however, the fall is rather sudden, and the spectrum soon dies out. In β the maximum intensity is between H and H_ϵ, and the ultra-violet up to H_ϵ is fairly strong. Beyond H_ϵ the intensity of the spectrum drops rather suddenly, and then continues with a gradual decrease for some distance.

No. 14. There is no great difference in the lengths of these two spectra, the one of α Geminorum (fourth stage), the other α Aurigæ (second stage), but the red portion of the second stage spectrum is most decidedly more intense than that of the fourth stage, the latter, in fact, being scarcely visible at all. In α Aurigæ the maximum intensity is at about λ 454, but in α Geminorum it must be placed at or near to H_δ (λ 410).

No. 37. In the taking of this negative the spectrum of Capella (second stage) received the advantage of exposure and is consequently much stronger in the H_γ—H_β region than that of Castor (fourth

stage), and yet the latter extends as far into the ultra-violet as the former.

Furthermore, the intensity of the blue part of the spectrum of Capella rises to its maximum immediately at H_β and commences to decline towards the violet at H_γ, whereas the maximum region of Castor does not commence at once after leaving the green gap, and attains its centre at about λ 422.

It will be observed that whether we take Arcturus or Capella to represent Stage 2, the spectra of stars of higher stages have relatively longer ultra-violet and reduced red radiation. It has to be noted, however, that there are indications that Stage 2 will, as a result of further work, have to be divided, for Capella is certainly hotter than Arcturus as determined in the manner now under discussion.

Stage 4 *to Stage* 6.

No. 11. On examination of this negative it is seen that the detached red portion of the spectrum of Sirius (fourth stage) is decidedly more intense than the same portion of the Rigelian spectrum (sixth stage). In the ultra-violet, however, we find that although both stars are fairly high on the temperature curve, and, therefore, both spectra extend far into the ultra-violet, the extension of the spectrum of Rigel is more intense and greater than that of Sirius.

Stage 6 *to Stage* 9.

No. 35. By reason of its greater exposure the spectrum of Rigel (sixth stage) is generally much stronger than that of κ Orionis (ninth stage), and especially is this so in the red portions of the two spectra. This inequality notwithstanding, the spectrum of κ extends practically as far into the ultra-violet as that of Rigel.

Various Intervals.

No. 34. In α Ursæ (second stage) the red part of the spectrum is comparatively very bright, nearly as bright as the region between G and F. In γ Ursæ (sixth stage) this red portion is barely visible. Again, in the α Ursæ spectrum the maximum occurs at λ 460, and then the intensity gradually declines to K, beyond which it is very faint. The maximum intensity of the γ Ursæ spectrum is situated at about λ 422, and it extends without becoming greatly impaired to H_ϵ.

No. 36. In this comparison we have a very striking case. The spectrum of Capella (second stage) is compared with that of η Ursæ Majoris (eighth stage); Capella has been over-exposed, so that the red portion is abnormally intense, η Ursæ Majoris received the correct exposure, and the *red part* of the spectrum is rather faint.

Notwithstanding this difference the spectrum of η Ursæ extends further into the ultra-violet than does that of Capella, and not only does it extend further, but the maximum intensity is extended much further into the ultra-violet than is that of Capella, which drops off rapidly beyond K.

No. 23. In this comparison the red part of the Procyon (third stage) spectrum is much brighter than that of Alnitam (ninth stage). The intensity in the longer portion of the spectrum of Procyon rises at once to its maximum at H_β, has its centre of maximum at about λ 460, and at H_γ commences to diminish towards the violet. In the spectrum of Alnitam, however, the maximum is delayed until the region about λ 426 is reached, and is then sustained up to H_ζ, finally extending to the ultra-violet with a marked superiority, comparatively, over that of the Procyon spectrum.

Extreme Interval.

No. 33. This comparison of two type stars respectively situated near the extremities of the temperature curve is naturally one of the most striking pieces of evidence in support of this method of temperature classification. The spectrum of Alnitam (ninth stage) is nowhere so intense as the red and blue parts of the Aldebaran (second stage) spectrum, and yet it extends more than twice as far towards the ultra-violet, from H_β, as the hydrogen series, whilst the more refrangible limit of the Aldebaran spectrum only extends to K. The maximum intensity of the blue portion of the Alnitam spectrum occurs much nearer the ultra-violet than that of the Aldebaran spectrum, the latter attaining its maximum at about λ 465.

8. *Conclusions.*

It may be pointed out that the temperature classification, confirmed by this research, does not agree with that published by Sir William and Lady Huggins who, in their "Atlas of Representative Stellar Spectra" containing "a Discussion of the Evolutional Order of the Stars," place the solar stars on a higher temperature level than the white stars.

A reduction of intensity in the continuous spectrum beyond the hydrogen series to which attention has been called by more than one observer, Schumann* among others, does not affect the results which I have stated. Another paper dealing with this and similar po in course of preparation.

The result of the research may be stated as follows :—

Taking the stars assumed to be hottest in the chemical classifica we find that in all cases the relative length of the spectrum is redu

* 'Smithsonian Contributions to Knowledge,' No. 1413, 1903, p. 23.

VEGA
ARCTURUS
STAGE 2
STAGE 4
CASTOR
CAPELLA
STAGE 2
STAGE 4
CASTOR
CAPELLA
STAGE 2

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ASTOR, LENOX AND
TILDEN FOUNDATIONS.

STAGE 6
RIGEL

SIRIUS
STAGE 4

STAGE 9
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STAGE 6
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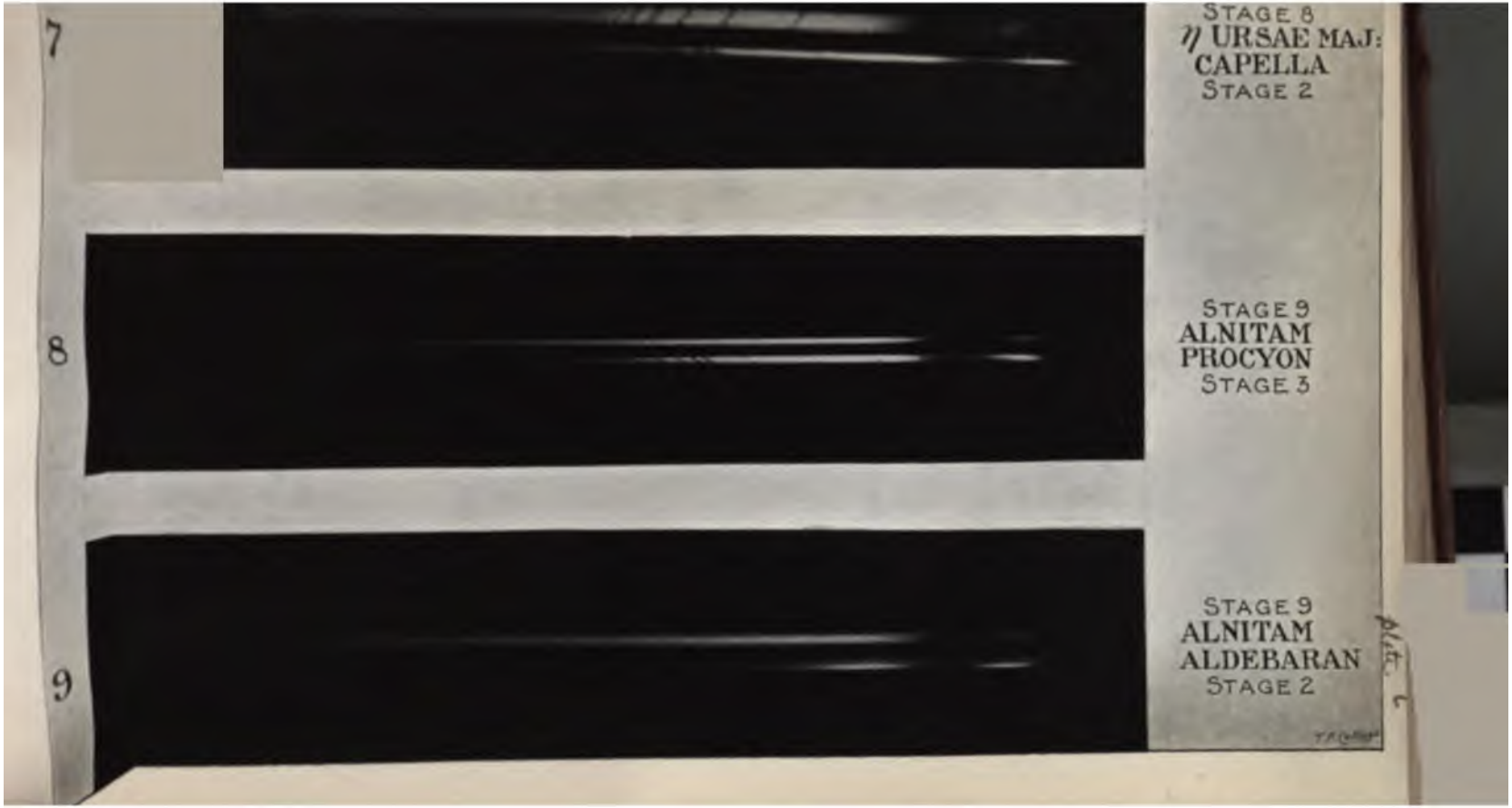
7
STAGE 8
η URSAE MAJ:
CAPELLA
STAGE 2
8
STAGE 9
ALNITAM
PROCYON
STAGE 3
9
STAGE 9
ALNITAM
ALDEBARAN
STAGE 2
Plate C

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and the relative intensity of the red is increased, as a lower temperature is reached. That is to say that where two spectra having their intensities about the region H_δ — H_γ equal are compared, we find that in the cooler stars, according to the chemical classification, the emissions in the red preponderate, whilst in the hotter star the ultra-violet is more extended and intense.

My best thanks are due to Messrs. Rolston and Goodson, who took the various photographs to which I have referred, the former also helping me in the preparation of this paper, and to Mr. Wilkie for preparing the enlargements of the negatives.

9. *Description of Plates.*

No.	No. of negative.	Stars.	Stage.	Type.
		Plate [4].		
1	19	Vega	4	Sir.
		Arcturus	2	Arct.
2	14	Castor	4	Sir
		Capella	2	Arct.
4	37	Castor	4	Sir.
		Capella	2	Arct.
		Plate [5].		
1	11	Rigel	6	Rig.
		Sirius	4	Sir.
5	35	κ Orionis.....	9	Alnit.
		Rigel.....	6	Rig.
6	34	γ Ursæ Maj. ..	6	Mark.
		α Ursæ Maj. ..	2	Arct.
		Plate [6].		
7	36	η Ursæ Maj. ..	8	Cruc.
		Capella.	2	Arct.
8	23	Alnitam	9	Alnit.
		Procyon	3	Proc.
9	33	Alnitam	9	Alnit.
		Aldebaran	2	Aldeb.

In producing these plates the original negatives have been enlarged about 3½ times.

HARRISON AND SONS, *Printers in Ordinary to His Majesty*, St. Martin's Lane.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXIV. MAY 13, 1904. No. 7

Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

Zia Uddin Ahmad, D.Sc., Trinity College, Cambridge;
Daniel Buckney, 61 Strand, W.C., and 51 Highbury Park, N.; and
Robert James Wallace, Yerkes Observatory, Williams Bay, Wis., U.S.A.

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:

Major C. F. Close, C.M.G., R.E., Brompton Barracks, Chatham (proposed by Sir David Gill); and
Thomas Andrew Common, 63 Eaton Rise, Ealing, W. (proposed by W. H. M. Christie).

Eighty presents were announced as having been received since the last meeting, including, amongst others:

S. C. Chandler, Revision of Elements of third Catalogue of Variable Stars (presented by the author); Royal Observatory, Greenwich, Astrographic Catalogue, vol. i. (presented by the Observatory); Wislicenus, W. F., Astronomischer Jahresbericht, 1903 (presented by the author).

Note on the Instrumental Errors affecting Observations of the Moon. By F. W. Dyson, M.A., F.R.S.

1. In connection with Mr. Cowell's comparison of the observed positions of the Moon with the tabular positions Professor Turner contributed a note to the *Monthly Notices* of 1904 March in which he urged the necessity for more elaborate discussion of the instrumental errors to which observations of the Moon are liable, and suggested the possible utility of the observations made with the old altazimuth at Greenwich for this purpose. My reason for offering any remarks on the subject is that I have had opportunities of discussing the matter with Mr. Cowell and may be able to remove some misconceptions.

The ordinary sources of systematic errors in star catalogues such as a period in the adopted clock-star list, the adopted value of the equinox correction, flexure, the adopted system of refractions, &c., are of secondary importance in the discussion of short-period terms in the Moon's tables.

(a) Because the Moon moves so rapidly in both right ascension and declination.

(b) Because they are largely overwhelmed (a) by the personal errors in observations of limbs, (β) the errors arising from observations being sometimes made in a bright and sometimes in a dark sky, and (γ) the large accidental error arising from the limb being sometimes ill-defined and hazy.

2. The errors in transit-circle observations which are not peculiar to the Moon being capable of independent determination should be allowed for wherever their effect is likely to be appreciable. They will very rarely affect the terms of short period, but may be of importance in the terms of longer period.

3. Professor Turner's note as far as it criticised Mr. Cowell's procedure had reference to the errors peculiar to the Moon marked (b) in § 2. The errors of this class largely increase the accidental errors of an observation of the Moon over that of a star. Their effect will, however, be of an entirely accidental character on the coefficients of all short-period terms except those whose period is a lunation or an exact part of a lunation. Certainly in the long series of observations from 1750 to 1901 which Mr. Cowell is discussing we may expect them to be entirely eliminated.

4. Errors of this class have, as Mr. Cowell has been careful to note (*Monthly Notices*, lxiv. p. 94), a systematic effect on the parallactic inequality and on the variation. Mr. Cowell proposes to determine the coefficient of sin D and sin 2D separately for

each period of 400 days from 1750–1901, and to obtain from the accordance of the separate determinations a measure of their accuracy. In this long period there have been a good many observers and several instruments at Greenwich. Errors of a personal character which are systematic in a short series of observations may be fairly considered as accidental in a long one extending from 1750–1901. The detailed examination of these personal errors would probably increase the accordance of the results derived from the separate periods but not seriously affect the mean. It should be noted that such a discussion would not be possible or would, at least, be very rough till the tabular errors which Mr. Cowell is considering are allowed for. At the best personal errors are troublesome to investigate, and the results never free from the uncertainty that observers may have slowly changed their habits of observing.

5. Mr. Cowell's method of giving the coefficients as determined separately for each period of 400 days gives a ready method of comparing the results as given by the different instruments used since 1750. If discrepancies are shown larger than can be legitimately attributed to accidental error they will be put clearly in evidence and their amounts will be known. But there are, as Mr. Cowell remarks, no means available for determining the instrumental correction which may be required by any particular instrument to the Moon's diameter at different ages and different conditions. The agreement of the observed with the theoretical variation and of the solar parallax obtained from the observations 1847–1901 with values obtained in other ways shows that the errors of this class are not very large for the present transit circle, and affords ground for supposing that their effect on other terms, except the parallactic inequality, is extremely small.

6. With regard to the altazimuth observations it is no doubt true that the systematic errors to which they are liable are probably different from those inherent in meridian observations. Unfortunately they have a large accidental error, and quite probably large systematic errors, depending directly on the Moon's altitude, and thus indirectly on its age. The comparison of the observed with the tabular places might be analysed. This would have the advantage over Professor Turner's comparison with the transit circle that all the observations could be used. Whether the results would repay the labour is, I think, a very open question.

7. Professor Turner uses the altazimuth observations as a standard to compare the old transit circle in use from 1847–1851 with the present one, 1851–1861. This comparison is, it seems to me, made better *via* the more uniform tabular places, and can be taken from Mr. Cowell's paper (*Monthly Notices*, vol. lxiv. p. 526, col. 6). The periods 86, 87, 88 in Mr. Cowell's paper correspond to the years 1847–1851, and the periods 90–99 to the years 1851–1861. I extract the figures for these periods.

Period.	Obs.–Calc. of no. 2.	Period.	Obs.–Calc. of no. 2.	Period.	Obs.–Calc. of no. 2.
86	+0″2	90	−0″5	95	−0″9
87	−0·1	91	+0·5	96	−0·2
88	−0·1	92	−0·8	97	−0·5
		93	0·0	98	−0·9
		94	−0·5	99	−0·1

The mean for the whole period 1847–1911 is 0″·0. It does not seem to me that these figures give any evidence—though their weight is small—of a systematic difference between the observations with the old transit circle from 1847–1851 and those with the present instrument.

The negative values for periods 94-98 suggest some systematic error of the transit-circle observations during this period, but viewed with the whole series of periods to 1911 it is quite possible that the agreement of sign for these years is accidental.

8. The figures which Professor Turner gives (Table V., p. 422) show that a very liberal allowance must be made for accidental error. The figures he gives are as follows:

Day.	Old.	New.	Day.	Old.	New.
16	0″0	+0″1	21	−0″7	−0″6
17	−0·1	−0·1	22	−0·2	+0·2
18	+0·1	0·0	23	+0·7	+0·8
19	−0·2	+0·4	24	+0·1	+0·8
20	−0·1	+0·2	25	−0·5	+0·2

The figures in the column "Old" seem wholly accidental, although except for day 25 each of these figures is derived from the differences of from 25 to 50 observations made with the two instruments. The systematic character of the figures in the column "New," as far as it is caused by the transit circle, seems to agree with the error noted in § 7 as existing in Mr. Cowell's periods 94-98, and in that case would not occur in a comparison extending over a longer period.

9. In conclusion, although the value Mr. Cowell gave for the solar parallax seems to me to be true within the limits he assigned, I do not think any discussion of the errors of the observations will materially narrow the limits. Observations of the Moon, whether meridian, altazimuth, or occultations, will always have the "limb" difficulty.

The agreement between the value he finds and the value obtained in other ways seems to me to show that in a long series of observations the errors largely balance one another, and that as the parallactic inequality and the variation are the terms which are affected systematically, the accuracy of the determination of short-period terms not so affected will be very great.

Comments on Mr. Dyson's "Note on the Instrumental Errors affecting Observations of the Moon." By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

(*a*) Mr. Dyson courteously submitted the preceding paper to me in MS., and I offer the following comments upon it which it may be convenient to have in the same number as the paper itself.

(*b*) Paragraphs 1, 2, 3 of his note call for no remark from me.

(*c*) In § 4 our views begin to diverge, but only in the phrase "to obtain from the accordance of the separate determinations a measure of their accuracy." The danger of this test is well known even when all conceivable systematic errors have been removed. When there is an obvious source of systematic error still affecting the observations, I doubt whether the test can be applied. But we shall come across this point more definitely later.

(*d*) On § 5 I would urge that if there really are "no means available for determining the instrumental correction, &c., &c." —that is, if the solar parallax as determined from observations of the Moon is affected with an entirely unknown systematic error, then it is a pity to publish the result. To do so is quite likely to make trouble, from the value being carelessly used irrespective of the limitation on some occasion or another; and that it is so near the probable truth only increases this likelihood. And what good is it? We know that the solar parallax is there or thereabouts; what we want to know now is, if possible, where *exactly* it is.

(*e*) On § 6 I have no comment.

On § 7 I have not yet before me the paper of Mr. Cowell's quoted, but I gather that the column referred to only gives the values of sin D. This only gives information concerning a very small part of the instrumental error for two reasons.

Firstly, if the instrumental error be expressed as a series

$$a_1 \sin D + b_1 \cos D + a_2 \sin 2D + b_2 \cos 2D + \&c.$$

the coefficient b_1 at any rate is likely to be large. For the instrumental error is almost certain to be quite different at new moon and full moon, *i.e.* at $D = 0°$ and $180°$. Now *sin D* has the same value at these points, and thus terms like $a_1 \sin D$ alone cannot represent errors of this kind. To trace changes in instrumental errors we must have at any rate the coefficients of cos D as well as those of sin D, and probably the higher coefficients also.

Secondly, *for expressing instrumental error* I do not trust the coefficients found in this way at all, for the reason that we have so few observations near new moon. It must be remembered that the case is totally different from that of the determi-

nation of a function whose form is known, in which case we can find its coefficient from a single value even. Suppose, for instance, we had two observed functions f_1 and f_2 thus:

D = 0°	30°	60°	90°	120°	150°	180°
f_1 = 0	1	2	3	(2)	(1)	(0)
f_2 = 0	1	2	3	(4)	(5)	(6)

the first rather like 3 sin D and the second rather like 3(1 − cos D); and suppose that we had no observations beyond D = 90°, so that we were in entire ignorance of the values except from 0° to 90°, where the functions coincide. Whatever analytical process we employed to represent them by series of sin D and cos D &c., we should of course get the same result for f_1 as for f_2; and yet the accordance would be quite illusory. This is only exaggerating for illustration what Mr. Cowell has done, if I understand him rightly. Observations near full moon where the instrumental errors are probably nearly constant (because the conditions remain nearly the same) receive most weight and largely control the analytical results. Thus, to adapt the above illustration a little nearer to the facts, suppose we had two quite different instrumental errors

D = 0°	30°	60°	90°	120°	150°	180°
f_1 = 3	3	2	1	0	(−1)	(−2)
f_2 = 3	3	4	5	6	(7)	(8)
Weight = 10	10	6	2	1	0	0

The great weight (of course purposely exaggerated) assigned to the observations near 0° would give nearly similar coefficients for the periodic terms sin D and cos D &c. and thus entirely mask the real and serious instrumental difference.

(*f*) With § 8 I entirely agree. Perhaps the large accidental errors of the altazimuth observations were scarcely sufficiently emphasised in my paper.

(*g*) With regard to § 9, as also to § 4 preceding, it seems to me very important to exclude any idea that the length of the series of observations discussed can improve the value of the solar parallax thus obtained. I do not see how a possible systematic error can be in any way diminished by multiplying observations, unless it is equally likely to occur with positive and negative signs; and in the case before us this is unlikely. At any rate the *onus probandi* is on those who make this assumption. If it is thoroughly understood that the value of the solar parallax is put forward, not as an exact value, but as one affected with an unknown systematic error, my objection is so far met, though I think it a pity to multiply imperfect values.

(*h*) But I will say in conclusion that I do not feel sure that Mr. Dyson and Mr. Cowell are right to despair of determining

this error, or at least of narrowing its limits. The altazimuth observations could be cleared of the error for high and low moon, with some trouble perhaps, but still it could be done, since observations in the same phase but at different altitudes could be compared. And then they could be used to throw some light on the errors of the meridian instruments. This would be one way. Another way would be to try the effect of restricting the determination of sin D to the days near full moon, where the instrumental or observational errors may be supposed with more legitimacy to remain constant, and where sin D is nevertheless varying rapidly. This would be easy to do, and a comparison of the results with those previously obtained from the whole material would perhaps tell us something valuable about the limits of error.

P.S.—After the above had been sent to the printers the following note was received from Professor Newcomb. In an accompanying letter he leaves open the question of printing it; but as it contains at least one important correction to what has already been said, there can be no doubt of the propriety of publishing it here. I refer to the question of the *division errors* of the azimuth circle of the altazimuth, to which Professor Newcomb draws attention. For my own part I was quite under the impression that these had been applied, so that Professor Newcomb's remark would have lost point; but on submitting this opinion to Mr. Dyson he replied (1904 May 20):

> "No division errors were applied before 1880, when the circle was redivided. This I have verified from the computing books. A determination was made in 1847, but apparently never used. I don't know how good it is."

Hence some preliminary work, which I had overlooked, is apparently necessary before the altazimuth observations can be used to discuss systematic errors.

On another point a word may be added to Professor Newcomb's note. The observations of altitude are not *all* made on the lower limb; in the second and third quarters the upper limb was sometimes observed near rising and setting respectively, as may be seen in the printed volumes. Whether there is enough material to determine the error of diameter from these I can scarcely say. Further, as I have remarked in a footnote on p. 405 of this volume, the combination of the altitude observations with those of azimuth may reduce the effect on the longitude in all cases to a simple error of semi-diameter.

Finally, I am glad to have confirmed by such an authority the view expressed in the last sentences of my remarks above—that in Professor Newcomb's words "the best result of meridian observations is to be obtained by throwing out all the observations made in daylight and eliminating the correction to the semi-diameter from the equations themselves." This experiment would in any case be most interesting.

Remarks on the Determination of the Parallactic Inequality of the Moon. By Simon Newcomb.

The following remarks are suggested by the papers of Professor Turner and Mr. Cowell in the December and March numbers of the *Monthly Notices* (vol. lxiv., Nos. 2 and 5). They relate to sources of possible systematic differences between the results of meridian and altazimuth observations of the Moon and possible systematic errors in a determination of the parallactic inequality which are not fully considered in the papers referred to.

Beginning with the meridian observations, it is necessary, in order to have a determination of the parallactic inequality free from systematic error, that all the observations used should be free from such systematic errors as are a function of the age of the Moon. The practical semi-diameter of the Moon, as it enters into the meridian observations, should not be regarded as a constant. Saying nothing of personal equation, which we assume to be eliminated from the mean during any half-lunation, the practical semi-diameter of the Moon is the true semi-diameter augmented by radiation. The latter is a function of the contrast between the brightness of the Moon's limb and that of the sky. It is therefore greater after the end of twilight and least when the Sun is above the horizon. It will also be less at the full moon on account of the greater obliquity at which the rays strike, and from the fact that the illumination of the extreme limb will be deficient, owing to many points on it being thrown into shadow when the observer is not practically in line with the Sun. For this reason we must expect the semi-diameter determined from transits to be too small. Under these circumstances it seems to me the best result of meridian observations is to be obtained by throwing out all observations made during daylight, and eliminating the correction to the semi-diameter from the equations themselves. I am inclined to think that we should get the best results if the discussions were confined to observations between November 1 and February 15. Each observation would give an equation of condition between the corrections to the semi-diameter and to the parallactic inequality.

The altazimuth observations are liable to errors, which are a function of the azimuth, unless the errors of graduation are determined and the instrument is so constructed as to be perfectly free from wabble. Observations with this instrument near and before the time of first quarter will commonly be made in the south-west or west, while those near the third quarter will be made in the south-east or east. We must also recognise the possibility of systematic errors in altitude as well as those of semi-diameter. A little consideration will show the following results of these errors :—

1. A constant error in altitude will have opposite effects

on the Moon's longitude at first and third quarters. It will therefore enter with its full effect into the parallactic inequality, but will be eliminated from the mean longitude.

2. Since all the observations of altitude must be made on the Moon's lower limb, the effect of error of semi-diameter will be the same as that of the altitude.

3. The errors in azimuth will practically have the same effect as those in right ascension, both as regards the limb and the errors which are a function of the azimuth itself.

For these reasons we must not expect the two methods to agree. Until we free the altazimuth observations from the source of error in question, it seems to me not safe to rely upon them for the parallactic inequality. To do this requires a more careful study of the instrument and the method of using it than I am aware of having been made.

Washington: 1904 *May* 11.

Methods of Correcting Moon's Tabular Longitude.

By P. H. Cowell.

In previous papers I have given a list of about a dozen corrections applied to Hansen's tabular longitudes from 1847·0 onwards. I now give a list of corrections to Airy's tabular longitudes, 1750–1851, that have already been entered, though the additions have not as yet been completed.

Terms whose arguments are multiples of the same angle are combined in one table, and in the following list are given under a single reference number; for example, the first table applies the correction

$$-2''{\cdot}58 \sin D - 0''{\cdot}20 \sin 2D - 0''{\cdot}37 \sin 3D - 0''{\cdot}46 \sin 4D$$

all at once.

The only exception to this is the terms in g' and $2g'$, where the secular variation of the former made it necessary to treat the latter separately.

The first list contains terms of short period; the second list terms of longer period:

Ref. No.	Argument.	Coefficient of Correction Applied.	Coefficient used by Airy *plus* Coefficient of Correction.	Remarks.
1	D	$-2''{\cdot}58$	$+ 124''{\cdot}68$	Parallactic Inequality
1	2D	−0·20	− 2370·10	Variation
1	3D	−0·37	+ 0·53	
1	4D	−0·46	+ 13·94	

Ref. No.	Argument.	Coefficient of Correction Applied.	Coefficient used by Airy *plus* Coefficient of Correction.	Remarks.
		"	"	
2	$-2g-g'$	+0·38	+ 7·68	
3	$-g-g'$	−1·20	+ 109·90	
4	$g-2g'$	+0·49	+ 2·59	
5	$g+2\omega-2\omega'$	−0·45	− 2·45	
6	$2g-g'+2\omega-2\omega'$	−0·88	24·48	
7	$-g-2g'+2\omega-2\omega'$	+0·44	+ 13·24	
8	$g-3g'+2\omega-2\omega'$	−3·30	+ 206·40	
9	$2g-3g'+2\omega-2\omega'$	−0·38	+ 165·52	
10	$3g-3g'+2\omega-2\omega'$	+0·63	+ 14·63	
11	$4g-3g'+2\omega-2\omega'$	+0·56	+ 1·16	
12	$3g-4g'+2\omega-2\omega'$	+0·44	+ 0·74	
13	$2g+g'+2\omega$	+0·42	+ 0·42	Term omitted by Airy.
14	$g+2\omega$	−2·36	− 39·56	
15	$2g+2\omega$	−0·62	− 411·62	
16	$-g+3g'+2\omega'$	+0·38	+ 0·38	Term omitted by Airy.
17	$2g+2g'+2\omega'$	+0·35	+ 0·55	
18	$g+\omega-\omega'$	+0·70	+ 17·90	
19	$-g-g'+\omega-\omega'$	−0·78	− 1·58	
20	$g-3g'+3\omega-3\omega'$	−0·61	− 1·21	
21	$g-4g'+4\omega-4\omega'$	+0·60	+ 1·10	
22	$3g-4g'+4\omega-4\omega'$	+0·46	+ 38·46	
23	$5g-4g'+4\omega-4\omega'$	+1·03	+ 1·93	
24	$2g-5g'+4\omega-4\omega'$	+1·55	+ 2·75	
25	$3g-5g'+4\omega-4\omega'$	+0·58	+ 4·38	
26	$4g-5g'+4\omega-4\omega'$	+0·58	+ 1·78	
27	$2g-2g'+4\omega-2\omega'$	+0·36	− 0·54	
28	$4g-2g'+4\omega-2\omega'$	−2·34	− 5·74	
29	$5g-2g'+4\omega-2\omega'$	−0·40	− 1·00	
30	$3g-3g'+4\omega-2\omega'$	−0·43	− 0·43	Term omitted by Airy.
31	$4g-6g'+6\omega-6\omega'$	+0·54	+ 0·54	" "
32	$2g-5g'+2\omega-4\omega'$	−0·40	0·00	Airy's wrong term.
33	$g+2\varpi-2\mathrm{J}$	−0·893	− 0·893	Jupiter term.
34	$g+2\varpi-3\mathrm{J}+7^\circ$	−0·316	+ 0·316	" "
35	g	−2·10	+ 22639·50	Anomaly.
35	$2g$	−0·46	+ 769·04	
35	$3g$	−0·58	+ 36·12	
36	$2\mathrm{D}-g$	+0·738	+ 4586·338	Evection.
36	$4\mathrm{D}-2g$	−3·75	+ 30·75	

Ref. No.	Argument.	Coefficient of Correction Applied.	Coefficient used by Airy *plus* Coefficient of Correction.	Remarks.
		"	"	
I.	$-g' + 2w - 2w'$	+ 3·80	+ 2·40	
II.	$-g' + w - w'$	− 0·60	− 18·60	
II.	$-2g' + 2w - 2w'$	− 0·65	+ 211·75	
III.	$-3g' + 2w - 2w'$	+ 0·86	+ 8·66	
IV.	$-g'$	+ 1·03	+ 669·63	Annual equation.
		− 1·63 T	− 1·63 T	T in centuries from 1800
V.	$-2g'$	− 0·40	+ 7·50	
VI.	$+2g' + 2w'$	− 0·30	− 55·20	
VII.	V − E	+ 0·30	− 0·80	Venus term
VIII.	2V − 3E + 85°	− 0·30	− 0·30	" "

Subsequently a term in cos g will probably be added to the above list; but I propose carrying out analyses for g and D without applying any further short period corrections.

I now proceed to explain the method employed.

The transits of the Moon, whether observed or not, are numbered consecutively, counting from 1750 September 13^d 10^h as No. 1; for convenience they are then divided into forties; and the 29th and 30th batch of forty are shown in the columns headed [29] and [30] respectively. The blank in the first line of column [29] informs us that the Moon was not observed at the corresponding transit which is clearly 28×40 lunar days later than column [1], line 1, which corresponds to 1750 September 13^d 10^h. If anyone is interested in expressing this date in terms of the civil calendar, it is easy to do so, by taking the lunar day as 1·03505 of a solar day. The entry −50 in the second line informs us that Airy's tabular longitude of the Moon exceeded the observed longitude at the corresponding transit by −50 units of one-tenth of a second of arc. This is the result of pure copying from Airy's Greenwich Lunar Reductions, with the following modifications only: (i) the second decimal of a second is omitted; (ii) the decimal point is omitted; (iii) the sign is changed, as Airy gives observed *minus* tabular; (iv) means are taken when Airy gives more than a single result, that is to say, when both first and second limbs have been observed. The third line shows that the corresponding transit was not observed; then follow three observations, and then seven unobserved transits (during which it is clear that new moon must have fallen) and so on.

TABLE 7.

	7	[29]	7	[30]	1	2
1	...	...	...	...	A+1	−4
2	−1	−50	...	...	+2	−4
3	=	...	−	−	+3	−4
4	+1	−20	−4	−72	+4	−3
5	+2	−46	−4	−74	+4	−2
6	+3	−38	...	...	+4	−1
7	...	...	−3	−28	+4	0
8	...	...	−2	−12	+4	+1
9	...	...	...	...	+3	+3
10	...	...	0	+63	+2	+3
11	...	...	+1	+78	+1	+4
12	...	...	+2	+71	−1	+4
13	...	...	+3	− 2	−2	+4
14	+1	−12	+4	−27	−3	+4
15	0	−29	...	...	−4	+4
16	...	...	+4	−31	−4	+3
17	−2	−45	...	...	−4	+2
18	−3	+19	...	...	−4	+1
19	...	...	...	...	−4	−1
20	—	...	...	...	−3	−2
21	−4	−13	...	...	−3	−3
22	−4	+64	0	0	−1	−4
23	−4	+87	−1	+31	0	−4
24	−3	+70	...	...	B+1	−4
25	...	...	...	...	+2	−4
26	...	...	...	...	+3	−4
27	...	...	...	...	+4	−3
28	...	...	...	...	+4	−2
29	...	...	...	...	+4	−1
30	...	...	...	...	+4	0
31	...	...	−3	−41	+4	
32	+4	−63	...	...	+3	
33	...	...	...	...	+2	
34	...	...	0	−83	+1	
35	...	...	+1	−89	0	
36	...	...	...	...	−1	
37	...	...	...	...	−2	
38	...	...	+4	−21	−3	
39	...	...	+4	+26	−4	
40	...	...	+4	+46	−4	

Precepts

	c	l
B	29	4
B	30	11
B	31	18
B	32	25
A	33	32
B	34	39

The computing form annexed is to be considered as consisting of three, or rather four, separate pieces of paper. The first piece contains column [29] as its extreme right-hand column, to the left lies the various corrections of which correction 7 is alone shown; the second piece shows column [30] and correction 7 in a similar manner; the third and fourth pieces are mere strips which may be referred to as table 7, columns 1 and 2, respectively.

The following is the process of entering the corrections: the last three entries of table 7, column 2, are held in juxtaposition to the first three lines of column [29]; that this is the right position is inferred from the fact that the preceding entry of table 7 has been previously found to apply to column [28] since 40. Holding the slip in his left hand and to the left of column [29], the computer then enters the correction whenever the transit has been observed, *e.g.* he does not copy −2 and 0 in the first and third lines, but he does copy −1 in the second line. Having now come to the end of table 7 he draws a double line as shown. When table 7 comes to an end it is always proper to begin again, either at the beginning (the first entry is marked A) or at the 24th entry, which is marked B. To ascertain at which of these two points to begin in the present instance, it is necessary to refer to the Precepts, an extract from which is annexed, stating that at column [29], line 4, the proper entry is B. This precept serves the further purpose of preventing mistakes, for, supposing for example that in turning over from column [28] to column [29] the computer slipped one line, he would discover the mistake on finding that the precept did not appear to come at the right line; he would then go back to the preceding precept, which has presumably come right, and make the necessary alterations, and the mistake is prevented from continuing undiscovered throughout an indefinite number of columns.

The plan of the table is obvious. The entries are the products of −4·4 by the sines of angles in an arithmetical progression, the common difference being 15°·322690, the movement of the argument in one lunar day. The corrections thus applied correspond to mean lunar noons, but no serious error is committed by applying them at apparent lunar noons.

To save a little labour in constructing the table the argument of the middle entry (in the present case the 35th) is taken as exactly zero. The second half of the table is then constructed, and the first half follows from it by a change of sign and reversing the order of the entries.

I give below the full calculation for the table:

Entry.	Argument.	Entry.	Argument.
35	0°·00	58	0° − 7°·35
46	180° − 11°·34	59	0° + 7°·98
47	180° + 3°·99	70	180° − 3°·36

n	$\log\left\{\frac{2n-1}{8\cdot8}\right\}$	Anti-log. sine.	Residual after subtracting multiples of 15°·332690.
1	0555	6°·52	6°·52
2	5326	19·93	4·60
3	7545	34·63	3 96
4	9006	52·70	6·70
		90·00	13·33

The above little computation is sufficiently explained by its headings; the last column informs us (second line for example) that $4\cdot4 \times \sin(15°\cdot332690+\theta) = 1$ if θ be less than 4·60 and $= 2$ if θ be greater than 4·60; the last line informs us that $5 \times 15°\cdot332690+\theta$ exceeds 90° if θ exceeds 13°·33. Table 7 is now constructed six entries at a time by merely referring to the last column of the above computation; for instance, the entries Nos. 46–41 in the reverse order are seen to be −1, −2, −3, −4, −4, −4, by merely comparing 11°·34 (see value of argument for entry No. 46) with the angles that stand in the above column.

I come now to the formation of the precepts. The movement of the argument in seventy lunar days is $3 \times 360° - 6°\cdot7117$; in forty-seven lunar days is $2 \times 360° + 0°\cdot63643$. From this it may be seen that to "Return to A" adds $-6°\cdot7117$ to the excess of the true argument (*i.e.* the true value of the argument at the nearest mean lunar noon) over the argument of the table; while to "Return to B" adds $+0°\cdot63643$ to the same excess. Consequently it follows that the difference between the true argument and the argument of table can always be kept less $\frac{1}{2}\{6°\cdot7117 + 0°\cdot63643\}$ or 3°·67, and the maximum error committed will be less than 0″·44 × circular measure of 3°·67, or less than 0″·03. Had it been considered desirable the error might have been confined to narrower limits by using a larger table. The numbers 47 and 70 are readily found by turning $\frac{15°\cdot332690}{360°}$ into a continued fraction, the first two convergents are $\frac{1}{23}$ and $\frac{2}{47}$; of the wo numbers 47 and 70, therefore, the former is the denominator of the last convergent, and the latter the sum of the denominators of the two last convergents.

It will be seen that the method is perfectly general; it can be applied to corrections of any size and to any degree of accuracy at the cost of enlarging the table. If the coefficient of the correction is N times the maximum permissible error, the table must consist of $2\pi N$ entries at least. If, however, the table had been required for an ephemeris instead of for a series of discontinuous dates, it would be reasonable to throw such operations as changing signs and copying backwards upon the computer, and the size of the table could be divided by 4, making $\frac{\pi}{2}N$ entries at the least.

The use of the table involves mere copying without interpolation, and it is clear that any table professing to tabulate a quantity

that may take all values from 1 to N without interpolation correctly to a single unit, must contain N entries. The coefficient of efficiency of the method may therefore be taken as $\frac{2}{\pi}$ (perfection being taken as unity).

This slight loss of efficiency is clearly not a high price to pay for reducing the entering to pure copying without interpolation, and abolishing the calculation of the arguments for the individual observations.

As I pointed out in the *Observatory* for 1903 July, in the case of large corrections interpolations may be grafted on. Suppose, for instance, it were required to tabulate $22639''{\cdot}50 \sin g$ correctly to $0''{\cdot}01$, the elliptic inequality for every ten minutes of mean solar time. A table of 1000 entries would give this quantity for equal increments of the argument corresponding to the movement in ten minutes from 0° to 90°. Pure copying of this table (backwards or forwards, with or without change of sign) would give all the quantities calculated with arguments wrong by the same constant Δg, say. It would then remain to calculate $22639''{\cdot}50\ \Delta g \cos g$, an operation not outside the limits of Crelle or Cotsworth's multiplication tables, if the correction be tabulated for the maximum value of Δg. The term in Δg^2 is not required. The modification is obvious, if the interval of the ephemeris is not to be 10 minutes, but say 12 hours; the table would then have to extend over 72 quadrants instead of one, and the number of entries in each quadrant will be one seventy-second part of the number (1,000) contemplated in the previous case.

There remains a special case to be alluded to, which is best illustrated by considering correction No. 23. The movement of the argument in one lunar day in this case is $63°{\cdot}995060$, and the successive convergents to $\frac{63°{\cdot}995060}{360°}$ are $\frac{1}{5}$, $\frac{1}{6}$, $\frac{2}{11}$, $\frac{3}{17}$, $\frac{8}{45}$, $\frac{355}{1992}$.

To form a table with $1592+45$ entries would be a labour altogether out of proportion to the end in view. A table, however, of $45+17$ entries is rather too small, as it would permit errors of $0''{\cdot}07$. Four tables of 45 entries each were therefore formed. The tables are lettered A, B, C, D. In each table the arguments are in arithmetical progression with a common difference of $63°{\cdot}995060$. The following extract from the value of the arguments

$-a$	A $1=180°-D45$	B $1=180°+D\ 1$	C $1=180°+A\ 1$
$180°+3°{\cdot}85-a$	$A14=180°-D32$	$B32=180°+D32$	$C14=180°+A14$
$-0°{\cdot}22-a-63°{\cdot}995$	$A45=180°-D\ 1$	$B45=180°+D45$	$C45=180°+A45$

shows that when one table has been constructed, the remaining tables follow by a change of sign, or a reversal of order, or both. It remains to explain the principles that determine the value of a, and the nature of the precepts.

When the computer reaches the end of any table, the precepts

usually direct him to recommence the same table. This, however, involves increasing the excess of the true argument over the argument of the table by $-0°{\cdot}22$, and cannot be indefinitely repeated.

Occasionally, therefore, the precepts direct him to substitute

A14 for D1	C14 for B1
B2 for A1	D2 for C1

In other words, from time to time he has to change his table, always following the order of the alphabet, and having changed to go round and round the same table till the time comes for the next change.

Now, as the precepts are written, before settling the value of a it will be seen that the excess of the arguments D1, B1 over A14, C14 respectively is $3°{\cdot}85-2a$; and the excess of the arguments A1, C1 over B2, D2 respectively is $2a$. Hence we are constantly increasing the excess of the true argument over the argument of the table by $-0°{\cdot}22$, and we have the power, whenever we choose, of affecting this excess by a further increment, $3{\cdot}85-2a$ or $2a$. It is now clear that we ought to take $a=0°{\cdot}96$ so as to make them two quantities each equal to $1°{\cdot}92$, when it will be seen that the power of adding $+1°{\cdot}92$ at will enables us to keep the difference between the true argument and the tabular argument below the numerical value $0°{\cdot}96$, corresponding to a maximum error $1''{\cdot}03\times0°{\cdot}96-0''{\cdot}02$ in the use of the table.

Moreover, the method of tabulating the errors is convenient for the analysis of errors as well as applying corrections. Suppose, for example, it is required to analyse the errors for possible terms $g+g'$, or $g+g'$ with the addition of any long-period argument such as $\omega-\omega'$. We may take as an auxiliary angle θ, whose movement in one lunar day is $+14°{\cdot}4$ and whose value is zero at each twenty-fifth day of each period of analysis. The ten sheets containing the 10×40 lunar days that go to one period of analysis are taken, and a cross is placed opposite columns 1 and 6, lines 25; columns 3 and 8, lines 20; columns 5 and 10, lines 15; columns 7 and 2, lines 10; columns 9 and 4, lines 5. The sheets are then placed so that the crosses fall in a row, whereupon the value of θ for all the observations in any row is the same. The sums are then taken down upon a convenient form for analysis into sine and cosine, and checked by the condition that the sum of the sums must be equal to the sum of all the errors for the whole period of analysis, a total that has been previously formed.

Now the movement of $g+g'-\omega+\omega'$ exceeds the movement of the auxiliary angle by $11°{\cdot}1221$ in a period of analysis. Hence an error with argument $g+g'-\omega+\omega'$ will discover itself by a periodicity in a little less than 33 periods of analysis in the coefficients of $\sin\theta$ and $\cos\theta$.

Further Analyses of Moon's Errors with Mean Elongation as argument, 1847–1901. By P. H. Cowell.

The present analyses are based upon the errors after thirteen corrections have been applied to Hansen's tabular places. These corrections are given in previous papers, where the periods of analysis are also defined.

In the annexed table the first column gives the number of the period of analysis. The next two columns are copied from last month's paper. The next three columns are fresh matter, and they are sufficiently distinguished by their headings. The meaning of the word "apparent" is that these are the corrections that are found by analysis upon the successive assumptions that each one alone exists. The subsequent columns, headed "resolved values," take account of the co-existence of the various separate errors. The relation between the "apparent" and "resolved" values is clearly dependent upon the distribution of the observations relatively to the age of the Moon. Relations were investigated, based upon the whole of the 48 periods of analysis (1847–1901). These relations should not strictly be applied to the individual periods, for any particular period may have an abnormal distribution of observations, but the error due to this cause has been allowed to coalesce with the accidental errors in the belief that they will not be so distributed as to sensibly affect the coefficients of deduced periodic corrections.

I now investigate the formulæ used. The notation is μ, δ_1, δ_2, Δ_1, Δ_2 stand respectively for the correction to semi-diameter, coefficients of sin D, sin 2D, cos D, cos 2D (resolved values). Accented letters denote apparent values. It will be seen that μ, δ_1, δ_2 fall into one group, Δ_1, Δ_2 into a second group, and that the two groups are kept separate.

First as to μ, δ_1, δ_2; in the December *Monthly Notices* I obtained from 5,647 observations normal equations:

$$5647\mu + 3788\delta_1 - 2327\delta_2 = \Sigma \pm \epsilon,$$
$$3788\mu + 3074\delta_1 - 1366\delta_2 = \Sigma \epsilon \,.\, \sin D,$$
$$-2327\mu - 1366\delta_1 + 2645\delta_2 = \Sigma \epsilon \,.\, \sin 2D,$$

ϵ being the error of any observation and the 48 times 400 lunar days being taken together.

For each of the present periods of analysis I put, n denoting the number of observations.

$$\mu' = \frac{1}{n} \Sigma \pm \epsilon$$

$$\delta'_1 = \frac{2}{n} \Sigma \epsilon \sin D.$$

$$\delta'_2 = \frac{5647}{2645 n} \Sigma \epsilon \sin 2D.$$

Analyses of Tabular minus *Observed Longitudes of Moon for Periods* 86–133 *in Tenths of a Second of Arc.*

Period.	Apparent Values of: Semi-Diameter Term.	Co-efficient of sin D.	Co-efficient of sin 2D.	Co-efficient of cos D.	Co-efficient of cos 2D.	Resolved Values of: Semi-Diameter Term.	Co-efficient of sin D.	Co-efficient of sin 2D.	Co-efficient of cos D.	Co-efficient of cos 2D.
86	+ 1	+ 2	0	− 5	+ 2	− 1	+ 3	+ 1	− 8	− 2
87	− 8	−11	+ 1	− 2	+ 1	−13	+ 2	−10	− 2	0
88	− 2	− 1	+ 3	−10	+ 4	− 7	+ 9	+ 1	−16	− 4
89	0	0	0	− 6	+ 4	0	0	0	− 3	+ 3
90	− 4	− 5	+ 3	− 7	+ 4	− 6	+ 2	− 1	− 6	+ 1
91	+ 3	+ 5	− 2	0	+ 2	0	+ 5	0	+ 8	+ 6
92	− 5	− 8	+ 6	− 3	+ 2	+ 2	− 8	+ 3	− 2	+ 1
93	− 2	0	+ 5	− 5	+ 4	− 9	+13	+ 4	0	+ 5
94	− 3	− 5	0	+ 1	− 1	− 2	− 4	− 4	− 1	− 1
95	− 6	− 9	+ 4	+ 7	− 4	− 4	− 4	− 1	+ 6	− 1
96	− 8	−12	+ 5	− 1	− 4	− 5	− 6	− 3	−19	−15
97	−11	−13	+11	+ 5	− 2	−16	+ 9	+ 1	+ 8	+ 2
98	− 6	− 9	+ 5	+ 1	+ 2	− 2	− 5	0	+11	+ 8
99	0	− 1	+ 1	− 4	+ 2	+ 5	− 6	+ 2	− 5	− 1
100	+ 3	+ 4	− 5	− 3	+ 2	+ 1	+ 1	− 4	− 2	+ 1
101	0	− 1	0	+ 1	− 1	+ 4	− 6	+ 1	− 1	− 1
102	+ 3	+ 4	− 5	+10	− 7	+ 1	+ 1	− 4	+ 3	− 5
103	+ 3	+ 4	− 4	+ 2	+ 1	+ 2	+ 1	− 2	+10	+ 6
104	− 3	− 6	+ 4	0	+ 1	+ 6	−12	+ 3	+ 4	+ 3
105	+ 1	+ 1	+ 2	− 4	+ 2	+ 5	− 4	+ 5	− 5	− 1
106	+ 1	+ 3	0	+ 3	− 1	− 5	+ 9	0	+ 6	+ 2
107	+ 1	+ 2	0	+ 5	− 2	− 1	+ 3	+ 1	+ 8	+ 2
108	+ 4	+ 6	0	+ 3	− 1	+ 5	+ 1	+ 5	+ 6	+ 2
109	+ 8	+13	− 4	0	0	+ 2	+10	+ 3	0	0
110	+ 3	+ 2	− 3	− 1	0	+11	−11	+ 1	− 3	− 2
111	− 2	− 3	0	− 3	+ 2	− 3	− 1	− 3	− 2	+ 1
112	0	− 1	0	− 5	+ 4	+ 4	− 6	+ 1	0	+ 5
113	+ 1	+ 2	− 3	+ 2	+ 1	− 4	+ 5	− 4	+10	+ 6
114	+ 2	+ 3	− 4	−12	+ 6	− 2	+ 3	− 4	−13	− 1
115	+ 3	+ 3	0	− 9	+ 6	+10	− 7	+ 5	− 4	+ 4
116	+ 3	+ 3	− 4	−14	+ 6	+ 6	− 5	− 2	−22	− 5
117	+ 4	+ 4	− 5	− 5	+ 3	+ 8	− 7	− 1	− 4	+ 2
118	+ 3	+ 4	− 2	− 2	+ 1	+ 4	0	+ 1	− 2	0
119	+ 1	+ 2	− 4	+ 5	− 3	− 5	+ 5	− 6	+ 4	− 2
120	0	0	− 3	− 4	0	− 3	+ 2	− 5	−13	− 7
121	+ 3	+ 5	0	0	+ 1	+ 2	+ 4	+ 4	+ 4	+ 3
122	+ 8	+12	− 5	− 3	+ 1	+ 5	+ 6	+ 3	− 6	− 2
123	+ 3	+ 5	− 3	− 2	+ 1	− 1	+ 6	− 1	− 2	0
124	+ 3	+ 5	− 2	− 6	+ 4	0	+ 5	0	− 3	+ 3
125	+ 1	+ 1	− 6	− 6	+ 4	− 3	+ 1	− 8	− 3	+ 3
126	− 4	− 6	+ 2	+ 1	− 1	− 3	− 3	− 2	− 1	− 1
127	+ 2	+ 2	− 2	− 4	+ 4	+ 4	− 4	0	+ 3	+ 6
128	0	− 1	0	0	− 1	+ 4	− 6	+ 1	− 4	− 3
129	− 4	− 4	+ 4	− 4	+ 2	− 9	+ 7	0	− 5	− 1
130	− 3	− 5	+ 2	− 5	+ 1	0	− 5	0	−12	− 5
131	− 5	− 6	+ 4	− 4	+ 2	− 8	+ 4	− 1	− 5	− 1
132	− 1	− 2	0	− 3	+ 2	+ 1	− 3	− 1	− 2	+ 1
133	− 1	− 2	+ 3	+ 4	+ 1	+ 4	− 5	+ 4	+17	+10

Subsequently I deduce μ, δ_1 and δ_2 from the formulæ

$$\begin{aligned} 5647\mu + 3788\delta_1 - 2327\delta_2 &= 5647\mu' \\ 3788\mu + 3074\delta_1 - 1366\delta_2 &= \tfrac{1}{2}5647\delta_1' \\ -1366\mu + 2327\delta_1 + 2645\delta_2 &= 2645\delta_2' \end{aligned}$$

It will be seen that μ', δ_1', δ_2' are purely arbitrary auxiliary quantities. The treatment is unfortunately not quite symmetrical. I wish that I had taken for δ_1', $\frac{1}{2} \cdot \frac{5647}{3074}$ times the value which I have taken. The explanation is that when I came to Δ_1, Δ_2 I was forced into definitions corresponding to that of δ_2'; I was in time to define δ_2' accordingly, but too late for δ_1'. Of course nothing is lost but symmetry.

Solving the equations for μ, δ_1, δ_2, in terms of μ', δ_1', δ_2', I obtain

$$\begin{aligned} \mu &= 7{\cdot}285\mu' - 3{\cdot}980\delta_1' + 1{\cdot}079\delta_2' \\ \delta_1 &= -7{\cdot}960\mu' + 5{\cdot}537\delta_1' - 0{\cdot}597\delta_2' \\ \delta_2 &= +2{\cdot}306\mu' - 0{\cdot}637\delta_1' + 1{\cdot}646\delta_2' \end{aligned}$$

and with the help of these equations the resolved values have been obtained from the apparent values.

It follows that

$$n\mu = \Sigma\epsilon\{\pm 7{\cdot}285 - 7{\cdot}960 \sin D + 2{\cdot}306 \sin 2D\}$$

$$\frac{n}{2} \cdot \delta_1 = \Sigma\epsilon\{\mp 3{\cdot}980 + 5{\cdot}537 \sin D - 0{\cdot}637 \sin 2D\}$$

$$\frac{n}{2}\delta_2 = \Sigma\epsilon\{\pm 1{\cdot}153 - 0{\cdot}637 \sin D + 1{\cdot}756 \sin 2D\}$$

Throughout any period of analysis the value of D is supposed to increase by $720° \div 57$ between one transit of the Moon and the next, and the small error of this assumption is prevented from accumulating by introducing the necessary discontinuity in passing from one period of analysis to the next. Only 57 values of D are therefore recognised—namely, multiples of $720° \div 57$; and the above factors merely change sign as we pass from the multiple by p to the multiple by $57 - p$.

I have therefore tabulated for values of D

$$= \frac{720°}{57} p \; (p = 3, 4 \ldots . 25)$$

the factors for $n\mu$, $\frac{n}{2}\delta_1$ and $\frac{n}{2}\delta_2$ respectively, viz.:

$$\begin{aligned} &\pm 7{\cdot}285 - 7{\cdot}960 \sin D + 2{\cdot}306 \sin 2D \\ &\mp 3{\cdot}980 + 5{\cdot}537 \sin D - 0{\cdot}637 \sin 2D \\ &\pm 1{\cdot}153 - 0{\cdot}637 \sin D + 1{\cdot}756 \sin 2D \end{aligned}$$

p	Factor for $n\mu_*$	Factor for $\frac{n\delta_1}{2}$	Factor for $\frac{n\delta_2}{2}$	p	Factor for $n\mu_*$	Factor for $\frac{n\delta_1}{2}$	Factor for $\frac{n\delta_2}{2}$
3	+4·7	−1·2	+2·5	15	−5·3	+2·9	−0·5
4	+3·4	−0·3	+2·4	16	−2·6	+1·4	+0·3
5	+2·1	+0·4	+2·0	17	−0·6	+0·2	+0·9
6	+0·8	+1·1	+1·4	18	+0·9	−0·8	+1·1
7	−0·5	+1·5	+0·6	19	+1·7	−1·4	+0·9
8	−1·4	+1·7	−0·1	20	+1·6	−1·6	+0·5
9	−1·8	+1·6	−0·7	21	+1·0	−1·7	−0·2
10	−1·4	+1·1	−1·0	22	0·0	−1·3	−1·0
11	−0·3	+0·3	−1·0	23	−1·3	−0·8	−1·7
12	+1·5	−0·8	−0·6	24	−2·7	−0·1	−2·2
13	+3·9	−2·1	+0·1	25	−4·0	+0·7	−2·5
14	+6·6	−3·6	+0·9				

For full Moon $p = 14\frac{1}{4}$.

Secondly as to Δ_1 Δ_2; I put

$$\epsilon = \Delta_1(\cos D - c_1) + \Delta_2(\cos 2D - c_2)$$

when c_1 c_2 are the average values of cos D, cos 2D respectively.

The normal equations are then

$$\Delta_1\Sigma(\cos D - c_1)^2 + \Delta_2\Sigma(\cos D - c_1)(\cos 2D - c_2) = \Sigma\epsilon(\cos D - c_1)$$
$$\Delta_1\Sigma(\cos D - c_1)(\cos 2D - c_2) + \Delta_2\Sigma(\cos 2D - c_2)^2 = \Sigma\epsilon(\cos 2D - c_2)$$

The coefficients on the left-hand side are

$$\Sigma(\cos D - c_1)^2 = \Sigma(\cos^2 D - c_1^2) = \frac{n}{2}\{1 + c_2 - 2c_1^2\}$$

$$\Sigma(\cos D - c_1)(\cos 2D - c_2) = \Sigma(\cos D_1 \cos 2D - c_1c_2) = \frac{n}{2}\{c_1 + c_3 - 2c_1c_2\}$$

$$\Sigma(\cos 2D - c_2)^2 = \Sigma(\cos^2 2D - c_2^2) = \frac{n}{2}\{1 + c_4 - 2c_2^2\}$$

where c_3, c_4 denote the mean values of cos 3D, cos 4D respectively.

My paper in 1903 December enables us to evaluate these quantities as

$$\frac{n}{2}\{1 - 0{\cdot}09 - 0{\cdot}46\} = \frac{n}{2} \cdot 0{\cdot}45 = \frac{n}{2}a$$

$$\frac{n}{2}\{-0{\cdot}48 - 0{\cdot}09\} = -\frac{n}{2}\ 0{\cdot}57 = \frac{n}{2}h$$

$$\frac{n}{2}\{1 + 0{\cdot}06 - 0{\cdot}01\} = \frac{n}{2}\ 1{\cdot}05 = \frac{n}{2}b$$

We shall subsequently require

$$ab = 0{\cdot}47,\ ah = -0{\cdot}26,\ bh = -0{\cdot}60,\ h^2 = 0{\cdot}32,\ ab - h^2 = 0{\cdot}148$$

The normal equations are therefore :

$$\left(\frac{n}{2}\Delta_1\right) + \frac{h}{a}\left(\frac{n}{2}\Delta_2\right) = \Sigma\epsilon\left(\frac{\cos D - c_1}{a}\right) = \frac{n}{2}\Delta'_1$$

$$\frac{h}{b}\left(\frac{n}{2}\Delta_2\right) + \left(\frac{n}{2}\Delta_2\right) = \Sigma\epsilon\left(\frac{\cos 2D - c_2}{b}\right) = \frac{n}{2}\Delta'_2$$

The apparent values Δ'_1, Δ'_2, which may be viewed as arbitrary auxiliary quantities, are defined as above ; and their values have been actually calculated from the formula

$$\Delta'_1 = \frac{2}{n}\Sigma\epsilon\,\frac{\cos D - c_1}{a} = \frac{2}{n}\Sigma\epsilon(\cos D + 0{\cdot}48) \times 2{\cdot}27$$

$$\Delta'_2 = \frac{2}{n}\Sigma\epsilon\,\frac{\cos 2D - c_2}{b} = \frac{2}{n}\Sigma\epsilon\,(\cos 2D + 0{\cdot}09) \times 0{\cdot}94$$

which correspond with sufficient accuracy to the algebraical formulæ.

Then solving for Δ_1, Δ_2 we have

$$\Delta_1 = \frac{ab}{ab-h^2}\Delta'_1 - \frac{bh}{ab-h^2}\Delta'_2 = +3{\cdot}2\Delta'_1 + 4{\cdot}1\Delta'_2$$

$$\Delta_2 = \frac{-ah}{ab-h^2}\Delta'_1 + \frac{ab}{ab-h^2}\Delta'_2 = +1{\cdot}7\Delta'_1 + 3{\cdot}2\Delta'_2$$

The following table is similar to a preceding one :

p.	Factor for $\frac{n}{2}\Delta'_1$.	Factor for $\frac{n}{2}\Delta'_2$.	Factor for $\frac{n}{2}\Delta_1$.	Factor for $\frac{n}{2}\Delta_2$.
3	+2·88	+0·32	+10·5	+5·9
4	+2·54	−0·09	+ 7·8	+4·0
5	+2·11	−0·47	+ 4·8	+2·1
6	+1·66	−0·74	+ 2·3	+0·4
7	+1·16	−0·86	+ 0·2	−0·8
8	+0·66	−0·79	− 1·1	−1·4
9	+0·18	−0·55	− 1·7	−1·4
10	−0·25	−0·20	− 1·6	−1·1
11	−0·61	+0·22	− 1·0	−0·3
12	−0·91	+0·60	− 0·4	+0·4
13	−1·09	+0·88	+ 0·1	+1·0
14	−1·18	+1·02	+ 0·4	+1·2
15	−1·16	+0·98	+ 0·3	+1·2
16	−1·02	+0·76	− 0·1	+0·7

p.	Factor for $\frac{n}{2}\Delta'_1$.	Factor for $\frac{n}{2}\Delta'_2$.	Factor for $\frac{n}{2}\Delta_1$.	Factor for $\frac{n}{2}\Delta_2$.
17	−0·77	+0·41	− 0·8	0·0
18	−0·45	0·00	− 1·4	−0·8
19	−0·05	−0·39	− 1·8	−1·3
20	+0·40	−0·69	− 1·6	−1·5
21	+0·91	−0·84	− 0·5	−1·1
22	+1·41	−0·82	+ 1·2	−0·2
23	+1·88	−0·62	+ 3·5	+1·2
24	+2·34	−0·29	+ 6·3	+3·0
25	+2·72	+0·11	+ 9·2	+5·0

The following table gives the weights of the various quantities when determined from n observations each of weight unity, and also the sum of their values for the 48 periods without regard to sign :

Quantity.	Sum of Numerical Values.	Weight.
μ'	146	n
δ'_1	208	$\frac{1}{2}n$ approximately
δ'_2	131	$\frac{1}{2}n$ approximately
Δ'_1	192	$\frac{1}{2}n \,.\, a = \frac{1}{2}n \div (1{\cdot}5)^2$
Δ'_2	113	$\frac{1}{2}n \,.\, b = \frac{1}{2}n$ approximately
μ	208	$n \times \frac{781}{5047} = n \div (2{\cdot}7)^2$
δ_1	235	$\frac{1}{2}n \times \frac{1030}{5047} = \frac{1}{2}n \div (2{\cdot}3)^2$
δ_2	117	$\frac{1}{2}n \times \frac{3228}{5047} = \frac{1}{2}n \div (1{\cdot}3)^2$
Δ_1	284	$\frac{1}{2}n \dfrac{ab-h^2}{b} = \frac{1}{2}n \div (3{\cdot}7)^2$
Δ_2	147	$\frac{1}{2}n \dfrac{ab-h^2}{a} = \frac{1}{2}n \div (1{\cdot}7)^2$

Judging from the last eight quantities alone, the probable error of a single observation is $\pm 1''{\cdot}4$. The first two quantities, however, give about double this result, a value which is clearly too large. The weighted mean is $\pm 1''{\cdot}7$ and agrees well with the estimates based upon other analyses.

Taking means for the 48 periods, I obtain

$$\begin{aligned} \mu &= -0''{\cdot}07 \\ \delta_1 &= 0{\cdot}00 \\ \delta_2 &= -0{\cdot}07 \\ \Delta_1 &= -0{\cdot}28 \\ \Delta_2 &= +0{\cdot}10 \end{aligned}$$

Of these quantities Δ_1 alone exceeds its probable error, and there is no gravitational explanation of a cos D term.

The three quantities μ, δ_1, and δ_2 are liable to probable errors of nearly 0″·10. Taking, however, the values as given above, the results of my Paper in December require slight corrections, and I now obtain

Correction to tabular semi-diameter	−0″·47
Observed Parallactic Inequality	−126″·46+1″·56 = −124″·90
Observed Variation	2370″·14+0″·07 = 2370″·21

The corrections to the December paper are,

$$\text{for } \mu - 0''\cdot12, \text{ for } \delta_1 + 0''\cdot12, \text{ for } \delta_2 - 0''\cdot04$$

and it will be seen that these numbers are approximately proportional to the factors for day 14. The values in the December Paper are probably slightly erroneous for the reason noted in *Monthly Notices*, December, p. 96, paragraph (i), but the cause there noted will only account for a small half of the discordances. The remaining part is, as far as I know, to be attributed to purely accidental differences. It must be remembered that several corrections have been applied to each observation in the interval between the two investigations. I incline, for the reasons stated, to the belief that the present values are more accurate.

The solar parallax, deduced on the assumption that there is no monthly term in the semi-diameter, has now been raised from 8″·76 by +0″·018 by Professor Brown's calculations (see *Monthly Notices*, 1904 April) and by +0″·009 on account of the revised result of the present paper. The value thus becomes

$$8''\cdot787 \pm 0''\cdot007.$$

Solving for $\omega - \omega'$ and Ω, I find no considerable terms except

$$+0''\cdot33 \sin \Omega \sin D.$$

This is nearly the same term, as I found a month ago, before taking account of the variation. There is no term in cos Ω cos D, and as the coefficient 0″·33 is only about three times the probable error, I have ceased to think that the term is certainly real.

There is reason to think that the coefficients of sin $(g - g')$ and sin $(g - g' + 2\omega - 2\omega')$ in Hansen's Tables (+148″·03 and −28″·60) require algebraic diminution by about 0″·10 each, subject to a probable error as large as the quantity itself. Hansen's coefficients are clearly more accurate than Delaunay's.

Newcomb's corrections are now applied in the *Nautical Almanac* in a manner equivalent to

$$N\{1 + \tfrac{1}{9}\cos g + \tfrac{3}{100}\cos 2D + \tfrac{3}{100}\cos(2D - g)\}$$

nearly. I have only applied the first two terms $N\{1 + \frac{1}{9}\cos g\}$

to the tabular places before 1883. Immediately preceding 1883, $N = -11''$, and the two omitted terms are, for a short time,

$$-0''\cdot2 \cos 2D - 0''\cdot2 \cos (2D - g).$$

There seems to be a very faint trace of this in the values of Δ_2, and a more pronounced trace in the values of Δ'_2 (see periods 110–117), but I find no trace whatever in the coefficients of $\cos (2D - g)$ published in a previous paper. This is not unnatural, for quantities of $0''\cdot2$ cannot be obtained from the analysis of a very short period.

On the Comparison between the purely Theoretical and Observed Places of the Moon. By E. Nevill.

The appearance in the last number of the *Monthly Notices* (1903 November) of Mr. Cowell's Note on the Errors of the Moon's Tabular Longitude recalls the result of a more complete comparison between observation and the theory of the Moon as developed by Delaunay's method which I had intended to send to the Society six or seven years ago, but was prevented by reasons no longer existing.

In his interesting note Mr. Cowell remarks that Hansen's *Tables de la Lune* may be brought into much better accord with observation by applying corrections to bring its data into accord with the theoretical results obtained by Professor Hill and M. Radau, and especially by making use of the correction to one of the *Venus* terms of long period suggested by Professor Newcomb. But Mr. Cowell does not give the requisite figures to show clearly to what extent the discord between observation and theory is reduced by these corrections, nor does he give the data for showing distinctly what is still more important, the systematic periodical discordance remaining outstanding.

This omission suggested the abstracting and forwarding to the Society the results of the comparison between pure theory and observation yielded by my earlier investigation, which does clearly show these most important points and afford material for testing some of the conclusions stated by Mr. Cowell in his published paper.

It has been already shown that as far as the perturbations are due to the direct disturbing action of the Sun, there is no systematic difference of any importance between the theory embodied in Hansen's Tables and that which would be obtained from Delaunay's theory.

But when the supplementary terms are considered which arise from the disturbing action of the planets, the figure of the Earth, &c., then there exists grave divergency between the

results obtained by Hansen's method and those obtained by Professor Hill, M. Radau, and others, by an extension of Delaunay's method to the calculation of these terms.

It is to the values determined by this extension of Delaunay's method that the term "pure theory" is here applied. So far as they go, they are accurate; whether they are complete remains to be shown.

Considering my own investigation, the place of the Moon according to pure theory which was to be compared with observation, was obtained from Hansen's Tables by removing the empirical terms, correcting certain of his coefficients which later research had indicated as incorrect, and supplementing these results by certain terms due to the disturbing action of the planets which had been omitted from Hansen's Tables.

The corrections applied by myself differed in some respect from those employed by Mr. Cowell, as will be seen by the following comparison, in which I have followed the notation employed in the first part of this paper.

I. Correction to the secular and semisecular terms:

$$\{-1''\cdot14-29''\cdot17[T-1800]-3''\cdot86[T-1800]^2 \\ -21''\cdot47\sin(8V-13E+274^\circ 14')-0''\cdot92\sin(18V-16E \\ -g+30^\circ 12')\}\times\{1+0\cdot1098\cos g\}$$

Presumably, Mr. Cowell, like myself, derived this correction from Professor Newcomb's *Researches on the Motion of the Moon*, page 268 (Washington, 1878), and in the correction to the secular acceleration the figure $-3''\cdot76$ given in his paper is a misprint for $-3''\cdot86$, as given by Newcomb.

II. Corrections to the tabular values of certain terms.

Mr. Cowell employs the corrections *

$$+0''\cdot63\ \cos\Omega$$
$$+0\cdot445\sin\Omega\,.\cos g$$
$$+0\cdot391\sin(2g+2\varpi-\Omega)^*$$
$$-0\cdot62\ \sin(2g-4g'+2\omega-4\omega')^*$$

My own corrections were

$$+0''\cdot400\cos\Omega$$
$$+0\cdot445\sin\Omega\,.\cos g$$
$$+0\cdot391\sin(2g+2\varpi-\Omega)^*$$
$$-0\cdot620\sin(2g-4g'+2\omega-4\omega')^*$$

* In the case of the term depending on the argument cos Ω Mr. Cowell's adopted correction does not seem accurate.

Apparently he derives the correction

$$-0''\cdot63\cos\Omega$$

and

$$+0''.240 \sin (V-E)$$
$$+0''.100 \sin (E-J)$$
$$-0''.100 \sin (2M-E+49°)$$
$$-0''.200 \sin (\quad -g'+2\omega-2\omega')$$
$$+0''.100 \sin (\quad +2\omega-2\omega')$$
$$-0''.185 \sin (\ g \quad +2\omega \quad)$$

III. Additional terms omitted from Hansen's Tables.
Mr. Cowell uses the terms

$$-0''.348 \sin (2V-3E+85°)$$
$$+0''.316 \sin (g+2\varpi-3J+7°)$$
$$-0''.881 \sin (g+2\varpi-2J)$$
$$-0''.681 \sin (g+2\varpi+3V-5E)$$

from the term in the expression of nz, on page 8 of the *Tables de la Lune*

$$+7''.76 \sin (\Theta+184° 12')$$

but that is not the complete value in the expression for the longitude, as a portion is derived from the auxiliary function of s.

The value really employed by Hansen in his tables is

$$-0''.706 \cos \Omega$$

a value which was reduced on revision in the *Darlegung* to

$$-0''.563 \cos \Omega$$

But, as M. Radau points out, Hansen's tabular value is too large, from his having omitted certain portions of the complete value, and the tabular value should be reduced to

$$-0''.311 \cos \Omega$$

Professor Hill finds the value

$$-0''.293$$

the greater part coming from the effects of the motion of the plane of the variable ecliptic.

Hence the correction necessary to bring Hansen's Tables into accord with theory is very approximately

$$+0''.40 \cos \Omega$$

and not

$$+0''.63 \cos \Omega$$

as adopted by Mr. Cowell.

An examination of the *Darlegung* will show Mr. Cowell that Hansen calculated his adopted value from theory, and it does not in any way depend on the coefficient of the term

$$-0''.97 \cos \Omega$$

deduced by Airy from the observations of the years 1750–1830. (Mr. Cowell attributing it to the value $-1''.06$ deduced from the observation of the years 1750–1851 is an accidental oversight, as that value did not appear till some years after the tables were finished.)

The additional terms employed by myself were

$$
\begin{aligned}
&-0''.348 \sin (2V-3E+85^\circ)\\
&+0''.316 \sin (g+2\varpi-3J+7^\circ)\\
&-0''.881 \sin (g+2\varpi-2J)\\
&-0''.681 \sin (g+2\varpi+3V-5E)\\
&-0''.181 \sin (3V-4E+88^\circ)\\
&+0''.143 \sin (g+2\varpi-20V+19E+272^\circ)\\
&+0''.112 \sin (2E-4M)\\
&+0''.206 \sin (\quad 2\varpi-2J)
\end{aligned}
$$

The terms marked with an asterisk are of little importance for the present purpose, where only mean errors for the entire year's observations will be discussed, as their effects will, to all intents, vanish from such means.

The differences between the system of corrections employed by myself and those employed by Mr. Cowell are of little importance, and with the view of obtaining results as comparable as possible with those employed in his investigation I modified my results by striking out the terms not included by Mr. Cowell.

Hansen's Tables, as thus modified, may be regarded as representing the pure theory of the Moon as developed according to Delaunay's method.

And as thus brought into accord with Delaunay's theory they absolutely fail to represent the observations in any degree or for any period.

To bring the modified tables into accord with observation it is absolutely essential to introduce some extra term of very long period and with a very considerable coefficient—a term for which the Lunar Theory, as developed by Delaunay's method, fails to afford the slightest justification.

Mr. Cowell, adopting the suggestion of Professor Newcomb, employs the empirical terms

$$-15''.50 \cos (18V-16E-g) \times (1+2e \cdot \cos g)$$

The adoption of this term as a correction is objectionable, because it is impossible that such a term can exist. There may be one or more terms with periods very similar to that suggested by Professor Newcomb; in fact, there are ; and it is possible that their combined coefficients might be as large as that of Newcomb's empirical term ; but it is impossible that there can exist a term depending on the argument $\cos (18V-16E-g)$ or $\sin (18V-16E-g+90^\circ)$ with a coefficient of fifteen seconds of arc. For even admitting that the neglected higher powers of the disturbing forces might give rise to additional terms sufficient when united to form a combined coefficient of fifteen seconds, the manner in which these extra terms must depend on the terms due to the

lower powers of the disturbing forces prevents any material change in the value of the constant part of the argument. A change of many degrees is impossible.

But as Mr. Cowell has adopted this empirical correction of Newcomb's, and appears to regard it as fairly representing the observations, it has been adopted in the present comparisons, as it was one of the provisional solutions tested in my earlier investigations.

The apparent error of Hansen's unaltered tables was obtained by an extension of the method adopted by Airy in his discussion of the Greenwich observations (*Memoirs R.A.S.*, vol. xxix., 1860), the unknown corrections to the other coefficients being eliminated by their values deduced from the observations of the same period. Airy united his results into nine-year groups in order to eliminate large rapidly fluctuating errors. In my case, as these errors do not exist when the observations are referred to Hansen's Tables, each year was considered independently, in the same manner as developed on pages 392–398 of the memoir on "The Corrections required by Hansen's 'Tables de la Lune'" (*Memoirs R.A.S.*, vol. xlviii.).*

The values as obtained are given as "Errors of Hansen." Then follow the corrections for the terms adopted by Mr. Cowell and specified above. By adding the correction to the preceding error there is obtained the value of the outstanding error of the tables as corrected.

The results are :—

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.		Error of Hansen's Tables.	Cowell's Correction	Outstanding Errors of Tables.
1750	+ 3″·24	+ ″·59	+ 3″·83	1756	+ 1″·93	− 1″·31	+ ″·62
51	+ 2·47	− ·11	+ 2·36	57	+ 3·10	− 1·49	+ 1·61
52	− ·17	− ·33	− ·50	58	+ 3·29	− 1·03	+ 2·26
53	+ 1·04	− ·80	+ ·24	59	+ 5·04	− ·84	+ 4·20
54	+ ·18	− ·53	− ·35	1760	+ 3·39	− 1·17	+ 2·22
55	− ·28	− ·13	− ·41	61	+ 3·61	− 1·15	+ 2·46

* Owing to the proofs of the Memoir being revised sheet by sheet in Natal from rough notes, whilst the MSS. and remainder of the proofs were in England, it contains a certain number of errors, which in no sense affect the computations or results, but are perplexing as they stand.

The most important is on page 360, where the numerical values of the argument of long period due to Venus have been interchanged.

Minor errors are :

Page 360. Where $a_{IV.}$ is printed for $l_{IV.}$

Pages 374–379. Where a has been printed throughout for a, where

$$a' = -a = A - A'.$$

Pages 319, 402, and 404, where the argument of the term depending on

$$f = 2A - 2A' + 3l_{,} - 5l_{,,}$$

should be increased by 200°.

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.
1762	+ 2″·14	− ″·60	+ 1″·54
63	+ 4·52	− ·40	+ 4·12
64	+ ·22	− ·84	− ·62
65	+ 1·30	− ·98	+ ·42
66	+ 2·50	− ·62	+ 1·88
67	+ 3·89	− ·67	+ 3·22
68	+ 1·63	− 1·36	+ ·27
69	+ 2·21	− 1·69	+ ·52
1770	+ ·63	− 1·40	− ·77
71	+ 1·43	− 1·60	− ·17
72	+ 2·84	− 2·25	+ ·59
73	+ 2·40	− 2·47	− ·07
74	+ 2·98	− 2·09	+ ·89
75	+ ·01	− 2·02	− 2·01
76	+ 3·40	− 2·36	+ 1·04
77	+ 3·80	− 2·27	+ 1·53
78	+ 3·21	− 1·61	+ 1·60
79	+ 2·76	− 1·31	+ 1·45
1780	+ 2·01	− 1·54	+ ·47
81	+ 1·39	− 1·41	− ·02
82	+ ·66	− ·77	− ·11
83	+ 1·52	− ·60	+ ·92
84	+ 2·37	− ·99	+ 1·38
85	+ 1·81	− 1·01	+ ·80
86	+ 1·88	− ·60	+ 1·28
87	+ 2·27	− ·62	+ 1·65
88	+ 1·14	− 1·21	− ·07
89	− ·76	− 1·35	− 2·11
1790	− 1·59	− ·99	− 2·58
91	+ ·02	− 1·00	− ·98
92	+ ·99	− 1·46	− ·47
93	+ 2·40	− 1·42	+ ·98
94	+ 1·05	− ·80	+ ·25
95	+ 1·79	− ·56	+ 1·23
96	+ 5·84	− ·74	+ 4·10
97	+ 4·48	− ·43	+ 4·05
98	+ 2·31	+ ·36	+ 2·67
99	+ 3·43	+ ·70	+ 4·03

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.
1800	+ 2″·47	+ ″·52	+ 2″·99
01	+ 1·87	+ ·70	+ 2·57
02	+ 2·01	+ 1·33	+ 3·34
03	+ 2·09	+ 1·39	+ 3·48
04	+ 4·67	+ ·95	+ 5·62
05	+ 4·26	+ ·90	+ 5·16
06	+ 1·82	+ 1·26	+ 3·08
07	+ 2·65	+ 1·14	+ 3·79
08	+ 2·25	+ ·55	+ 2·70
09	− ·07	+ ·49	+ ·42
1810	+ 1·43	+ ·83	+ 2·26
11	+ 4·48	+ ·90	+ 5·38
12	+ 2·87	+ ·61	+ 3·48
13	+ 2·54	+ ·84	+ 3·38
14	+ 4·35	+ 1·60	+ 5·95
15	+ 5·39	+ 1·90	+ 7·29
16	+ 4·74	+ 1·77	+ 6·51
17	+ 4·99	+ 2·12	+ 7·11
18	+ 3·25	+ 2·83	+ 6·38
19	+ 3·99	+ 2·96	+ 6·95
1820	+ 4·99	+ 2·61	+ 7·01
21	+ 2·24	+ 2·65	+ 4·89
22	+ 2·71	+ 3·07	+ 5·78
23	+ 1·23	+ 2·92	+ 4·15
24	+ ·73	+ 2·31	+ 3·04
25	+ 1·13	+ 2·19	+ 3·32
26	− 1·78	+ 2·47	+ ·69
27	+ ·82	+ 2·28	+ 3·10
28	+ ·91	+ 1·70	+ 2·61
29	+ 2·59	+ 1·77	+ 4·36
1830	+ 2·88	+ 2·27	+ 5·11
31	+ 4·44	+ 2·25	+ 6·65
32	+ 5·01	+ 1·92	+ 6·93
33	+ 1·39	+ 2·19	+ 3·58
34	− ·14	+ 2·80	+ 2·46
35	+ 3·13	+ 2·88	+ 6·01
36	+ 1·76	+ 2·54	+ 4·30
37	+ 1·09	+ 2·69	+ 3·78

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.		Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.
1838	+ 2″·93	+ 3″·12	+ 6″·05	1869	+ 3″·96	− 5″·67	− 1″·71
39	+ 2·94	+ 2·93	+ 5·87	1870	+ 4·65	− 5·50	− ·85
1840	− ·23	+ 2·36	+ 2·13	71	+ 4·65	− 5·91	+ ·74
41	− ·87	+ 2·24	+ 1·37	72	+ 7·09	− 6·61	+ ·48
42	− 2·35	+ 2·44	+ ·09	73	+ 8·20	− 6·75	+ 1·45
43	− 1·23	+ 2·05	+ ·82	74	+ 8·89	− 6·72	+ 2·17
44	− ·85	+ 1·32	+ ·47	75	+ 9·42	− 7·39	+ 2·04
45	− 2·62	+ 1·15	− 1·47	76	+ 9·73	− 8·32	+ 1·41
46	− 1·00	+ 1·38	+ ·38	77	+ 8·99	− 8·75	+ ·24
47	+ 1·75	+ 1·04	+ 2·79	78	+ 8·20	− 9·03	− ·83
48	+ ·99	+ ·43	+ 1·42	79	+ 9·62	− 9·95	− ·33
49	− ·95	+ ·42	− ·53	1880	+ 10·16	− 11·14	− ·98
1850	− 1·51	+ ·75	− ·76	81	+ 10·27	− 11·74	− 1·47
51	− 2·26	+ ·55	− 1·71	82	+ 12·07	− 12·09	− ·02
52	− 1·55	+ ·05	− 1·50	83	+ 13·25	− 13·07	+ ·18
53	− 1·90	+ ·14	− 1·76	84	+ 13·94	− 14·21	− ·27
54	− 2·37	+ ·53	− 1·84	85	+ 14·65	− 14·68	− ·03
55	− 1·32	+ ·34	− ·98	86	+ 15·51	− 14·87	+ ·64
56	− 1·52	− ·27	− 1·79	87	+ 16·14	− 15·65	+ ·49
57	− 2·55	− ·36	− 2·87	88	+ 17·09	− 16·55	+ ·54
58	− 2·42	− ·25	− 2·67	89	+ 17·14	− 16·80	+ ·34
59	− 2·74	− ·76	− 3·50	1890	+ 17·96	− 16·88	+ 1·08
1860	− 3·42	− 1·70	− 5·12	91	+ 19·60	− 17·60	+ 2·00
61	− 3·01	− 2·03	− 5·04	92	+ 20·65	− 18·54	+ 2·11
62	− 2·51	− 2·17	− 4·68	93	+ 20·38	− 18·89	+ 1·49
63	− 1·30	− 2·88	− 4·18	94	+ 20·02	− 19·16	+ ·88
64	+ ·29	− 3·85	− 3·56	95	+ 20·45	− 20·15	− ·30
65	+ ·96	− 4·22	− 3·26	96	+ 20·51	− 21·38	− ·87
66	+ 2·55	− 4·26	− 1·81	97	+ 20·80	− 22·02	− 1·22
67	+ 3·45	− 4·81	− 1·36	98	+ 21·65	− 22·54	− ·89
68	+ 4·05	− 5·56	− 1·51	99	+ 23·80	− 23·75	+ ·05

My original comparison ended in 1894, the few additional years are added now. Summing into the means of groups of nine years:

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.		Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.
1754	+ 1″·65	− ″·56	+ 1″·09	1781	+ 2″·18	− 1″·28	+ ″·90
63	+ 2·96	− ·91	+ 2·05	90	+ ·82	− 1·05	− ·23
72	+ 1·95	− 1·92	+ ·03	1799	+ 2·92	+ ·36	+ 3·28

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.		Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.
	″	″	″		″	″	″
1808	+2·71	+ ·85	+3·56	1853	− 1·76	+ ·24	−1·52
17	+4·05	+2·16	+6·21	62	− 1·29	− 2·46	−3·75
26	+1·25	+2·33	+3·58	71	+ 6·26	− 6·10	+ ·16
35	+2·50	+2·59	+5·09	80	+10·69	−10·92	− ·23
44	− ·71	+1·60	+ ·89	89	+17·68	−16·72	+ ·96

It is obvious that the system of correction adopted by Mr. Cowell entirely fails to represent the observed errors of Hansen's Tables; and their application leaves the outstanding errors far larger than before, except for the period 1865–1898. Nor can it be improved by any further permissible alteration in the epoch, motion in mean longitude, or secular acceleration.

In fact, the empirical alteration suggested by Professor Newcomb, though it serves well to represent the observations since 1870, and fairly well agrees with the uncertain observations prior to 1720, fails to properly represent those for the century 1760–1860. When it was suggested, their failure was not so apparent, as it was supposed that Hansen's Tables closely represented the observations between 1750–1850, instead of there being discordances between the tables and observations of several seconds.

Comparing the actual mean errors of Hansen's Tables with the value of Newcomb's empirical correction as given on p. 268 of his *Researches on the Motion of the Moon*, the discordances between them are:

	Error of Hansen's Tables.	Newcomb's Correction.	Outstanding Error.		Error of Hansen's Tables.	Newcomb's Correction.	Outstanding Error.
	″	″	″		″	″	″
1650	−39·3	+40·4	+1·4	1780	+ 1·8	− 1·7	+ ·1
60	−36·5	+37·0	+ ·8	90	+ ·9	− 1·2	− ·3
70	−35·0	+33·6	+1·4	1800	+ 3·0	− ·3	+2·7
80	−31·7	+29·2	+2·5	10	+ 1·9	+ ·5	+2·4
90	−26·9	+24·5	+2·4	20	+ 2·6	+ 1·4	+4·0
1700	−22·3	+19·8	+2·5	30	+ 1·3	+ 1·7	+3·0
10	−14·8	+15·0	+ ·2	40	+ ·1	+ 1·2	+1·3
20	− 9·0	+10·7	+1·7	50	− ·5	+ ·1	− ·4
30	− 6·0	+ 6·7	+ ·7	60	− 1·9	− 2·4	−4·3
40	− 5·0	+ 3·4	−1·6	70	+ 6·0	− 6·1	− ·1
50	− ·4	+ ·9	+ ·5	80	+11·3	−11·2	+ ·1
60	+ 2·5	− ·6	+1·9	90	+18·1	−17·3	+ ·8
70	+ 2·4	− 1·5	+ ·9	1900	+25·0	−24·6	+ ·4

Any system of corrections yielding residuals of from two to four seconds of arc in the modern observations since 1800 cannot

be regarded as other than a complete failure. Mr. Cowell, by diminishing the tabular coefficient of Hansen's Venus term depending on the argument

$$(18V-16E-g+30^\circ)$$

by nearly a second, has augmented the discordance between the corrected tables and observation by nearly the same amount.

This failure of Newcomb's empirical correction to represent the observations between 1760–1860 was pointed out on pages 372–373 of *Memoirs R.A.S.*, vol. xlviii., in the discussion of the "Correction required by Hansen's 'Tables de la Lune,'" and it is this failure that swamps the improvement introduced by some of the other corrections employed by Mr. Cowell, which are real corrections which should be introduced into the tables, and do diminish the existing discordances.*

Considering next the correction to the coefficients of the terms depending on the argument, *sine mean anomaly* and *cosine mean anomaly*, deduced by comparing Hansen's Tables with the observations, and eliminating the errors of the coefficients of the other terms in the tabular expression for the longitude, by their values deduced from the observations of the same year, in the same manner as in the memoir on the corrections required by Hansen's "Tables de la Lune" (*Mem. R.A.S.*, vol. xlviii. pp. 392–98). These are given under the heading "A_s" for the coefficient of the *sine anomaly* and "A_c" for the coefficient of the *cosine anomaly*, with the sub-heading of "Error of Hansen's Tables."

This is followed by the correction assumed by Mr. Cowell to be applicable on p. 25 of his paper, including the constant correction of $-0''.50$.

Lastly, the errors of the tables after being thus corrected are given under the heading of "Outstanding Errors of Tables."

	A_s.			A_c.		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
	"	"	"	"	"	"
1750	+·97	−1·62	−·65	+·05	−·53	−·48
51	+1·65	−1·78	− 13	+1·12	−1·28	− ·16
52	+ ·19	−1·55	−1·36	− ·84	−1·75	−2·59
53	−1·96	−1·03	−3·03	−2·63	−1·89	−4·52

* It must not be supposed that Newcomb's empirical correction is the only available means of reconciling the tables and observations. This would be serious if it were so, as Newcomb's is theoretically impossible. But there are several terms known to exist in the complete expression for the perturbations of the Moon which will represent the existing discordances between the tables and observation with far greater accuracy than Newcomb's empirical correction, by assigning them smaller coefficients than that of Newcomb's terms; coefficients which, if larger than that which can be deduced from the first power of the disturbing forces, are yet not larger than may be found when the complete values of the terms are calculated. In fact there are several combinations of such terms available for the purpose.

	As.			Ac.		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
	"	"	"	"	"	"
1754	+1·05	−·39	+·66	−·33	−1·61	−1·94
55	−·14	+·19	+·05	−·19	−1·08	−1·27
56	−·45	+·50	+·05	−1·80	−·49	−2·29
57	−1·67	+·54	−1·13	−1·64	+·10	−1·54
58	+·82	+·32	+1·14	−·22	+·31	+·09
59	+1·44	−·08	+1·38	−·55	+·37	−·18
1760	−1·41	−·43	−1·84	−·89	+·26	−·63
61	−·91	−·54	−1·45	+1·75	+·14	+1·89
62	−2·22	−·72	−2·94	+·81	+·10	+·91
63	+1·93	−·63	−1·30	−2·18	+·22	−1·96
64	−·75	−·56	−1·31	−6·78	+·46	−6·32
65	+·28	−·61	−·43	−2·16	+·66	−1·50
66	+3·97	−·84	+3·13	−3·35	+·71	−2·64
67	+1·81	−1·26	+·55	−·73	+·51	−·22
68	+3·16	−1·68	+1·48	+1·88	+·05	+1·93
69	+2·72	−2·01	+·75	+·72	−·54	+·18
1770	+·75	−2·04	−1·29	−·60	−1·15	−1·75
71	−·65	−1·75	−2·40	−2·47	−1·56	−4·03
72	+2·77	−1·13	+1·64	+2·62	−1·59	+1·03
73	−·45	−·34	−·79	+1·13	−1·27	−·14
74	−·57	+·37	−·20	−1·06	−·62	−1·68
75	+·25	+·96	+1·21	−·33	+·22	−·11
76	−2·39	+1·02	−1·37	−2·51	+·90	−1·61
77	−1·10	+·81	−·29	−·85	+1·34	+·49
78	−·11	+·28	+·39	+1·23	+1·43	+2·66
79	+1·65	−·18	+1·47	−·06	+1·22	+1·16
1780	−·22	−·69	−·91	−2·16	+·94	−1·22
81	−1·18	−·77	−1·95	−·44	+·43	−·01
82	+·01	−·77	−·76	−1·68	+·14	−1·54
83	+·27	−·61	−·88	−2·11	−·02	−2·13
84	+·03	−·42	−·39	−1·04	−·08	−1·12
85	+1·05	−·39	+·66	+·02	−·14	−·12
86	+1·39	−·52	+·87	−2·52	−·32	−2·84
87	+1·90	−·72	+1·18	−1·73	−·67	−2·40
88	−·47	−·98	−1·45	−·85	−1·14	−1·99
89	+1·10	−1·05	+·05	−·88	−1·69	−2·55

	Ar.			Ar.		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
	s	s	s	s	s	s
1790	+ ·11	− ·90	− ·79	+ 2·06	− 1·99	+ ·07
91	− ·31	− ·49	− ·80	+ ·58	− 1·98	− 1·40
92	+ 1·16	+ ·05	+ 1·21	+ ·62	− 1·61	− 1·19
93	− ·42	+ ·60	+ ·18	+ 1·46	− ·90	+ ·56
94	− ·67	+ ·95	+ ·28	+ 1·56	− ·03	+ 1·53
95	− 1·94	+ ·93	− 1·01	+ ·10	+ ·77	+ ·87
96	+ ·69	+ ·51	+ 1·20	− 4·82	+ 1·35	− 3·47
97	− 2·72	− ·12	− 2·84	− 2·53	+ 1·55	− ·98
98	+ 1·66	− ·90	+ ·76	− 4·21	+ 1·33	− 2·88
99	+ ·64	− 1·53	− ·89	− 3·52	+ ·76	− 2·76
1800	+ 3·51	− 1·88	+ 1·63	− 1·52	+ ·30	− 1·12
01	+ 3·59	− 1·94	+ 1·65	− ·49	− ·21	− ·70
02	+ 4·89	− 1·73	+ 3·16	− ·34	− ·48	− ·82
03	+ ·24	− 1·35	− 1·11	+ 4·46	− ·54	+ 4·12
04	+ ·90	− ·98	− ·08	+ ·32	− ·46	− ·14
05	+ 2·24	− ·99	+ 1·15	− 1·48	− ·36	− 1·84
06	+ ·91	− 1·09	− ·18	− 1·36	− ·38	− 1·74
07	+ 2·26	− 1·16	+ 1·10	− 2·59	− ·48	− 3·07
08	+ 2 52	− 1·08	+ 1·44	+ ·37	− ·63	− ·26
09	+ ·94	− ·77	+ ·17	+ 2·41	− ·76	+ 1·65
1810	+ 1·89	− ·26	+ 1·63	+ 1·90	− ·71	+ 1·19
11	+ 1·51	+ ·31	+ 1·82	+ 1·04	− ·34	+ ·70
12	+ 2·24	+ ·78	+ 3·02	+ 1·24	+ ·28	+ 1·52
13	+ ·70	+ ·95	+ 1·65	+ ·45	+ 1·03	+ 1·48
14	− 3·21	+ ·78	− 2·43	+ 4·36	+ 1·88	+ 6·24
15	− ·91	− ·30	− ·61	− 4·34	+ 2·25	− 2·09
16	+ ·20	− ·38	− ·18	− 2·56	+ 2·31	− ·25
17	+ 2·03	− 1·02	+ 1·05	− ·82	+ 1·93	+ 1·11
18	+ 2·49	− 1·46	+ 1·03	− 4·11	+ 1·24	− 2·87
19	+ 1·71	− 1·59	+ ·12	− 1·01	+ ·49	− ·52
1820	+ 2·00	− 1·38	+ ·62	− 1·93	− ·40	− 2·33
21	+ ·44	− ·92	− ·48	+ 2·34	− ·97	+ 1·35
22	+ 1·43	− ·40	+ 1·03	+ 3·62	− 1·20	+ 2·42
23	− 2·01	− ·01	− 2·02	+ ·50	− 1·15	− 65
24	+ ·33	+ ·16	+ ·49	− ·32	− ·98	− 1·30

	Δc.			Δc.		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
	"	"	"	"	"	"
1825	+2·32	+ ·14	+2·46	− ·04	− ·74	− ·78
26	+ ·17	− ·01	+ ·16	+1·89	− ·59	+1·30
27	− ·04	− ·16	− ·20	+ ·39	− ·58	− ·19
28	−2·50	− ·20	−2·70	+4·84	− ·56	+4·28
29	−1·89	− ·07	−1·96	− ·52	− ·52	−1·04
1830	−1·16	+ ·14	−1·02	+3·39	− ·29	+3·10
31	− ·72	+ ·36	− ·36	+ ·28	+ ·13	+ ·41
32	−2·74	+ ·38	−2·36	+1·62	+ ·71	−2·33
33	−1·51	+ ·14	−1·37	−1·85	+1·29	− ·56
34	+ ·23	− ·38	− ·15	−1·69	+1·68	− ·01
35	+3·84	−1·04	+2·80	+ ·06	+1·73	+1·79
36	+1·10	−1·94	− ·84	−1·33	+1·39	+ ·06
37	+1·99	−2·28	− ·24	+ ·52	+ ·73	+1·25
38	+ ·73	−2·35	−1·62	− ·63	− ·10	− ·73
39	+1·51	−2·09	− ·58	+1·05	− ·86	+ ·19
1840	+1·51	−1·53	− ·02	+3·17	−1·36	+1·81
41	·00	− ·81	− ·78	−1·55	−1·50	−3·05
42	+1·50	− ·23	+1·27	+1·30	−1·27	+ ·03
43	−1·00	+ ·15	− ·85	−1·10	− ·78	−1·88
44	−1·89	+ ·14	−1·75	− ·68	− ·20	− ·86
45	+ ·95	+ ·10	+1·05	+1·82	+ ·28	+2·10
46	−1·60	− ·15	−1·75	+1·21	+ ·58	+1·79
47	− ·48	− ·38	− ·86	−1·26	+ ·71	− ·55
48	− ·56	− ·46	−1·02	+1·39	+ ·69	+2·08
49	−1·28	− ·38	+ ·90	+1·14	+ ·70	+1·84
1850	+ ·45	− ·13	+ ·27	−2·47	+ ·51	−1·96
51	+ ·04	− ·05	− ·01	−1·87	+ ·66	−1·21
52	− ·31	− ·07	− ·38	−2·25	+ ·90	−1·32
53	+1·05	− ·30	+ ·75	−1·88	+1·04	− ·84
54	+1·79	− ·71	+1·08	−1·32	+1·00	− ·32
55	+1·23	−1·16	+ ·07	− ·72	+ ·71	− ·01
56	+1·83	−1·49	+ ·34	− ·55	+ ·14	− ·41
57	+2·32	−1·54	+ ·78	+1·46	− ·58	+ ·88
58	+ ·77	−1·21	− ·44	+1·21	−1·29	− ·08
59	+ ·59	− ·67	− ·08	+1·82	−1·76	+ ·06

	Δl.			Δc.		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
	"	"	"	"	"	"
1860	+ ·08	+ ·08	+ ·16	+ 1·73	− 1·88	− ·16
61	− ·42	+ ·73	+ ·31	+ 1·28	− 1·62	− ·34
62	− ·56	+ 1·10	+ ·54	+ 1·01	− 1·09	− ·08
63	− ·86	+ 1·17	+ ·31	+ 1·88	− ·50	+ 1·38
64	− ·66	+ ·83	+ ·17	+ ·67	+ ·02	+ ·69
65	− ·76	+ ·30	− ·46	+ ·05	+ ·26	+ ·31
66	− ·41	− ·29	− ·70	+ ·21	+ ·21	+ ·42
67	− ·20	− ·99	− ·92	− ·04	− ·06	− ·10
68	+ 1·22	− 1·06	+ ·23	− ·29	− ·45	− ·74
69	+ 1·22	− ·92	+ ·16	+ ·98	− ·77	+ ·21
1870	+ ·94	− ·92	+ ·02	+ ·37	− ·96	− ·59
71	+ 1·67	− ·87	+ ·90	+ ·28	− ·94	− ·66
72	+ 1·32	− ·93	+ ·49	+ ·69	− ·83	− ·14
73	+ 1·14	− 1·12	+ ·02	+ ·63	− ·77	− ·14
74	+ 2·12	− 1·39	+ ·73	+ ·85	− ·86	− ·01
75	+ 2·26	− 1·63	+ ·63	+ 2·32	− 1·18	+ 1·14
76	+ 2·22	− 1·69	+ ·53	+ 2·43	− 1·64	+ ·79
77	+ 2·64	− 1·45	+ 1·19	+ 3·04	− 2·09	+ ·95
78	+ ·26	− ·98	− ·72	+ 2·93	− 2·37	+ ·56
79	+ ·56	− ·34	+ ·22	+ 3·26	− 2·37	+ ·89
1880	+ ·03	+ ·33	+ ·36	+ 3·46	− 2·04	+ 1·42
81	− ·62	+ ·72	+ ·10	+ 3·82	− 1·44	+ 2·38
82	− ·17	+ ·79	+ ·62	+ 1·54	− ·73	+ ·81
83	+ ·05	+ ·48	+ ·53	+ 1·07	− ·12	+ ·95
84	+ ·02	− ·09	− ·07	+ 1·19	+ ·17	+ 1·02
85	− ·45	− ·72	− 1·17	− ·28	+ ·12	− ·16
86	+ 1·65	− 1·25	+ ·40	− ·43	− ·29	− ·72
87	+ 1·78	− 1·52	+ ·26	+ ·20	− ·91	− ·71
88	+ 1·97	− 1·49	+ ·44	+ 1·39	− 1·54	− ·15
89	+ 1·88	− 1·19	+ ·69	+ 1·88	− 2·00	− ·12
1890	+ 1·85	− ·79	+ 1·06	+ 2·29	− 2·26	+ ·03
91	+ ·34	− ·46	− ·12	+ 1·88	− 2·31	− ·43
92	+ ·45	− ·26	+ ·19	+ 4·11	− 2·26	+ 1·85
93	− ·91	− ·26	− 1·17	+ 3·67	− 2·28	+ 1·39
94	+ ·43	− ·35	+ ·08	+ 3·63	− 2·41	+ 1·22

	Δα			Δδ		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
	″	″	″	″	″	″
1895	+ ·34	− ·41	− ·07	+2·26	−2·73	− ·47
96	+ ·63	− ·34	+ ·29	+3·63	−3·12	+ ·51
97	+ ·98	− ·06	+ ·92	+3·34	−3·85	− ·51
98	+ ·70	+ ·35	+1·05	+4·17	−4·06	+ ·11
99	+ ·66	+ ·80	+1·46	+5·12	−3·96	+1·16

These values exhibit large and rapidly fluctuating variations especially in the earlier period; but it is to only a minor degree that these can be attributed to outstanding errors in the tables. But though by the change of tables there has been eliminated the greater part of the large rapidly fluctuating errors of Airy's values for these coefficients, yet the observed places remain unaltered and retain the full effect of any errors due to faulty reduction.

An examination of Airy's observed places indicates that there is much room for improvement in the reduction of the observations to the adopted observed places, and that they are affected by considerable errors varying with the seasons of the year, which exert exceptional influence on the values deduced for the apparent values of the coefficients of the Moon's *mean anomaly*. Though many of the exceptional values appear due to errors in the adopted elements of reduction of the observations with the transit when its faulty condition was not suspected and its pivots had become much worn, yet the main portion of the irregular fluctuations appear to be due to outstanding errors in the reduction of the quadrant observations. The north polar distances derived from this instrument show many irregular fluctuations, varying with the seasons, which can only be ascribed to uneliminated errors in the adjustment of the quadrant; but there are strong indications of systematic error in the elements used to reduce the observations.

In some instances exceptionally large discordances can be attributed to inexperienced observers having used the instrument for a fraction of the year.

In the absence of all means of referring to the details of the observations—and the printed volumes of reductions afford very little of the requisite data—nothing can be done to eliminate the effects of these irregular or semi-accidental errors, though something could be done to eliminate the effects of the more systematic errors.

An examination of the outstanding errors for the years 1851–1898, when the observations made with the Greenwich Transit Circle are all that can be desired, shows clearly that there remain outstanding considerable periodic errors beyond those eliminated by Mr. Cowell's system of corrections. The fact that

the adopted correction, though in the right direction, is insufficient in amount to eliminate the actual observed error, indicates very distinctly that Mr. Cowell has adopted the right arguments for the terms constituting his system of corrections, but that his coefficients are too small. By increasing all the coefficients of the terms due to the action of the planets by a quarter the observed values would be much better represented.

To better eliminate the errors in the observations themselves, and yet leave the data in a form which enables the character of the outstanding errors to be readily understood, take the mean of every three years in the preceding table of values. The results are—

	As.				Ac.			
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Outstanding Errors. Nine-year Mean.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Outstanding Errors. Nine-year Mean.
	"	"	"		"	"	"	
1751	+ ·94	−1·65	− ·71	...	+ ·11	−1·19	−1·08	...
54	− ·35	− ·41	− ·76	...	−1·05	−1·53	−2·58	...
57	− ·43	+ ·45	− ·02	...	−1·22	− ·03	−1·25	...
60	− ·29	− ·35	− ·64	−·55	+ ·10	+ ·26	+ ·36	−1·12
63	− ·35	− ·64	− ·99	...	−2·72	+ ·26	−2·46	...
66	+2·02	− ·90	+1·12	...	−2·08	+ ·63	−1·45	...
69	+2·22	−1·91	+ ·31	+·31	+1·67	− ·55	+1·12	− ·49
72	+ ·56	−1·07	− ·51	...	+ ·33	−1·47	−1·14	...
75	− ·90	+ ·78	− ·12	...	−1·30	+ ·17	−1·13	...
78	+ ·22	+ ·30	+ ·52	+·27	+ ·11	+1·33	+1·44	− ·19
81	− ·46	− ·74	−1·20	...	−1·43	+ ·54	− ·89	...
84	+ ·27	− ·47	+ ·20	...	−1·04	− ·08	−1·12	...
87	+ ·94	− ·74	+ ·20	−·17	−1·70	− ·71	−2·41	−1·61
90	+ ·30	− ·81	− ·51	...	+ ·58	−1·88	−1·30	...
93	+ ·02	+ ·53	+ ·55	...	+1·15	− ·85	+ ·30	...
96	−1·32	+ ·45	− ·87	+·06	−2·42	+1·22	−1·20	−1·06
1799	+1·94	−1·44	+ ·50	...	−3·08	+ ·80	−2·28	...
1802	+2·91	−1·67	+1·24	...	+1·32	− ·41	+ ·91	...
05	+1·35	−1·02	+ ·33	+·82	− ·84	− ·40	−1·24	− ·30
08	+1·91	−1·00	+ ·91	...	+ ·06	− ·62	− ·56	...
11	+1·88	+ ·28	+2·16	...	+1·39	− ·26	+1·13	...
14	− ·53	+ ·68	+ ·13	+·98	+ ·16	+1·72	+1·88	+ ·78
17	+1·59	− ·95	+ ·64	...	−2·50	+1·83	− ·67	...
20	+1·40	−1·30	+ ·10	...	− ·20	− ·29	− ·49	...
23	− ·08	− ·08	− ·16	+·25	+1·27	−1·11	+ ·16	− ·07
26	+ ·82	− ·01	+ ·81	...	+ ·75	− ·64	+ ·11	...

	Δs.				Δc.			
	Errors of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Outstanding Errors. Nine-year Mean.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Outstanding Errors. Nine-year Mean.
1829	−1·85	− ·04	−1·89	...	+2·57	− ·46	+2·11	...
32	−1·66	+ ·29	−1·37	−·89	+ ·03	+ ·71	+ ·74	+1·15
35	+1·72	−1·12	+ ·60	...	− ·99	+1·60	+ ·61	...
38	+1·41	−2·22	− ·81	...	+ ·31	− ·08	+ ·23	...
41	+1·01	− ·86	+ ·15	−·39	+ ·97	−1·38	− ·41	− ·13
44	− ·65	+ ·13	− ·52	...	+ ·01	− ·23	− ·22	...
47	− ·88	− ·33	−1·21	...	+ ·45	+ ·66	+1·11	...
50	+ ·59	− ·20	+ ·39	−·11	−1·07	+ ·62	− ·45	− ·08
53	+ ·84	− ·36	+ ·48	...	−1·88	+ ·99	− ·89	...
56	+1·79	−1·40	+ ·39	...	+ ·06	+ ·09	+ ·15	...
59	+ ·48	− ·60	− ·12	+·20	+1·59	−1·65	− ·06	+ ·25
62	− ·69	+1·03	+ ·34	...	+1·19	− ·52	+ ·67	...
65	− ·61	+ ·28	− ·33	...	+ ·31	+ ·16	+ ·47	...
68	+ ·75	− ·92	− ·17	−·03	+ ·22	− ·43	− ·21	− ·07
71	+1·31	− ·91	+ ·40	...	+ ·45	− ·91	− ·46	...
74	+1·51	−1·38	+ ·13	...	+1·27	− ·94	+ ·35	...
77	+1·71	−1·37	+ ·34	+·23	+2·80	−2·03	+ ·77	+ ·89
80	− ·01	+ ·24	+ ·23	...	+3·51	−1·95	+1·56	...
83	− ·33	+ ·39	+ ·36	...	+1·27	− ·23	+1·04	...
86	+ ·99	−1·16	− ·17	+·31	− ·17	− ·36	− ·53	+ ·16
89	+1·89	−1·16	+ ·73	...	+1·85	−1·87	− ·02	...
92	− ·04	− ·33	− ·37	...	+3·22	−2·28	+ ·94	...
95	+ ·47	− ·37	+ ·10	+·29	+3·19	−2·75	+ ·44	+ ·54
98	+ ·79	+ ·36	+1·15	...	+4·21	−3·96	+ ·25	...

It will be seen that the outstanding errors of the three-year groups are considerable and periodical; so that a closer examination shows that, in addition to a term with a period equal to that of the lunar perigee, there are other considerable terms with periods of about four, six, and eleven years. All signs of these disappear when the observations are divided into nine-year groups and the means taken so as to obtain the residuals headed "Nine-year Means." But these latter show very clearly, in turn, that there exist considerable inequalities of about twenty and forty years' period.

It is not a case of tracing one or two missing terms, as Mr. Cowell appears to suggest, but of disentangling and determining the values of at least seven or eight. Some are associated in pairs which, being of similar period, are difficult to determine,

and when deduced from a short period of observations like those for the years 1883–1898 come out as a single term, but with a compound coefficient very different from the theoretical value of either term. Thus Mr. Cowell assumes Radau's theoretical value for the term of the form

$$-0''\cdot68 \sin (g+2\varpi+3V-5E)$$

The observations for the period 1862–1888 yield the values

$$-0''\cdot23 \sin (g+2\varpi+3V-5E)$$

(corrections to Hansen's "Tables de la Lune," *Memoirs R.A.S.*, vol. xlviii.).

In fact if the value of this coefficient be approximately calculated it will be found that the coefficient yielded by the observations is

1896–1882	Coefficient	$=-0''\cdot32$
1881–1866	,,	$=-0\cdot20$
1865–1851	,,	$=-0\cdot54$

But if the earlier years are similarly treated the values will be found to rise to over $-1''\cdot20$; the mean of the whole series of years from 1751–1896 being approximately

$$-0''\cdot88 \sin (g+2\varpi+3V-5E)$$

It had been my intention to copy out from my early investigation the values of the coefficients of the mean anomaly derived from the expression

$$\text{Observed Errors of Airy's Corrected Table} = a \cos g + b \sin g$$

where Airy's Tables have been corrected for the errors in the coefficients of the terms which in Mr. Cowell's list bear the numbers

1 7 8 10 14 15 16 17 18 21 22 23 24 29
30 39 50 51 53 54 55 56 59 65 66 68 70

all of which have some influence, though in cases only indirect, on the values of the coefficients a and b; instead of merely the three terms 15, 39, 51 used by Mr. Cowell; and so have afforded more accurate data for the basis of investigation. But further consideration makes it doubtful whether these comparisons could be put into a form readily available for comparison with the results obtained by Mr. Cowell by means of his fourteen-month groups. For this reason it has been thought sufficient to give the means for each three-year group, so that the nature of the outstanding errors of less than nine years' period can be better seen before taking the mean of the nine-year groups, from which these irregularities disappear, for comparison with the values given by Mr. Cowell.

Apparently the corrections applied by Mr. Cowell for the "five planetary terms (or figure of the Earth)" are the following:

$$+0''.316 \sin (g + 2\varpi - 3J + 7°)$$
$$-0.881 \sin (g + 2\varpi - 2J)$$
$$-0.681 \sin (g + 2\varpi + 3V - 5E)$$

and

$$+1'.562 \sin (A + 30°) \cos g$$
$$+1.039 \sin \Omega . \cos g$$

No correction seems to have been applied by Mr. Cowell for the effects of the errors in tabular mean longitude.

	b=Coeff. of sin Anomaly.			a=Coeff. of cos Anomaly.		
	Airy's Values.	Cowell's Corr.	Corr. Values.	Airy's Values.	Cowell's Corr.	Corr. Values.
	"	"	"	"	"	"
1751	+2·55	−1·28	+1·27	+ ·03	−1·13	−1·10
54	−1·04	+ ·11	− ·93	+ ·54	−1·07	− ·50
57	+2·45	+1·04	+3·49	−1·07	+1·07	− ·00
60	+1·82	+ ·07	+1·89	+ ·28	+1·39	+1·67
63	−1·86	− ·13	−1·99	−1·15	+ ·88	− ·27
66	+3·32	− ·34	+2·98	− ·05	+ ·74	+ ·69
69	+3·61	−1·51	+2·10	− ·84	− ·80	−1·64
72	+ ·75	− ·63	− ·12	+2·38	−1·61	+ ·77
75	− ·16	+1·36	+1·20	−2·62	+ ·68	−1·94
78	+1·61	+ ·78	+2·39	−2·71	+2·04	− ·67
81	− ·19	− ·27	− ·46	+ ·64	+ ·64	+1·28
84	+2·37	+ ·08	+2·45	−1·24	− ·55	−1·59
87	+1·02	− ·22	+ ·80	− ·39	−1·54	−1·93
90	+1·74	− ·40	+1·34	+1·34	−2·76	−1·42
93	+2·17	+1·10	+3·27	− ·30	−1·24	−1·54
96	−2·07	+1·01	−1·06	−1·52	+1·22	− ·30
1799	+2·95	−1·03	+1·92	−1·18	+ ·41	− ·76
1802	+4·70	−1·23	+3·47	+2·22	−1·61	− ·61
05	+ ·77	− ·49	+ ·28	+1·14	−2·01	− ·87
08	+4·12	− ·58	+3·54	+1·83	−2·27	− ·44
11	+2·88	+ ·81	+3·69	+2·14	−1·53	+ ·61
14	+ ·18	+1·28	+1·46	+1·23	+1·04	+2·27
17	+3·42	− ·52	+2·90	− ·25	+ ·94	+ ·69
20	+2·17	− ·88	+1·29	+ ·45	−2·00	−1·55
23	+ ·12	+ ·49	+ ·60	+3·10	−3·30	− ·20
26	+1·78	+ ·49	+2·29	+2·81	−2·79	− ·02

	b=Coeff. of sin Anomaly.			a=Coeff. of cos Anomaly.		
	Airy's Values.	Cowell's Corr.	Corr. Values.	Airy's Values.	Cowell's Corr.	Corr. Values.
	"	"	"	"	"	"
1829	+1·12	+ ·43	+1·56	+3·14	−2·26	+ ·88
32	+ ·42	+ ·88	+1·30	+2·79	− ·50	+2·29
35	+4·79	− ·54	+4·26	−1·12	+ ·52	− ·60
38	+4·29	−1·85	+2·44	+2·14	−1·76	+ ·38
41	+1·11	− ·31	+ ·80	+3·92	−3·70	+ ·22
44	+ ·56	+ ·74	+1·30	+2·62	−2·47	+ ·15
47	+1·68	+ ·12	+1·80	+ ·79	−1·06	− ·37
1850	+1·00	+ ·32	+1·32	+ ·27	− ·34	− ·07

These values show clearly that there are outstanding considerable corrections having arguments of less than twelve years period, besides those considered by Mr. Cowell.

Summing into the means of nine-year groups, so as to eliminate the greater part of the effect of the outstanding terms of less than twelve years' period, there are obtained the results

	b=Coeff. of sin Anomaly corrected.	a=Coeff. of cos Anomaly corrected.		b=Coeff. of sin Anomaly corrected.	a=Coeff. of cos Anomaly corrected.
	"	"		"	"
1757	+1·48	+ ·36	1811	+2·70	+ ·81
66	+1·03	− ·40	20	+1·60	− ·36
75	+1·24	− ·61	29	+1·71	+1·06
84	+ ·93	− ·82	38	+2·50	+ ·00
1793	+1·15	−1·09	47	+1·47	− ·07
1802	+1·56	− ·35			

It is not possible to give for comparison the corresponding values obtained by Mr. Cowell in the absence of sufficiently exact information as to the mean date of his groups.

The two mean differences between the results is that the more completely corrected tables show a much smaller correction to the coefficient of the sine anomaly than the imperfectly corrected tables of Mr. Cowell, or −1″·58 instead of −2″·03, and indicate a decided correction to Damoiseau's mean anomaly as far as can be judged by the use of these coefficients.

Of course these coefficients a and b, as determined from year to year, are not the same as the true coefficients A_c and A_s of the terms depending on the cosine and sine of the *mean anomaly* in the expression for the Moon's longitude, though Mr. Cowell appears to use them as if they were, but are really complex quantities made up of the errors of the *equation of the centre*, *annual equation*, *minor parallactic terms*, *evection*, *parallax*, and a good many other factors which do not disappear from even the nine-year groups. The difference between is clearly shown by the following comparison between the values of the coefficients a

and b and the true coefficients of the terms depending on the *mean* anomaly derived from the observations for the years 1892–1896 by the method detailed in the memoir on the "Corrections to Hansen's 'Tables de la Lune'" (*Memoirs R.A.S.*, vol. xlviii. pp. 301–310).

	Coefficient of sine Anomaly.		Coefficient of cosine Anomaly.	
	b	Δb	a	Δa
	"	"	"	"
1892	− ·96	+ ·45	+ 3·90	+ 4·11
1893	− 1·58	− ·91	+ 2·88	+ 3·67
1894	− ·24	+ ·44	+ 2·98	+ 3·63
1895	− ·23	+ ·34	+ 2·01	+ 2·26
1896	− ·18	+ ·63	+ 2·60	+ 3·64

Mr. Cowell employs the value of the coefficient a as if it truly represented the value indicated by the observations of the coefficient of the mean anomaly on the expression for the Moon's longitude, and suggests that the application of Newcomb's empirical term

$$-15''\cdot50 \cos (A) (1 + 2e \cos g)$$

to the expression for the Moon's longitude would reconcile the observed and calculated values, or approximately

1750	$a = +0''\cdot41$	$(-1''\cdot72 \cos A) = +0''\cdot86$	corrected value $= +1''\cdot27$
1800	$= -0\cdot45$	$= +1\cdot70$	$= +1\cdot25$
1850	$= +0\cdot50$	$= +0\cdot58$	$= +1\cdot08$
1891	$= +2\cdot16$	$= -0\cdot96$	$= +1\cdot20$

This result if sound would indicate that Damoiseau's epoch of mean anomaly was about 12″ too large, but its secular motion nearly correct. Mr. Cowell apparently regards this as some confirmation of the accuracy of Newcomb's empirical correction.

But the result can only be regarded as satisfactory by disregarding the enormous discordances between theory and observation which are introduced by these assumptions employed by Mr. Cowell. This is clearly shown by the following comparison between the observations and Damoiseau's Tables after correcting them so as to bring the coefficient of the perturbations by the Sun into accord with theory. The table shows, first, the results of the comparison with Damoiseau's Tables without any long-period term; secondly, after the addition of the theoretical value of Hansen's direct Venus term with the argument sin (A + 30°); and thirdly, the result after adding also Newcomb's empirical term with the argument cos A.

	(I)–(Obs.)				(I)–(Obs.)		
1750	+″·50	+″·04	+″·79	1850	−″·1	−″·01	−″·29
60	+·25	+·63	+·74	40	−·0	−·04	−·01
70	−·02	+·2	+·49	30	−·07	−·03	−·08
80	−·35	−·36	+·38	60	−·37	−·09	−·03
1790	−·44	−·37	+·36	70	−·24	−·08	−·08
1800	−·05	−·37	+·37	80	−·35	−·07	−·01
10	−·1	−·08	+·41	1890	−·31	−·05	−·01
20	+·19	−·05	+·18				

That Damoiseau's tabular value for the mean anomaly does not represent the observations is shown by the following comparison between the observations and Damoiseau's Tables as corrected.

	(I)–(I)	Cowell's Corr.	Damoiseau corrected.	Airy's Values.
1750	+″·15	−″·21	−″·06	—
60	+·06	+·30	+·36	+·34
70	+·10	−·54	−·44	−·46
80	+·24	+·60	+·84	+·16
1790	+·05	−·22	−·17	−·30
1800	−·32	+·37	−·35	+·61
10	+·31	−·87	+·04	+·30
20	+·05	−·10	+·43	+·25
30	+·33	−·31	+·32	+·04
40	+·28	−·70	+·28	+·38
50	+·55	+·21	+·76	+·80
60	+·69	−·52	+·17	—
70	+·03	+·74	+·77	—
80	+·45	−·67	+·78	—
90	+·31	+·74	+·65	—

These results are in accord with those obtained by Airy, whose results are approximately corrected to reduce them to the same basis, only using the correction for the terms depending on the Moon's node derived from observation. The improved consistency due to using the larger values yielded by observation for the terms with periods close to that of the node is marked.

The improvement in the values deduced by Mr. Cowell by applying Newcomb's empirical term simply means that having

thrown the tabular values into discord with observations by applying to the tables the term

$$+1''\cdot562 \sin (A+30^\circ) \cos g$$

to rectify the evil, and remove the discordances thus introduced, and to neutralise the effects of Damoiseau's erroneous value for the motion in mean anomaly, he has been obliged to introduce the consequent empirical term

$$-1''\cdot722 \cos (A) \cos g$$

for which theory affords no justification.

The only satisfactory value for the coefficient of the cosine mean anomaly is that obtained by eliminating the errors in the tabular value of the mean longitude by means of their observed errors. If that is done in the case of the observed coefficients of Damoiseau's mean anomaly, the apparent agreement at the different epochs at once disappears, and the need for introducing a considerable correction becomes manifest. Nor can it be assumed that the corrections which ought to be introduced for the errors of the tabular longitude are obviated by the existence of corresponding opposite errors in the tabular longitude of *perigee*; because if that were so, corresponding symmetrical errors would be found in the coefficient of the cosine of the evection, and these do not exist.

There is only one further point to which I would draw Mr. Cowell's attention, and that is to the value assigned by him to the correction required by Airy's value of the principal figure of the Earth term. Mr. Cowell combines this term with that due to the *nutation*, and apparently deduces a correction to the combined coefficients by comparing them with the sum of the values assigned respectively by Hill and Hansen to these terms.

Thus

Figure of Earth.		Nutation.		Sum.
Airy	$= +6''\cdot60 \sin \Omega$	Airy	$= -16''\cdot78 \sin \Omega$	$= -10''\cdot18 \sin \Omega$
Hill	$= +7\cdot67 \sin \Omega$	Hansen	$= -17\cdot33 \sin \Omega$	$= -9\cdot66 \sin \Omega$
			Difference ...	$= +0\cdot52$

This difference, which apparently Mr. Cowell takes as being only $+0''\cdot45$, is adopted as the correction required by Airy's value.

But on reconsideration Mr. Cowell will see that this requires revision. For Airy determines the observed R.A. of the Moon by direct comparison with the apparent R.A. of certain standard stars in deducing whose apparent place he uses the value of the nutation employed in the *Tabulæ Regiomontanæ*. As this is $= -16''\cdot78 \sin \Omega$, he is bound to use the same value for the

place of the Moon, and any error in the adopted value of the nutation practically disappears from the result. This point has been already dealt with by Airy in his *Reduction of the Greenwich Lunar Observations* (1831–1851), p. xxiv.

Hence the correction to be applied to Airy's value is the difference between Airy's value and Hill's value for the figure of the Earth Term or

$$+1''\cdot27 \sin \Omega$$

and not

$$+ \cdot25 \sin \Omega$$

the value employed by Mr. Cowell.

By greater skill and ingenuity Mr. Cowell may be more successful than myself in dealing with the investigation of the corrections required by the lunar tables in the rather fragmentary manner he has adopted. I failed, and I fear that further experience will lead him to the same conclusion as myself—that no successful result can be expected without taking the problem up as a whole and simultaneously taking into consideration the inequalities of all kinds and periods.

Natal Observatory:
1904 January 21.

Photographic Chart of the Heavens to Argelander's scale 1° = 20 mm., with special reference to the Milky Way. By J. Franklin-Adams; *with description of Lenses and Mounts* by H. Dennis Taylor and Alfred Taylor.

Preliminary Notes by J. Franklin-Adams.

To Sir David Gill am I indebted for the suggestion of photographic research into the structure of the Milky Way. The tentative and experimental method by which the work has been carried out may be interesting in detail.

During the winter of 1897–8 Mr. Dennis Taylor, of Messrs. Cooke & Sons, was asked to design a 6-inch lens of short focus and considerable field-focus, $\frac{f}{4}$. During the autumn of 1898 the lens was temporarily mounted upon an 8-inch German equatorial, also by Cooke & Sons: it gave fair definition over a field, 20° × 20° upon a plate 10 in. × 10 in., working with full aperture. This was in the observatory at Machrihanish House, Argyllshire, with the conveniences of exact time, electric current, developing rooms, and workshops. After working for a

year upon various problems of diffusion of focus, speed, diaphragm, exposure, &c., I had the good fortune to receive a month's visit—in January 1900—from Professor Barnard, who had heard of the lens, and had travelled to Machrihanish to examine it. After scrutinising the results to date he worked for ten nights at experiments of his own, especially comparing the lens with his celebrated Petzval as to rapidity. For this purpose we made a rough camera for his lens and placed the two side by side.

We then journeyed together to York, taking with us a set of our photographs—upon patent plate—mostly of *Orion*, as giving stars of many magnitudes, and two important nebulæ. After discussion with Mr. Dennis Taylor it was decided to recompute curves and alter the ratio to $\frac{f}{4{\cdot}5}$, thus reducing the field, and make a new experimental lens of 4 in. diameter upon these new lines; if successful the 6 in. to be refigured in accordance with experience gained in making the 4 in.

This 4-in. lens was finished in time to take to Santa Pola in Spain with the Scottish Eclipse (1900 May 28) Expedition, to which I was attached; some photographs of the corona were taken with it, and, after the eclipse, also some of stars, *Jupiter*, and the Moon. The results were excellent, the plates were taken to Mr. Dennis Taylor, and the re-figuring of the 6 in. put in hand.

The experiments with the 6-in. lens in its original state were now repeated with the lens in its altered form, and were so successful that a 10-in. lens was ordered, a detailed description of which, by Mr. Dennis Taylor, is appended to these notes. I would wish to say that I alone am responsible for the choice between a field of $12° \times 12°$ with good images everywhere, or a field of $15° \times 15°$ with the images at the corners only "fair." I chose the latter partly because it would reduce the magnitude of the undertaking before me and partly because 120 of the 206 plates would each cover exactly an hour of R.A.

On receiving the 10-in. lens I found that its weight in brass and aluminium mount was about 100lb.: it was mounted temporarily for experimental work on the 8-inch equatorial with 4-inch guider, and prints were sent to Professor Kapteyn, of Gröningen, and, in consideration of his admirable work on the Milky Way, to M. Easton, of Brussels. Professor Kapteyn gave much kindly advice, criticism, and approval of the work. In one square (equatorial) degree in *Cygnus* he counted 670 stars—exposure 2 hours. This gives in a space $2° \times 2°$ more stars than are visible to the naked eye in a hemisphere. The exposure adopted was 2 hours 20 minutes for British stations, 2 hours for southern stations.

Mount.—After visits to every important observatory in Great Britain and to several on the Continent—and I would especially acknowledge great kindness from Dr. Wolf, of Heidelberg, and Professor Becker, of Glasgow, and from all the staff of

Pyrafam—I decided upon an English mount with modifications. A description of this, by Mr. Alfred Taylor, of Messrs. Cooke & Sons, follows this paper. I will only mention the points for which I am specially responsible. (a) Two 6-in. guiding telescopes instead of one. (b) A new eyepiece carrier for rapid finding of guiding stars and fine adjustment when found. As the instrument will in charting always be set to an arbitrary point, *e.g.* 3^h-60°, there may not be, and as a fact there generally is not, a catalogue star in the field. When only small stars are available the colour decides which shall be used. (c) Against much good advice I chose a circle instead of a sector, and a circle for slow motion in declination, driving both slow motions by motors. I do not know whether I can lay claim to being the first to use motors for these controls, but they have been successfully used by me for some years. Especially is it useful in alteration of declination necessitated by the varying refraction, the touching of the telescope in second-controlled clocks often causing a dropped second. With the luck of varying atmospheric densities happening at the right time and in the right direction, some plates have been run by the Hepsold clock for the two hours without hand interference. (d) An arrangement by which either of the guiding telescopes can be thrown 10 degrees or less away from the direction of the camera. By this means, photographs of comets can be guided by the head in the corner of the plate, thus utilising the whole of the plate—15 to 25 degrees—for the tail. I am indebted for this suggestion to my friend Professor Barnard. (e) I made a full-sized schematic model of the mount and a small model of the house, which latter Messrs. Cooke developed and copied.

Expedition to the Cape.—In 1903 June my assistant, Mr. G. N. Kennedy, sailed for Cape Town, taking with him the complete instrument and its house. I followed five weeks later, found the house erected and the piers built, and within a fortnight of arrival all was ready for final adjustment of mount and for focussing and centering of the lenses and réseaus. The réseau for the 10 in. was the only exception to all being in order: it had been engraved and packed whilst I was ill in bed, and unfortunately some lines, through not being deeply engraved, are missing upon the early prints 19^h to 23^h, after which a new réseau arrived. It is immaterial, however, as transparent scales are to be used with the proposed atlas, with the degrees of declination marked exactly in hyperbolas or approximately in circles as may be decided; whilst for counting stars on the negatives the measuring machine can be adapted to bridge the missing lines. There were personal reasons why it was convenient to finish the charting of the southern hemisphere first, but the following extract from M. Easton's work, *Distribution de la Lumière Galactique*, is perhaps sufficient without these:

"La distribution des grandeurs stellaires trouvée par Kapteyn n'est pas en contradiction avec la supposition que nous avons faite.

Ce serait un travail intéressant de pousser plus loin cette comparaison, mais je n'ose décider si les données fournies par la présente étude ne sont pas trop incomplètes, presque rien n'étant connu sur la distribution des étoiles dans l'hémisphère austral, ni si elles comportent la précision nécessaire pour des recherches pareilles."

The equatorial plates were compared and identified with Argelander and Schönfeld, but comparison with Gould for the plates south of these was hopeless. For this reason and for the count of the lucid stars I fell in with Sir David Gill's suggestion to take also "triangle plates" with the 10-in. lens. The working scheme is therefore as follows :

91	plates	from	$+7° 30'$ to $+90°$
91	„	„	$-7° 30'$ to $-90°$
24	„	„	$+7° 30'$ to $-7° 30'$
206	„	„	15 in. × 15 in., 2 hours.
206	„	„	15 in. × 15 in., 7 minutes ; 3 times in triangle form.
206	„	„	12 in. × 10 in., 2 hours.
618			
618 transparencies.			
1,236 plates.			

The scheme for the Cape includes :

91 + 24 = 115	plates	15 in. × 15 in., 2 hours'	exposure.
115	„	15 in. × 15 in., 7 mins. 3 times	„
115	„	12 in. × 10 in., 2 hours'	„
345			
345 transparencies.			
Total 690 plates.			

Besides these there will be about fifty duplicates, and some picture plates of long exposures—four to five hours.

Each 15 in. × 15 in. is compared with its fellow triangle plate, and with the 12 in. × 10 in. of the same region.

When the 10-inch proved such a success, I at once communicated with Messrs. Cooke as to a duplicate, but, alas ! the melting from which my discs were made had all been used, and another melting could not be depended upon to give the same focus and therefore the same scale. I decided therefore to mount the 6-inch for duplicate plates. If I had been able to arrange for twin lenses of 10 in. I should not have required the transparencies for sending home in a separate steamer, and should

have saved considerable time in being obliged to take second exposures of 15 in. × 15 in. and 12 in. × 10 in. alone for spoiled plates. Duplicate plates are absolutely necessary for identifying nebulæ, perhaps a nova, meteors, faults of vision, or blemishes in film.

The scale of the 10-in. lens is 20 mm. = 1° on the equator

" " 6-in. " 11 mm. = 1° " "

this latter only approximately.

The plates of the 10 in. are 15 in. × 15 in. (approximately 2nd-plate)

" " 6 in. " 15 in. × 10 in., of which only 10 in. × 10 in. will be used, and these will give a good overlap, which the large plates do not.

Both lenses are worked at full aperture.

The good and fair field on each plate is 15° × 15°.

The developer throughout is amidol; at some important observatories the labour of development is much reduced by *time* and not by *visual* development. The developer is applied at a certain strength and temperature for a given number of minutes. The plates under consideration have been developed visually. When safe, ample, and equally distributed light is not available this time system has much to recommend it, but it would probably mean a fresh developer for each plate.

The chart plates are Cadett Lightning; the triangle plates are Imperial flash-light; the transparencies are ordinary plates of both makers.

The driving of the instrument in R.A. by a Repsold clock is a great success. Some plates have been exposed for the two hours without once touching the R.A. motor, but as a rule the varying refraction in so long an exposure and so large a field makes a slight alteration, say three or four times an hour, both in R.A. and decl. advisable. It is quite remarkable that with the varying driving power required in different positions—owing again to the long exposure—this clock drives steadily on with hardly any variation of pace, whereas a Russell Mouse-control clock would stop. If, too, as occasionally happens, a little grit in the driving worm or some other unknown cause should slow down the motion, it is only a slight slow down, and not the miss of a whole second of time as in controlled clocks. The circle instead of sector—a particularly fine one—is a great convenience.

I have been asked why I have deviated from my first intention of photographing the Milky Way only. The reply is (*a*) that the ramifications and extensions of the Milky Way are so undefined that it seemed advisable to chart all regions; (*b*) that as nine months' work, approximately, would be required for the Milky Way alone the whole of the hemisphere could be charted in the same time, although with, of course, more work.

Many points—*e.g.* length of exposures—have been decided because of their bearing on the Milky Way. The intention is to publish the work as an atlas in triplicate *form*: (1) the

15 in. × 15 in.; (2) the 12 in. × 10 in. as a check upon false nebulæ and false stars, both upon paper with gelatine surface; (3) the triangle plates upon writing paper, and for this reason, besides being a second check upon faults in film, they should be a valuable help to workers in special branches of stellar work. A double-star worker, for instance, could number all known double stars according to the several catalogues, S 1004, B 191, I 34, and so on. The same with variables, nebulæ, &c. The expense of such publication would, however, be so serious that I must wait until better times in South Africa for a millionaire to help me with the cash, and in the meantime I propose to print photographically about five copies for public institutions. Any printing, whether photographic or mechanical, will, however, be absolutely *untouched*, and will show all faults, scratches, &c. It seems to me that a single interference of only one spot in a thousand plates would cast a doubt upon the pictorial integrity of any given region.

To record my sense of thanks to Sir David Gill for his kindness in allotting me a splendid site, and for his ever-kindly help and advice, is a great pleasure. Each one of the staff of the Royal Observatory at Cape Town has done his best to render the work as easy and as agreeable as possible.

Professor Barnard has had a 10-inch lens and mount made in America, the lens by Brashear and the mount by Warner & Swasey. It will be interesting to compare the two instruments. I have reason to think that of the two his will work upon a rather smaller field, and that his lens will be more rapid. I do not know the direction in which he proposes to work, nor do I know the scale.

In these days of many observatories in Great Britain, understaffed though they may be; of many more on the continent of Europe, supplied liberally with Government funds; and of still more in America, where wealthy citizens give large sums to build, house, and endow instruments of record size, it may seem difficult to find an unoccupied field of research. That there are such fields, however, is certain, and among them it seems likely that many will be found in that domain which contains the secrets of the structure of the universe.

Description of the Lenses. By H. Dennis Taylor.

Mr. Franklin-Adams has asked me to give a condensed description of his 10-inch aperture Cooke lens, with a short explanation of the theory of its construction.

It is a particular case of the Cooke photographic lens modified for celestial purposes. The essential aim of the Cooke lenses was to obtain a flat image substantially free from astigmatism, besides being rectilinear and achromatic, with the minimum number of lenses possible. This result was obtained with only three simple lenses, two outer positive lenses and an inner

negative lens, which latter had to perform simultaneously the following functions:

1st. To correct the spherical aberration of the two positive lenses, on the axis as well as obliquely.

2nd. To correct the chromatic aberration of the two positive lenses, not only along the axis but obliquely, and also be so placed in position as to cause the final images in various colours to be of the same size and free from curvilinear distortion.

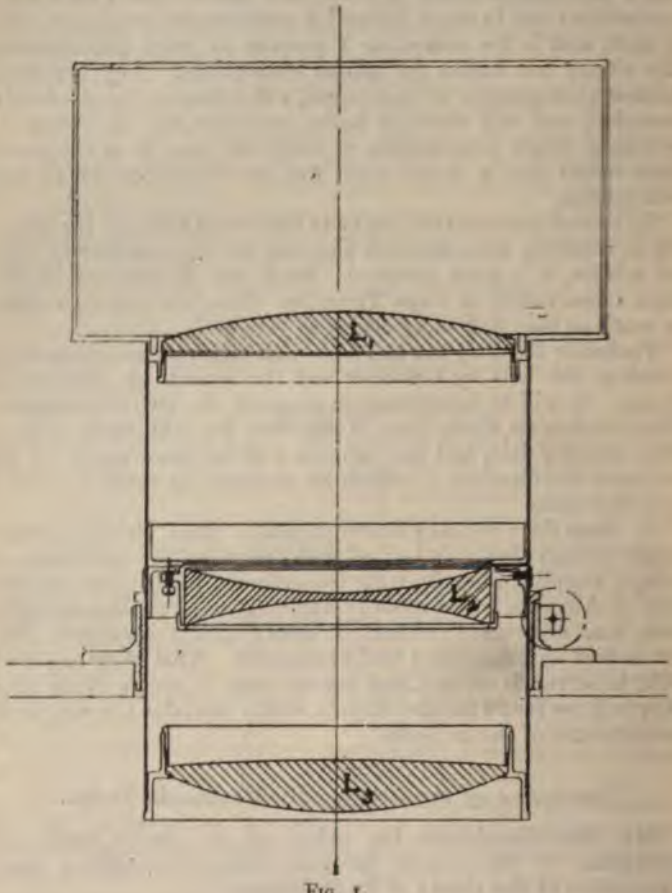

Fig. 1.

3rd. To correct the curvature of image of the two positive lenses, if possible both in primary and secondary sections of the same oblique pencil simultaneously, so as to bring about a final flat image free from astigmatism.

4th. To correct the coma, if any, produced in the oblique pencils by the two positive lenses.

All the above corrections may be carried out simultaneously, although very much interlocked, by a suitable choice of materials, relative powers, and shapes of the lenses, together with suitable separations. A longitudinal section of the complete lens is given in fig. 1.

Spherical Aberration.

It is obvious that we have the power of varying the collective spherical aberration of the two positive lenses between the extreme values $(a+a)$ and 2^3a or $8a$ for any constant aperture, combined power, and separation between the two positive lenses. If they are of equal powers and equal aberrations we have the sum of both aberrations or $(a+a)$, whereas if the power of one lens is about vanished and the power of the other is correspondingly about doubled then the collective aberrations become $(0+8a)$. Thus the collective spherical aberration is capable of being varied to suit any desired negative lens.

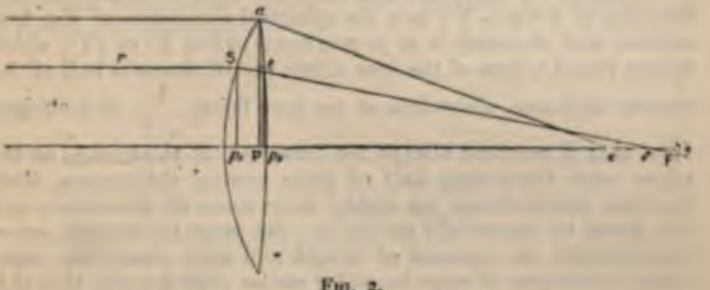

FIG. 2.

There remains for lenses of large relative aperture the question of zonal aberration, a defect of no particular consequence for camera lenses, but of immense importance in the case of lenses for celestial photography which are required to give the sharpest possible images of stars.

Fig. 2 represents a double convex lens on which falls an axial pencil of parallel or divergent rays.

Let the curves be carried out to a sharp edge a, and let the semi-aperature aP (a—P being perpendicular to the axis) be called A. Let a ray be traced through the sharp edge. It meets with a certain amount of spherical aberration, which is a function of A^2, and is refracted to the point e considerably nearer to the lens than F, the point where the ultimate axial rays come to focus. Then e—F is the longitudinal spherical aberration. Let an intermediate ray r—s—t—f be considered. It strikes the first surface at s at a perpendicular distance s—p_1 (= y), from the axis and it meets with a certain amount of spherical aberration, which is a function of y_1^2. After refraction the ray converges

towards the axis, and after traversing the thickness of glass—(it meets the second surface at s nearer the axis, so that s_1 or s_2 is appreciably less than y_1. It is this disparity between the two y's at the two surfaces at positions in the lens between the axis and the sharp edge which gives rise to zonal aberration.

If the aberration of such a lens is examined by projecting its sections of the cone of rays on to a screen or by an eyepiece then the zone of aberration is so disguised by the much greater general aberration as to be invisible; but supposing the general aberration is neutralised by the general aberration of a negative lens, then the zonal aberration shows up, and we get an imperfect focus; for while we may get the extreme edge ray to focus at the same point as the ultimate axial rays, yet there is a zone in the lens from which the rays cut the axis at a point considerably inside of the focus for edge and centre rays. If a section of the cone of rays be taken at a point a little inside of that focus the patch of light takes the form of a disc with a bright zone about half-way between the centre and the edge, while outside of the focus the edge and centre of the disc of light are the brightest. If Y stands for either y_1 or y_2 then the zone of aberration is a function of $Y^2(A^2 - Y^2)$ into the spherical aberration of the first surface, and obviously is at a maximum when $A^2 = 2Y^2$, which occurs round a zone of the lens where the thickness is half of the central thickness, the radius of the zone being $\frac{A}{\sqrt{2}}$. It is obvious then, that if we could always use lenses whose thicknesses at the edges were more than half of their central thicknesses, their aperture would always be inside their zones of aberration, and the latter be practically got rid of; but large thicknesses are so objectionable on account of weight and light absorption, especially in the case of large lenses for stellar photography, that it is scarcely to be thought of, and other methods have to be devised. We have to deal with the case of the rays impinging upon the first lens being parallel, and those impinging upon the third lens being divergent. In the case of the first lens the zone of aberration is a function of the aberration of the first surface, and (supposing shape of the lens is kept constant) that aberration is a function of $\left(\frac{1}{r}\right)^2 \left(\frac{1}{F}\right)^2$, or the cube of the reciprocal of the radius of curvature into the square of the focal power.

Therefore the zone of aberration of the first lens can be enormously reduced by keeping down its power, while, since in the back lens the rays traverse its substance more nearly in a state of parallelism than in the case of the first lens, an increase in the power of the back lens balancing the decrease in power of the front lens does not augment the zone of aberration of the back lens in anything like the same degree. For this reason the power of the back lens was made preponderant. In the first model the two positive lenses were equal in power, and the zone of aberration was so bad that the lens was practically useless for

stellar work in large sizes, and at any aperture over $\frac{F}{5}$, nor could the zone of aberration be got rid of by special figuring, whereas the present arrangement of making the power of the back lens about 1½ times the power of the front lens so reduces the zone of aberration as to render it possible to get rid of it entirely by special retouching; a somewhat troublesome but quite practicable process. In the case of negative lenses which grow thicker as the centre is left and have no sharp edge it is obvious that zones of aberration do not arise.

Of course the state of the chromatic correction of the final image along the axis is simply a matter of the mutual adjustment of the relative dispersive powers of the positive and negative lenses.

But whether or not the oblique image of stars shall also be achromatic, or whether they shall be drawn out into spectra, pointing to the centre of the field, depends upon the negative lens being placed at a certain position between the two positive lenses. Should this condition be fulfilled then the other highly important condition of freedom from distortion or rectilinearity is simultaneously attained.

Fig. 3.

The diagram fig. 3 shows one of these lenses considered as reducing or copying in the ratio 3 to 2. OP is the original and ip the image, O being a particular point in OP and i its image. As the two positive lenses are of the same shape, but turned in opposite ways, and the distance $PL_1 : pL_2 :: F_1 : F_2$, and we place the diaphragm point at D, such that $L_1D : L_2D :: F_1 : F_2$ also; then it is obvious, without stating the formulæ for distortion (which fully confirm this case), that all is symmetrical on each side of D and $ip : OP :: L_2p : L_1P$ exactly. Hence there is no distortion, and the lens gives a rectilinear image. Moreover, all the variously coloured oblique principal rays from O will pass through D, although traversing each lens L_1 and L_2 at different distances from the axis. For instance, the solid line shows the path of a yellow principal ray from O, while the dotted line shows the path of a blue principal ray from O. If, therefore, the centre of the

negative lens, of properly chosen dispersive power, is also placed at D, the intercrossing or diaphragm point, then we secure that the various colours come to final focus in the same plane, while at the same time the conditions of oblique achromatism also hold good, for the variously coloured principal rays go straight through the centre of the negative lens as if it were not there, and traverse L_1 and L_2 at heights from the axis always proportionate to their focal lengths. Hence perfect symmetry prevails. But supposing the original OP is removed to a much greater distance or infinitely far off, and p moves up towards the principal focal plan $f f$, then the conditions of oblique achromatism for the image of the point O no longer strictly hold good. It is obvious that the blue image of O at i will fall a little further away from the axis than the yellow image at the same time that positive distortion will arise. For the image of the diaphragm point D as viewed through the back lens L_2 is subject to spherical aberration, the principal ray or line of projection dk radiating from k, whereas the image point of D formed by L_2 would be at K were it not for its spherical aberration. Thus, k—K is the longitudinal spherical aberration of L_2. Therefore the line of projection k—i strikes the principal focal plane f—f too far from the axis, and there is positive or pincushion distortion of the images of straight lines. In order to correct this it is necessary to move the negative lens nearer to L_1 by such an amount as is necessary to again make the various coloured rays from O or a star all strike the principal focal plane f—f at the same distance from the axis, when it will be found that the lens will also become quite rectilinear again. In the case of Mr. Franklin-Adams's lens the amount of movement required is about ·39 inch, or 3 per cent. of the whole separations. While in certain other combinations of separated simple lenses, such as the Huygenian eyepiece, and four lens-erecting eyepiece, the conditions of freedom from distortion and oblique achromatism are more or less independent of one another; in these Cooke lenses, provided that the two positive lenses are of similar but reversed shapes, the two conditions of rectilinearity and oblique achromatism are necessarily simultaneously attained.

The actual state of colour correction in these lenses as designed for stellar work is not such as to bring the actinic blue rays about G and the most luminous D rays to the same focus as in the case of camera lenses, but to bring the most highly actinic blue rays on the F side of G to the minimum, and therefore most intense focus, the most luminous yellow green and yellow rays being refracted to foci considerably beyond, and thus the most distinct image from a purely photographic point of view is obtained at the sacrifice of the visual distinctness. The two positive lenses are made of heavy barium silicate crown, having a refractive index of about 1·57 for the D ray, and the negative lens of flint glass having a refractive index of about 1·623 for the D ray.

Astigmatism and Flatness of Field.

3. The means whereby flatness of field combined with almost complete absence of astigmatism is attained really constitutes the most novel feature in the construction of these lenses.

If F = the principal focal length of any thin lens whatever, positive or negative, μ = its refractive index, and ϕ the angle of obliquity of any pencil of rays traversing the lens centrally, U the axial distance from the lens of any plane object placed perpendicular to the axis, and V the corresponding conjugate distance of the image from the lens, and y = the semi-aperture of the pencil when it traverses the lens, then

$$\frac{1}{V}=\frac{1}{F}-\frac{1}{U}+\frac{1}{8F^3}\omega y^2+\frac{\tan^2\phi}{2F}\cdot\frac{3\mu+1}{\mu}$$

wherein $\frac{1}{8F^3}\omega y^2$ = the spherical aberration to which all the pencils are subject and $\frac{\tan^2\phi}{2F}\cdot\frac{3\mu+1}{\mu}$ is the error of curvature of image as formed by the rays in the primary plane of the oblique pencil or in that plane containing the principal or central ray of the oblique pencil and the optic axis. Whereas the error of curvature for rays in the secondary plane of the same pencil or the plane at right angles to the former and also containing the principal ray is $\frac{\tan^2\phi}{2F}\cdot\frac{\mu+1}{\mu}$.

Thus the two curvature errors are

$$\frac{\tan^2\phi}{2F}\cdot\frac{3\mu+1}{\mu}\text{ and }\frac{\tan^2\phi}{2F}\cdot\frac{\mu+1}{\mu}$$

in the two planes, and, if $\mu = 1{\cdot}5$, are in the ratio 5·5 to 2·5 or 2·2 to 1, while their difference is $\frac{\tan^2\phi}{F}$ simply, which represents the astigmatic error of the lens. Thus the astigmatism depends solely upon the obliquity and the power of the lens and nothing else, while the two curvature errors depend also upon the refractive index, but are independent of U or V.

More than fifty years ago Petzval promulgated his now well-known theorem to the effect that if a system of thin positive and negative lenses combined in contact, and so arranged as to give a final image free from astigmatism away from the axis, then the image, if anastigmatic or free from astigmatism, will be curved to a radius whose reciprocal value is equal to $\Sigma\frac{1}{P\mu}-\Sigma\frac{1}{N.M}$

wherein $\Sigma\frac{1}{P}$ is the sum of the powers of the positive lenses and μ

their refractive index, and $\Sigma\frac{1}{N}$ is the sum of the powers of the negative lenses and M their refractive index. Supposing $\mu = M$; then obviously, if $\Sigma\frac{1}{P} = \Sigma\frac{1}{N}$, then the reciprocal value of the radius of curvature of the final anastigmatic image becomes zero, that is, the image is flat as well as free from astigmatism. But, unfortunately, as the focal powers of the negative lenses would in that case completely neutralise the focal powers of the positive lenses, there could be no real image. But if the refractive index for the positive lenses would be made higher than that for the negative lenses, which is contrary to the usual state of things, and in Petzval's day not feasible consistently with achromatism, then obviously the above formula equates to zero when $\Sigma\frac{1}{P} = \Sigma\frac{1}{N}\frac{\mu}{M}$ which leaves the powers of the positive lenses in excess, so that the combinations will yield an actual image. Petzval did not fully make known how he arrived at his theorem, nor did he state the conditions under which the oblique pencils of rays would have to traverse the lens system in order to form an anastigmatic image on a flat surface. As a matter of fact, if the Petzval condition is fulfilled in a system of thin lenses in contact having $\mu = M$ and having the diaphragm point at their centre, so that the oblique pencils all traverse the *centre* of the system, then the final curvature of image for rays in primary sections of the oblique pencils is given by $\Sigma\frac{1}{P}\frac{\tan^2\phi}{2}\frac{3(\mu-M)}{\mu}$, and for rays in secondary planes by $\Sigma\frac{1}{P}\frac{\tan^2\phi}{2}\cdot\frac{\mu-M}{\mu}$, that is, the two errors are in the ratio 3 to 1 instead of the usual 2.2 to 1 or so in the case of simple lenses. What is now further required in order to uncurve both these curved images and throw them back simultaneously on to the plane passing through the axial focus is that the lenses must be given certain shapes, and that the oblique pencils of light must be constrained to pass *eccentrically* through the system. This latter condition is secured by placing a circular aperture, considerably smaller than the lens aperture, at some distance either behind or in front of the system. The oblique pencils of rays are thus compelled to traverse the system eccentrically and then become subject to further corrections which I call diaphragm corrections. It can be proved that any diaphragm correction equal to + or −1 affecting the rays in secondary sections of oblique pencils is invariably accompanied by diaphragm corrections equal to + or −3 affecting rays in primary sections of the same pencils. Therefore by suitable diaphragm corrections introduced into any contact system of positive lenses of high refractive index combined with negative lenses of low refractive index fulfilling the formula $\Sigma\frac{1}{P_\mu} - \Sigma\frac{1}{N}\frac{1}{M} = 0$ we can get the image formed by

secondary rays (of curvature = 1), and the image formed by primary rays (of curvature = 3) simultaneously thrown back upon the plane passing through the final axial focus, and thus obtain a positive image that is both flat and free from astigmatism.

Supposing we confine our attention to combinations of two simple thin positive lenses and one simple thin negative lens placed between and in contact in accordance with the above conditions we may go further than Petzval did, and consider what will happen, supposing the positive lenses are more and more separated from the negative lens. When in contact the power of the combinations on parallel rays is simply $\frac{1}{P_1}-\frac{1}{N}+\frac{1}{P_2}$; but it can be shown that when separated the power of the combinations becomes

$$\frac{1}{P_1}-\frac{1}{N}+\frac{1}{P_2}+\left\{\frac{S_1(P_2-N)+S_2(P_1-N)-S_1S_2}{P_1NP_2}\right\}$$

in which formula S_1 stands for the separation between first lens and negative lens, and S_2 the separation between the back positive lens and the negative lens.

The formula in brackets is the increment to the power due to separation alone.

Now while the power of a simple lens inevitably carries with it the proportional curvature errors

$$\frac{\tan^2\phi}{2F}\cdot\frac{3\mu+1}{\mu} \text{ and } \frac{\tan^2\phi}{2F}\cdot\frac{\mu+1}{\mu}$$

on the other hand it can be proved that the great increment to power brought about by separation in these triplets does *not* carry with it any curvature errors whatever, and is an unqualified gain. A contact triplet combination of the power 1 and aperture $\frac{F}{16}$ may, when separated, give a power equal to 2 or 3, with respective apertures of $\frac{F}{8}$ or $\frac{F}{5\frac{1}{3}}$, while its image will under certain conditions remain flat and free from astigmatism if the equation $\frac{1}{P_1\mu}-\frac{1}{N}\frac{1}{M}+\frac{1}{P_2\mu}=0$ is fulfilled. The next question that occurs is, supposing this equation, which we may call the Petzval condition, is not strictly fulfilled, either in that the negative lens is of a less power than the equation specifies, or that its refractive index is higher instead of lower than that of the positive lenses, as is the case in this Cooke lens for stellar photography; then can the image still be got sufficiently free from astigmatism if flat? The answer is in the affirmative. For although in the case of a contact combination giving a flat mean image, there is sure to be more or less considerable astig-

matism should the Petzval condition be departed from, yet since the resultant focal power is very largely increased by separation of the lenses, these errors are relatively decreased. If the focal power is doubled or trebled by separation, then the errors consequent upon not fulfilling the Petzval condition are relatively reduced to $\frac{1}{2}$ or $\frac{1}{3}$ respectively.

There are as often as not practical difficulties in the way of strictly following the Petzval condition, so that a method of construction which enables one, as it were, to drown the errors due to its non-fulfilment, and therefore becomes to a large extent independent of it, has much to recommend it. As in Mr. Franklin-Adams's type of lens the refractive index of the two positive lenses is 1·5853 for the G ray, and that of the negative lens about 1·64615, then the Petzval condition demands that the power of the negative lens should be $\frac{1·646}{1·585}$ or 1·04 times the sum of the powers of the two positive lenses. As a matter of fact it is only about ·9475, a deviation of ·09, or 9 per cent. from the Petzval condition, which would be fatal in the case of a contact combination of lenses, but is of vanishing importance in this widely separated combination, 86 per cent. of whose equivalent focal power is due solely to separation, and whose residual astigmatism for the G ray is thus reduced to an exceedingly small amount, namely, to one-seventh part.

For the *total* actual power of the lens $= \frac{1}{4·46} = ·0224$, while the sum of the powers of the three lenses $=$

	$+·039305$ for L,
	$+·058417$ for L,
	$·097722$
	$-·094545$ for L,
	$+·003177$ total

Thus the actual power is seven times the simple sum of the powers of the constituent lenses, and therefore the astigmatic error as expressed by $\frac{1}{\mu P_1} - \frac{1}{MN} + \frac{1}{\mu P_2}$, wherein the Petzval condition is not fulfilled, is reduced or diluted seven times.

It is also obvious, since the power of the negative lens is so nearly equal to the sum of the powers of the positive lenses, that the mean curvature errors of the latter as expressed by $\frac{\tan^2\phi}{2}\left(\frac{1}{P_1}+\frac{1}{P_2}\right)\frac{2\mu+1}{\mu}$ are almost completely neutralised by the corresponding opposite errors of the negative lens or

$$\frac{\tan^2\phi}{2}\cdot\frac{1}{N}\cdot\frac{2M+1}{M}$$

Anyway, the remaining positive curvature errors are so small (in this case only about 5 per cent.) compared to the total curvature errors of either the positive lenses or negative lens that any diaphragm corrections left in the system for correcting these residual curvature errors are also merely residual. Hence the lens may be described as composed of three simple lenses each of which is so shaped as to be about free from diaphragm corrections when the rays first incident on the first lens have a certain assigned degree of divergence. In this particular case the lenses are symmetrically disposed to two conjugate focal planes, one being at a distance from the front lens L_1, equal to $1\frac{1}{2}$ times the distance of the conjugate focus from the back lens L_3. Also the first separation S_1 is about $1\frac{1}{2}$ times the second separation S_2, and the diaphragm point is supposed to be at the centre of the negative lens. As the oblique pencils pass centrally through the negative lens, no appreciable diaphragm corrections arise in its case, whatever its shape may be, but the two positive lenses have to be so shaped that the presence of the diaphragm a long way behind L_1 and in front of L_3, causing the oblique rays to traverse those lenses eccentrically, shall have a disturbing effect equal to -5 per cent. of the natural mean curvature errors of the two lenses as expressed by

$$\frac{\tan^2\phi}{2}\left(\frac{1}{P_1}+\frac{1}{P_2}\right)\frac{2\mu+1}{\mu}$$

In other words, the diaphragm corrections of the two lenses are simultaneously approximately eliminated or reduced to nearly zero. It is almost the same thing as saying that the two positive lenses are so shaped as to be about free from coma under the conditions in question. Thus, the diaphragm corrections being about eliminated from the system under the above conditions, it can be shown that the system as a whole will remain about free from diaphragm corrections should the plane object presented to L_1 on the left be removed by an infinite distance. For slight outward coma will then arise in L_1, giving rise to positive diaphragm corrections, but outward coma will simultaneously arise in L_2, giving rise to negative diaphragm corrections neutralising the former, so that the final image will retain its flatness, while at the same time inward coma arises in the negative lens, which neutralises the outward coma of the two positive lenses.

Also in order to retain the oblique achromatism of the lens under the new conditions, the negative lens should be moved nearer to L_1, as we have seen before when dealing with the chromatic correction and rectilinearity of the lens. The amount of this axial traverse of the negative lens required between the symmetrical case of copying or reducing 3 to 2 and the case of photographing infinitely distant objects varies roughly as the cube of the separation between the two

positive lenses, other things being equal, and in Mr. Franklin-Adams's 10-inch lens is actually about ·39 inch. The adjustments of a large lens of this type are necessarily of a very delicate nature, and means are provided both for tilting the negative lens in any direction with respect to the optic axis and also for moving it laterally in any direction parallel to itself. Perfect symmetry of field is thus more at command, and after that has been satisfactorily secured the negative lens is carefully fixed in position.

The actual photographs taken and exhibited by Mr. Franklin-Adams will sufficiently show the capabilities of the lens for stellar photography in cases where large regions of the sky are required to be depicted in one view.

1903 *July* 10.

Description of the Mount. By Alfred Taylor.

The mounting was designed to utilise two existing cameras with lenses of unequal sizes and focal lengths. Both cameras having to be in use at the same time, the results from one serving as a check on the other, perfect parallelism and sympathy of motion are of great importance. Rigidity in the mounting, with great ease of working, had therefore to be carefully kept in view in its construction.

The mounting is of the English type, there being no central pillar to interfere with a complete twelve hours' run for all positions of the telescope. This is an important factor where star photography is concerned. The polar axis consists of a cross-braced steel frame, square in section, the width at the middle being determined by a suitable spread of bearings to give length to the declination axis. Both ends of the polar axis run in double ball bearings, mounted in swivelling frames, which take the side thrust due to the inclination of the axis, while another ring of balls at the lower end takes the end thrust there. The swivelling frames much reduce the difficulty of adjusting the polar axis, and the risk of friction from error in alignment there would be with plain bearings. Ball bearings have been successfully adopted by T. Cooke & Sons for some time on equatorial mountings carrying telescopes anything over 6 inches in diameter, the reduction of friction and consequent ease of movement greatly conducing to regularity in running, the work required to be overcome by the driving clock being more uniform and considerably less than with plain bearings.

The hour-circle, tangent wheels for both axes, and declination circle are all of gun-metal.

The R.A. tangent wheel is 43 inches diameter, and has 1080 teeth. It is loose on its axis and can be clamped by a screw worked by bevel wheels and loose cords led conveniently near to the eye-ends of the guiding telescopes.

The hour-circle is 24 inches diameter, both edges divided on

silver to minutes of time, with fixed and movable verniers reading to two seconds of time.

It being Mr. Franklin-Adams's intention to do work in both hemispheres, the hour-circle and verniers have been figured to read both ways round, one set of the engraved figures being filled in with red wax for the southern hemisphere, and the other in black for the northern hemisphere.

The driving clock is by Messrs. Repsold, of Hamburg, and will run 2½ hours without rewinding.

The spindle which communicates the motion of the clock to the mounting makes thirty revolutions in one minute. At the top end of this spindle a worm engages with a small wheel on the tangent shaft, so that between the clock and the large tangent wheel the number of driving wheels is kept small—a matter of some importance where accuracy of drive is concerned.

The slow motion consists of a train of mouse gear, driven by a small reversible electric motor, which can run up to 3,000 revolutions per minute. The slow motion is unlimited, *i.e.* the telescope can be run through a complete circle by means of the motor if desired.

The socket for the declination axis is of cast iron bushed at the ends with gun-metal and formed with large square flanges bolted to two sides of the polar axis frame. The length across the bearings is 3 feet.

The axis is also of cast iron and hollow. The end flanges carry the two cameras and brackets for the two guiding telescopes. These brackets have specially large flanges, which allow the guiders to be temporarily set 10° out of parallelism to view any corner of the rectangular field covered by the cameras. Each guider has a 6-inch achromatic object-glass of 6 feet focal length. The eye-end has a sliding stage allowing a rough circular movement of ⅝ inch, and a sub-stage with a fine adjustment of $\frac{1}{10}$ inch, such as is used with microscopes.

The declination circle is fixed to the side of the polar axis frame. It is 2 feet 6 inches diameter, divided on silver to 5 minutes of arc, and reading by verniers (which turn with the cameras) to 10 seconds of arc. One of the verniers is read from alongside the eye-end of the adjoining guider by a telescope with a diagonal eyepiece, which can be turned in any convenient direction. The rim of the declination circle is divided and engraved with large figures for rough setting without using the reading microscope. The slow motion in declination is effected by means of a mouse feed gear, driven by a reversible electric motor—similar to that for the R.A. slow motion—driving a worm which works into a complete wheel. This gives an unlimited range to the slow motion and avoids the annoyance caused by coming to the end of the motion when the amount is limited. The driving wheel is 27 inch pitch diameter, and has 1,080 teeth. It is loose on the axis and may be clamped from the eye-end of the guider alongside.

Starting and reversing switches hang from near the eye-ends of both guiders, and the slow motions in both R.A. and declination are from thence under easy control.

The guiding telescopes have light and dark field electric illuminations to the cross wires, the leads for these and for the slow-motion motors passing up through the centre of the declination axis and down the guiders to the switches, the usual outside festoons of electric wires, and the consequent risk of their breakage, being thereby avoided.

The upper and lower end brackets for the polar axis will be supported on concrete, or stone, and brick piers.

The telescope will be erected in a wood shed 12 feet 6 inches wide × 20 feet 6 inches long, both inside measurement, resting on a brick foundation raised a little above the ground. The ridged roof is in two parts, each with four rollers, running on rails laid upon the upper side beams of the house. These beams are continued 10 feet beyond each end of the house ; the two halves of the roof run along these, and thus allow a clear overhead view for the instrument ; or by both halves being drawn to one end of the house, a clear view may be obtained at the other end down to the horizon.

The sides and roof are covered with Willesden 4 ply waterproof paper.

The total weight of the mounting with cameras is about $2\frac{3}{4}$ tons, 100 lb. being the weight of the 10-in. lens.

On the New Greenwich Micrometer for measurement of Photographs of Eros.

(Communicated by the Astronomer Royal.)

The photographs of *Eros* taken for the determination of the solar parallax need to be measured with greater refinement than was necessary for the astrographic plates. A new micrometer was accordingly obtained from Messrs. Troughton and Simms early in 1903 in which the principle adopted by Mr. Hinks in the Cambridge micrometer (*Monthly Notices*, lxi. p. 444) of a divided glass scale in the focal plane moved by micrometer screws was employed. The new features in the Greenwich micrometer are (i) the microscope, which is arranged so that it can be used to give magnification 2 or 1, so as to adapt it to photographs taken either with the Astrographic telescope, focal length 11 ft. 3 in. (3·40 metres), or with the Thompson refractor, focal length 22 ft. 6 in. (6·80 metres) ; (ii) the glass scale is ruled with pairs of lines so spaced as to admit of very accurate pointing on the réseau line placed between them.

ROYAL OBSER

a circle of 8·1 mm. of the object-glass is used. The following figures give the optical details of the microscope :

	mm.
Diameter of object-glass	17
Focal length	78
Diameter of stop	7

With magnification 1 the object-glass is 156 mm. from the plate and at an equal distance from the focal plane. With magnification 2 it is 117 mm. from the plate and 234 mm. from the focal plane.

FIG. 1.—Arrangement of Scales on Glass Diaphragm.

The field to be measured extends 5 mm. from the centre on the plate, so that the image extends 5 mm. from the centre on each side for magnification 1 and 10 mm. for magnification 2.

The Diaphragm.

A glass diaphragm with divided scales in perpendicular directions is mounted on cross slides in the focal plane of the microscope, the slides being moved by micrometer screws. The

scales, which extend 10 mm. from the centre in the four directions, divide each 10 mm. into thirty equal parts by pairs of lines, the distance from the centre of one pair of lines to the next being $\frac{1}{3}$ mm., and equal to one revolution of either of the micrometer screws. In measuring the position of a star image within a réseau square the readings of the micrometer screws are first set at zero, and the image bisected on the central cross by means of the slow motions. The micrometer screws are then moved so that the réseau lines are bisected by the nearest pairs of lines of the scale. If there were no correction for runs the two réseau lines in each direction would be simultaneously bisected. In practice this is not the case, and the difference between the two micrometer readings supplies the correction for runs. The separation of each pair of lines is $\frac{1}{6}$ mm. or $\frac{1}{5}$ of the distance of the centre of each pair from the centre of the next. This separation is a very suitable one in relation to the breadth of a réseau line on a photograph. It also simplified the division of the scales, as, with the exception of the centre, 90 equidistant divisions were required with every third division omitted. The division of the diaphragm was made to agree exactly with the actual value of one revolution of the screw. For convenience of computation, therefore, the scale may be regarded as giving the integral part of the reading of the screw. With the magnification 2, used for the plates taken with the Astrographic telescope (scale 1 mm.$=1'$)

1 réseau interval$=300''=30$ divisions of the scale ;

so that in this case

1 rev. of screw$=1$ div. of scale$=10''$.

With magnification unity, used for the Thompson plates (scale 1 mm.$=30''$)

1 réseau interval$=150''=15$ divisions of the scale ;

so that again

1 rev. of screw$=1$ div. of scale$=10''$.

Micrometer Screws.

The total range through which the screws are used is one revolution. The value of one revolution was found sensibly equal to a mean division of the scale.

Division Errors of the Glass Diaphragm.

To determine the division errors of the diaphragm free from the optical distortion of the object-glass of the microscope the diaphragm was removed from the microscope and mounted in the

position of the photographic plate. A second similar diaphragm was substituted for it in the focal plane of the microscope. It was thus possible to compare different lengths of the diaphragm to be measured with a standard length at the centre of the field of the microscope. In this way the total length of the scale was subdivided into five lengths of twelve divisions each. These were further subdivided in a similar manner into four lengths of three divisions, and these again into single divisions.

The errors of the corresponding divisions on the left and right, or above and below the centre of the scale, are combined in the same way as the readings of the two réseau lines in forming the measured coordinate of a star's image. The weights thus given to the two errors in forming the mean are inversely proportional to the distance of the divisions from the centre of the scale. The following table gives the corrections for each division and the corrections to the mean for the two réseau lines :

Corrections for Division Errors of Glass Diaphragm, and the Correction to the Mean applicable when Magnification 2 is used.

Division.	x Coordinate. Left of Centre.	x Coordinate. Right of Centre.	Correction to Mean.	y Coordinate. Below Centre.	y Coordinate. Above Centre.	Correction to Mean.
	″	″	″	″	″	″
0	·000	−·031	−·031	·000	+·059	+·059
1	+·026	−·020	−·018	+·033	+·029	+·029
2	+·008	−·015	−·013	+·001	+·022	+·021
3	−·007	−·021	−·020	+·006	+·032	+·029
4	−·013	−·047	−·042	·000	+·005	+·004
5	+·003	−·032	−·026	+·015	+·018	+·016
6	+·009	−·006	−·003	+·030	+·044	+·041
7	+·047	+·049	+·049	+·070	+·084	+·080
8	+·048	+·032	+·036	+·072	+·078	+·076
9	+·036	+·018	+·023	+·060	+·053	+·055
10	+·012	+·016	+·015	+·037	+·051	+·046
11	+·001	+·006	+·004	+·026	+·053	+·043
12	+·010	+·005	+·007	+·048	+·049	+·049
13	+·038	+·015	+·025	+·060	+·065	+·063
14	+·008	+·009	+·009	+·040	+·051	+·046
15	−·011	−·007	−·009	+·012	+·040	+·026
16	−·082	−·079	−·081	−·052	−·041	−·047
17	−·083	−·103	−·092	−·061	−·062	−·061
18	−·119	−·106	−·114	−·081	−·068	−·076
19	−·109	−·096	−·104	−·072	−·046	−·062
20	−·081	−·113	−·092	−·052	−·051	−·052
21	−·093	−·121	−·101	−·058	−·059	−·058
22	−·107	−·126	−·112	−·081	−·052	−·073

Division.	*x* Coordinate. Left of Centre.	Right of Centre.	Correction to Mean.	*y* Coordinate. Below Centre.	Above Centre.	Correction to Mean.
	"	"	"	"	"	"
23	−·101	−·101	−·101	−·050	−·032	−·053
24	−·067	−·089	−·071	−·022	−·018	−·021
25	−·024	−·050	−·028	+·006	+·016	+·008
26	−·031	−·027	−·030	·000	+·030	+·004
27	−·029	−·023	−·028	+·005	+·029	+·007
28	−·033	−·055	−·034	−·014	−·004	−·013
29	−·040	−·069	−·041	+·003	−·019	+·002
30	−·031	·000	−·031	+·059	·000	+·059

With the magnification unity only the central half of the diaphragm is used. Thus if one réseau line is on division 5, to the right of the centre, the adjacent one falls on 20, to the left of the centre. The figuring to the right and top gives the integer set down in the measures. The corrections for division errors are formed by combining those given above, as in the following table:

Corrections for Division Errors of the Glass Diaphragm and Corrections to Mean applicable when Unit Magnification is used.

Division.	*x* Coordinate. Left of Centre.	Right of Centre.	Correction to Mean.	*y* Coordinate. Left of Centre.	Right of Centre.	Correction to Mean.
	"	"	"	"	"	"
15, 0	−·011	−·031	−·031	+·012	+·059	+·059
16, 1	−·082	−·020	−·024	−·052	+·029	+·024
2	−·083	−·015	−·024	−·061	+·022	+·011
3	−·119	−·021	−·041	−·081	+·032	+·009
4	−·109	−·047	−·064	−·072	+·005	−·016
5	−·081	−·032	−·048	−·052	+·018	−·005
6	−·093	−·006	−·041	−·058	+·044	+·003
7	−·107	+·049	−·024	−·081	+·084	+·007
8	−·101	+·032	−·039	−·060	+·078	+·004
9	−·067	+·018	−·033	−·022	+·053	+·008
10	−·024	+·016	−·011	+·006	+·051	+·021
11	−·031	+·006	−·021	·000	+·053	+·014
12	−·029	+·005	−·022	+·005	+·049	+·014
13	−·033	+·015	−·027	−·014	+·065	−·003
14	−·040	+·009	−·037	+·003	+·051	+·006
15	−·031	−·007	−·031	+·059	+·040	+·059

Distortion of the Field of the Object-glass.

The distortion of the field was determined for the magnification 2 by comparing intervals on the diaphragm (whose errors had

already been determined) with a standard interval. For this purpose the diaphragm was replaced in the focal plane of the microscope and the subsidiary scale mounted in the position of a photograph to be measured. The scale from end to end was divided into ten equal intervals, which were successively compared with the same interval of the subsidiary scale. There was some difficulty in the measures at the edge of the field owing to the lens being slightly over-corrected for flatness of field. In the following table are given the differences of each interval from the standard interval after correction for the errors of the scale. The heading gives the reading of the scale as measured from the centre :

				0^d–6^d	6^d–12^d	12^d–18^d	18^d–24^d	24^d–30^d
				″	″	″	″	″
From centre to	right		...	+·077	+·002	·000	−·003	−·063
,,	,,	left	...	+·004	+·038	+·051	−·008	−·093
,,	,,	top	...	+·054	+·050	+·034	−·002	−·050
,,	,,	bottom	...	+·017	+·031	+·010	−·036	−·108
	Mean		...	+·037	+·031	+·023	−·012	−·079

Applying a correction −″·035 per six divisions, which is equivalent to taking the scale at the centre, the correction for distortion is found to be :

0^d	6^d	12^d	18^d	24^d	30^d
″·000	+″·002	−″·002	−″·014	−″·061	−″·175

The reading 30^d corresponds to an angular distance of the object of $2\frac{1}{2}°$ from the centre of the object-glass. The above numbers indicate that the distortion is roughly proportional to the fifth power of the distance from the centre of the field. It is practically insensible up to 18^d, and increases rapidly after 24^d, or 240″ on the astrographic plate.

For the photographs taken with the Thompson equatorial the microscope is used with the magnification unity, and the field used can only extend to 15^d from the centre, within which range the distortion is quite insensible.

The amount of the distortion at different distances from the centre is shown in the diagram by the curve above the horizontal line. The *correction* for distortion, which is negative on the positive side of the centre, is shown by the dotted curve below the line. On the negative side the correction changes sign and is positive, being represented by the full-line curve. Practically the whole effect of the distortion appears in alteration of the "runs" of the micrometer, and the effect on the mean of the measures of a star image is very small, for in measuring the position of an image it is bisected at the centre of the field C, and in effect the distances from the two réseau lines between which it falls, *i.e.* CA and CB, are read. The error of runs is eliminated by combining these measures with weights in inverse

proportion to their distances from the centre. Measuring now in one direction, from right to left, the correction for distortion will be positive on one side of the centre and negative (indicated by the dotted curve) on the other. In the diagram the correction to the measure of the réseau line on the left is AM, to the réseau line on the right is zero, and the correction to the mean is CR.

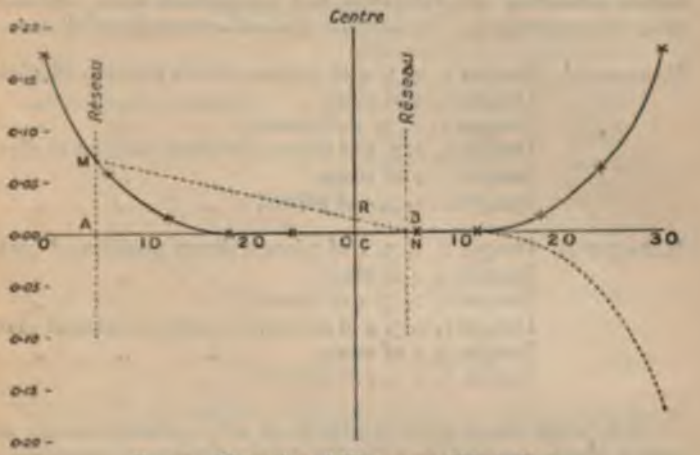

FIG. 2.—Distortion of Microscope Object-glass.

The position in the diagram corresponds to the maximum effect. The same result is shown in the following table:

Correction for Distortion.

Reading of Scale.	Left.	Right.	Mean.
d	"	"	"
0	+·175	·000	·000
6	+·061	+·062	+·012
12	+·014	−·002	+·003
18	+·002	−·014	−·003
24	−·002	−·061	−·012
30	·000	−·175	·000

Measurement of the Photographs of Eros.

On each photograph taken with the Astrographic equatorial measures are made of the planet, ten to twelve reference stars and six comparison stars. The reference stars are selected from M. Loewy's list within 50′ of the centre of the field, and, as far as practicable, symmetrically placed with respect to *Eros*. The comparison stars are approximately of the same brightness as *Eros*, within 25′ of the centre (so as to be well within the field of the Thompson plates) and symmetrically placed as regards *Eros*. On photographs with the Thompson equatorial the planet and

six comparison stars are measured. Four images are obtained on each photograph, and these are all measured, two by one observer and two by a second, in both direct and reversed positions of the plate. Four times as many measures are made of *Eros* as of the stars, each measurer measuring all four images of the planet in direct and reversed positions of the plate twice over, once before measuring the reference and comparison stars, and once after these measures. The scheme of measurement is as follows :

Measurer A.	Images 1, 2, 3, 4 of planet,	direct	position	of plate.
	Images 1, 2 of stars,	,,	,,	,,
	Images 1, 2, 3, 4 of planet,	,,	,,	,,
	Images 1, 2, 3, 4 of planet,	reversed	position	of plate.
	Images 1, 2 of stars,	,,	,,	,,
	Images 1, 2, 3, 4 of planet,	,,	,,	,,
Measurer B.	Images 1, 2, 3, 4 of planet,	direct	position	of plate.
	Images 3, 4 of stars,	,,	,,	,,
	Images 1, 2, 3, 4 of planet,	,,	,,	,,
	Images 1, 2, 3, 4 of planet,	reversed	position	of plate.
	Images 3, 4 of stars,	,,	,,	,,
	Images 1, 2, 3, 4 of planet,	,,	,,	,,

The large number of measures of *Eros* (thirty-two in all) makes them comparable with the total number of measures of reference and comparison stars. It is considered that the above system of measurement is amply sufficient to obtain the position of *Eros* with all the accuracy of which the photographs admit.

The progress made with the measurement of the photographs is as follows :

Of the 144 photographs with the Astrographic equatorial between 1900 October 1 and December 31, 121 have been completely measured, and ten by one measurer only.

Of the 102 photographs with the Thompson equatorial between the same dates fifty-five have been completely measured.

The stars used as comparison stars by micrometric observers have not yet been measured. This work will be taken in hand when the measures requisite for the reduction of the Greenwich photographs are completed.

The Accuracy of the Measures.

From measures made on a réseau and others made on specks on the film of a photograph it appears that the probable error of a pointing on a perfectly defined speck is $\pm$"020. This result is obtained with the magnification used in the actual measurement, and the probable error would doubtless be reduced if a higher power eyepiece were used.

For the determination of the accuracy with which the planet *Eros* is bisected, comparisons have been made between the two

pointings on *Eros* in the same position of the plate by the same measurer. As explained in the last paragraph these pointings were made after an interval of time during which other images were measured. The actual differences between Mr. Davidson's (C.D.) measures on plate 5306 are given as a specimen.

Plate 5306. *Measurer C.D.*

	Difference of 1st and 2nd Measures of Eros.	
	x	y
	"	"
Image 1 measured direct	− ·06	+ ·02
" 2 " "	− ·02	+ ·14
" 3 " "	+ ·08	+ ·04
" 4 " "	+ ·02	+ ·02
" 1 " reversed	+ ·01	− ·22
" 2 " "	− ·10	+ ·18
" 3 " "	+ ·23	+ ·04
" 4 " "	+ ·22	+ ·01
	± ·093	± ·084

Results of this nature, each depending on eight comparisons, are given below for a number of plates for the three measurers, Mr. Davidson (C.D.), Mr. Edney (D.E.), and Mr. Burkett (W.B.).

PLATES TAKEN WITH ASTROGRAPHIC EQUATORIAL.

Mean Differences between Two Pointings on Eros by the same Measurer and in the same Position of the Plate.

	C.D.		D.E.		W.B.	
No.	x	y	x	y	x	y
	"	"	"	"	"	"
5275	± ·111	± ·073	± ·075	± ·118	...	...
5276	·080	·070	·133	·081	...	...
5278	·126	·064	·124	·133	...	...
5280	·121	·156	·131	·124	...	...
5283	·151	·183	...	...	± ·189	± ·183
5285	·131	·073	·100	·106	...	...
5286	·108	·132	...	...	·040	·090
5287	·064	·058	·103	·050	...	...
5288	...	...	·115	·080	·134	·071
5289	·101	·149	·078	·101	...	...
5290	·059	·150	...	...	·078	·146
5294	...	...	·113	·131	·090	·081
5297	·085	·184	...	...	·135	·119
5299	...	...	·120	·083	·155	·129

No.	C.D. x	C.D. y	D.E. x	D.E. y	W.E. x	W.E. y
	"	"	"	"	"	"
5300	·080	·173	·158	·079	...	...
5304	...	...	·070	·120	·098	·106
5306	·093	·084	·101	·166	...	...
5307	...	...	·164	·158	...	...
	±·101	±·120	±·113	±·109	±·115	+·116
			Mean = ±·110	±·115		

PLATES TAKEN WITH THOMPSON EQUATORIAL.

Mean Differences between Two Pointings on Eros by the same Measurer and in the same Position of the Plate.

No.	C.D. x	C.D. y	D.E. x	D.E. y	W.E. x	W.E. y
	"	"	"	"	"	"
845	±·086	±·068	±·069	±·093	...	...
846	...	...	·101	·071	±·094	±·079
847	...	...	·105	·053	·123	·075
848	·069	·075	·096	·073	...	...
849	·048	·081	·081	·091	...	...
850	·051	·084	·069	·090	...	...
851	...	...	·079	·058	·100	·104
852	...	...	·081	·110	·038	·078
853	·068	·065	·071	·084	...	...
854	·140	·128	·065	·118	...	...
855	...	...	·108	·095	·065	·090
857	...	...	·086	·078	·140	·086
858	·064	·096	·085	·055	...	...
859	...	...	·084	·051	·054	·070
860	·132	·097	...	...	·067	·095
861	...	...	·103	·125	·066	·068
863	·063	·037	·070	·062	...	...
864	...	...	·070	·054	·064	·073
865	...	...	·073	·079	·111	·111
	±·080	±·088	±·083	±·080	±·084	±·084
			Mean ±·082	±·083		

The above tables show that the accidental error of measurement is the same for the three measurers, and that it is considerably smaller for the Thompson than for the Astrographic photographs. The probable accidental error of a single pointing on *Eros* is

$\pm''\cdot112 \times \frac{1}{\sqrt{2}} \times \cdot85 = \pm''\cdot067$ for the Astrographic photographs

and

$$\pm''\cdot082 \times \frac{1}{\sqrt{2}} \times \cdot85 = \pm''\cdot049 \text{ for the Thompson photographs.}$$

The comparison of the results shows that the accidental errors of measurement are only a trifling part of the final error. No attempt has been made to analyse the sources of error in detail, though it seems clear that a considerable error is inherent in the star images.

In the following table the results obtained from different plates on October 26 and 27 with the two instruments are collected to show what accuracy may be expected in the results. These are the first two long nights as yet reduced, and may be taken as fairly representative nights. For the photographs with the Astrographic equatorial two results are given. In the first of these the position of *Eros* is obtained directly from twelve reference stars (within 50′ of the centre of the plate) taken from M. Loewy's Circular; the second result is obtained from six comparison stars of about the same magnitude as *Eros*, whose positions are determined from the mean of the seven photographs. The same six comparison stars were used for the photographs with the Thompson Equatorial.

Residuals of Measured Places of Eros, 1900 October 26.

Astrographic Equatorial.

Plate No.	G.M.T.	Right Ascension. Parallax Factor.	R.A. T−O. Ref. stars.	R.A. T−O. Comp. stars.	R.A. Residuals. Ref. stars.	R.A. Residuals. Comp. stars.	Declination. Parallax Factor.	Dec. T−O. Ref. stars.	Dec. T−O. Comp. stars.	Dec. Residuals. Ref. stars.	Dec. Residuals. Comp. stars.
	h m s		s	s	s	s		"	"	"	"
5183	6 17 37	+1·44	+0·242	+0·227	+0·004	−0·011	−1·05	−1·28	−1·09	−0·02	+0·17
5184	7 0 1	+1·41	·240	·251	+ ·002	+ ·013	−0·83	1·53	1·32	− ·27	−0·06
5185	7 35 48	+1·34	·241	·224	+ ·003	− ·014	−0·66	1·33	1·16	− ·07	+ ·10
5186	11 25 5	+0·26	·244	·245	+ ·006	+ ·007	+0·04	1·07	1·22	+ ·19	+ ·04
5187	12 20 19	−0·09	·244	·242	+ ·006	+ ·004	+0·06	1·21	1·31	+ ·05	− ·05
5189	17 18 1	−1·41	·216	·230	− ·022	− ·008	−0·81	1·02	1·13	+ ·24	+ ·13
5190	17 39 31	−1·45	+ ·240	+ ·249	+ ·002	+ ·011	−0·97	−1·39	−1·63	− ·13	− ·37
Means			+ ·238	+ ·238	±·0064	±·0097		−1·26	−1·26		
Mean discordance for one plate ...					±″·06	±″·09				± ·14	± ·13
Thompson Equatorial.											
787	6 43 14	+1·43	+ ·220	...	− ·011	...	−0·92	−1·23	...	+ ·01	...
788	7 5 42	+1·40	·238	...	+ ·007	...	−0·80	1·15	...	+ ·09	...
789	7 31 19	+1·35	·235	...	+ ·004	...	−0·68	1·27	...	− ·03	...
790	11 19 20	+0·30	·229	...	− ·002	...	+0·04	1·21	...	+ ·03	...
791	11 58 18	+0·05	·231	...	·000	...	+0·06	1·32	...	− ·08	...
792	17 6 21	−1·41	·238	...	+ ·007	...	−0·80	1·30	...	− ·06	...
793	17 27 15	−1·44	+ ·225	...	− ·006	...	−0·91	−1·21	...	+ ·03	...

Astrographic Equatorial.

Plate No.	G.M.T.	Right Ascension: Parallax Factor.	R.A. T−O. Ref. stars.	R.A. T−O. Comp. stars.	R.A. Residuals. Ref. stars.	R.A. Residuals. Comp. stars.	Declination: Parallax Factor.	Dec. T−O. Ref. stars.	Dec. T−O. Comp. stars.	Dec. Residuals. Ref. stars.	Dec. Residuals. Comp. stars.
	h m s		s	s	s	s		″	″	″	″
5191	6 36 27	+1·44	+0·235	+0·231	+0·003	−0·001	−0·92	−1·31	−1·26	−0·13	−0·08
5192	7 2 1	+1·41	·227	·229	− ·005	− ·003	−0·80	1·37	1·30	− ·19	− ·12
5193	7 23 53	+1·37	·225	·216	− ·007	− ·016	−0·69	1·20	1·08	− ·02	+ ·10
5200	11 9 48	+0·32	·239	·240	+ ·007	+ ·008	+0·05	1·02	1·17	+ ·16	+ ·01
5201	11 30 28	+0·19	·240	·242	+ ·008	+ ·010	+0·06	1·14	1·17	+ ·04	+ ·01
5202	16 56 55	−1·42	·226	·224	− ·006	− ·008	−0·79	1·12	1·16	+ ·06	+ ·02
5203	17 25 1	−1·45	+ ·235	+ ·245	+ ·003	+ ·013	−0·93	−1·07	−1·10	+ ·11	+ ·08
Means	... ...		+ ·232	+ ·232	± ·0056	± ·0084		−1·18	−1·18		
Mean discordance for one plate				...	±″·05	±″·08				± ·10	± ·06

Thompson Equatorial.

794	7 18 1	+1·39	...	+0·234	...	−0·008	−0·71	...	−1·12	...	+0·04
795	7 49 30	+1·30	...	·232	...	− ·010	−0·56	...	−1·19	...	− ·03
796	11 37 52	+0·14	...	·248	...	+ ·006	+0·07	...	−1·12	...	+ ·04
797	12 15 6	−0·10	...	·248	...	+ ·006	+0·07	...	−1·17	...	− ·01
798	17 0 27	−1·42	...	·240	...	− ·002	−0·80	...	−1·19	...	− ·03
799	17 25 27	−1·46	...	·249	...	+ ·007	−0·93	...	−1·20	...	− ·04
Means	... ...		...	+ ·242	...	± ·0063		...	−1·16	...	
Mean discordance for one plate				...	...	±″·06					± ·03

The mean discordances of the above observations are $\pm''\cdot089$ for the Astrographic plates and $\pm''\cdot048$ for the Thompson plates. The probable error of a position of *Eros* derived from an Astrographic plate is $\pm''\cdot089 \times \cdot85 \times \sqrt{\frac{7}{8}} = \pm''\cdot082$, and for a Thompson plate $\pm''\cdot048 \times \cdot85 \times \sqrt{\frac{9}{8}} = \pm''\cdot045$. The result for the Thompson photographs is entirely satisfactory. Some of the photographs with the Astrographic telescope on October 26 are not very good, and it seems likely that the probable error would be smaller than $\pm''\cdot082$ on good nights.. There is, however, an undoubted superiority of the Thompson photographs over those taken with the Astrographic equatorial.

As illustrating the accuracy which may be expected in the determination of the solar parallax from photographs of *Eros*, the following provisional results for October 26 and 27 may be of interest, but it is to be noted that they are not definitive, no correction for change of error in the ephemeris of *Eros* having been applied.

Instrument.	Mean Epochs G.M.T.		Deduced Solar Parallax.
	h m	h m	
Astrographic	Oct. 26 6 58	Oct. 26 17 29	8″·783
,,	Oct.-26 17 29	Oct. 27 7 1	8·756
,,	Oct. 27 7 1	Oct. 27 17 11	8·771
Thompson	Oct. 26 7 7	Oct. 26 17 17	8·798
,,	Oct. 26 17 17	Oct. 27 7 34	8·805
,,	Oct. 27 7 34	Oct. 27 17 13	8·763

Royal Observatory, Greenwich:
1904 *May.*

Note on the Optical Distortion of the Microscope of one of the Oxford Machines for Measuring Astronomical Photographs. By H. C. Plummer, M.A.

The measures of the plates for the Oxford share of the Astrographic Chart Catalogue being now finished, a beginning has recently been made at the University Observatory in measuring with a modified form of instrument the photographs of the planet *Eros* which were taken in the autumn of 1900. The higher degree of accuracy which is demanded by parallax work has revealed defects in the optical arrangement of the apparatus which call for some examination. A brief note on the matter may be useful in drawing attention to a source of error which is possibly of general occurrence.

The type of measuring machine which has been in use at Oxford was described in *Monthly Notices*, vol. lv. p. 102. The modification which has been added recently consists in the pro-

vision of micrometer screws by means of which the graduated glass diaphragm in the eyepiece of the microscope can be moved parallel to the two perpendicular scales.

The objective of the microscope is a simple achromatic combination having an aperture of about 1·2 cm. and a focal length of nearly 5 cm. The distance between the plate and the scale is about 20 cm. and the magnification is 1·5, the réseau space being 0·5 cm. and 100 divisions of the scale being equal to 0·75 cm. The angular field which should be covered has thus a radius of $3\frac{1}{2}°$. It is clearly of great importance that there should be sharp definition without distortion over this field. The detection of the distortion which exists in the Oxford machine was due to the marked variation in the measured "runs," which was found on examination not to be due to irregularities in the réseau. For instance, a pair of réseau lines were set successively on the positions ·025 and ·525 on the scale. Several settings for "runs" were made in the two positions, and gave as a result −·0005 and +·0038 in the two cases, representing −0″·15 and +1″·14 respectively. The conditions under which these measures were obtained were identical except as regards the position of the réseau lines on the scale.

A partial calibration of the scale by comparing its divisions with distances measured by the micrometer screws was enough to show that neither the scales nor the screws could be held responsible for errors of this magnitude. The Zeiss scale leaves nothing to be desired in point of accuracy or clearness. The screws, on which the precision of the measures depends only in a minor degree, are also quite satisfactory. The experiments were therefore continued on the supposition that the discrepancies were due to optical distortion alone.

Systematic measures were made of the "runs" for two particular lines of a 5 mm. réseau at distances of ·05 along the X scale. The scale was first carefully centred, and measures were made along the whole length of the scale. Then the scale was shifted by turning the micrometer screws through five revolutions. In this way it was possible to prolong the measures beyond the range of the scale in its central position. The measured "runs" were then plotted as ordinates with abscissæ corresponding to the positions of the réseau lines on the scale. No distinction was made between equal distances from the centre to right or left, as the object was to find a distortion symmetrical about the centre. The points so obtained were smoothed by a graphical interpolation. The curve to which they approximated was parabolic in form, and corresponded fairly closely to the formula

$$E = 0^{1}\cdot012\ (s-0\cdot5)^{2} = 3''\cdot6\, f^{2},$$

where E is the effect of distortion on the "runs" and $s = f + 0\cdot5$ is the scale reading of either end of the réseau line expressed as a fraction of the 100 scale-divisions which cover a réseau interval. Thus the effect, which vanishes when the scale is symmetrically

placed with respect to the two réseau lines, reaches the value $0^{t}\cdot0030 = 0''\cdot9$ at the extreme positions. If the effect is to be attributed to distortion symmetrical about the centre of the field, then that distortion must be represented by the formula

$$D = 0^{t}\cdot004\ s^3 = 1''\cdot2\ s^3.$$

The effect of the distortion is in the direction away from the centre of the field, so that the magnification increases in the outward direction.

It must be concluded then that distortion exists in the microscope now examined, and that it is of considerable amount. The effect of "runs" and distortion is that, instead of the true reading s, we obtain the reading*

$$r_1 = s + as + ds^3$$

for the preceding réseau line, where d is the distortion coefficient and a the value which the "runs" would have if there were no distortion. The reading for the following réseau line will be

$$r_2 = 1 - [1 - s + a(1-s) + d(1-s)^3],$$

whence the measured "runs" are found to be

$$r_1 - r_2 = a + d(1 - 3s + 3s^2).$$

The mean "runs" derived from the measures will therefore be $a + \frac{1}{2}d$, while the "runs" corresponding to $s = 0\cdot5$ will be $a + \frac{1}{4}d$. There are then three ways which suggest themselves of correcting the reading r_1: (*a*) by following the usual method and adopting the value $(1-s)r_1 + sr_2$; (*b*) by correcting r_1 for the mean "runs;" (*c*) by correcting r_1 for the "runs" corresponding to $s = 0\cdot5$. The effect of the distortion on the final measure will be in the three cases:

$$(a)\ ds(1-s)(2s-1);\ (b)\ \tfrac{1}{2}ds(2s^2-1);\ (c)\ \tfrac{1}{4}ds(4s^2-1).$$

These effects are here tabulated, the unit being $0^{t}\cdot0001 = 0''\cdot03$, with the value of d given above:

s	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
(*a*)	0	−3	−4	−3	−2	0	+2	+3	+ 4	+ 3	0
(*b*)	0	−2	−4	−5	−5	−5	−2	0	+ 4	+11	+20
(*c*)	0	−1	−2	−2	−1	0	+2	+7	+12	+20	+30

* Since symmetrical distortion alone is looked for, no term ks^2 is included. But such a term might be produced by a sensible tilt of the scale. This effect has been noticed in the case of the Cambridge machine, and is remedied by mechanical readjustment. It may be noticed that the effect of terms depending on even powers of s will be eliminated by measuring in direct and reversed positions of the plate, while the effect of radial distortion will not be so eliminated.

With the method (*a*), usually adopted for weighting the readings of the two réseau lines, the effect of distortion is much reduced, and its influence on the measures will scarcely be appreciable when these are only made to $0^{1}\cdot001$. But in more accurate work the defect is more serious. If a method like (*b*) or (*c*) be adopted, in which the reading of one réseau line alone is corrected for "runs," these being taken to be constant, it will always be possible to choose the nearer line to the measured object. Hence no attention need be paid to the large numbers occurring in the second half of the above table, and it is clear that method (*c*) is better than method (*b*), and even preferable, so far as the effect of distortion is concerned, to the ordinary method. In method (*c*), by adopting the "runs" as measured in the centre of the scale, we use the minimum measured "runs." Even so the measures are clearly over-corrected, and if we seek further for the most advantageous value to use over the range $s = 0$ to $s = 0\cdot5$ we arrive at the number $a + \frac{1}{4}d$, for this will make the mean error over the range zero. The outstanding error in the final coordinate is given here with the same units as in the above table :

s	0·0	0·1	0·2	0·3	0·4	0·5
Error	0·0	−0·5	−0·6	−0·4	+0·6	+2·5

The distortion arises from want of flatness in the field, and to this again is due a marked absence of good definition at a distance from the centre. Any appreciable distortion within the field to be covered, even when not complicated by bad definition, is inconvenient as requiring corrections which might be obviated by better optical means. That this can be done may be inferred from the performance of photographic lenses. It may be necessary to use a doublet, as in the Cambridge instrument, or at least a triple lens. It is to raise this question that the present note has been written. Can objectives be obtained which are sensibly free from distortion,* and is it necessary to substitute for the single objective an optical combination of greater complexity ?

Since the adoption of the réseau as an essential aid in the work of measuring, the machines employed for measuring astronomical photographs seem to fall under three distinct classes :

(*a*) The perpendicular threads in the focus of the microscope are fixed during the measurement. Relative motions in perpendicular directions between the plate and the microscope are produced by micrometer screws, so that the star and the four sides of the containing réseau square can be brought under the microscope threads. The mechanical arrangement of the apparatus

* The answer to this first question must be taken to be in the affirmative, for the Cambridge machine appears to be entirely free from distortion of an amount which can be detected. It is not so clear whether the capacity of a properly designed single objective is necessarily inadequate to the requirements.

must be of a high order of perfection, and the cost of the instrument must therefore be considerable. Accuracy will then depend on the quality of the screws, and will be affected by their deterioration with continued use. On the other hand little depends on the quality of the microscope. All that is required of it is good definition at the centre of the field.

(*b*) The microscope and plate are fixed during the measurement of the stars within a réseau square, and the threads in the focus of the microscope are moved by micrometer screws. The dependence on the quality of the screws is the same as in the former case, while the mechanical requirements are probably easier to satisfy. The centre of the réseau square is brought to the centre of the field of the microscope, and good definition without sensible distortion is required over an area corresponding to the square. If fixed threads, forming a square in the focus of the microscope, be added, we have the form of instrument adopted by Sir David Gill for the astrographic work at the Cape Observatory. By bringing the image of the réseau square into approximate coincidence with the fixed square a certain rapidity of measurement is attained, though at some sacrifice of accuracy owing to the partial neglect of "runs."

(*c*) The microscope is provided with two rectangular crossed scales, ruled on a glass plate and placed in the focal plane. The star to be measured is brought to the intersection of the scales, and the positions of the containing réseau lines on the scales must be determined. As this can be done with considerable accuracy by direct estimation, and a rigid connection between the plate and microscope while the measure is made is the only mechanical requirement, a useful instrument of this class can be made in the cheapest possible form (*British Association Report*, 1900, p. 676). Since a star may be anywhere within a réseau square, each arm of the crossed scale must have a length equal to a réseau space, and the microscope must give a field, free from distortion and of good definition, of a width equal to two réseau spaces.

The Oxford machine now examined belongs to this last type. The substitution of the graduated scale for the screw as the chief means of measurement is seen to increase the demand for optical efficiency in the microscope. Hence the importance of the question raised above as to the relative qualities of different types of objective. Perhaps it may be pointed out that the effective field required of the microscope may be reduced by extending the principle of the crossed scale and developing it into a reticle, i.e. a ruled réseau. If it were possible to rule such a reticle without sacrificing the clearness possessed by the simple scale the field might be reduced to little more than a single réseau space, while the rapidity of measuring all the stars within a single square might be increased.

Note on the Influence of the Plate Constants on the Accuracy of the Position of an Object measured on a Photograph. By H. C. Plummer, M.A.

When a preliminary solution is made for the constants of a plate on which the corrected positions of all the stars are required, as in the case of plates for the Astrographic Chart catalogue, the manner of proceeding is sufficiently obvious. It is then simply necessary to employ as large a number of comparison stars as possible, and to use the most accurate positions which can be found for them. The case is, however, rather different when the object is to find, not a solution which is to be applied over the whole extent of the plate, but one which is to be used to correct the measured coordinates of a particular object. This is what is required in parallax work, for example. Here questions arise which concern the choice and number of the comparison stars, and it is important from the point of view of economy to decide the relative degree of accuracy with which their coordinates should be measured. Mr. Filon has discussed the matter very thoroughly* and his main inferences are probably well known. The subject seems sufficiently important to justify discussion in a slightly different way, even if little that is new be added.

The equations from which the plate constants are calculated are of the form

$$ax+by+c=\Delta x\,;\ dx+ey+f=\Delta y$$

where Δx, Δy are the differences between the computed coordinates and the measured coordinates of the comparison stars, *i.e.* the projections on the axes of the distances Δr between the computed and the measured positions. It is assumed that the comparisons are of equal weight for both coordinates and for all the stars. The six-constant solution will be considered first. The normal equations are

$$\begin{aligned}
a[x^2]+b[xy]+c[x]&=[x\Delta x]\,;\ d[x^2]+e[xy]+f[x]&=[x\Delta y]\\
a[xy]+b[y^2]+c[y]&=[y\Delta x]\,;\ d[xy]+e[y^2]+f[y]&=[y\Delta y]\\
a[x]+b[y]+cn&=[\Delta x]\,;\ d[x]+e[y]+fn&=[\Delta y]
\end{aligned}$$

where n is the number of comparison stars. But it is true, and formal proof might be given if necessary, that had the comparison been made according to any other axes the results would have been equivalent. Hence for theoretical purposes we may take as axes the principal axes of the momental ellipse at the centroid of particles of unit mass occupying the positions of the comparison

* *Monthly Notices*, vol. lxii. p. 561.

stars. The equations for the plate constants then take the simple form

$$\begin{aligned} a[x^2]&=[x\Delta x]\,;\quad d[x^2]=[x\Delta y]\\ b[y^2]&=[y\Delta x]\,;\quad e[y^2]=[y\Delta y]\\ cn\ &=[\Delta x]\,;\quad fn\ =[\Delta y]. \end{aligned}$$

If p is the probable error of a difference Δx or Δy, and p_a is the probable error of a, &c., then

$$\begin{aligned} p_a^2[x^2]&=p_b^2[y^2]=p_c^2n\\ &=p_d^2[x^2]=p_e^2[y^2]=p_f^2n=p^2 \end{aligned}$$

Now if x_o, y_o are the coordinates of the object whose position is required; δx_o, δy_o the errors in x_o, y_o due to errors $\delta\alpha$ in a, &c.; and p_x, p_y the probable errors of these coordinates so far as the effect of erroneous plate-constants is concerned,

$$\delta x_o=x_o\delta a+y_o\delta b+\delta c\,;\quad \delta y_o=x_o\delta d+y_o\delta e+\delta f.$$

Hence,

$$\begin{aligned} \delta x_o&=\Sigma\left\{\delta\Delta x\left(\frac{x_ox}{[x^2]}+\frac{y_oy}{[y^2]}+\frac{1}{n}\right)\right\}\\ \therefore p_x^2&=p^2\Sigma\left(\frac{x_ox}{[x^2]}+\frac{y_oy}{[y^2]}+\frac{1}{n}\right)^2\\ &=p^2\Sigma\left(\frac{x_o^2x^2}{[x^2]^2}+\frac{y_o^2y^2}{[y^2]^2}+\frac{1}{n^2}\right)\\ &=p^2\left(\frac{x_o^2}{[x^2]}+\frac{y_o^2}{[y^2]}+\frac{1}{n}\right)\quad\ldots\quad\ldots\quad\ldots\quad(1)\\ &=p_y^2, \text{ similarly.} \end{aligned}$$

Again, if the star places be corrected for refraction and aberration, we have

$$e=a,\ d=-b.$$

Then a four-constant solution is possible and the normal equations are

$$\begin{aligned} a[x^2+y^2]&=[x\Delta x+y\Delta y]\,;\quad b[x^2+y^2]=[y\Delta x-x\Delta y]\\ cn&=[\Delta x]\,;\qquad\qquad fn=[\Delta y] \end{aligned}$$

Hence

$$p_a^2[x^2+y^2]=p_b^2[x^2+y^2]=p_c^2n=p_f^2n=p^2$$

In this case

$$\begin{aligned} \delta x_o&=\Sigma\left\{\frac{x_o(x\delta\Delta x+y\delta\Delta y)}{[x^2+y^2]}+\frac{y_o(y\delta\Delta x-x\delta\Delta y)}{[x^2+y^2]}+\frac{\delta\Delta x}{n}\right\}\\ \therefore p_x^2&=p^2\Sigma\left\{\frac{x_ox+y_oy}{[x^2+y^2]}+\frac{1}{n}\right\}^2+p^2\Sigma\left\{\frac{x_oy-y_ox}{[x^2+y^2]}\right\}^2\\ &=p^2\Sigma\left\{\frac{x_o^2x^2+y_o^2y^2}{[x^2+y^2]^2}+\frac{1}{n}\right\}+p^2\Sigma\left\{\frac{x_o^2y^2+y_o^2x^2}{[x^2+y^2]^2}\right\}\\ &=p^2\left(\frac{x_o^2+y_o^2}{[x^2+y^2]}+\frac{1}{n}\right)\quad\ldots\quad\ldots\quad\ldots\quad\ldots\quad\ldots\quad(2)\\ &=p_y^2, \text{ similarly.} \end{aligned}$$

The formulæ (1) and (2) have been obtained without making any assumption as to the configuration of the comparison stars. Moreover, by basing the calculation directly on the errors of the coordinates of the comparison stars, instead of using the probable errors of the plate constants, the necessity of assuming an absence of correlation between these constants has been avoided.

The most important inference is that the object required should be at, or very near, the origin—*i.e.* should coincide as closely as possible with the centroid of the comparison stars. If this condition be satisfied, the configuration of the stars and the method of reduction (by four or six constants) are practically immaterial. The probable errors, so far as they are due to erroneous constants, are given by

$$p_x = p_y = p/\sqrt{n}$$

If the object does not coincide with the centroid of the comparison stars it is desirable to make the moments of inertia about both axes in the plate large—that is, to have the stars widely scattered in both directions—if the six-constant solution is used; but if the four-constant solution is used it is only necessary that one of the principal moments should be large. The four-constant solution is more valuable when the solution is to apply to the whole extent of the plate, particularly when the distribution of the comparison stars is bad.*

University Observatory, Oxford:
1904 *May* 12.

Note on the formulæ connecting "Standard Coordinates" with Right Ascension and Declination. By F. W. Dyson, M.A., F.R.S.

The simplest and most interesting formulæ from a mathematical point of view for determining Standard Coordinates are those given by M. Trépied in the Introduction to the Algiers Section of the *Astrographic Catalogue* (p. v), and all others are readily deduced from them.

* Mr. Filon, to whom I have had the advantage of showing the above note in MS., points out the desirability of stating definitely the assumptions which underlie the argument—namely, that "there is no correlation between the errors in the residuals of two given stars," and also that "there is no correlation between errors of measurement in the two coordinates" for each star. He adds that "distortion or ellipticity of the images may correlate the errors in x, y; and, further, that if this distortion affect similarly all the star discs, it may introduce correlation into the x-measures of different stars." But in general, in work of the kind to which this note is intended to apply, the distortion of the images can be considered so slight that no serious effect of this nature need be apprehended.

Let x and y be the Standard Coordinates α, δ; the Right Ascension and Declination of a star; and A, D the Right Ascension and Declination of the centre of the photograph:

$$\left.\begin{aligned} \text{Then}\quad \sqrt{1+x^2+y^2}\,\sin\delta &= \sin D + y\cos D \\ \sqrt{1+x^2+y^2}\,\cos\delta\,\sin(\alpha-A) &= x \\ \sqrt{1+x^2+y^2}\,\cos\delta\,\cos(\alpha-A) &= \cos D - y\sin D \end{aligned}\right\} \begin{matrix} \text{i} \\ \text{ii} \\ \text{iii} \end{matrix}$$

The second of these formulæ gives

$$x = \sin(\alpha-A)\cos\delta + \tfrac{1}{2}x\,(x^2+y^2) + \&c.$$

If $x = y = 1°$

$$\tfrac{1}{2}x\,(x^2+y^2) = 1''{\cdot}09.$$

Thus the term $\frac{1}{2}x\,(x^2+y^2)$ may be readily tabulated as a small correction, or a diagram may be formed to give it. It is to be noted that $\frac{1}{2}x\,(x^2+y^2)$ is independent of the position of the plate's centre and very rough values of x and y are sufficient to give the correction accurately.

The first equation gives y in the terms of the declination. When $x = 0$ the equation becomes

$$\sqrt{1+y^2}\,\sin\delta = \sin D + y\cos D$$

and as $y = \tan(\delta-D)$ is known to be a solution of this equation, the following transformation of equation (i) is suggested.

$$\{y-\tan(\delta-D)\}\,\{y+\tan(\delta+D)\} = \frac{x^2\sin^2\delta}{\cos(\delta-D)\cos(\delta-D)},$$

$$\text{or}\quad y-\tan(\delta-D) = \frac{\tfrac{1}{2}x^2\tan\delta}{1+\tfrac{1}{2}\,\dfrac{y-\tan(\delta-D)}{\tan\delta}\cdot\dfrac{\cos(\delta-D)\cos(\delta+D)}{\cos^2\delta}}$$

For brevity put $y-\tan(\delta-D) = \tfrac{1}{2}x^2\tan\delta\,.\,z$

$$\text{and}\qquad \frac{\cos(\delta-D)\cos(\delta+D)}{\cos^2\delta} = \lambda^2$$

Then $z\,(1+\tfrac{1}{4}\lambda^2x^2\,.\,z) = 1$

$$\text{or}\qquad z = 2\,\frac{\sqrt{1+\lambda^2x^2}-1}{\lambda^2x^2}$$

$$= 1-\frac{\lambda^2x^2}{4}+\frac{1\cdot3}{4\cdot6}\lambda^4x^4\,\frac{1\cdot3\cdot5}{4\cdot6\cdot8}\lambda^6x^6+\&c.$$

Therefore

$$y = \tan(\delta - D) + \tfrac{1}{2}x^2 \tan\delta - \tfrac{1}{8}x^4 \tan\delta \frac{\cos(\delta - D)\cos(\delta + D)}{\cos^2\delta} + \tfrac{1}{16}x^6 \tan\delta \frac{\cos^3(\delta - D)\cos^3(\delta + D)}{\cos^3\delta} - \&c.$$

This formula shows explicitly the magnitude of the correction at different declinations to the approximate formula

$$y = \tan(\delta - D) + \tfrac{1}{2}x^2 \tan\delta.$$

For example, take the case where $\delta = 85°$, $D = 84°$, and $x = 1°$.

$$-\tfrac{1}{8}x^4 \tan\delta \frac{\cos(\delta - D)\cos(\delta + D)}{\cos^2\delta} = +\tfrac{1}{8}\frac{\sin^3 1° \cdot \cos 1° \cdot \cos 11°}{\sin^3 5°} \times 3600'' = 3''{\cdot}55.$$

For $\delta = 70°$ the correction can only amount to $0''{\cdot}06$.
This correction can be readily tabulated as far as $\delta = 85°$.

An Analysis of the Distribution of Stars on the 1,180 Plates in Zones +25° to +31° allotted to the University Observatory, Oxford, in connection with the International Astrographic Survey. By F. A. Bellamy.

1. The completion, on 1904 February 17, of the measurement in two positions of the 1,180 plates required for the zones +25° to +31°, which were allotted to the University Observatory, Oxford, in connection with the International Astrographic Catalogue of Stars to the eleventh magnitude, has afforded the means of revising the figures given by me in a paper (*Monthly Notices*, vol. lx. p. 12) concerning the distribution of stars. It is my intention now only to refer to that portion of the sky to which the resources of the observatory have been mainly devoted during the past twelve years, reserving consideration of the distribution of stars outside +25° to +31° to a future communication.

The plates discussed in the paper referred to were 513 in number, each with exposures of 6^m, 3^m, 20^s, and were exposed between 1892 January and 1899 June; other plates with exposures not precisely of these durations were omitted. In the present paper all plates are included, but only one of the three exposures has been dealt with throughout.

2. In selecting plates considered good, and rejecting others which did not show enough stars, constant use has been made of

the numbers shown on Argelander's charts for the areas covered by the plates. To obtain these numbers the outline of the area was ruled on glass and laid over the proper portion of the charts, when the number of stars included could be quickly counted. These numbers have been used to form Table II., which corresponds in all details with Table I. As a general rule a plate has always been accepted for measurement if it showed three times as many stars as Argelander. It was at once recognised that this rule would admit plates of inferior quality in the region of the Milky Way ; but this was accepted as an advantage from the point of view of limiting the work in these regions. Further, this rule was not adopted until a good deal of the early work had been done and many plates had been measured for which the Argelander ratio was less than 3·0. It may be advisable to repeat these plates, but it was determined to finish the whole work first, since a region not measured at all has stronger claims than one where measures only fall somewhat short of a definite standard. Among the 1,180 plates the Argelander ratio falls below 3·0 for 280 plates ; but for most of these it is greater than 2·5.

3. Table I. gives the total number of stars measured. Under R.A. 1^h, Dec. $+25°$ are given the total numbers on plates with centres at declination $+25°\ 0'$ and R.A.'s $1^h\ 0^m$, $1^h\ 8^m$, $1^h\ 16^m$, $1^h\ 24^m$, $1^h\ 32^m$, $1^h\ 40^m$, $1^h\ 48^m$, $1^h\ 56^m$; and since each plate in this zone includes about $4\frac{1}{2}$ min. each way from the centre this total will include about $4\frac{1}{2}^m$ of 0^h and about $\frac{1}{2}^m$ of 2^h ; in declination it contains the belt from $+23°\ 55'$ to $+26°\ 5'$. Hence all stars are included at least twice, and on the average about 2·6 times.

TABLE I.

Total Number of Stars Measured in each Hour and Zone.

Hours.	+25°.	+26°.	+27°.	+28°.	+29°.	+30°.	+31°.	Totals.	Stars measured to 1899 June.
0	2879 7	1936 8	1623 7	1582 7	1671 7	1233 7	1653 7	12577 50	9283 35
1	2214 8	1533 7	2440 8	1711 7	2097 6	1606 7	1813 6	13414 49	7227 28
2	1738 7	2052 8	1605 7	1261 6	2152 7	1544 6	1974 7	12326 48	5784 25
3	2051 8	1871 7	1864 8	2002 7	2793 7	1269 7	1308 7	13158 51	5428 21
4	1248 7	1330 8	445 7	1022 7	1388 6	1617 7	1245 6	8295 48	3224 18
5	3290 8	2890 7	2241 8	2392 6	1959 7	3977 6	5289 7	22038 49	6578 23
6	2975 7	4470 8	2823 7	4616 7	3450 7	4019 7	2991 7	25344 50	9322 20
7	3480 8	2005 7	1934 8	2630 7	2167 6	2931 7	2506 6	17653 49	6533 23
8	1828 7	1575 8	1420 7	2100 6	2643 7	1453 6	1367 7	12386 48	4751 22
9	1042 8	1323 7	1429 8	2908 7	1362 7	1526 7	1137 7	10727 51	4216 25
10	968 7	1557 8	1160 7	1630 7	994 6	1670 7	1290 6	9269 48	2733 19
11	696 8	967 7	1393 8	1366 6	1036 7	945 6	1210 7	7613 49	1263 12
12	565 7	1110 8	1189 7	2070 7	1065 7	1322 7	1295 7	8616 50	2407 18
13	617 8	622 7	882 8	1574 7	1064 6	1712 7	1318 6	7789 49	2574 23
14	742 7	1461 8	898 7	1059 6	1122 7	1499 6	1725 7	8506 48	2718 19
15	948 8	1042 7	1256 8	1401 7	1391 7	1992 7	1976 7	10006 51	1668 12
16	1256 7	2189 8	995 7	1845 7	1382 6	3449 7	1919 6	13035 48	2208 13
17	2084 8	6239 7	3020 8	1997 6	1735 7	4791 6	2942 7	22808 49	3024 12
18	2067 7	5185 8	5892 7	3459 7	2465 7	3891 7	4078 7	27037 50	5764 16

Hours.	+25°.	+26°.	+27°.	+28°.	+29°.	+30°.	+31°.	Totals.	Stars measured to 1899 June.
19	2747 8	4536 7	5937 8	5104 7	7687 6	8473 7	9127 6	43611 49	13189 17
20	5011 7	6805 8	5209 7	3034 6	4375 7	5517 6	7622 7	37573 48	12877 23
21	2857 8	2982 7	3229 8	2399 7	3138 7	5074 7	4373 7	24052 51	10673 27
22	2066 7	2829 8	2500 7	2423 7	1738 6	3143 7	3419 6	18118 48	9768 29
23	2059 8	1703 7	1857 8	1341 6	2398 7	2247 6	2249 7	13854 49	8327 23
Totals & Plates	47,428 180	60,212 180	53,241 180	52,926 160	53,272 160	66,900 160	65,826 160	399,805 1180	141,539 513

TABLE IV.

Oxford. Average Number of Stars measured on each Plate, and the Ratio of Oxford Numbers to Argelander.

Hour.	+25° Stars.	+25° Ratio.	+26° Stars.	+26° Ratio.	+27° Stars.	+27° Ratio.	+28° Stars.	+28° Ratio.	+29° Stars.	+29° Ratio.	+30° Stars.	+30° Ratio.	+31° Stars.	+31° Ratio.	Final Results. Stars.	Final Results. Ratio.	Final Results. Stars per Sq. Deg.	Previous Results. Stars.	Previous Results. Ratio.	Previous Results. Stars per Sq. Deg.
0	411	6·7	242	3·8	232	3·7	226	3·7	239	3·9	176	2·7	236	3·6	252	4·0	53·7	265	4·6	56
1	277	4·7	219	3·6	305	5·1	244	4·3	350	6·0	229	3·7	302	4·6	274	4·6	58·4	258	4·3	55
2	248	4·9	256	4·9	229	4·1	210	4·3	307	6·3	257	4·2	282	4·3	257	4·7	54·8	231	4·8	49
3	256	4·2	267	4·0	233	4·1	286	5·9	399	8·8	181	3·5	187	3·3	258	4·7	55·0	259	4·4	55
4	178	4·7	166	4·2	63	1·7	146	3·8	231	2·5	231	3·6	208	3·5	173	3·3	36·7	179	4·4	38
5	411	3·8	413	3·9	280	3·0	399	4·5	280	3·1	663	5·6	756	6·2	450	4·3	96·0	286	3·5	61
6	425	2·9	559	3·7	403	3·1	659	5·9	493	4·7	574	4·6	427	3·8	507	4·0	108·1	466	3·7	99
7	435	4·7	287	3·0	242	2·8	376	4·6	361	4·9	419	4·9	418	4·7	360	4·2	76·8	284	3·3	60

Hour.	+25° Stars.	Ratio.	+26° Stars.	Ratio.	+27° Stars.	Ratio.	+28° Stars.	Ratio.	+29° Stars.	Ratio.	+30° Stars.	Ratio.	+31° Stars.	Ratio.	Final Results. Stars.	Ratio.	Stars per Sq. Deg.	Previous Results. Stars.	Ratio.	Stars per Sq. Deg.
8	261	4·1	197	2·9	203	3·2	350	6·5	378	7·2	242	3·7	195	2·7	258	4·1	55·0	216	3·4	46
9	130	2·4	189	3·3	180	3·4	415	8·6	195	4·3	218	4·0	162	2·7	210	4·0	44·8	169	3·3	36
10	138	3·0	194	4·0	166	3·5	233	5·3	166	4·1	239	4·9	215	4·0	193	4·1	41·2	144	3·1	31
11	87	2·2	138	3·4	174	4·3	228	5·6	148	4·1	158	3·5	173	4·2	155	3·9	33·1	105	2·6	22
12	81	1·8	139	3·1	170	3·9	296	7·8	152	4·3	189	4·0	188	3·8	172	4·0	36·7	134	3·2	29
13	77	2·0	89	2·1	110	3·1	225	5·9	177	4·8	245	5·3	220	4·9	159	4·0	33·9	112	2·9	24
14	106	2·3	183	3·9	128	2·7	176	4·4	160	4·4	250	5·1	246	5·4	177	4·0	37·7	143	3·4	30
15	119	2·5	149	2·9	157	3·4	200	4·6	199	4·5	270	5·5	282	5·7	196	4·1	41·8	139	2·9	30
16	179	3·1	274	4·8	142	2·4	263	4·8	230	4·5	493	7·8	320	5·0	272	4·7	58·0	170	3·0	36
17	261	3·4	891	11·8	378	5·3	333	4·4	248	3·2	799	9·6	420	6·0	465	6·2	99·4	252	3·3	54
18	295	3·0	648	6·5	842	9·1	494	5·4	352	3·9	556	5·4	583	5·5	541	5·5	115·4	360	3·7	77
19	343	2·8	648	5·2	742	5·9	729	5·6	1281	5·2	1210	7·4	1521	8·8	890	5·9	189·9	776	6·3	165
20	716	6·2	851	7·2	744	6·2	506	4·2	625	4·8	920	5·9	1089	6·8	783	6·0	166·9	560	4·7	119
21	357	4·4	426	5·0	404	4·1	343	3·4	448	4·7	725	6·5	625	5·7	472	4·9	100·6	395	4·4	84
22	295	4·0	354	4·9	357	4·4	346	4·4	290	3·5	449	4·6	570	5·9	327	4·6	80·4	337	4·4	72
23	257	4·0	243	3·6	232	3·4	224	3·4	343	5·3	375	4·7	321	4·1	283	4·1	60·3	252	4·0	54
Average	263·5		334·5		295·8		330·8		333·0		418·1		411·4		338·8		72·3	276		58·7
Plates	180		180		180		160		160		160		160		1180			513		
Ratio ...		3·74		4·64		4·24		4·96		4·63		5·30		5·18		4·66			4·0	

TABLE II.

Total Number of B.D. Stars included in the same Areas as in Table I.

Hours.	+25°.	+26°.	+27°.	+28°.	+29°.	+30.	+31°.	Totals.
0	431	512	443	429	430	461	462	3168
1	467	420	482	403	349	437	392	2950
2	352	419	391	292	340	365	461	2620
3	493	465	451	339	316	358	390	2812
4	267	315	268	269	563	452	359	2493
5	872	748	739	534	627	711	850	5081
6	1019	1201	897	776	741	865	781	6280
7	740	666	689	570	446	596	531	4238
8	448	545	443	323	368	389	503	3019
9	434	396	420	338	318	383	413	2702
10	326	388	329	306	245	339	324	2257
11	321	287	321	242	252	269	290	1982
12	306	356	306	265	248	328	341	2150
13	311	289	287	265	224	324	267	1967
14	327	378	327	242	256	293	322	2145
15	384	359	368	302	308	364	348	2433
16	401	457	413	385	304	440	381	2781
17	618	527	573	450	549	498	494	3709
18	688	803	648	639	638	722	747	4885
19	972	869	998	908	1468	1141	1036	7392
20	806	941	839	729	910	942	1114	6281
21	653	597	794	706	662	778	773	4963
22	514	575	572	555	494	685	575	3970
23	517	470	544	397	446	473	548	3395
Total ...	12,667	12,983	12,542	10,664	11,502	12,613	12,702	85,673

4. Features worthy of remark are the fairly uniform totals, about 2,000 and 3,000, for the hours outside the Milky Way, and the similarity of the average of the three hours densest in stars in the two parts of the Milky Way which cut the Oxford zones about 6^h and 19^h—namely, 5,200 and 6,200. In the Oxford results the numbers are between 8,000 and 13,000 for similar hours in the former case, but in the Milky Way (5^h to 8^h, and 18^h to 21^h) the average Oxford numbers are 21,700 and 36,100; and, as I shall show later in this paper, the average for the 6^h portion might be increased to 23,900, while the 19^h portion would become 47,100. For Argelander the increase from 6^h to 19^h (each mean of three hours) is 19 per cent.; for the Oxford measures the increase is as much as 66 per cent., and might readily be increased to more than 100 per cent. without increasing

the exposure of 3 min. I have always had a difficulty in obtaining a plate that showed 1,000 stars with a 3 min. exposure in the 6^h part of the Milky Way, but have numerous plates containing 2,000, 3,000, 4,000, and one nearly 5,000 stars with this exposure in the 19^h part.

This is an independent proof of the statement that has been made more than once that Argelander did not observe all the 9·5 mag. stars in the Milky Way, owing to the enormous number that had to be dealt with, but unconsciously decreased his scale by 0·2 mag. The difference between the Oxford and Argelander percentages would be much less if he had added a thousand stars in these two tenths of a magnitude; even a tenth of a magnitude fainter would have added a large number of stars where the increase in number is so rapid.

5. Now this seems to have a very important bearing on Kapteyn's conclusion, which is also supported by Professor S. Newcomb (*Astronomical Journal*, vol. xxi. p. 155), that "the stars of the Milky Way are in general bluer than the stars in other regions of the sky" (*C.P.D.* vol. i. p. [22]). For here the comparison is not between the Milky Way and other regions, but simply between two parts of the Milky Way. If the effect is to be attributed to a physical characteristic of the stars, then we must further conclude that there are variations of stellar luminosity in different parts of the Milky Way itself, in addition to the broad distinction between the Galaxy and other parts of the sky. It is surely simpler and more natural to infer a want of uniformity in the *Bonner Durchmusterung*, and thereby to explain the facts on the single hypothesis that Argelander's limiting magnitude in each region of the sky is a function of the star density.

6. Table III. exhibits all the instances when over 1,000 stars occur on those plates adopted for measurement.

TABLE III.

Dec.	R.A.	No. of Stars in 3 Min.	Plate.	Dec.	R.A.	No. of Stars in 3 Min.	Plate.
°	h m			°	h m		
+26	5 44	1158	1169		17 51	1489	1958
	5 52	1456	1170	+26	18 40	1267	1086
	7 28	1020	1536	+27	18 44	1146	1107
+28	6 18	1287	1734	+28	18 9	1489	1567
+31	5 37	1079	2298	+28	18 36	1165	1560
					18 45	1574	1561
				+29	18 22	1396	1578
+26	17 36	1223	1078		18 40	1401	1579
	17 44	1436	1079	+30	18 27	1064	1595
	17 52	1058	1080		18 36	1042	1596
+30	17 42	1224	1957		18 45	1084	1597

Dec.	R.A.	No. of Stars in 3 Min.	Plate.	Dec.	R.A.	No. of Stars in 3 Min.	Plate.
°	h m			°	h m		
+26	19 36	1763	1117		19 43	2210	2008
	19 52	1428	1119		19 52	2118	2001
+27	19 8	1398	1109	+25	20 12	1181	2289
	19 16	1230	1103	+26	20 0	1808	1120
	19 48	1176	1112		20 8	1774	1121
+28	19 12	1027	1223		20 16	1549	1212
	19 21	1100	474	+30	20 6	2097	1568
	19 48	1158	858		20 15	1496	1569
+29	19 7	2723	1562		20 24	2310	1564
	19 16	2856	1563		20 33	1541	1570
+29	19 34	1996	851		20 42	1823	1571
	19 43	2440	852		20 51	2703	1565
	19 52	1270	853	+31	20 1	1341	2016
+30	19 12	1057	1876		20 10	1228	1835
	19 39	4920	1582		20 19	1325	1834
	19 48	4097	1583		20 28	1310	1836
	19 57	2561	1584		20 46	1017	2015
+31	19 7	1178	1605	+30	21 9	1680	1566
	19 16	1342	1999		21 18	1802	1585
	19 25	1923	2000				
	19 34	2180	1606				

From this it can be seen that whilst we have in 5^h, 6^h, and 7^h only five plates with over 1,000 in 3 min. exposure we have fifty-five plates in the later part of the Milky Way, where it is more expanded, with more than 1,000 stars; of these twelve have more than 2,000. The greatest number is obtained on plate 1582, with the centre at R.A. $19^h\ 39^m$ +30°.

7. Taking the numbers in Table I. and dividing them by the subscript figures, the average numbers of stars for each hour, each zone, and all zones combined are obtained, and are given in Table IV. (see p. 652) on the left-hand side of each zone; when the numbers in Table I. are divided by the corresponding numbers in Table II. the ratios for Oxford to Argelander are found, and these form the quantities given on the right-hand side of the different zones.

8. Though the ratio of three times Argelander is well exceeded, except in 3^h, in the average for each hour of the seven zones combined and for each zone of twenty-four hours, some hours of all the zones, except +28°, show that this minimum has not been maintained; this is particularly the case in zone +25°, in the hours where there are least stars per square degree. This zone was that first photographed—mostly eleven

years ago—when the plates used were less sensitive, and a number of plates had been measured previous to the adoption of the standard, as mentioned in the beginning of this paper. A considerable portion of +26° and +27° had been re-photographed before the measurement of the plates had advanced very far; this especially applied to the latter part of the Milky Way in +26°. In the case of some of the hours in +31° the falling off in numbers was largely due to the quality of the plates supplied; though many were retaken, too much time had been spent over the first plates of the region to justify the whole being rejected.

9. The average given at the foot of Table IV. (see p. 652) indicates the improvement in the sensitiveness of the plates. The most southern zone was first photographed; the most northern zones were only photographed within the last two or three years.

The Argelander ratio for the whole 1,180 plates is 4·7; and the average number of stars measured on a plate is 339.

10. So far, except § 6, we have dealt with *actual measures* executed at Oxford. But it has been remarked (see § 2) that there is some lack of uniformity of procedure. The main object kept in view was to measure a fair number of stars all round the sky, and not necessarily to reach the same limit of magnitude in all R.A.'s. To do this we should have to measure an enormous number of stars in the Milky Way, which would have extended the work unduly. It is, however, a simple matter to *count* these plates, which gives all the information required for the study of stellar distribution; and for such counts I proceed to determine the corrections to the above tables necessary to represent the same limits of stellar magnitude round the sky. I will first state exactly how the work was limited in the rich regions.

11. As a first experiment only those stars showing all three images (6^m, 3^m, 20^s) were measured (ten plates only), when in this way at least three times Argelander's number could be obtained. This was found in practice to be too severe a limitation, and the 20 sec. exposure was accordingly increased to 60 sec., which was generally found sufficient to give an Argelander ratio exceeding 3·0; forty-five plates—mostly in the Milky Way—were so measured, *i.e.* those photographed in one minute. In addition to these about eighty-three plates had exposures differing slightly from the regulation 6^m 3^m 20^s; all the remaining 1,042 plates received these exposures.

Table V.

Zone. +	Centre. R.A. h m	Plate.	Number Measured.	3m Images that might have been Measured.	Additional Stars.
29	2 55	1860	†117	468	351
25	3 40	1266	†221	422	201
28	3 45	1317	†297	705	408

Zone. +	Centre. R.A.	Plate.	Number Measured.	3m Images that might have been Measured.	Additional Stars.
°	h m				
26	5 44	1169	†687	1158	471
26	5 52	1170	†538	1456	918
29	5 28	1309	†252	628	376
29	5 55	1311	†261	729	468
28	6 0	1733	†329	1875	1546
26	6 8	1171	†455	989	534
28	6 45	1497	†358	882	524
29	6 31	1314	†242	592	350
29	6 58	1315	†312	657	345
26	7 28	1536	†368	1020	652
28	7 3	1499	†324	836	512
26	12 48	1186	†164	276	112
27	17 40	1095	†197	800	603
30	17 51	1958	†345	1489	1144
25	18 4	1198	†410	870	460
25	18 28	1199	†443	943	500
25	18 44	1200	†290	916	626
28	18 9	1567	†463	1375	912
28	18 36	1560	†551	1165	614
28	18 45	1561	†686	1574	888
29	18 22	1578	†611	1396	785
29	18 40	1579	†449	1401	952
30	18 27	1595	†554	1064	510
30	18 36	1596	†496	1042	546
30	18 45	1597	†582	1084	502
27	19 16	1103	†273	1230	957
26	19 36	1117	†292	1763	1471
26	19 44	1118	†203	702	499
26	19 52	1119	†387	1428	1041
28	19 3	1222	†357	974	617
28	19 12	1223	†506	1027	521
29	19 7	1562	†1133	2723	1590
29	19 16	1653	†1097	2856	1759
29	19 52	853	†365	1270	905
30	19 39	1582	†2102	4920	2818
30	19 48	1583	†1469	4097	2628
30	19 57	1584	†1120	2561	1441
31	19 7	1605	†591	1178	587

Zone.	Centre. R.A.	Plate.	Number Measured.	3m Images that might have been Measured.	Additional Stars.
+	h m				
31	19 34	1606	†943	2180	1237
26	20 16	1212	†683	1549	866
27	20 20	1225	†395	802	407
30	20 6	1568	†916	2097	1181
30	20 15	1569	†852	1496	644
30	20 24	1564	†909	2310	1401
30	20 33	1570	†754	1541	787
30	20 42	1571	†872	1823	951
30	20 51	1565	†1214	2703	1489
27	21 32	1230	†236	615	379
28	21 9	1240	†242	524	282
28	21 27	1215	†272	738	466
30	21 18	1585	†792	1802	1010
29	22 34	1466	†229	550	321

12. Table V. contains the data relating to the 45 plates with 6^m, 3^m, 1^m exposures, and the 10 plates with 6^m, 3^m, 20^s, upon which only those stars showing all three exposures were measured. The number actually measured is preceded by this symbol, †; the number that would have been measured in the ordinary way is given in the next column; the difference or additional stars—those showing the 3^m, but not the 20^s or 1^m exposure—that might be measured at any time are in the last column. If these extra stars be incorporated in the *measured* totals given in Tables I. and IV. we should obtain the amended results exhibited in Table VI., with a total addition of 44,462 stars.

13. To measure these stars it would require nearly 90,000 bisections in each coordinate to be made. This would be an additional year's work for comparatively little advantage, for it will be noticed that nearly all these 44,000 stars are within the region covered by the Milky Way, where, for star *positions*, they can be very well spared, as most of them are probably fainter than the 10·5 magnitude; but when considering the distribution of stars they should not be omitted in discussing the results. In the latter case any discussion of photographic results is rendered uncertain by that extremely variable quantity the climatic conditions. A small variation of exposure is almost insignificant compared to those conditions and to the varying chemical properties in the emulsion from the time the plate is coated till the finished negative is obtained.

TABLE VI.

Difference between the Stars Measured that show the Three Images and those Counted for the 3m Exposures.

Hour.	Sums for Hours and Zones. 25°.	26°.	27°.	28°.	29°.	30°.	31°.	Additional Stars *Counted*, 3m Exposure.	Stars *Measured* added to previous Column.	Ratio.
0	...	...	...	...	...	...	...	0	12577	3·97
1	...	...	...	...	...	...	...	0	13414	4·55
2	...	...	...	...	351	...	...	351	12677	4·84
3	201	...	...	408	...	...	...	609	13767	4·91
4	...	...	...	...	...	...	...	0	8295	3·33
5	...	1389	...	...	844	...	...	2233	24271	4·78
6	...	534	...	2070	695	...	...	3299	28643	4·56
7	...	652	...	512	...	...	...	1164	18817	4·44
8	...	...	...	...	...	...	...	0	12386	4·11
9	...	...	...	...	...	...	...	0	10727	3·96
10	...	...	...	...	...	...	...	0	9269	4·11
11	...	...	...	...	...	...	...	0	7613	3·85
12	...	112	...	...	...	...	...	112	8728	4·06
13	...	...	...	...	...	...	...	0	7789	3·96
14	...	...	...	...	...	...	...	0	8506	3·97
15	...	...	...	...	...	...	...	0	10006	4·14
16	...	...	...	...	...	...	...	0	13035	4·69
17	...	...	...	...	...	1144	...	1144	23952	6·46
18	1586	...	...	2414	1737	1558	...	7295	34332	7·03
19	...	3011	957	1138	4254	6887	1824	18071	61682	8·34
20	...	866	407	...	...	6453	...	7726	45299	7·21
21	...	...	379	748	...	1010	...	2137	26189	5·28
22	...	...	...	...	321	...	...	321	18439	4·64
23	...	...	...	...	...	...	...	0	13854	4·09
Sums	1787	6564	1743	7290	8202	17052	1824	44462	444267	5·19 Mean

14. A comparison of the final results with those upon which the previous paper communicated to the Society (*Monthly Notices*, vol. lx. p. 12) in 1899 were based may be of interest. It will be noticed (Table VII.) that while more than twice as many plates have been used in the final results nearly three times the number of stars have been included. The number of stars (those showing both the 6m and 3m images) per square degree of ranges from 56·2 in +25° zone to 89·2 in +30° zone; if the extra stars had been measured the range would be 57·8 to 111·8. The value for the whole of the seven zones photographed at Oxford being 71·3

for the stars measured, this is very similar to that found (71·2) at the Royal Observatory, Greenwich, for zones +65° to +69° (inclusive).

15. The last six columns in Table IV. give the details for the separate hours in a similar manner to the results given in Table VII. for each whole zone; the earlier results are added for comparison. In plotting these new values on plate 1 of the former paper I find almost precisely the same *form* of curve results, but generally at a higher level. It may be remarked that the maximum number of stars measured per square degree is 189·9 at 19^h (R.A. $19^h\ 0^m$ to $20^h\ 0^m$), and taking only those numbers that are above the mean (72·3) the Milky Way may be considered to extend from R.A. $16\frac{1}{2}^h$ to $22\frac{1}{2}^h$; the width about the 6^h portion is very much narrower, and, according to the Oxford survey, of about half the star density, presumably to the eleventh magnitude stars, the extent in R.A. is apparently $4\frac{1}{2}^h$ to $7\frac{1}{2}^h$ (in reality it is $5^h\ 20^m$, as there is an extraordinary scarcity of faint stars during R.A. $4^h\ 0^m$ to 5^h, with a very precipitous rise—a perfect wall of stars—at $5^h\ 15^m$); thus, $6\frac{1}{2}^h$ against 2^h. The minimum is at 11^h and 13^h; there is a small yet distinct rise at 12^h, with an "accidental" minimum at 4^h, possibly a small or confined stellar vacuity and peculiar to the Oxford zones.

TABLE VII.

Dec.	Number of Plates.	Stars Measured.	Stars in a Square Degree.	Average Number of Stars each Plate.	Number measured plus the Extra Stars indicated in Table VI.	Stars in a Square Degree.
+25°	138	34141	52·6			
	180	47428	56·2	263·5	49215	57·8
+26	118	31877	57·4			
	180	60212	71·5	335·5	66776	78·9
+27	106	26545	53·3			
	180	53241	63·1	295·8	54984	65·1
+28	56	18103	68·8			
	160	52926	70·5	330·8	60216	80·2
+29	97	30853	67·6			
	160	53272	71·0	333·0	61474	81·8
+30	0	0	...			
	160	66900	89·2	418·1	83952	111·8
+31	0	0	...			
	160	65826	87·7	411·4	67650	89·9
Sum or Mean	513 / 1180	399805	72·3	338·8	444267	80·2

I wish to express my thanks to Mr. E. A. Gray, Mr. F. H. Scragg, and Mr. F. F. Lovegrove for assistance in collecting the data upon which the tables are based.

University Observatory, Oxford:
1904 *April* 7.

A New Cluster in Cygnus, with Right Ascensions and Declinations of 103 *Stars included in it.* By F. A. Bellamy.

1. At the end of last year I exposed a plate for the purpose of obtaining a photograph of the region of Dr. Max Wolf's new variable B.D. +37° 3876 (No. 59, 1903, *Cygni*) in order to measure the positions of stars surrounding it. The plate (2294) was exposed for 10^m and 16^m on 1903 October 9, and the adopted centre is +37°, 20^h 16^m: the hour angle was about 4^h west. The plate probably contains stars fainter than the 13th magnitude.

2. Whilst proceeding with the measurement of stars close to the variable I was impressed with the appearance of a well-defined and compact cluster of stars about $1\frac{1}{2}^m$ away and became interested in ascertaining its name. Upon reference to Dreyer's "New General Catalogue of Nebulæ and Clusters" (*Memoirs, R.A.S.*, vol. xlix. p. 1) I failed to find any mention made about it, nor could I find it referred to in his additional catalogue (*Memoirs, R.A.S.*, vol. li. p. 185). Such being the case it occurred to me that it was desirable to fix the stars' positions, and the results form the basis of this communication.

3. Without further description, it need only be said that the plate was exposed in the 13-inch astrographic telescope, which was moved in decl. about 45″ between the two exposures, that I measured both images in two positions of the plate 180° apart, using the glass-ruled scale in the microscope, the means of these four measures in each coordinate were carried to the fourth decimal—*i.e.* to a unit ·0001 = 0″·03. The calculations were made according to the formulæ published by Professor Turner.

4. The reference stars for the purpose of determining the plate constants have been selected from the Lund A.G.C. These positions have been brought to the epoch 1900·0 with precessions given in that catalogue, but no proper motions have been applied. The standard coordinates ξ and η referred to the plate centre +37°, 20^h 16^m, were computed for fifteen stars chosen in the SW, NW, NE, SE corners of the plate and about 45′ from the plate centre. If we put ξ' and η' equal to $\xi+13$ and $\eta+13$ respectively we shall obtain the coordinates referred to the corner of the réseau; they will always be positive, and are easily compared with the measured x and y. Putting $ax+by+c=x-\xi$ and $dx+ey+f=y-\eta$, the values of a, b, &c., were found to be

a	b	c	d	e	f
−·00068	+·00601	+·0863	−·00609	−·00073	−·0360

These are derived from the mean of the two exposures. It is not considered that any appreciable error is introduced in the present case by proceeding in this way rather than by treating the exposures separately, and a great deal of work is saved

without sacrificing accuracy. Reference may be made to *Monthly Notices*, vol. lxiii. pp. 513, 514.

5. The plate constants were applied to the mean measured x and y, and the standard coordinates ξ' and η' are thus obtained; they are given in columns 7 and 8 of Table I. By means of tables existing in MS. at the Observatory these ξ''s and η''s were readily converted to differences in R.A. and Decl., and when applied to the adopted plate centre ($+37°$, $20^h\ 16^m$) the R.A.s and Decls. for 1900·0, as given in columns 9 and 10, are obtained.

6. The magnitudes given in columns 5 and 6 consist, in the former case, of the *sum* of the four measured diameters; and in the next column I have given the approximate magnitude in the usual notation. These were derived by plotting the B.D. mags., comparing them with the measured diameters, assuming that the sum of the four measures of the smallest star visible was equal to 10 units (13th mag.), and then drawing a curve through these points. They must be considered as quite approximate, though I think they are not more than half a magnitude in error.

Grouping them, I find there are two stars brighter than 8·0, six stars from 8·0 to 9·9 (inclusive), fifty-one from 10·0 to 11·9, and forty-four from 12·0 to 13·0.

TABLE I.

Oxford Ref. No.	Potsd. No. on Plate 922.	B.D. +37° No.	Magnitude. B.D.	Magnitude. Oxford Diameter (sum of 4).	Magnitude. Inferred.	$\xi + 13'0000$. 1900·0.	$\eta + 13'0000$. 1900·0.	Deduced. R.A. 1900·0.	Deduced. N. Dec. 1900·0.
						R.I.	R.I.	$20^h +$ m s	$37° +$ ′ ″
1	...	...	...	27	12·3	3·8339	17·4099	12 9·37	21 49·0
2	...	...	...	13	12·9	3·9234	15·2629	12 12·15	11 5·2
3	...	...	...	23	12·5	3·9586	15·7206	12 12·92	13 22·7
4	...	...	...	28	12·3	3·9855	17·9992	12 13·03	24 46·2
5	...	...	...	46	11·6	4·0044	17·0919	12 13·73	20 14·1
6	212	...	...	57	11·1	4·0224	15·4606	12 14·59	12 4·8
7	...	...	...	19	12·6	4·0989	16·8512	12 16·17	19 2·2
8	...	...	...	19	12·6	4·0949	15·8247	12 16·32	13 54·3
9	222	...	...	80	10·2	4·2649	16·0572	12 20·53	15 4·5
10	...	...	...	25	12·4	4·3602	16·8854	12 22·73	19 13·2
11	...	...	...	28	12·3	4·3768	15·6205	12 23·45	12 53·8
12	...	...	...	48	11·5	4·3964	17·3520	12 23·53	21 33·2
13	225	3855	9·3	117	8·8	4·3961	16·9167	12 23·62	19 22·7
14	...	...	...	18	12·7	4·3956	16·6273	12 23·68	17 55·9
15	231	...	...	80	10·2	4·5084	17·9302	12 26·21	24 27·1

Oxford Ref. No.	Potsd. No. on Plate 922.	B.D. +37° No.	Magnitude. B.D.	Magnitude. Oxford Diameter (sum of 4).	Magnitude. Inferred.	ξ+13°0000. 1900·0.	η+13°0000. 1900·0.	Deduced. R.A. 1900·0.	Deduced. N. Dec. 1900·0.
						B.I.	B.I.	20h+ m s	37°+ ′ ″
16	230	3856	9·5	85	10·0	4·5002	16·0264	12 26·45	14 55·9
17	...	...	...	29	12·2	4·5054	16·3780	12 26·50	16 41·4
18	...	...	...	25	12·4	4·5141	16·3450	12 26·72	16 31·5
19	...	...	...	45	11·6	4·5641	16·3923	12 27·97	16 45·8
20	...	...	...	44	11·6	4·5854	17·7116	12 28·20	23 21·6
21	...	...	...	37	11·9	4·6453	15·4176	12 30·24	11 53·7
22	238	...	...	57	11·1	4·6625	15·8728	12 30·56	14 10·3
23	...	...	...	34	12·0	4·7154	17·7746	12 31·45	23 40·9
24	...	...	...	41	11·8	4·7196	15·5316	12 32·08	12 28·1
25	...	...	...	43	11·7	4·7503	16·0329	12 32·73	14 58·6
26	...	...	...	31	12·2	4·9068	16·7792	12 36·49	18 42·8
27	...	...	...	39	11·8	4·9245	15·3821	12 37·25	11 43·8
28	...	...	...	52	11·4	5·0770	17·3378	12 40·65	21 30·8
29	...	...	...	26	12·4	5·1042	17·5352	12 41·30	22 30·1
30	259	...	...	62	10·9	5·2228	16·3553	12 44·53	16 36·5
31	260	...	...	53	11·3	5·2448	16·6291	12 45·03	17 58·7
32	...	...	...	27	12·3	5·2506	16·6810	12 45·16	18 14·3
33	...	...	...	29	12·2	5·2931	16·8770	12 46·19	19 13·2
34	263	...	...	54	11·3	5·3090	15·8604	12 46·80	14 8·3
35	...	...	...	(13)	12·4	5·3197	16·9427	12 46·84	19 33·0
36	264	...	...	52	11·4	5·3333	17·1075	12 47·15	20 22·4
37	...	...	...	27	12·3	5·3392	16·7315	12 47·38	18 29·7
38	...	...	...	33	12·1	5·3524	16·8209	12 47·69	18 56·5
39	267	3859	9·2	105	9·2	5·3645	17·0508	12 47·95	20 5·5
40	268	...	...	45	11·6	5·3706	17·0289	12 48·10	19 59·0
41	271	...	...	51	11·4	5·4001	16·8110	12 48·89	18 53·6
42	274	...	...	67	10·7	5·4220	17·3410	12 49·33	21 32·7
43	275	3860	8·0	143	7·8	5·4246	17·0366	12 49 46	20 1·4
44	276	...	...	40	11·8	5·4255	17·1031	12 49·47	20 21·3
45	279	3862	9·2	118	8·8	5·4674	17·0731	12 50·53	20 12·5
46	...	...	...	35	12·0	5·4897	17·4039	12 51·02	21 51·8
47	...	...	...	(19)	11·9	5·4861	16·9266	12 51·03	19 28·6
48	...	...	...	37	11·9	5·4980	17·2611	12 51·26	21 8·9

Oxford Ref. No.	Potsd. No. on Plate 922.	B.D. +37° No.	Magnitude. B.D.	Magnitude. Oxford Diameter (sum of 4).	Magnitude. Inferred.	ξ+13°0000. 1900·0.	η+13°60000. 1900·0.	Deduced. R.A. 1900·0.	Deduced. N. Dec. 1900·0.
						R.L.	R.L.	20h+ m s	37°+ ′ ″
49	282	...	...	61	11·0	5·4944	15·9543	12 51·44	14 36·9
50	280	...	...	54	11·3	5·5057	17·0584	12 51·50	20 8·2
51	283	...	...	67	10·7	5·5262	17·0414	12 52·01	20 3·1
52	...	...	...	46	11·5	5·5342	16·9546	12 52·23	19 37·1
53	...	...	...	41	11·8	5·5498	17·3140	12 52·55	21 24·9
54	...	...	...	30	12·2	5·5562	16·8713	12 52·80	19 12·2
55	289	...	...	54	11·3	5·5784	17·3640	12 53·26	21 40·0
56	...	...	...	44	11·6	5·5902	17·0413	12 53·62	20 3·2
57	...	...	...	24	12·4	5·5916	16·4699	12 53·78	17 11·9
58	290	...	...	50	11·4	5·5989	16·1612	12 54·02	15 39·3
59	...	...	...	20	12·6	5·6191	15·4738	12 54·67	12 13·1
60	...	...	...	43	11·7	5·6350	17·1721	12 54·72	20 42·6
61	...	...	...	21	12·6	5·6334	16·2634	12 54·87	16 10·0
62	292	...	...	115	8·9	5·6459	17·5726	12 54·92	22 42·8
63	...	...	...	24	12·4	5·6551	16·9066	12 55·28	19 23·0
64	...	...	...	34	12·0	5·6796	15·8092	12 56·12	13 53·9
65	...	...	...	32	12·2	5·7157	16·9144	12 56·81	19 25·5
66	...	...	...	28	12·3	5·7701	16·3175	12 58·29	16 26·6
67	...	...	...	48	11·5	5·7890	17·5987	12 58·52	22 50·9
68	...	...	...	28	12·3	5·7928	17·3080	12 58·67	21 23·7
69	299	3864	8·8	131	8·3	5·8017	15·7340	12 59·20	13 31·6
70	...	...	...	46	11·6	5·8250	16·7212	12 59·59	18 27·8
71	303	...	...	54	11·2	5·8706	17·1674	13 0·65	20 41·7
72	...	...	...	30	12·2	5·9288	16·7298	13 2·20	18 30·6
73	307	...	...	82	10·1	5·9729	17·4677	13 3·17	22 12·0
74	312	...	...	46	11·6	6·0329	17·6432	13 4·64	23 4·8
75	...	...	...	25	12·4	6·0530	18·0341	13 5·07	25 2·1
76	...	...	...	44	11·6	6·0470	16·9992	13 5·12	19 51·7
77	...	...	...	23	12·5	6·1495	17·1029	13 7·68	20 23·0
78	...	...	...	30	12·2	6·1585	16·8592	13 7·95	19 10·0
79	...	...	...	22	12·6	6·1816	16·9166	13 8·52	19 27·2
80	...	...	...	38	11·9	6·1965	16·4708	13 8·98	17 13·5

Oxford Ref. No.	Potsd. No. on Plate. 922.	B.D. +37° No.	Magnitude. B.D.	Oxford Diameter (sum of 4).	Inferred.	$\xi+13^{\circ}0000$. 1900·0.	$\eta+13^{\circ}0000$. 1900·0.	Deduced. R.A. 1900·0.	Deduced. N. Dec. 1900·0.
								$20^{h}+$	$37+^{\circ}$
						R.I.	R.I.	m s	′ ″
81	...	...	...	46	11·6	6·2058	17·2562	13 9·07	21 9·1
82	...	...	...	22	12·5	6·2119	17·0270	13 9·26	20 0·4
83	...	...	...	25	12·4	6·2409	15·9445	13 10·19	14 35·8
84	318	3865	9·0	110	9·0	6·2395	15·5333	13 10·24	12 32·4
85	...	...	...	42	11·7	6·2657	16·9085	13 10·64	19 25·0
86	...	...	...	27	12·3	6·3026	16·9153	13 11·56	19 27·1
87	...	...	...	24	12·4	6·3633	17·8313	13 12·92	24 2·0
88	323	...	...	74	10·4	6·3688	17·8779	13 13·05	24 16·0
89	...	...	...	25	12·4	6·4080	15·9192	13 14·40	14 28·5
90	...	...	...	27	12·3	6·4228	17·6979	13 14·44	23 22·1
91	...	...	...	23	12·5	6·4487	17·3122	13 15·17	21 26·5
92	...	...	...	22	12·5	6·4607	17·4992	13 15·44	22 22·6
93	332	3866	8·0	165	7·0	6·5215	17·4903	13 16·97	22 20·1
94	...	...	...	43	11·7	6·5105	15·4744	13 17·05	12 15·3
95	...	...	...	38	11·9	6·5223	17·0303	13 17·07	20 2·1
96	...	...	...	22	12·5	6·5426	17·7584	13 17·45	23 40·5
97	335	...	...	59	11·0	6·5783	16·7143	13 18·53	18 27·5
98	...	...	...	46	11·6	6·6396	17·9095	13 19·86	24 26·1
99	...	...	...	34	12·0	6·6463	17·6439	13 20·08	23 6·4
100	336	...	...	55	11·2	6·6446	16·3118	13 20·27	16 26·9
101	...	...	...	49	11·4	6·7036	16·9905	13 21·64	19 50·5
102	...	...	...	46	11·6	6·7046	17·0518	13 21·65	20 8·9
103	338	...	...	48	11·5	6·7990	16·6175	13 24·10	17 58·9

Nos. 35 and 47.—The magnitude in column 5 for these two stars depends upon the first exposure only. The quantities given in columns 7 to 10 also depend upon the first exposure; in the former case the second image is completely involved in the first exposure of another star, in the latter case the second image is too doubtful to be measured.

It should be stated that these two stars have been corrected for plate constants, which had already been determined for another purpose, from the first exposure. But the values for a, b, d, e might have been used from the mean of the two exposures (§ 4), as there is a fairly close agreement between these two sets of constants. Of course c and f must differ.

a	b	c	d	e	f
−·00077	+·00605	+·0874	−·00618	−·00067	−·1114

7. After these stars had been measured and nearly the whole of the computations had been completed, I became acquainted

with the fact that the region containing this cluster had already been measured at Potsdam, and the results are given in *Photographische Himmelskarte*, Band II., page 221, plate 922, where twelve stars are indicated with an asterisk as forming a small cluster. Having reached this stage it seemed to me that it would be of interest to compare these results with those from the Oxford plate 2294.

8. The plate centre of Potsdam plate 922, exposed in 1896 July 13 by Dr. Clemens, when it was 7^m west of the meridian, is given as $20^h\ 14^m\ 47^s{\cdot}2$, $+37°\ 0'\ 30''$, but the x and y coordinates given have not been corrected for plate constants, and none are indicated. To make the results comparable with those I have deduced from Oxford plate 2294 I have computed the positions of fifteen stars from the Lund A.G.C., adopting $+37°\ 0'\ 0''$, $20^h\ 15^m\ 0^s$ as a convenient centre, and, after a comparison of these places, reduced to coordinates ξ' and η' (1900·0), with the Potsdam measured x and y have solved the equations and obtained the following constants:

a	b	c	d	e	f
−·00001	−·00014	+·5125	+·00027	+·00004	−·1009

These corrections have been applied to the measures (as printed) in a similar manner as was performed for the Oxford measures (§§ 4 and 5), and the results (ξ' and η') transformed to differences of R.A. and decl. from the adopted plate centre ($+37°\ 0'\ 0''$, $20^h\ 15^m\ 0^s$). A comparison of the plates is given in Table II.

9. In this table I have only given the number, R.A., and decl. for Oxford and Potsdam. The corrected Potsdam measures (X, Y) can be found by inference from the Oxford R.A.'s and decl.'s (centre $+37°$, $20^h\ 16^m$). In the fifth and eighth columns are the differences O−P, the means being $-^s{\cdot}002$ and $+0''{\cdot}59$, indicating a correction required to the constant f.

Owing to the difference between the centres of the two plates, only seven of the fourteen reference stars are common to the two plates.

TABLE II.

Oxford No.	Potsdam No.	Oxford.	Potsdam.	O−P	Oxford.	Potsdam.	O−P
		h m s	s	s	° ′ ″	″	″
6	212	20 12 14·59	14·57	+0·02	+37 12 4·8	4·3	+0·5
9	222	20·53	20·52	+ ·01	15 4·5	4·3	+0·2
13	225	23·62	23·60	+ ·02	19 22·7	22·5	+0·2
15	231	26·21	26·24	− ·03	24 27·1	26·2	+0·9
16	230	26·45	26·41	+ ·04	14 55·9	55·5	+0·4
22	238	30·56	30·55	+ ·01	14 10·3	9·4	+0·9
30	259	44·53	44·54	− ·02	16 36·5	36·1	+0·4
31	260	45·03	45·01	+ ·02	17 58·7	58·2	+0·5
34	263	46·80	46·81	− ·01	14 8·3	7·7	+0·6

Oxford No.	Potsdam No.	Oxford.	Potsdam.	O−P	Oxford.	Potsdam.	O−P
		h m s	s	s	° ′ ″	″	″
36	264	20 12 47·15	47·15	·00	+37 20 22·4	22·1	+0·3
39	267	47·95	47·91	+ ·04	20 5·5	4·7	+0·8
40	268	48·10	48·04	+ ·06	19 59·0	60·8	−1·8
41	271	48·89	48·90	− ·01	18 53·6	52·9	+0·7
42	274	49·33	49·34	− ·01	21 32·7	31·5	+1·2
43	275	49·46	49·42	+ ·04	20 1·4	1·1	+0·3
44	276	49·47	49·50	− ·03	20 21·3	20·8	+0·5
...	277	...	49·59	...	19 ...	55·2	...
45	279	50·53	50·50	+ ·03	20 12·5	11·8	+0·7
49	282	51·44	51·52	− ·08	14 36·9	37·4	−0·5
50	280	51·50	51·50	·00	20 8·2	7·3	+0·9
51	283	52·01	52·03	− ·02	20 3·1	2·6	+0·5
55	289	53·26	53·33	− ·07	21 40·0	39·5	+0·5
58	290	54·02	54·08	− ·06	15 39·3	38·8	+0·5
62	292	54·92	54·88	+ ·04	22 42·8	41·6	+1·2
69	299	59·20	59·18	+ ·02	13 31·6	31·3	+0·3
71	303	13 0·65	0·66	− ·01	20 41·7	40·8	+0·9
73	307	3·17	3·19	− ·02	22 12·0	11·2	+0·8
74	312	4·64	4·66	− ·02	23 4·8	3·6	+1·2
84	318	10·24	10·19	+ ·05	12 32·4	30·6	+1·8
88	323	13·05	13·08	− ·03	24 16·0	14·9	+1·1
93	332	16·97	16·93	+ ·04	22 20·1	19·4	+0·7
97	335	18·53	18·57	− ·04	18 27·5	27·0	+0·5
100	336	20·27	20·27	·00	16 26·9	26·4	+0·5
103	338	24·10	24·15	− ·05	17 58·9	58·6	+0·3
			Mean	−·002		Mean	+0·56

Potsdam 277 has a double note of interrogation against the printed measures. I cannot find anything on the Oxford plate to agree with this, but on the original measures I have the following note written opposite to Oxford No. 43: "? another star coalesced in the image of the bright star in the position *s.f.*" It is therefore very probable that Potsdam 277 is a star, as the position given would agree with my note; the semi-diameter of the image would be quite sufficient to overspread the faint star.

Potsdam 268 (Oxford 40). On the Oxford plate the images are partly involved in those of No. 39.

10. At the moment of completing this paper an advance copy of the first volume of the Greenwich Astrographic Catalogue was received by Professor Turner. In view of what has just been written in § 6 about the magnitudes it will be of interest to apply the Astronomer Royal's method for converting photographic into actual magnitudes to the measured diameters given

for this plate (2294) in column 5 ; it will, however, be necessary to divide those sums by 2 to reduce them to the same scale as adopted at Greenwich, *i.e.* the sum of D and R diameters.

In the formula given by Dr. Christie

$$\text{Mag.} = C - n\sqrt{d}$$

where C is a constant to be determined, d is the measured diameter, the unit being $0''.15$; and n a constant which varies slightly with the duration of exposure. The mean value of n is found to be at Greenwich 0·73 for 20 sec., 0·77 for 6 min., and 0·84 for 40-min. exposures. In making the present comparison I have adopted 0·82 as the value of n for plate 2294, and have used B.D. magnitudes for twenty-two stars in the comparison with the Oxford measures, when divided by 2, and obtain 14·78 as the value of C. From this data I have formed Table III.

TABLE III.

Diameter.	$0{\cdot}82\sqrt{d}$.	$C-0{\cdot}82\sqrt{d}$. mag.	Diameter.	$0{\cdot}82\sqrt{d}$.	$C-0{\cdot}82\sqrt{d}$. mag.
5	1·84	12·94	40	5·18	9·60
10	2·59	12·19	50	5·80	8·98
15	3·17	11·61	60	6·36	8·42
20	3·67	11·11	70	6·86	7·92
25	4·10	10·68	80	7·33	7·45
30	4·49	10·29	90	7·78	7·00
35	4·85	9·93			

University Observatory, Oxford :
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The Definitive Places of the Standard Stars for the Northern Zones of the Astronomische Gesellschaft. By A. M. W. Downing, D.Sc., F.R.S.

In *Astron. Nachrichten*, Nos. 3927–9, Professor Auwers has published his definitive corrections to the provisional places of the standard stars for the northern zones of the *Astronomische Gesellschaft*, which have been adopted in the reduction of the German zone observations. The Appendix to the *Berliner Jahrbuch* for 1906 supplies the corresponding corrections, for 1906·0, to the places of these stars as given in the body of that work.

It may be of some interest, therefore, to exhibit the results of a comparison of the places of the *Berliner Jahrbuch* stars, thus corrected, with their places as derived from Newcomb's "Fundamental Catalogue," which are given in the *Nautical*

Almanac for 1906; thus furnishing the systematic differences between the two Fundamental Catalogues from the North Pole to −25° Decl. for the epoch 1906.

The stars available for the comparison have been arranged in order of declination, and then combined in convenient groups. The further comparison in R.A. of stars within 13° of the pole has not been carried out. In forming the groups the following stars have also been omitted as being unsuitable for the purpose, viz. 61 *Cygni*, *α Canis Minoris*, *γ Virginis*, and *α Canis Majoris*.

The individual comparisons for these omitted stars, taken in the sense *Nautical Almanac*−*Berliner Jahrbuch*, are as follows:

Name of Star.	Approx. Decl.	Δα.	Δδ.
	° ′	s	″
λ Ursæ Minoris... ...	+89 0	−·436	...
α Ursæ Minoris... ...	88 48	−·083	...
51 H. Cephei	87 12	−·314	...
δ Ursæ Minoris... ...	86 37	−·028	...
ε Ursæ Minoris... ...	82 12	−·025	...
ζ Ursæ Minoris... ...	78 5	+·098	...
γ Cephei...	77 6	+·062	...
61′ Cygni	38 17	−·044	+·21
α Canis Minoris... ...	+ 5 28	−·003	−·66
γ′ Virginis	− 0 56	+·061	+·29
α Canis Majoris... ...	−16 35	−·093	+·11

The following Table exhibits the results for the groups, taken in the sense already mentioned, and the number of stars in each group. The total number of stars used for the comparison is in R.A. 255, and in Decl. 262.

Mean Decl.	Mean Δα.	Mean Δδ.	No. of Stars.	Mean Decl.	Mean Δα.	Mean Δδ.	No. of Stars.
° ′	s	″		° ′	s	″	
+87 54	...	+·04	4	22 34	+·005	−·03	17
79 8	...	−·04	3	17 26	+·006	+·13*	15
70 50	+·052	−·01	5	12 41	+·006	+·10	19
62 40	+·048	+·03	8	7 46	+·004	+·15	24
57 32	+·047	+·15	9	+ 3 8	+·002	+·28	12
52 40	+·019	+·17	5	− 1 58	+·003	+·28	18
47 41	+·011	+·11	6	7 56	+·001	+·17	19
42 45	+·017	+·15	9	12 21	+·002	+·21	10
38 8	+·006	+·04	9	16 50	−·002	+·24	17
32 25	+·001	+·01	12	22 31	−·001	+·10	15
27 41	+·005	+·05	16	−28 35	−·003	−·14	10

* Or +″·08 (adopted for curve) omitting α *Delphini*, for which Δδ = +″·86.

The above quantities have been graphically represented on cross-ruled paper, and curves drawn in the usual manner, from which the $\Delta\alpha$ and $\Delta\delta$, representing Newcomb—Auwers for the epoch 1906 have been read off for each 5° of declination. The results are exhibited in the columns headed I. in the last Table below. The proper quantities, interpolated from those given in these columns, have been then applied, with reversed signs, to the $\Delta\alpha$ and $\Delta\delta$ for each individual star (omitting the four stars mentioned above) which lies between the limits of Decl. +50° and −28°. These stars have been arranged in order of R.A., and the residual differences combined in groups of 1^h each. The results, which are exhibited in the following Table, are assumed to represent the systematic differences depending on R.A.

By proceeding exactly as before in the case of the differences depending on declination, those depending on R.A. have been read off from the appropriate curves for the beginning of each hour of right ascension. The results are exhibited in the next Table but one.

Mean R.A. h m	Mean $\Delta\alpha$. s	Mean $\Delta\delta$. "	No. of Stars.	Mean R.A. h m	Mean $\Delta\alpha$. s	Mean $\Delta\delta$. "	No. of Stars.
0 31	+·013	−·02	10	12 27	+·001	−·06	9
1 37	·000	·00	9	13 32	−·007	+·04	8
2 29	+·003	−·09	8	14 25	+·010	−·05	7
3 31	+·006	+·03	12	15 34	−·004	+·07	10
4 35	+·002	+·07	10	16 29	−·008	+·07	10
5 27	+·003	·00	17	17 24	−·002	−·05	9
6 32	+·001	+·08	10	18 33	−·011	+·01	8
7 28	−·001	·00	6	19 25	−·003	−·05	11
8 30	·000	+·02	9	20 29	+·003	·00*	11
9 33	−·008	+·02	7	21 28	·000	·00	7
10 26	−·003	+·09	6	22 30	+·001	−·01	11
11 20	·000	−·09	9	23 24	−·002	+·01	8

N − A (1906).

R.A. h	$\Delta\alpha$. s	$\Delta\delta$. "	R.A. h	$\Delta\alpha$. s	$\Delta\delta$. "
0	+·003	·00	7	·000	+·04
1	+·007	−·01	8	−·001	+·01
2	+·005	−·03	9	−·002	+·02
3	+·006	−·03	10	−·005	+·03
4	+·005	+·05	11	−·002	−·01
5	+·004	+·03	12	+·002	−·05
6	+·003	+·04	13	+·002	−·01

* Or −".07 (adopted for curve) omitting α *Delphini*, for which $\Delta\delta = +''.76$.

R.A. h	$\Delta\alpha_{\alpha}$. s	$\Delta\delta_{\alpha}$. "	R.A. h	$\Delta\alpha_{\delta}$. s	$\Delta\delta_{\delta}$. "
14	·000	·00	19	−·009	−·02
15	−·001	+·01	20	+·001	−·06
16	−·007	+·07	21	+·002	−·03
17	−·005	−·01	22	+·002	−·01
18	−·008	−·02	23	·000	−·01

The proper quantities, interpolated from the immediately preceding table, have then been applied, with reversed signs, to the differences for each individual star arranged in order of declination. The residuals give a second approximation to the systematic differences depending on declination, which (proceeding exactly as in the first approximation) are found to be those given in the columns headed II. in the final Table.

N−A (1906).

Dec.	$\Delta\alpha$s. I.	$\Delta\alpha$s. II.	$\Delta\delta$" I.	$\Delta\delta$" II.
°	s	s	"	"
+85	...	...	+·01	+·01
80	...	...	−·03	−·04
75	...	...	−·03	−·03
70	+·052	+·053	−·01	·00
65	+·049	+·050	+·01	+·02
60	+·047	+·047	+·09	+·10
55	+·033	+·032	+·16	+·17
50	+·013	+·014	+·14	+·14
45	+·012	+·011	+·13	+·13
40	+·010	+·009	+·10	+·10
35	+·001	+·001	+·02	+·02
30	+·002	+·003	+·03	+·03
25	+·005	+·005	·00	−·01
20	+·005	+·004	·00	−·01
15	+·006	+·006	+·10	+·10
10	+·005	+·006	+·11	+·11
+ 5	+·003	+·002	+·22	+·22
0	+·002	+·003	+·28	+·27
− 5	+·002	+·002	+·20	+·21
10	+·001	·000	+·18	+·18
15	·000	−·001	+·23	+·23
20	−·001	−·001	+·18	+·18
−25	−·002	·000	·00	·00

It is very satisfactory to find a considerably closer agreement between the two systems of fundamental star places, due to the application of the definitive corrections deduced by Professor Auwers, than formerly obtained. The great improvement that has been effected in this respect is made obvious by comparing the reduction tables given above, for 1906, with those in Dr. Cohn's paper published in *Astron. Nachrichten*, No. 3742, which exhibit similar quantities applicable to the provisional fundamental catalogues of the *Astronomische Gesellschaft*, for the epochs 1875 and 1900. In particular it will be noticed that the right ascensions of stars of moderate declination are now in very close accord indeed; due not only to the agreement in the adopted position of the equinox in the respective systems, but also to the agreement (at the epoch considered) in the mean positions of groups of stars extending over a considerable range of declination.

H.M. Nautical Almanac Office:
1904 *May* 9.

Note on a Suggested Method of Determining the Declination of Stars. By A. E. Conrady.

The values of the declination of stars and of the latitude of observatories depend directly on the readings of graduated circles, and are limited to the accuracy with which the errors of graduation and of flexure have been, or can be, determined, besides involving errors of the refraction tables.

The following method of determining the declination of stars not too near the equator requires circle readings of a lower order of accuracy than that of the resulting declinations. I believe it is novel; whether it is of practical value remains an open question.

When the idea first occurred to me (1899 March 21) I began my note of it as follows:

"The clock is without doubt the most accurate and cheapest instrument for measuring angles; and in the case of many observations, made at different periods of the day and of the year, it is entirely free from constant errors.

"The question arises, whether declinations and latitude might not be obtained by transit observations based on observations of time only. This is indeed possible."

The method calls for a transit instrument in the meridian firmly united with a second telescope which has its optical axis parallel to the plane defined by the axis of rotation and the optical axis of the principal telescope, and is capable of being

fixed in any direction intermediate between that of the axis of rotation and that of the principal telescope.

When the principal telescope describes the meridian the auxiliary telescope describes a small circle parallel to the meridian. Then the angular distance between the two telescopes, for which the symbol C seems appropriate (for it really represents a very large intentional collimation error), is connected with the hour-angle t, at which a star of declination δ passes the small circle by the equation

$$\sin C = \sin t \cos \delta \quad \ldots \quad \ldots \quad (1)$$

For stars near the equator $\cos \delta$ differs but slightly from unity, and an error in the assumed value of δ has but very little effect on the resulting value of C; hence C may be determined by transits of equatorial stars without requiring an accurate knowledge of their declination, and if the stars are also observed in the principal telescope, errors in the tabular right ascension, as well as small errors in the position of the instrument, will also be eliminated.

C being thus determined, the telescopes are directed towards the pole and six-hour circle, when stars of declination nearly equal to C can be observed either by transit or, preferably, by micrometrical bisections in the manner which is practised with the transit instrument in the prime vertical. A transposition of equation (1) then gives the declination as

$$\cos \delta = \frac{\sin C}{\sin t} \quad \ldots \quad \ldots \quad (2)$$

and as t is now nearly 90° this observation is practically free from errors in the time or in the R.A. of the observed stars. The principal telescope will, of course, be used in both positions to control the instrumental constants.

It would at first sight seem that these observations would be affected by refraction even more strongly than measurements with the meridian circle, but in reality the refraction is to a very great extent eliminated; for if we consider the triangle between zenith, star, and the spherical perpendicular C, calling the angle at the star p, we shall have the differential relation

$$\partial C = \partial \zeta \cos p$$

$\partial \zeta$, being the refraction, is very nearly equal to a constant R multiplied by the tangent of the zenith distance, or

$$\partial C = R \operatorname{tg} \zeta \cos p.$$

But the same triangle also gives

$$\cos p = \frac{\operatorname{tg} C}{\operatorname{tg} \zeta}$$

and, introducing this, we get

$$\partial C = R \operatorname{tg} C \quad \ldots \quad \ldots \quad (3)$$

that is to say, the refraction uniformly raises the small circle without interfering with its parallelism to the corresponding vertical great circle. Therefore the effect of refraction in these observations is limited, besides second-order terms, which are amenable to computation, to the comparatively slight departures from the tangent law and to the effects of changes in the density of the atmosphere, both of which are more accurately known within the narrow limits which come into question than the total amount of the refraction for similar zenith distances.

Flexure of the instrument next suggests itself as a source of error. This also will be eliminated to a very great extent if, after observing the equatorial stars, the instrument is reversed in its bearings for the observations on the six-hour circle, as the instrument will then remain very nearly in the same position with regard to the direction of gravity.

It is obvious that the method could be most advantageously used in an observatory on the 45th parallel, where the elimination of errors would be most complete, and that it could not be used at all either near the equator or near the poles.

The declination of suitable stars having been determined, the latitude of the place of observation could be obtained by the zenith telescope or by a transit instrument in the prime vertical, again without involving close readings of graduated circles.

It would seem that this method could be advantageously carried out photographically, photographs of equatorial and six-hour regions being taken on the same plate, the resulting star traces forming approximately right angles with each other and being thus clearly distinguished.

Bedford Park, W.: 1904 *May* 10.

Micrometrical Measures of Double Stars made with the 17½*-inch Reflector. Second Series.* By the Rev. T. E. Espin, M.A.

In the following list Column 1 gives the number in Σ, OΣ, &c., Column 2 the approximate R.A. and decl. for 1880, Column 3 the position angle, Column 4 the distance, Column 5 the number of nights, Column 6 the magnitudes, Column 7 the date, and Column 8 any notes. In addition to the usual symbols, A.G. refers to stars found double in the Catalogue of the *Astronomische Gesellschaft*. Under the heading Various Stars will be found several new pairs that have been detected since the last list was published, or have been accidentally omitted.

Σ *Stars.*

Σ	B.A. 1880 R.A. h m	Decl. ° ′	P. °	D. ″	Nights.	Mags.	Date.	Notes.
38	0 28·7	+58 0	145·4	17·25	1	...	04·06	
70	46·7	+52 2	244·6	7·91	1	6·5, 9·4	01·89	
162	1 41·8	+47 18	205·1	1·58	1	...	00·99	AB
			179·6	20·16	1	...	00·99	AC
163	42·6	+64 15	33·8	34·47	2	6·0, 8·3	03·43	
182	47·9	+60 42	304·9	3·38	2	...	04·06	*Comes* 13 mag 71° ± 32″ ±
230	2 6·5	+57 55	258·2	23·49	2	...	03·87	
272	24·4	+57 56	38·9	2·07	1	8·0, 8·0	03·88	
392	3 1·4	+52 29	346·1	25·09	1	...	01·00	
550	4 22·5	+53 36	306·7	10·24	2	...	01 58	1 Cameli
587	38·5	+52 54	183·6	21·04	2	...	01·51	
603	45·1	+49 23	239·2	8·28	1	...	01·04	
617	52·3	62 50	122·5	12·76	3	8·5, 9·0	04·05	
618	52·5	62 54	210·6	32·44	3	...	04·05	
718	5 23·1	49 19	76·0	7·69	1	6·5, 6·8	99·88	
865	6 4·9	51 12	66·9	5·28	1	...	01·04	
884	11·5	47 11	271·2	9·17	1	8·3, 8·5	01·07	
946	34·2	59 34	129·7	4·34	2	...	04·07	
948	35·6	59 34	112·8	1·44	2	...	04·07	AB 12 Lyncis
			305·9	8·67	2	...	04·07	AC
958	38·2	+55 50	256·2	5·16	2	6·0, 6·0	01·56	
1006	56·0	+62 56	71·8	29·83	1	...	04·05	
1091	7 20·8	+50 13	330·3	28·51	1	...	00·11	
1174	56·1	+47 36	215·0	5·40	1	...	01·11	
1184	8 1·5	+38 14	340·8	26·50	1	8·0, 8·3	00·10	*Comes* 14 mag. 18″ ± 210° ±
1217	15·9	+45 21	241·7	29·07	1	...	00·11	
1263	37·3	+42 10	21·1	53·75	1	...	00·11	
1346	9 17·1	+54 32	312·6	5·45	2	...	01·17	21 Ursæ
1815	14 7·9	+45 45	152·4	8·84	1	...	01·38	
1830	11·9	+57 13	292·1	7·02	2	8·5, 9·0	02·33	AB
			80·5	35·84	2	C = 11·8	02·33	AC
1831	12·3	+57 15	140·3	5·84	2	6·5, 8·5	02·33	
1982	15 45·8	+43 8	301·7	5·42	2	8·9, 9·1	01·44	
2006	58·0	+59 15	194·3	1·42	2	7·5, 9·3	00·96	AB
			218·8	45·04	2	C = 8·7	00·96	AC
2047	16 19·7	+47 55	331·5	2·24	1	...	00·40	
2054	22·2	+61 57	356·2	1·37	4	6·8, 7·2	03·60	

Σ	R.A. 1880 h m	Decl. ° ′	P. °	D. ″	Nights.	Mags.	Date.	Notes.
2065	28·5	+40 14	217·1	31·00	1	8·1, 8·6	00·45	
2092	37·4	60 56	2·8	8·49	3	7·8, 8·4	03·57	
2116	54·3	63 42	4·6	19·52	2	8·5, 9·0	03·61	
2124	58·8	65 22	87·1	15·01	2	8·5, 9·0	03·61	
2155	17 14·6	60 50	112·3	9·75	2	6·5, 8·9	03·56	
2180	26·5	50 57	262·9	3·30	2	...	02·57	
2199	36·4	55 48	84·3	1·58	3	7·5, 7·9	02·93	
2218	39·5	63 43	345·2	1·72	3	6·6, 7·8	03·57	
2219	40·1	61 39	99·8	17·61	2	7·6, 8·5	03·56	
2225	41·9	51 59	313·0	8·82	1	9·0, 9·2	03·69	
2273	58·6	64 9	284·1	20·45	2	6·5, 7·0	03·56	
			266·4	24·1	2	C=12	03·56	(New)
2357	18 32·1	63 37	268·8	4·87	1	...	03·56	
2368	36·1	52 14	326·8	1·74	2	...	03·70	
2420	49·4	59 15	333·9	32·32	3	...	03·65	*o* Draconis
2440	56·1	62 14	122·2	16·78	2	6·5, 8·0	03·58	
2481	19 7·1	38 34	221·6	4·32	1	7·2, 7·4	00·52	
2487	9·7	38 55	83·1	28·13	1	...	00·52	η Lyræ
2554	32·1	59 59	195·4	17·97	2	8·3, 8·6	03·62	
2573	38·3	60 12	27·2	18·13	1	...	03·60	
2604	51·5	63 49	184·3	27·38	4	6·5, 8·5	02·97	
2611	55·2	47 2	206·0	5·40	1	8·1, 8·1	01·64	Both very yellow
2640	20 3·2	63 32	19·6	5·18	3	7·0, 10·0	03·88	
2691	25·2	37 43	32·3	17·22	2	8·0, 8·2	01·57	
2764	21 2·9	61 40	302·2	6·67	2	8·0, 8·3	03·88	
2846	50·1	45 12	266·4	3·32	2	...	01·76	
2896	22 14·6	62 46	241·8	20·95	2	...	02·87	
2906	21·4	36 50	1·1	3·86	1	6·7, 9·2	00·62	
2985	23 4·4	47 18	253·3	15·57	1	6·3, 6·9	01·81	Yellow; yellowish white
3049	52·9	55 6	325·9	3·05	3	...	03·80	σ Cassiopeiæ
3053	56·4	65 26	70·5	14·69	2	...	03·78	

O Σ *Stars.*

59	3 32·3	45 38	350·0	2·72	1	8·0, 8·7	00·11	
158 rej.	6 43·9	51 41	303·5	16·39	1	7·0, 12·5	01·04	
291 rej.	14 56·5	47 45	156·5	35·94	1	...	01·38	
312	16 22·3	61 46	141·5	5·25	3	...	03·60	η Draconis

Σ	R.A. 1880 h m	Decl. ° ′	P. °	D. ″	Nights.	Mags.	Date.	Notes.
329	17 20·1	37 3	13·6	32·75	3	7·3, 8·2	00·35	
366 rej.	19 9·8	34 1	230·6	21·28	1	7·5, 8·7	00·57	AB
			141·0	26·1	1	11·7	00·57	AC
			21·7	13·2	1	13·2	00·57	CD

OΣΣ Stars.

15	1 13·3	64 0	348·1	52·63	2	6·0, 7·5	03·35	Yellow white, red
26	2 11·0	59 29	199·9	63·06	1	...	03·86	
39	3 39·7	56 45	75·1	57·57	2	...	02·51	
44	4 8·6	45 53	321·8	58·49	1	...	01·06	
46	4 11·3	55 13	188·6	34·40	1	7·0, 12·0	01·07	AB (New)
...	...	...	160·2	98·88	1	C 7·2	01·07	AC
87	7 30·4	42 46	178·0	65·10	2	...	00·10	
156	17 29·9	55 14	312·1	61·84	2	...	03·66	ν^1 ν^2 Draconis
163	55·5	62 37	40·1	58·10	2	7·0, 7·1	02·69	
186	19 31·2	59 53	287·0	75·76	2	...	03·62	
194	51·3	59 23	359·6	75·37	1	...	03·64	
242	23 1·0	46 16	31·0	80·34	1	6·9, 7·5	01·81	
247	24·3	57 53	268·9	75·80	1	...	00·86	
...	...	...	254·9	10·92	1	11·0, 11·2	00·86	*h* 1889
...	...	...	114·5	42·79	1	6, 12		*h* 1888
248	40·1	50 0	339·8	22·94	2	6·5, 12·0	02·84	AB (New)
...	...	...	139·9	52·68	2	C = 9·1	02·84	AC
251	47·5	50 52	199·6	43·35	2	...	03·86	

h's Stars.

1066	0 59·0	62 0	297·3	7·32	3	8·6, 11·6	03·57	*h* 302·2
2069	1 35·8	52 41	240·4	20·11	2	„ 9·2, 9·3	00·02	
1091	40·3	61 14	152·3	27·66	2	8·5, 8·9	04·05	
2085	41·9	52 12	60·8	10·18	2	9·3, 9·8	00·47	
2307	6 11·8	54 6	92·0	22·40	1	8·6, 11·5	00·12	
417 756	7 13·8	34 55	69·4	17·60	1	8·5, 13·5	00·10	
2478	8 55·1	56 9	199·4	17·96	2	9·6, 10·7	01·21	
1231	13 19·9	41 5	6·2	22·39	1	8·5, 12·0	01·39	
554	14 27·4	35 15	294·3	12·11	2	8·9, 9·1	02·33	
1261	50·7	58 3	16·3	9·13	2	9·2, 9·3	01·35	B.D. + 58°·1538
2771	15 9·2	54 27	279·7	37·88	1	8·8, 12·5	00·45	*h's* R.A. 1ᵐ

Σ	R.A. 1880 h m	Decl. ° ′	P. °	D. ″	Nights.	Mags.	Date.	Notes.
2784	24·7	50 3	229·4	17·28	1	9·0, 13·0	00·34	
587	16 35·7	37 46	300·5	16·00	1	8·8, 13·5	00·52	
2802	53·0	39 18	123·5	11·32	1	9·3, 14·0	00·54	
263	57·2	38 8	113·3	18·04	1	9·3, 11·5	00·52	
264	17 3·4	36 10	197·4	12·54	2	9·5, 12·0	00·56	
1342	18 40·3	43 19	185·7	18·80	1	9·0, 12·0	02·65	
1356	52·9	45 17	343·7	29·23	2	8·3, 9·0	01·63	
1357	53·6	45 37	212·1	26·23	2	7·9, 11·7	01·64	
1395	19 20·9	36 53	65·2	3·09	1	9·0, 9·2	01·55	
1439	42·3	55 33	199·6	30·21	2	7·0, 9·0	02·67	A 195°·0
2905	46·2	60 57	196·9	8·61	2	9·1, 11·6	02·69	
1457	53·4	37 36	44·2	11·58	1	9·1, 9·2	00·57	
1505	20 17·2	43 14	116·9	16·84	1	8·3, 10·5	00·53	A 110°·3
1580	44·2	55 26	246·8	10·70	1	8·9, 12·0	02·68	
1606	56·8	54 4	184·0	17·60	2	9·0, 11·0	03·19	
1654	21 25·5	61 5	30·8	3·98	3	9·1, 9·3	02·99	
1659	29·3	58 8	300·4	6·68	1	8·4, 9·0	00·62	A 312°·2
939	30·5	30 28	172·7	14·81	1	8·0, 12·0	00·71	
940	30·5	30 31	290·6	40·72	1	8·0, 10·0	00·71	
1673	34·5	43 48	82·8	5·76	1	8·3, 8·8	00·75	
1687	39·9	45 37	51·0	5·44	1	9·4, 9·9	00·75	A 238°·5
1697	44·9	34 18	260·4	8·86	2	8·0, 9·6	00·66	
3072	50·4	58 53	76·8	13·83	1	...	01·01	
1714	56·5	44 46	252·6	16·68	1	8·6, 8·8	00·75	
1788	22 29·6	40 57	300·3	3·18	1	8·4, 9·2	00·62	
1796	33·8	56 10	6·6	29·59	1	5·0, 11·0	02·83	A 22°·1
1808	41·2	48 25	317·4	10·06	1	9·5, 9·6	01·76	A 133°·1
1860	23 8·5	62 0	355·8	7·75	1	9·0, 11·5	04·02	A 14°·9
1876	20·0	36 10	211·6	10·12	1	9·4, 9·6	00·71	
1896	33·3	61 26	115·6	16·33	3	6·8, 11·2	02·77	

Various Stars.

pin	0 16·4	53 40	191·7	10·23	3	8·4, 9·3	02·91	B.D. + 53°·54
pin	3 28·1	63 49	171·9	3·71	3	9·9, 10·4	04·05	B.D. + 63°·435
pin	4 12·7	59 20	58·9	32·14	1	6·0, 8·8	04·07	B.D. + 59°·793
pin	5 18·5	62 35	222·9	2·32	1	8·7, 10·5	04·06	B.D. + 62°·756
961 rej.	6 39·9	41 15	48·6	8·04	1	9·2, 10·5	00·14	
	53·5	40 1	247·7	6·59	2	9·5, 11·5	00·51	BC (New)
			151·3	8·47	2	9·4, 9·5	00·51	AB B D. + 40°·1776

Σ	R.A. 1880 h m	Decl. ° ′	P. °	D. ″	Nights.	Mags.	Date.	Notes.
Σ 2069 rej.	16 31·7	34 5	71·3	27·27	1	6·8, 9·4	00·54	
KR. 47	17 51·7	64 16	204·9	8·10	2	8·9, 9·3	03·59	
Σ 2334 rej.	18 24·5	62 51	205·2	14·48	2	...	03·58	
Espin	31·5	58 41	260·0	13·98	2	8·9, 11·9	02·81	
Σ 2353 rej.	31·6	58 36	224·8	11·75	2	8·2, 13·7	02·81	
Σ 2365 rej.	34·4	63 36	23·7	19·71	4	7·5, 9·6	03·24	
Σ 2377 rej.	37·5	63 25	340·7	15·55	3	6·9, 9·8	03·14	
A.G.	19 9·4	47 34	228·9	5·24	2	8·5, 8·7	01·65	B.D. + 47° 2782
A.G.	44·7	51 36	258·4	13·47	3	8·4, 9·3	03·00	B.D. + 51° 2683
—	20 1	35 35	149·1	96·47	2	...	01·59	Sh. 314, Sh. 315
Espin	42·5	57 9	162·7	68·65	3	5·0, 8·7	02·83	B.D. + 57° 2240
A.G.	47·2	44 36	251·3	9·48	1	8·8, 9·1	00·75	B.D. + 44° 3603
A.G.	48·0	53 36	361·1	9·47	2	8·7, 8·8	02·73	AB B.D. + 53° 2506
			250·5	13·04	2	C = 12·5	02·73	AC
Σ 2772 rej.	21 5·5	53 52	230·5	11·44	1	...	00·75	
A.G.	14·2	38 39	112·8	5·78	2	8·4, 9·3	00·76	B.D. + 38° 4442
Holmes	16·6	58 11	244·2	12·75	2	9·0, 9·1	02·78	B.D. + 58° 2252
A.G.	27·0	44 37	181·8	4·08	1	9·0, 9·3	00·79	B.D. + 44° 3852
Σ 2808 rej.	30·5	30 28	321·6	19·12	...	...	00·71	
Σ 2844 rej.	48·6	64 20	258·5	11·16	2	8·0, 10·0	03·74	
Espin	22 51·9	64 9	330·5	2·76	1	11·0, 12·0	02·73	BC
			296·4	19·13	1	A = 10	02·73	AB (à 1833)
Espin	23 15·6	61 45	...	4 ±	...	11·5, 12·5	04·02	BC
				30 ±	...	A = 8·0	04·02	AB B.D. + 61° 2430

The Society has received the following circular letter from the Weather Bureau of the United States Department of Agriculture. The Council has directed that the letter should be published in the *Monthly Notices*.

U.S. Department of Agriculture:
Weather Bureau, Washington, D.C.
April 15, 1904.

Sir,—You may have noticed that records have been published by Messrs. C. Dufour, of Lausanne; H. H. Kimball, S. P. Langley, and C. G. Abbot, of Washington; Max Wolf, of Heidelberg; and Gorczynski, of Warsaw, showing that an appreciable general diminution of the transparency of the Earth's atmosphere took place some time during the year 1902, but disappeared at some time during 1903. This is an important

matter, and may possibly be made the basis of an explanation of other meteorological phenomena. I beg to ask whether you have any records that will assist in defining the dates of beginning and ending, and the extent of this change in transparency. Such records may consist of photometric or photographic observations of the brightness of the stars; changes in the solar or stellar spectra; unusual prevalence of halos, large Bishop's ring, or haze; observations of heat received from the sun, as made with actinometers or pyrheliometers; observations of the polarisation of the blue sky light and of scintillation of the stars.

Undoubtedly this diminution and increase of transparency began and ended at different dates in different places, as the phenomena spread gradually over the world during the years 1902 and 1903; additional records are therefore desired in order to trace its progress. Will you not kindly examine your records from this point of view and send me the result for publication in a general article on this subject?

Very respectfully,

CLEVELAND ABBE,
Professor and Editor.

Errata.

Annual Report of the Council, page 333.

The Report of the Kodaikanal Observatory was communicated by Mr. C. P. Butler, who was in charge of the Observatory during the Director's absence in England.

Prof. Turner's Paper on Time of Sunset.

Page 193, line 18, *for* Belfast *read* Southport.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXIV. JUNE 10, 1904. No. 8

Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

Henri Deslandres, D.-ès-Sc., F.R.A.S., Observatoire d'Astronomie Physique, Meudon, S.-et-O., France;
Charles D. Perrine, Lick Observatory, Mount Hamilton, California, U.S.A.; and
George W. Ritchey, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.,

were balloted for and duly elected Associates of the Society.

Edward Barlow, Harewood, Andover, Hants;
Otto J. Klotz, LL.D., Department of the Interior; and 437 Albert Street, Ottawa, Canada; and
Major C. Leigh-Lye, care of Messrs. Holt & Co., 3 Whitehall Place, S.W.

were balloted for and duly elected Fellows of the Society.

The following Candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

William Allan, M.A., B.Sc., 88 Leamington Terrace, Edinburgh (proposed by P. S. Hardie);
Ernest Cuthbert Atkinson, M.A., formerly Assistant Curator of the Temple Observatory, Rugby, Erwood, Beckenham, Kent (proposed by H. H. Turner);
Arthur Henry Bagnold, Colonel R.E., Superintendent of the Building Works Department, Royal Arsenal, Warren Road, Shooter's Hill, S.E. (proposed by Major P. A. MacMahon);

The Rev. D. B. Marsh, D.Sc., Hamilton, Ontario, Canada (proposed by John A. Brashear); and
William Newbold, Tonbridge School, Tonbridge, and 7 Broadwater Down, Tunbridge Wells (proposed by Edward Weldon).

Fifty presents were announced as having been received since the last meeting, including amongst others:—

Twenty-one prints of the Astrographic Chart (Zones +65°, +66°, +67°), presented by the Royal Observatory, Greenwich; two plates of the Harvard photographic map of the sky (to complete series), presented by the Harvard Observatory.

Analyses of Errors of Moon's Longitude for Inequalities of Longer Periods. Methods and Results. By P. H. Cowell.

Up to the present I have confined myself to short-period terms, and I have taken 400 lunar days as the period of analysis. To this there have been two exceptions. In order to properly discuss the terms $\sin g$ and $\sin(2D-g)$ it was necessary to analyse for $\sin(D-g)$; and similarly for terms $g' \pm D$ it was necessary to analyse for $\sin g'$.

Although the analysis for short-period terms will not be finally left in its present state, I have now gone on to terms of longer period. The whole fifty-four years, 1847–1901, has now been taken as a single period of analysis.

The errors to be analysed are shown in Table I.; the Hansen period 1847–1901 has been divided into 48×400 lunar days, or 480 times 40 lunar days, numbered from 891 to 1330 inclusive, in order to permit the Airy period beginning with No. 1. Each period of forty lunar days I refer to as a column, as the original errors have been arranged in columns, each column containing the errors of such observations as occurred in the corresponding forty lunar days. Table I. exhibits the mean error for each column, each individual observation having been corrected for thirteen corrections, as described in previous papers; but the constants for each period of 400 lunar days, which was removed for the purpose of the short-period analyses, have now been restored, and a correction $+1''.3$ applied throughout the whole period, reducing the sum of the 480 quantities here tabulated to $+1''.1$. It is further necessary to state that there were no observations in column 1244—that is to say on line 34, strip 10, of Table I.—and the number there entered has been supplied by inspection.

Mean Error of Moon's Longitude for each of 480 Columns of Forty Lunar Days each in Tenths of a Second of Arc.

	Columns											
	851 to 890.	891 to 930.	931 to 970.	971 to 1010.	1011 to 1050.	1051 to 1090.	1091 to 1130.	1131 to 1170.	1171 to 1210.	1211 to 1250.	1251 to 1290.	1291 to 1330
	1	2	3	4	5	6	7	8	9	10	11	12
1	+27	− 3	−18	−34	−21	0	+25	− 2	+13	+12	+28	− 1
2	+29	− 8	−30	−56	−19	+ 6	+32	− 8	+15	+17	+27	− 4
3	+29	−14	−21	−60	−11	−10	+30	−34	+12	+ 9	+33	− 6
4	+16	−22	− 3	−61	−18	+ 7	+32	− 5	+20	+19	+22	− 8
5	+16	− 9	−21	−53	−19	− 5	+29	+17	+29	+20	+19	− 8
6	+17	− 6	−12	−61	−23	−17	+16	0	+22	+48	+26	0
7	+21	−23	−17	−65	− 8	− 4	+21	+ 4	+11	+17	+ 2	+ 4
8	+27	−27	−17	−35	−18	+ 5	+21	+11	+12	+32	+17	− 4
9	0	+ 5	−27	−33	−19	+10	+30	+21	+25	+40	+21	− 9
0	+ 4	+ 9	−31	−39	−14	+ 2	+19	− 2	+19	+21	+35	−13
1	+14	+ 1	−28	−37	+ 5	0	+26	+ 3	+28	+30	+16	−14
2	+15	−21	−15	−52	− 4	+24	+20	+ 3	+14	+22	+26	−10
3	+ 1	−10	−21	−52	−10	+19	+29	− 5	+20	+11	+22	−11
4	+ 7	−18	−19	−58	−11	+16	+26	−10	+24	+ 4	+18	−14
5	+16	−17	−33	−53	0	+12	+26	+ 6	+ 7	+14	+ 9	+ 3
6	+24	−23	−31	−61	− 8	+12	+21	+ 6	−14	+ 6	+ 7	+ 5
7	+ 6	−15	−48	−35	0	+16	+24	+19	+ 2	+20	+13	+ 2
8	− 2	−11	−50	−32	−15	+10	+27	+12	+17	+20	+ 9	− 6
9	− 6	−12	−32	−31	− 1	+20	+23	+12	+21	+25	0	− 1
0	+17	− 6	−27	−44	− 9	+10	+25	+ 7	+31	+16	+14	− 2
1	+ 3	− 8	−27	−46	+ 1	+14	+24	+ 6	+35	+19	− 3	−14
2	+ 9	−14	−22	−35	+ 5	+12	+18	− 3	+29	+19	+26	+ 7
3	+13	−18	−27	−48	+10	+ 7	+22	− 1	+32	+ 9	+14	−23
4	+ 2	−20	−24	−34	+ 8	+ 2	+13	− 6	+ 3	+ 5	+ 3	+ 5
5	0	−25	−42	−38	+ 2	+13	+12	+ 1	+ 7	+25	+ 1	− 5
6	0	−16	−46	−28	− 4	+21	+13	+10	+ 3	+15	+10	+ 6
7	−14	−10	−46	−45	− 6	+14	+14	− 1	+17	+22	+10	+ 6
8	−16	−30	−42	−29	+10	+22	+ 5	+ 3	+28	+29	+14	− 5
9	+ 6	−13	−24	−32	+ 7	+25	+15	+ 6	+26	+31	+13	+ 3
0	+ 8	− 7	−40	−35	− 4	+15	+24	+19	+21	+23	+ 1	− 4
1	+ 7	−14	−27	−27	+13	+10	+10	+10	+26	+20	+10	− 4
2	+17	−12	−33	−19	− 8	+17	+ 9	+ 1	+25	+25	− 5	+ 6
3	0	−21	−35	−35	− 3	+28	0	+14	+ 3	+49	− 8	− 5
4	− 7	− 9	−36	−25	−14	+21	− 5	+ 8	+22	[+33]	+ 2	−16
5	− 6	−12	−55	−26	−14	+13	+ 2	+25	+21	+37	− 4	−25
6	−12	0	−46	−29	−13	+12	+ 2	+15	+17	+35	+ 4	− 6
7	−21	−12	−46	−23	−10	+16	+ 3	+16	+23	+30	+ 9	−12
8	−20	−12	−55	−14	+ 6	+17	− 2	+16	+28	+29	+21	− 1
9	− 2	−24	−37	− 9	− 1	+ 8	+11	+18	+33	+23	+ 8	− 5
0	− 5	−15	−28	−32	+ 7	+22	− 3	+13	+25	+27	− 5	+12

Taking $\pm 1''.7$ as the probable error of a single observation, the quantities in Table I. will be subject to an accidental error of $\pm 0''.5$. This is approximately the accidental error of a star.

It will be noticed that I refer to the columns of Table I. as strips. This is partly because I use the word "column" as just explained to denote forty lunar days and partly with reference to my method of analysis, which I now explain in detail for the argument E−J, a Jupiter term. The columns of Table I. are used by me in MS. written on separate strips of paper, 2 inches wide, with the numbers 1–40 printed down them. A strip blank, except that it is marked with the consecutive numerals, is then placed on the table, and column 1091, that is to say, the first entry of the seventh strip, is placed in a line with the "1" on the blank strip; column 1092 will then fall opposite the "2," and so on. Column 851, the first of the first strip, is placed opposite the "2" of the blank strip; and the first column of the ninth strip opposite the "4" of the blank strip, the complete arrangement being indicated by the annexed table :—

Column.	Number of Blank Strip.	Column.	Number of Blank Strip.
851	2	1091	1
891	13	1131	12
931	24	1171	4
971	16	1211	15
1011	27	1251	7
1051	19	1291	18

The arrangement is so contrived that entries opposite the same number of the blank strip correspond as nearly as possible to the same value of the argument E−J, and may properly be added for an E−J analysis. Moreover, the value of the argument E−J for any line is found by multiplying the number on the blank strip by the "unit," an exact fraction if possible of 360° or a low multiple of it. It is clearly necessary that the "unit" shall be a close approximation to the movement of E−J in 40 lunar days. In the present instance $\frac{3 \times 360^\circ}{29}$ has been taken as the unit, so that the values of E−J recur every twenty-ninth line of the blank strip. I postpone an account of the calculations justifying the precepts of the foregoing table until the rest of the analysis has been described. The numbers on the strips are then added horizontally and the results shown in the following table :—

No. on Blank Strip.	Sum.	No. of Quantities added.	No. on Blank Strip.	Sum.	No. of Quantities added.	No. on Blank Strip.	Sum.	No. of Quantities added.
1	+ 25	1	30	− 18	12	59	−49	2
2	+ 59	2	31	− 30	12	60	−60	2
3	+ 59	2	32	+ 6	12	61	−69	2
4	+ 74	3	33	+ 8	12	62	−50	2
5	+ 60	3	34	− 27	12	63	−38	2
6	+ 44	3	35	− 40	12	64	+ 6	1
7	+ 86	4	36	− 72	12	65	− 1	1
8	+ 98	4	37	− 14	12	66	+ 7	1
9	+112	4	38	+108	12	...	...	...
10	+ 52	4	39	− 36	12	...	...	...
11	+ 61	4	40	− 97	12	...	...	...
12	+ 83	5	41	− 19	11	...	...	...
13	+ 54	6	42	− 19	10	...	...	...
14	+ 30	6	43	+ 15	10	...	...	...

No. on Blank Strip.	Sum.	No. of Quantities added.	No. on Blank Strip.	Sum.	No. of Quantities added.
29	−64	12	58	−41	3
28	−80	12	57	− 3	4
27	−65	12	56	−27	4
26	+ 7	11	55	−43	5
25	−24	11	54	+ 1	6
24	+ 1	11	53	−30	6
23	+79	10	52	−41	7
22	+27	10	51	−38	8
21	+ 8	10	50	−11	8
20	+28	10	49	− 4	8
19	−28	10	48	−12	8
18	+22	9	47	+31	8
17	+32	8	46	− 2	9
16	+70	8	45	−34	9
15	+61	7	44	− 1	9

Owing to the recurrence every twenty-ninth line, the entries for Nos. 1, 30, 59 may properly be added. Moreover, since the sum of the values of E−J for the pth and $29-p$th lines is 360°, the values of cos (E−J) are the same throughout the whole of a horizontal line of the preceding table; while the values of sin (E−J) are the same numerically, but the last two are of changed sign.

Hence we obtain the following table, sufficiently explained by its headings :—

Multiples of Unit of $\frac{360°\times 3}{29}$	Value of E−J.	Sum for Sine Analysis.	100 × sin E−J.	Product.	Sum for Cosine Analysis	100 × cos (E−J).	Product.
0	0·0	...	0	...	−105	+100	−105
1	37·2	+41	+ 60	+25	−125	+ 80	−100
2	74·5	+61	+ 96	+59	−123	+ 27	− 33
3	111·7	+32	+ 93	+30	− 40	− 37	+ 15
4	149·0	+55	+ 51	+28	+ 9	− 86	− 8
5	186·2	+24	− 11	− 3	− 34	− 99	+ 34
6	223·4	−28	− 69	+19	+ 48	− 73	− 35
7	260·7	+24	− 99	−24	+ 2	− 16	0
8	297·9	+94	− 88	−83	+ 88	+ 47	+ 41
9	335·2	−20	− 42	+ 8	+ 28	+ 91	+ 25
10	12·4	+56	+ 21	+12	− 24	+ 98	− 23
11	49·6	−89	+ 76	−67	+ 17	+ 65	+ 11
12	86·9	+34	+100	+34	+ 94	+ 5	+ 5
13	124·1	− 1	+ 83	− 1	+ 71	− 56	− 40
14	161·4	−15	+ 32	− 5	+105	− 95	−100
Sums	...	...	...	+32	+ 11	...	−313

In the last line the sum +11 should obviously be the total of the entries of Table I., which, as previously noted, is +11 ; this checks nearly all the arithmetic. The other two sums must be divided by

$$\Sigma \sin^2 (E-J) \text{ and } \Sigma \cos^2 (E-J)$$

respectively : each of these quantities is 240. Dividing, however, by 2400 we get the result expressed in seconds of arc.

It is as follows :—

Hansen's tabular places exhibit an excess over the observations of

$$+0''\cdot01 \sin (E-J) - 0''\cdot13 \cos (E-J).$$

Both these terms may be discarded as accidental.

Before proceeding to give the results of similar analyses, I must explain the calculation of the table of precepts for the arrangement of the twelve strips for the additions.

From the table published at the end of my paper in the *Monthly Notices* for March I obtain E−J = 21°·4980 at the middle of my forty-fourth period of analysis, and its movement in forty lunar days = 37°·36605.

The latter quantity suggested the unit

$$\frac{3\times 360^\circ}{29} = 37^\circ\cdot 24$$

and in terms of this unit E−J = 1·73178÷3 at the middle of the forty-fourth period of analysis and the movement in forty lunar days is 3·010044÷3. It is convenient to keep the "3" in evidence in the manner indicated.

The interval from the middle of the forty-fourth period of analysis to the middle of the first strip is 435 columns of forty lunar days, and the movement in this time is

$$1309\cdot 36914\div 3,$$

or discarding multiples of 29

$$4\cdot 36914\div 3.$$

Hence the value for the middle of strip 1 (that is to say, strip 1, line 20½) is

$$6\cdot 10092\div 3.$$

It is convenient to add 58 to this, making 64·10092, so that on division by 3 the fraction lies between ⅓ and ⅔. Thus, for strip 1, line 20½ E−J = 21·37 units. Therefore, strip 1, line 1, or column 851, E−J = 2 units.

Continuing, in one strip or forty columns, the movement of E−J = 120·40÷3.

Adding ·40 to ·10 (from 64·10092 above), the unit is unchanged, the division by 3 leaves the fraction between ⅓ and ⅔; there is no break in continuity, and column 891 corresponds to unit 2+40−29 = 13.

Adding another ·40, making ·90, there is still no break of continuity, and column 931 corresponds to 13+40−29 = 24.

Adding another ·40, making 1·30, we now have to add 29, making 0·30 on casting out threes.

A discontinuity of 10 is therefore introduced, and column 971 corresponds to

$$24+40-58+10 = 16$$

and so on.

It is clear that since 360° = 9⅔ units, the discontinuity of 10 units is really a discontinuity of ⅓ unit, and at the middle of each strip the true and adopted value of E−J never differ by more than ⅙ unit or 6°·2.

Almost every figure used in the E—J analysis has now been set down, and it will be seen that the analysis for fifty-four years is reduced to a quite short computation when once the method has been reduced to routine. I have carried through the necessary computations for four different arguments in one day.

The following table gives the outline of the various analyses performed :—

Argument	g'	$g'-w+w'$	$(2g'-w+w')$	$(2g'+2w')$	$(\mathrm{V}-\mathrm{E})$	$(2\mathrm{V}-3\mathrm{E}+85^\circ)$	
Unit	$\frac{360^\circ}{9}$	$\frac{360^\circ}{10}$	$\frac{3\times360^\circ}{14}$	$\frac{5\times360^\circ}{21}$	$\frac{360^\circ}{14}$	$\frac{360^\circ}{35}$	
Movement in 40 lunar days							
in degrees	40°·80584	36°·19540	77°·00524	86°·00040	25°·52522	10°·24425	37°
in units	1·020146	1·005428	2·994648÷3	5·016690÷5	0·992647	0·995969	3
Value at middle of 44th period of analysis							
in degrees	4°·1375	57°·8240	61°·9615	141°·1374	47°·5763	76°·5091	21°
in units	0·10344	1·60622	2·40961÷3	8·23301÷5	1·85019	7·43848	1
Value in units corresponding to columns							
851	1	9	5	7	8	1	
891	6	10	12	1	6	6	
931	2	10	10	20	4	11	
971	7	10	8	14	1	16	
1011	3	10	6	8	13	21	
1051	7	1	4	6	11	25	
1091	3	1	11	21	8	30	
1131	8	1	9	15	6	35	
1171	4	1	7	13	4	5	
1211	9	1	5	7	2	10	
1251	4	2	12	1	13	15	
1291	9	2	10	20	11	19	
tab—obs. apparent coefficients of							
sin (arg.)	−0″·35	−0″·09	+0″·02	+0″·25	−0″·34	+0″·29	
cos (arg.)	−0·28	+0·32	−0·08	−0·02	−0·02	0·00	
sin (twice arg.)	+0·05	−0·17	+0·20	−0·03	+0·10		
cos (twice arg.)	−0·04	+0·01	−0·04	−0·06	−0·01		

The arguments $\omega-\omega'$, Ω, $2\mathrm{M}-\mathrm{E}+49^\circ$ move so slowly that 5×40 lunar days were taken as the unit of time, and the following table was formed from Table I. by throwing two columns into one, and adding five lines at a time. Each entry is therefore the mean for 200 lunar days expressed in units of 0″·02.

	A.	B	C.	D.	E.	F.
1	+117	− 93	−88	+148	+ 89	+129
2	+ 69	−104	−82	+107	+ 89	+101
3	+ 53	−116	−20	+127	+ 93	+ 91
4	+ 39	−188	−33	+120	+ 57	+ 43
5	+ 27	−142	+26	+ 89	+106	+ 41
6	− 16	−198	+ 3	+ 71	+ 95	+ 48
7	+ 11	−186	−26	+ 16	+ 97	− 5
8	− 60	−212	−11	+ 11	+126	+ 37
9	− 56	−264	− 2	− 32	+ 77	− 27
10	− 42	−233	− 4	+ 34	+158	− 22
11	− 65	−252	+71	− 3	+ 81	− 46
12	− 67	−203	+68	+ 56	+ 87	− 2
13	− 85	−201	+48	− 3	+ 77	− 30
14	− 76	−169	+97	+ 37	+120	+ 6
15	− 68	−132	+89	+ 58	+164	− 44
16	− 63	−107	+75	+ 78	+144	− 12

The same plan of analysis was adopted, and the six strips are referred to by the letters A to F. The outline is given in the tables below :—

Argument.	$2M-E+49^\circ$.	$w-w'$.	$-\Omega$.	$2w$.
Unit	$\frac{360^\circ}{28}$	$\frac{360^\circ}{16}$	$\frac{360^\circ}{33}$	$\frac{3\times360^\circ}{10}$
Movement in 100 lunar days in degrees	12°92915	23°05215	10°96205	68°0478
in units	1·005601	1·024540	1·004855	3·02435÷3
Value at middle of 44th period of analysis in degrees	54°4667	306°3135	326°9251	25°4894
in units	4·2363	13·6139	29·9681	1·1329÷3
Value in units corresponding to A 1	4	3	15	2
B 1	20	4	31	13
C 1	8	4	14	13
D 1	25	5	30	13
E 1	13	5	13	8
F 1	1	5	29	8
Tab.−obs. apparent coefficients of sin (arg.)	−0″33	+0″60	−1″18	+0″10
cos (arg.)	−0·06	−0·31	−0·28	−0·15
sin (twice arg.)	−0·01	0·00	...	...
cos (twice arg.)	−0·05	−0·03	...	...

It should be noticed that $-\mathfrak{g}$ and not $\mathfrak{g}$ has been taken as the argument.

All the long-period analyses at present completed have now been exhibited. It is clear that many of the coefficients obtained may be treated as accidental. Further it is to be remembered that all coefficients given are "apparent" (see *Monthly Notices*, March, p. 413). I proceed now to the explanations. The quantities are always spoken of in the sense *tabular* minus *observed*

(i.) $$-0''\cdot35 \sin g'$$

On p. 419 I found from a former analysis

$$+0''\cdot24 \sin (g'-\mathrm{D})-0''\cdot83 \sin g'+0''\cdot81 \sin (g'+\mathrm{D})$$

I have subsequently applied the correction $+0''\cdot55 \sin g'$, whose apparent effect, according to the ideas of p. 413, will be

$$+0''\cdot55 \sin g'-0''\cdot27 \sin (g'-\mathrm{D})-0''\cdot27 \sin (g'+\mathrm{D})$$

We ought now to have

$-0''\cdot28 \sin g'+0''\cdot54 \sin (g'+\mathrm{D})$. The former term is here verified to within $0''\cdot07$. It is clearly the apparent effect of the latter term which alone is real.

(ii.) $-0''\cdot28 \cos g'$. On p. 419 $-0''\cdot33 \cos g'+0''\cdot46 \cos (g+\mathrm{D})$ was found. Again the latter term is alone real.

We have therefore real terms

$$+0''\cdot54 \sin (g+\omega-\omega')+0''\cdot46 \cos (g+\omega-\omega')$$

As I said on p. 420, the true argument may be $g+\varpi$, or the *Venus* term $g+2\varpi+3\mathrm{V}-5\mathrm{E}$, and it is necessary to wait till the Airy period is reduced. My present supposition is—but it is no more than a supposition—that the term is partly $+0''\cdot4 \cos (g+\varpi)$, with possibly a correction to the coefficients of $\sin (g+\omega-\omega')$ and the *Venus* term. A term $+0''\cdot4 \cos (g+\varpi)$ might arise in this way. Suppose the observed declinations on the average $1''$ too small. Remove the constant correction $-1''$ that Hansen applies to all latitudes. If, then, $+1''$ be applied to all tabular latitudes, and $+1''$ to all observed declinations, the difference "tabular *minus* observed" longitude receives a correction $-0''\cdot4 \cos (g+\varpi)$, which would cancel part of the terms that result from my analysis of the errors. Errors of refraction or division error may account for part of the supposed error of $1''$ in the observed declinations, but not, I think, for all of it. The outstanding part may possibly be due to an erroneous parallax of the Moon, which is, as far as I can judge, a not very accurately determined quantity.

(iii.) $-0''\cdot09 \sin (g-D)$ must be taken with $+0''\cdot17 \sin g$ (see p. 419, l. 13). The latter is the real term, the former its apparent effect.

(iv.) $+0''\cdot32 \cos (g-D)$. On p. 417 I gave $+0''\cdot34 \cos (g-D)$. It there appears as if this is the real term, and $\cos (g-2D)$, $\cos g$ merely the apparent terms. Of course $\cos (g-D)$ cannot be exactly the real term, but there may possibly be some term with an argument of nearly the same speed.

(v.) $-0''\cdot17 \sin (2g'-2\omega+2\omega')$. This is for the most part the apparent effect on an error of $0''\cdot22$ in the coefficient of the allied term $(-g-g'+\omega-\omega')$.

(vi.) $+0''\cdot20 \sin (4g-2\omega+2\omega')$. This term is nearly what we might expect. Hansen's tabular places contain the term $-0''\cdot134 \sin (4g'-2\omega+2\omega')$ (see *Monthly Notices*, January, p. 164), and the true coefficient is $-0''\cdot28$ (see Newcomb's Transformation, *Astron. Papers, Amer. Eph.* vol. i.).

(vii.) $+0''\cdot25 \sin (2g'+2\omega')$. This term is confirmed in the same way as (vi.). The tabular coefficient is $-54''\cdot98$, and the true $-55''\cdot2$.

(viii.) $-0''\cdot34 \sin (V-E)$.

Hansen's term is	$-1''\cdot10 \sin (V-E)$
Radau gives	$-0''\cdot86 \sin (V-E)$

Radau is therefore more nearly right, and is within the possible errors of observation.

(ix.) $+0''\cdot10 \sin (2V-2E)$. Hansen gives $+0''\cdot43$, Radau $+0''\cdot28$. Again Radau's term seems the better.

(x.) $+0''\cdot29 \sin (2V-3E+85°)$. Hansen gives no term. Radau gives $-0''\cdot35$, and is thus confirmed.

(xi.) $-0''\cdot33 \sin (2M-E+49°)$. This inequality is entirely masked by the term in Ω, which separates from it at the rate of one revolution a century. It follows, therefore, that fifty-four years is inadequate for a thorough discussion. In fact, if the terms subsequently found $-1''\cdot18 \sin (-\Omega) -0''\cdot28 \cos (-\Omega)$ be taken out and the analysis repeated, the present term would approximately change sign. About 1875 the two arguments $2M-E+49°$ and $-\Omega$ are equal; for a short time $-1''\cdot18 \sin (-\Omega)$ is indistinguishable from $-1''\cdot18 \sin (2M-E+49°)$, and although this effect is reduced by extending the analysis over fifty years, it still comes in with about two-thirds of its full effect, changing the sign as I have said.

Of course, if another century of observations were not so soon to be available it might be worth while to try and separate the two terms. Each coefficient could, however, only be obtained with a probable error several times as large as the probable error in ordinary cases. The process would bear some analogy to the separation of the parallactic inequality from the semi-diameter, or the cos D term from the mean error.

(xii.) $+0''\cdot60 \sin(\omega-\omega')$.

Hansen's tables give $+1''\cdot58$
Hansen's theory gives $+1''\cdot33$
Delaunay gives $+0''\cdot87$

The evidence of the observations is therefore in favour of Delaunay.

(xiii.) I am unable to explain $-0''\cdot31 \cos(\omega-\omega')$.

(xiv.) $-1''\cdot18 \sin(-\Omega) - 0''\cdot28 \cos(-\Omega)$. The coefficients are uncertain to about $0''\cdot3$ on account of the *Mars* term just discussed. It is clear, however, that Hansen's figure of Earth terms require considerable diminution. On the other hand, the term in $\sin\Omega \cos g$ apparently requires a large increase. Hansen's tables are equivalent to $+0''\cdot59 \sin\Omega \cos g$; I have already applied $+0''\cdot445 \sin\Omega \cos g$, bringing the coefficient up to $+1''\cdot04$ in accordance with Hill's calculations. On analysing the coefficients of $\cos g$ and $\sin g$, given on pp. 415, 416 (last two columns), I obtain

$$(-0''\cdot22 \sin\Omega + 0''\cdot29 \cos\Omega)\cos g$$
$$+ (+0''\cdot13 \sin\Omega + 0''\cdot05 \cos\Omega)\sin g$$

The coefficient of $\sin\Omega \cos g$ should, therefore, apparently be $+1''\cdot26$, or about double Hansen's value.

I ought to add that any at present undiscovered term with a period between fourteen years and twenty-five years (a very wide range) will affect an analysis for Ω based upon fifty-four years. This range will shortly be considerably reduced by taking the Airy period into the discussion. As no two discussions have hitherto given the same coefficient of $\cos\Omega$, I suspect the existence of such an undiscovered term.

The Parallactic Inequality: A Reply. By P. H. Cowell, M.A.

On p. 567 of the present volume of the *Monthly Notices* Professor Turner writes: "If the solar parallax as determined from observations of the Moon is affected with an entirely unknown systematic error, then it is a pity to publish the result." I thought I had made it clear on p. 96, in a passage that Professor Turner has quoted on p. 406, that I saw no means of separating the errors of observation from the error of tabular parallactic inequality, and that my results were given on the assumption that the errors of observation were zero. I see no objection to publishing a result if its limitations are clearly set down.

I said last December, and I still say now, that I am unable to determine how much of the coefficient of the parallactic in-

equality is to be attributed to errors of observation. I did say, however, that I thought probably not more than $\pm 0''.3$. If this estimate is correct the lunar observations give the solar parallax almost free from accidental error (that is to say, more observations under similar conditions are not required), but subject to an unknown systematic error possibly amounting to $0''.02$. I still adhere to the $0''.3$ given above, partly because I have since found a term $0''.28 \cos D$, which must be attributed to errors of observation, and partly because of the investigations of which I give some account at the end of the present paper, and which were undertaken in consequence of the suggestions of Professor Newcomb on p. 570.

The fundamental idea of Professor Turner was that the observations with another instrument should be examined with a view to seeing whether they gave the same parallactic inequality as the transit circle. It will be noticed that whatever the answer to this question may be it does not tell us much. For if the two instruments agree, they may both be wrong ; and if they disagree, there is nothing to show which is wrong.

Again, Professor Newcomb has suggested the exclusion of the daylight observations. I have carried out the necessary calculations with this modification of Professor Newcomb's plan, that because the six o'clock observations were rejected in summer, it was absolutely necessary to reject them in winter as well, for reasons which I set out later. It is a matter of judgment only, and not of absolute certainty, whether the six o'clock observations are so far inferior to the rest that it is better to leave them out altogether.

Following out his own idea, Professor Turner set out to examine fifteen years of the Old Altazimuth. He was looking for a discordance of possibly $0''.3$—I do not think he can have expected to find anything much larger than this. Under these circumstances he was, in my opinion, bound to keep his accidental error under a probable value of $0''.2$, or even less ; but let us say $0''.2$. On p. 96 I stated that the weight of the parallactic inequality found from 5647 observations of unit weight was 515. It would therefore require 1000 observations at the least, with an instrument as good as the transit circle, to obtain a difference of parallactic inequality with a probable error of $0''.2$. And, moreover, these 1000 observations must be carefully worked up without unnecessary waste in the working. Now Professor Turner has chosen the altazimuth as his comparison instrument ; he has thrown aside the observations of what he calls "day 15" (or full moon) and smoothed out days 14 and 16. These are the most important observations of all, because the parallactic inequality is then most different from its mean value, and we are at once left with an amount of material insufficient for its purpose. It was not necessary to confine the investigation to fifteen years, and astronomers, like lawyers, ought to adopt the doctrine of "best evidence."

This being my view about a comparison with fifteen years of the altazimuth, *a fortiori* I can give no credit to a result based upon four years, 1847–1851. To my judgment the figures given by Professor Turner (p. 402, Table V., column 2, $+0''.10$, $-0''.71$, $+0''.21$, $-0''.28$, $-0''.21$, $-0''.36$, $-0''.28$, $+0''.36$, $+0''.21$, $-0''.71$) are palpably of an accidental character. He implicitly admits this himself of the two largest by smoothing them out, and I was astonished that he should have considered the smaller figures unimpeachable.

In his second paper (May 1904) Professor Turner seems in some apprehension as to whether I have mistaken a cosine term for a sine term. I can assure him he need not be in the least anxiety. His argument is based upon the assumption that we only have one quarter available; an assumption that seems to owe its origin to the fact that in his first paper he quite properly subtracted the results for any day before full moon from the corresponding day after full moon, as he was then dealing with an odd function of D. When it is a question of cosines the corresponding results should be added. If the table in which he has set out so clearly the similarity between $\sin D$ and $2+\cos D$ in the second quarter of the Moon (D is usually measured from new moon) be extended to the third quarter, it will be seen that the two functions are of opposite signs. Analytically the cross term in the normal equations, when the errors are equated to $2, \sin D - 2, \cos D$, is proportional to the mean value of $\sin 2D$, the value of which is found on p. 94 to be $-0''.04$, and has been neglected throughout by me.

It was so obvious to me that no preliminary remarks would avoid the necessity of resorting to normal equations that I omitted the remarks and went straight to the equations. Perhaps now, though somewhat late, I had better give the remarks. I do so in a perfectly general form.

Let ϕ be a function whose coefficient a we wish to determine. Let χ_1, χ_2 &c., be other functions, such that they are liable to be confounded with ϕ. To give a precise meaning to this, consider the method of finding a. The errors are grouped according to the values of ϕ, which we will call ϕ_1, ϕ_2 &c. For the typical group of n_r observations we have

$$n_r a\phi_r = \Sigma\epsilon;$$

and its contribution to the normal equation is

$$n_r a\phi_r^2 = \Sigma\epsilon.\phi_r;$$

and the complete normal equation is

$$a\Sigma n_r\phi_r^2 = \Sigma\epsilon.\phi.$$

Now a function χ is liable to be confounded with ϕ if

$$\Sigma\chi\phi = 0.$$

and it will be seen that this is a reciprocal relationship; if χ may be confounded with ϕ, so may ϕ with χ.

Having, then, chosen the function ϕ, whose coefficient I wish to investigate, I pick out all possible forms of the function χ, and the value of my analysis will depend upon the completeness with which I have picked out these functions. In particular—

when $\phi=\sin D$ we have $\chi_1=\mu$	$\chi_2=\sin 2D$	(p. 94)
when $\phi=\cos D$ we have $\chi_1=1$	$\chi_2=\cos 2D$	(p. 582)
when $\phi=\sin A$ we have $\chi_1=\sin(A+D)$	$\chi_2=\sin(A-D)$	(p. 413)

this last illustration being, as far as I am aware, original; and it, moreover, introduces an important correction (0″·27) into the value of the eccentricity when investigated by comparison with Hansen's Tables (see p. 418). On pp. 420 and 693 I have pointed out cases where it would be necessary to resort to the same process when only fifty years' observations are under discussion, but when the process can be avoided by waiting for the reduction of the Airy period, 1750 to 1851.

For simplicity I now consider one fraction χ only. The disentanglement of ϕ from χ involves of course the disentanglement of χ from ϕ. The errors are therefore equated to

$$a\phi+b\chi.$$

Now if $\Sigma\phi\chi = k\Sigma\phi^2$, it follows that a term $\beta\chi$ is liable in an analysis for ϕ alone to be mistaken for $\beta k\phi$. I therefore in the first place solve for $a+kb$ on the supposition that this is the coefficient of ϕ. Next I try to distinguish between

$$a\phi+b\chi \quad \text{and} \quad (a+kb)\phi$$

that is to say, I try from the errors to measure b considered as the coefficient of $\chi-k\phi$.

Now in most cases it is natural to take for the forms of χ and ϕ functions that vary between the limits ± 1. When ϕ, χ are liable to be confounded, it will generally happen that $\chi-k\phi$ varies between smaller limits, $\pm\lambda$ say. Writing, then, $b\lambda\,\frac{\chi-k\phi}{\lambda}$ instead of $b(\chi-k\phi)$, so that $\frac{\chi-k\phi}{\lambda}$ varies between the limits ± 1, it will be seen that $b\lambda$ can be determined with approximately the same accuracy as the coefficient of any sine or cosine term that does not present any special difficulty. Also it will be seen that, when it comes to fitting the formula found, in its form

$$(a+bk)\phi+b\lambda\,\frac{\chi-k\phi}{\lambda}$$

on to the errors, the usual degree of precision may be expected.

It is only when we exhibit the values of a and b separately, for the latter multiplying $b\lambda$ by $\frac{1}{\lambda}$, that we multiply discordances by $\frac{1}{\lambda}$ and get values with weights reduced in the ratio λ^2 to 1.

If we examine Professor Turner's first paper in the light of these remarks, it appears that there has been no adequate attempt to disentangle the quantities I call μ and δ_1. Professor Turner evidently recognises that δ_1 cannot be determined without determining μ; he omits, however, to see that the relationship is reciprocal and that μ cannot be determined apart from δ_1. His method of determining δ_1 is therefore based upon giving two alternative guesses at μ (p. 407); and it will be seen that these alternative values of μ are in reality values not of μ but of $\mu+0{\cdot}7\ \delta_1$ and of $\mu+0{\cdot}4\ \delta_1$ respectively.

I come now to the calculations that I have performed in consequence of Professor Newcomb's suggestion that the daylight observations should be omitted.

I have already stated that it thereupon became necessary to omit the six o'clock winter observations. It is for this reason. Suppose we took the December and January observations only. For these months g' is nearly zero. We are therefore unable to distinguish between

$$\sin (g-g'+\omega-\omega') \text{ or } \sin D$$

the parallactic inequality and

$$\sin (g+\omega-\omega')$$

Now on p. 419 I obtained corrections

$$-0''{\cdot}57 \sin (g+\omega-\omega')-0''{\cdot}46 \cos (g+\omega-\omega')$$

and although I then stated that the arguments were uncertain, and might be $g+\varpi$ or $g+2\varpi+3V-5E$, still the fact remains that as long as errors of this period, or approximately of this period, exist, the parallactic inequality is not to be found from observations in the months of December and January alone. And the same objection applies to any scheme whereby the observations discussed are not symmetrically arranged with regard to the argument g' or time of year.

Consequently I cut down the material to days 10–18 and 39–47 in the notation of my December paper, full moon being at $14\frac{1}{2}$ and $42\frac{1}{2}$ days approximately, so that the observations extend for four days on each side of full moon, approximately.

With the help of Table III. pp. 90–95 *Monthly Notices*, Dec. 1903 the coefficients of the normal equations can be formed on the assumption that every error is equated to

$$\mu+\delta_1 \sin D+\delta_2 \sin 2D.$$

The left-hand sides are

$$\begin{aligned} &3052\mu + 1344\delta_1 - 2097\delta_2 \\ &1344\mu + 768\delta_1 - 1144\delta_2 \\ -&2097\mu - 1144\delta_1 + 1734\delta_2 \end{aligned}$$

Consider the quadric

$$a\mu^2 + b\delta_1^2 + c\delta_2^2 + 2f\delta_1\delta_2 + 2g\delta_2\mu + 2h\mu\delta_1$$

where

$$\begin{aligned} a &= 3052, & b &= 768, & c &= 1734 \\ f &= -1144, & g &= -2097, & h &= 1344 \end{aligned}$$

The quadric may be put into the form

$$a\left\{\mu + \frac{h}{a}\delta_1 + \frac{g}{a}\delta_2\right\}^2 + \frac{ab-h^2}{a}\left\{\delta_1 - \frac{gh-af}{ab-h^2}\delta_2\right\}^2 + \frac{\Delta}{ab-h^2}\delta_2^2$$

where $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 9152000$

$$\begin{aligned} ab - h^2 &= 537600 \\ gh - af &= 673120 \end{aligned}$$

Consequently the quadric becomes

$$\begin{aligned} &3052\{\mu + 0{\cdot}44\delta_1 - 0{\cdot}68\delta_2\}^2 \\ +\ &176\{\delta_1 - \tfrac{5}{4}\delta_2\}^2 \\ +\ &17\delta_2^2 \end{aligned}$$

and therefore
$\mu + 0{\cdot}44\delta_1 - 0{\cdot}68\delta_2$ can be determined with a weight equal to 3052 single observations

$$\begin{aligned} &\delta_1 - \tfrac{5}{4}\delta_2 \text{ with a weight of } 176 \\ &\delta_2 \text{ with a weight of } 17. \end{aligned}$$

This is practically equivalent to saying that δ_2 cannot be determined at all from the material.

The same results follow, with approximate accuracy, from geometrical reasoning, as follows:—

By changing the sign of all second limb errors, we get a determination, with weight 3052, the total number of observations employed of

$$\begin{aligned} &\mu + \delta_1 \times (\text{mean numerical value of sin } D = 0{\cdot}44) \\ &+ \delta_2 \times (\text{mean numerical value of sin } 2D = -0{\cdot}68) \end{aligned}$$

Next, if we try to measure the difference between

$$\mu+\delta_1 \sin D+\delta_2 \sin 2D$$

and

$$\mu+\delta_1\times 0{\cdot}44-\delta_2\times 0{\cdot}68$$

we find that we are approximately always measuring a multiple of $\{\delta_1-\frac{5}{4}\delta_2\}\times 0{\cdot}4$, and that therefore δ_1, δ_2 cannot be separated.

Lastly, as the factor of $(\delta_1-\frac{5}{4}\delta_2)\times$, 0·4 ranges between the values ± 1, the above quantity can be determined from 3052 observations with a weight of $3052\div 3$ approximately, and therefore $\delta_1-\frac{5}{4}\delta_2$ with a weight $(0{\cdot}4)^2\times 3052\div 3 = 162$, the more accurate weight being 176, as stated above.

I have grouped the forty-eight periods of analysis into six groups of eight, the group of eight being taken as a unit in order to get rid of inequalities with argument $\omega-\omega'$, whose period is 8×400 lunar days nearly. The results are

Periods.	$\mu+0{\cdot}44\delta_1-0{\cdot}68\delta_2$.	$\delta_1-\frac{5}{4}\delta_2$.
	"	"
86–93	−0·30	−0·24
94–101	−0·40	−0·34
102–109	+0·13	−0·48
110–117	+0·29	−0·20
118–125	+0·18	−0·10
126–133	−0·26	−0·20
Means	−0·06	−0·26

Considering the superior weight of $\mu+0{\cdot}44\delta_1-0{\cdot}68\delta_2$, the discordance of the separate results is striking, and indicates marked changes in semi-diameter. On the other hand, the accordance of the six separate values of $\delta_1-\frac{5}{4}\delta_2$ is remarkable, for I expected a probable accidental error in each value of $\pm 0''{\cdot}25.(\pm 1''{\cdot}7\div\sqrt{176}\div 6)$. This is perhaps evidence of the superior character of the night observations.

From the mean we have

$$\delta_1-\tfrac{5}{4}\delta_2 = -0''{\cdot}26$$

and to obtain δ_1 we must adopt a value for δ_2 from other sources —with this inconvenience, that any error in δ_2 introduces a larger error into δ_1.

The truth probably lies between the following two extremes :—

(1) The variation should have its theoretical value or $\delta_2 = +0''{\cdot}39$ leading to $\delta_1 = +0''{\cdot}23$.

(2) The variation should have its value as determined from all the observations, or $\delta_2 = -0''{\cdot}03$ and $\delta_1 = -0''{\cdot}30$, or, in other words, I assume that the variation apparent from the

observations between 8^h and 16^h will be intermediate between the true variation and the variation apparent from all the observations.

Consequently

$$\delta_1 = -0''.03 \pm 0''.27$$

the $\pm 0''.27$ indicating the probable *extreme* limits to the unknown systematic error.

The solar parallax corresponding to $\delta_1 = -0''.03$ is $8''.79$, as found last month.

I take this opportunity of stating that my principal object is to analyse the Moon's errors with a view to obtaining empirical coefficients to be compared with Professor Brown's theoretical ones. From time to time there will be digressions into the values of astronomical constants, accompanied, I hope, by a discussion of the accidental error and a notification of undetermined systematic errors.

Reduction of 295 Photographs of Eros made at Nine Observatories during the period 1900 November 7–15, with a determination of the Solar Parallax. By Arthur R. Hinks, M.A.

§ 1. *Introduction.*—In two previous papers (*Monthly Notices*, 1901 November and 1902 June, vol. lxii. pp. 22 and 551) there is an account of the experimental reduction of certain photographs of *Eros* made at Cambridge, Lick, and Minneapolis. Its object was to test a method of reduction in rectangular coordinates; and the conclusion was that the method was simple, convenient, and worthy of a more extended trial. I therefore ventured to propose that we should undertake at Cambridge the reduction of so much of the photographic material obtained during the period 1900 November 7–15 as might be placed at our disposal by the kindness of the directors of the different observatories taking part in the cooperation to observe the planet *Eros* at that opposition. At the time this proposal was made we had very little information as to the real accuracy of the photographic method when pushed to the limit, and especially little knowledge of what systematic discordances might be found in the work of different observatories. It was hoped that the discussion of the material of these nine days, considerable in itself, but only a small part of the whole, might lead to a preliminary value for the solar parallax of weight equal to that of the best existing values, and at the same time be some guide in the operations which must eventually be undertaken to combine the whole of the observations in one definitive solution.

§ 2. The first step was to choose the stars which were to form the standard of comparison stars.

For this system the *étoiles de repère* selected by M. Loewy for special meridian observation are not suitable. They are chosen for plates 2° square, and in great part lie outside the limits of the smaller fields of many photographic telescopes; and they are on the average considerably brighter than the planet. A set of stars was chosen, including a few of these *repère* stars which lay near the track of the planet, and a number of others which were comparable in brightness with *Eros*, and evenly distributed, so that one might count on getting at least ten suitably distributed stars in any field with a radius of 15′. Photographic copies of a chart of these stars were sent to all the observatories who had photographs within the selected dates, with the request that in measuring their plates they should select from the list about ten stars, with the planet near the centre of gravity of the group, and should send to Cambridge a copy of the measured rectangular coordinates of stars and planet, uncorrected for anything except errors of the measuring machine and réseau.

§ 3. I must make grateful acknowledgment of the kindness with which this request was received. The directors of eight observatories have done me the honour of placing in my hands measures made under their direction, fulfilling in somewhat various ways the proposed conditions.

It will be generally convenient in what follows if each separate set of measures as communicated to me is called a plate, though in most cases a number of exposures were actually made on the same plate. The contributions were as follows:—

Algiers ...	measures of	40 plates
Lick... ...	... ...	28
Minneapolis	... ...	17
Northfield	... ...	23
Oxford (University Obs.)	...	32
Paris ...	... ...	21 (means of 3)
San Fernando	... ...	9
Tacubaya ...	... ...	15

These, added to

Cambridge ...	...	110

make up the total of 295 discussed in the present paper.

Some more Oxford plates arrived unfortunately just too late to be included.

Each Paris plate is the mean of three separate exposures. On it are measured, in accordance with the Paris programme, all the *étoiles de repère*, and all the stars lying in a square of 20′ with the planet at its centre. A number of my standard comparison stars are found in this square, and some others are

among the fainter *repère* stars. The average number of standard comparison stars on the Paris plates was 14·5.

The Algiers plates are measured on the same plan, but the exposures are single. The number of standard comparison stars measured on the Algiers plates was on the average 12.

On the San Fernando plates all the *repère* stars and all the stars in my list were measured. On the Tacubaya plates a large number of the stars in my list were measured. On the Lick, Minneapolis, Northfield, Oxford, and Cambridge plates groups of about ten stars were chosen from this list and measured.

The Algiers, Paris, and San Fernando plates are impressed with a réseau, but measured in millimetres; the Oxford, Cambridge, and Tacubaya plates are measured in réseau intervals (5 mm.); the Lick, Minneapolis, and Northfield plates are measured in millimetres without a réseau. The Tacubaya plates were measured by estimation on an eyepiece scale, and the results will be comparatively rough.

§ 4. *Standard Centre.*—A feature of the method to be used is the transformation of every plate to the standard centre and standard axes of rectangular coordinates. The standard centre used in the experimental reductions was in R.A. $1^h\ 57^m\ 8^s{\cdot}0$, Decl. $+54°\ 22'\ 0''$ (1900·0), and it has been found convenient to retain this throughout. The standard axes in the tangent plane to the sphere at this point are respectively at right angles to and along the projection of the meridian through the standard centre.

§ 5. *Construction of the Standard System of Comparison Stars.*—In the Paris circulars Nos. 8 and 9 the results of the meridian observations of the *repère* stars made at a number of observatories are given in the form of a table of mean places found by each observatory. As the number of observations at different observatories varied very much it seemed well to weight the results by the square root of the number of observations on which each depends.

It should be noticed that the system of weights adopted is by no means unimportant. I have remarked in the *Observatory* (vol. xxvi. p. 342) that the final places of the *repère* stars adopted at Bordeaux, Paris, and Cambridge differ systematically from one another, and that in consequence *absolute* places determined by these three observatories would not be homogeneous, and could not properly be used as they stand in a determination of the solar parallax. As, however, I have ventured to contend that it is a mistake to attempt to deduce absolute places of the planet for use in the parallax equations, and shall throughout this paper consider the deduced places of the planet as relative only to my adopted system of stars, it is not necessary to insist further on this point.

Rectangular coordinates for the standard centre and axes were calculated from the adopted places of the *repère* stars for 1900·0, and these will, as usual, be called standard coordinates.

The places of the standard comparison stars were obtained from Paris and Cambridge plates. The Paris plates contained a number of the standard stars in the 20′ square; and a set of Cambridge plates was specially measured with all the available *repère* stars. These measures, referred to many different centres, were all transformed approximately to the standard centre and axes by the method given in § 8.

The six constants of the ordinary linear reduction were determined from the *repère* stars on each selected Paris and Cambridge plate, and the comparison stars were thus reduced to standard. On collecting the results it appeared that the places deduced from the two series of plates agreed fairly well with one another. The Cambridge x coordinates averaged $0''\cdot1$ greater than the corresponding Paris places, which is probably an effect of the nature of magnitude equation due to the fact that the Paris plates were reduced with many more bright *repère* stars than the Cambridge plates; it has been shown that there is a sensible magnitude equation in the meridian places (F. Cohn, *A. N.* 3952). But when the differences Cambridge minus Paris were classified with respect to magnitude there was little if any trace of relative magnitude equation between the two. It is possible, therefore, that the zero of the adopted places of the standard stars, the simple mean of all the individual results, may be affected by magnitude equation, but the internal smoothness of the system is unaffected by it. The number of places included in each mean averaged 6·5. It was estimated that the probable divergence of any adopted place from a smooth system (not necessarily from its absolute place) would be less than $0''\cdot1$; and it will be shown later that the internal errors of the system are actually a good deal smaller than was estimated (see § 10).

The resulting catalogue of standard places of forty-eight comparison stars has served for the complete reduction to standard of all the plates (with the exception of one series which will be mentioned later).

§ 6. *Choice of Scale Unit and Reduction to Scale.*—The greater part of the measures sent to me were expressed in millimetres on the plate, the remainder in terms of the 5 mm. réseau. As the focal lengths of the telescopes employed varied from 9 to 20 feet it was necessary to adopt some standard scale to which everything should be reduced. As proposed in the second paper, I have converted all the measures into a scale whose unit is the one-thousandth part of the focal length of the telescopes employed respectively in making the photographs. One unit in the fourth place of decimals is then very nearly $0''\cdot02$; the probable error of a single measure on the best plates is about three times this quantity, so that my unit is just about the right size to give a suitable degree of accuracy in the computations, without any superfluous figures, if the reductions of the stars are carried out to four places of decimals, and the reductions of the measures of the planet only to five.

The first step in the treatment of all the measures was to reduce them approximately to this standard scale of one-thousandth the radius of projection, by multiplication on the arithmometer.

§ 7. *Second Order Terms in the Differential Refraction.*—The next step was to apply to the measures such part of the correction for differential refraction as involves the squares of the coordinates on the plate. These corrections rarely amount to more than a few units in the fourth place; but since they consist of the sum of six small terms they are troublesome to compute. To save this labour I have devised a graphical method of determining them, which is described in *Monthly Notices*, lxiii. 138, 1903 Jan. This method has been used throughout, and has been found very convenient.

§ 8. *Transformation to the Standard Centre.*—The next step is to transform the measured coordinates to the standard centre.

Let the R.A. and Decl. of the original centre of the plate be A_1, D_1; and of the standard centre be A_2, D_2; and let $A_2-A_1=a$.

Then if x_1, y_1 be the measured coordinates of a star, and x_2, y_2 those coordinates transformed to the standard centre, it is easy to show that

$$x_2 = \{x_1 . \cos a + y_1 . \sin D_1 \sin a - \cos D_1 \sin a\} / N$$

$$y_2 = \{-x_1 . \sin D_2 \sin a + y_1 (\cos D_1 \cos D_2 + \sin D_1 \sin D_2 \cos a) + \sin D_1 \cos D_2 - \cos D_1 \sin D_2 \cos a\} / N$$

where the denominator N is equal to

$$x_1 . \cos D_2 \sin a + y_1 (\cos D_1 \sin D_2 - \cos D_2 \sin D_1 \cos a) + \sin D_1 \sin D_2 + \cos D_1 \cos D_2 \cos a.$$

A rigorous transformation by these formulæ is exceedingly cumbersome; it is, however, unnecessary. The transformation differs from a linear transformation owing to the presence of the denominator, and consequent introduction of terms of the second order in x_1 and y_1. But when the distance between centres amounts to only a few degrees these terms are small; and when the distance is only a few minutes they are negligible.

Consider the following equations:

$$Mx_2 = (x_1 + By_1 + C)(1 - Kx_1 - Ly_1)$$
$$My_2 = (-Bx_1 + y_1 + D)(1 - Kx_1 - Ly_1)$$

where $$B = \sin D_2 \sin a.$$

C and D are approximate mean values of x_2-x_1 and y_2-y_1, obtained by inspection of the measured and standard coordinates.

$$K = \cos D_2 \sin a / M.$$
$$L = (\cos D_1 \sin D_2 - \sin D_1 \cos D_2 \cos a)/M$$

and $$M = \sin D_1 \sin D_2 + \cos D_1 \cos D_2 \cos a.$$

For a range of centering such as I have used M never differs from unity by more than 0·0005. We can neglect it in the calculation of K and L, and need not trouble to multiply x_2 and y_2 by it, since the small change of scale value introduced will be eliminated in the further reductions. The numerators differ slightly from the rigorous expressions, but are sufficiently close to it to give the terms of the second order. If, therefore, we transform by these approximate relations, omitting M, instead of by the exact expressions given before, we introduce the second order terms accurately, but leave outstanding some not very large terms of the first order, which are included in the linear reduction which follows.

When our unit is the one-thousandth of the radius of projection, and $D_2 = 54° 22'$, we have

$$B = a \times 0{\cdot}000\,236$$

$$K = a \times 0{\cdot}000\,000\,169$$

$$L = (D_2 - D_1)' \times 0{\cdot}000\,000\,291 + a^2 \times 0{\cdot}000\,000\,000\,020,$$

where a is $(A_2 - A_1)$ expressed in minutes of arc, and K, L have been adjusted to the unit in which the x's and y's are expressed.

This method of approximate transformation of centres has been used throughout, and has proved very simple and convenient. When the plates are in pairs—that is, two exposures close together on one actual plate—these computations are easily checked by differencing.

§ 9. *Reduction to Standard.*—The six constants of the linear formulæ for reduction to standard were determined for each plate in the usual way by the method proposed by Dyson. When there are many plates with the same group of comparison stars it saves work to invert the process and reduce standard to observed; only the numerical terms are then different from plate to plate. Afterwards the constants thus found must be reversed, and when their values are large the reversal involves a little more than mere change of sign. It is easily shown that if a, b, c, d, e, f are the constants for reduction of standard to observed, and A, B, C, D, E, F the constants for reduction of observed to standard, then

$$A = -a + a^2 + bd \qquad D = -d + ad + de$$

$$B = -b + ab + be \qquad E = -e + bd + e^2$$

$$C = -c + ac + bf \qquad F = -f + cd + ef$$

These expressions were used in a paper by the author in the *Astronomical Journal*, No. 475 (vol. xx. p. 151); but it should be noted that a misprint occurs there of c for e in two places.

All the comparison stars as well as the planet have been reduced to standard, and the residuals formed from the adopted standard places. The condition that the sum of the residuals in each coordinate should be zero is a valuable final check on a reduction of which the greater part has already automatically checked itself.

§ 10. *Accuracy of the Adopted Standard System of Comparison Stars.*—The residuals in the reduction of each star, in the sense standard minus observed, may be considered as apparent corrections to the adopted standard places. They have been collected from the Cambridge, Lick, and Paris reduction sheets, and an apparent mean correction formed for each star that had been used more than six times. The mean of all, without regard to sign, was

$0''{\cdot}04$ in x $\qquad$ $0''{\cdot}05$ in y

Of course these individual apparent corrections are really corrections relative to the mean of the group of about ten stars used in the reduction of the plate, so that they will average a little less than the corrections relative to the mean of the whole system. But the groups are thoroughly interwoven and overlapped, and the uniform character of the residuals is a sufficient guarantee against any sudden discontinuities in the system, which alone could do serious harm.

Moreover the residuals used in the above discussion are not altogether free from the effects of certain systematic errors which will be discussed later, and whose elimination would have some tendency to reduce discordances. I think that we may conclude that the P.E. of an adopted standard place is at least as small as $\pm 0''{\cdot}04$, and that the effect of errors in the standard places on the resulting places of the planet will be very small.

§ 11. *Examination of the Series of Photographs for Systematic Error.*—If it had been possible for each observatory to secure observations of *Eros* symmetrically disposed with regard to the meridian there would have been comparatively little reason to fear that systematic errors running through a series of photographs would prejudicially affect the value of the solar parallax deduced from them. But it was actually the case that bad weather made large and unsymmetrical gaps in the work of all observatories. Moreover some observatories have a large preponderance of evening observations in the days about opposition, and all have of necessity a great excess of post-meridian observations throughout the period some weeks after opposition, when the planet was nearest and its parallax greatest. This inevitable dissymmetry gives every opportunity to systematic errors to exert a baneful effect, and it seems to me not possible to accept as axiomatic the statement, which has been made more than once, that they may be trusted to eliminate one another completely in the combination of a large series of observations. On the contrary, two of the series of photographs which we shall

have to discuss contain systematic errors so large that they would ruin any determination of the solar parallax into which they were introduced. The errors can, however, be detected by an examination of the residuals in the comparison star reductions. Our next step will be to show that a critical examination of the residuals in each series of plates must be made an essential part of any discussion of the solar parallax from photographs.

It will be convenient to summarise the kinds of error to which attention has at various times been directed.

Errors of Measurement.—All experience goes to show that these are relatively unimportant. The Cambridge machine has no sensible errors of scale or screw, and it will be shown later that the réseau is nearly perfect. The evidence for freedom from error of other machines whose results enter here is not complete, but it may be assumed that there are no large errors. There seems good reason to believe that personality in measurement is very nearly eliminated by reversal of the plate, which has been done in all series except one.

Real Errors of the Image.—A large part of the error in a measured coordinate is due to real error in the position of the image. Evidence is accumulating that a set of images close together may be affected by a quite large common error, which is probably local distortion of the gelatine film within the réseau square. When no réseau is used this error may of course be much greater.

Constant optical distortion of the field may be expected in reflector photographs, but not otherwise. There seems good reason to suppose *a priori* that it should be symmetrical about the centre of the field, but it will evidently depend very much upon the brightness of the stars.

Hour-angle Error.—Under this name Kapteyn conveniently includes the effect of atmospheric dispersion, and also optical distortions and tilt of plate, varying with the hour angle, due to flexure of objectives, mirrors, and mountings.

Magnitude Equation and Guiding Error.—If the driving is not perfect, but there is a tendency to accumulate error in one direction until it is corrected by hand or automatic control, the brighter stars may be displaced relatively to the faint, and show a kind of magnitude equation varying from plate to plate, but on any one plate depending on the magnitude of the star alone. This is probably one of the most important and subtle errors of the photographic method. It is distinguished from other kinds of magnitude equation, due to an imperfectly symmetrical diffraction pattern of the image, or to personality of measurement, by the fact that its appearance will be capricious.

We shall have occasion to deal with most of these sources of error in the discussion that follows.

A. *The Lick Plates.*—The Lick plates were taken with the Crossley reflector of 36-inch aperture and 17·6 feet focal length, with short exposures averaging 15ˢ. They were measured on

the Repsold machine at Columbia University Observatory, N.Y. Upon reduction to standard the comparison stars on many of the plates had unduly large residuals, amounting in some cases to a second of arc. There was plenty of evidence from other sources that the standard places were of a high order of accuracy, and it became clear that the error was to be found in the plates or the measures. It was chiefly in the x coordinates, appearing in this way. The x residuals, in the sense adopted standard minus reduced measure, were positive for stars near the middle region in x, and negative for stars towards both sides, the sum of the residuals being necessarily zero. This arrangement of signs indicated an apparent bodily displacement of the central images relative to the outer, and there was consequently every reason to fear that the positions of the planet deduced from the plates might be systematically wrong. The effect was not constant; on a few plates it seemed to be absent, and in one or two cases changed sign. It had no apparent relation to the hour angle; was almost exactly the same for a pair of exposures on the same plate, but varied capriciously from plate to plate.

In general character it resembled the effect of a tilt of the plate relative to the optical axis, such as might be caused by an error in squaring on the plate, or by defective collimation of the mirrors. But it needed only a small computation of the amount of tilt required to explain the magnitude of the error, to show that this explanation was grotesque: a tilt of several degrees is entirely out of the question.

The asymmetry of the error with respect to the centre seemed to make it improbable that it was due to optical distortion.

In order to elucidate this perplexing question the Director of the Columbia University Observatory very kindly had measures made of a large number of stars on certain overlapping plates, and the reduction of these confirmed the reality of the error without throwing any light on its cause. I then asked Professor Campbell to be so good as to let me examine some of the plates at Cambridge, and he very kindly sent me three of them. Inspection showed that there was no possibility of a large tilt of the plate; the optical centre was strongly marked by the perfection of the images around it and their regular degradation as they departed from it. It appeared to me, however, that many stars had been measured which I should myself have judged unfit for measurement on account of the familiar umbrella-shaped character of the images, so I decided to try rejecting all the measures of stars more than 20 mm. or 13′ from the centre—that is to say, all stars whose discs were not sensibly round. The effect of doing so was remarkable. When a new solution was made with the stars that were left the residuals were quite small, and the resulting place of the planet was very much altered; and when the outer rejected stars were reduced to standard with the constants derived from the inner stars they gave residuals up to 1″·5.

It is clear that the error was due to the use of stars too far from the centre, and that it is unsafe to measure the Crossley plates where the star discs are not sensibly round. But it remains a mystery why the error should take the shape it does. The images are distorted quite symmetrically with respect to the centre, and there seems no reason why the observer should make an error of measurement always more or less in the same direction on images oppositely distorted. I have tried to go further into this matter, and to discover something more definite about the law of the error, but have failed. This is, perhaps, not surprising when one considers that the shape of the distorted image depends very much on the magnitude of the star as well as on its position. It is unlikely that one will ever be able in such a complicated case to devise a system of corrections applicable to measures of distorted discs, and the safe course will be to confine measurement of these plates to a field of about 12′ radius, within which the images are small and beautifully defined.

The places of *Eros* to be used in the present reductions depend entirely on these central stars.

B. *Algiers Plates.*—A very puzzling error exists in the Algiers plates, which are taken with the standard Astrographic Refractor. I had hoped to be able to use the Algiers measures in forming the system of standard stars, for many of those stars were to be found in the 20′ square about the planet. But when they were reduced to standard with the aid of the *repère* stars in the usual way they were systematically discordant from the results of the Cambridge and Paris plates, and could not be used.

At first sight the error looked like a magnitude equation making the x's of faint stars too large; but I am now convinced that magnitude equation is not the chief error. It would be hard to go into details without taking up too much space; the question is much complicated by the fact that the stars measured near the centre of the plate are mostly faint, and the stars further out most of them brighter; they give respectively negative and positive residuals. To examine this question properly it would be necessary to measure both bright and faint stars distributed uniformly over the plate. But I have found as a rule that a bright star among faint ones near the centre gives a negative residual very much like theirs, while a faint *repère* star far out gives a positive residual as large as that for neighbouring brighter stars. Further, if very few stars in the centre of the field have been measured at all, the residuals immediately become small, and there is no marked magnitude equation between bright and faint outer stars. The conclusion seems to be that the central stars are displaced upon the plate, not because they are faint, but because they are central; and it was entirely consistent with this to find that Algiers places of the planet differed systematically from the Paris and Cambridge places by quantities of the order of half a second of arc.

This is very remarkable. One is accustomed to expect certain difficulties with a reflector; but it is disquieting to find that a standard astrographic refractor, when devoted to solar parallax work after long use on the astrographic chart and catalogue, can give results which are wrong by amounts quite serious when judged from the standard of the catalogue, and hopeless as they stand for parallax work. It is not easy to imagine what may be the cause. We cannot suppose that the plates were tilted enough to explain it, and we have not material enough to find a general expression of the second degree to represent it.

It is hard to decide what to do in a case like this. If the error is like the error in the Lick plates, and develops suddenly as one gets away from the centre, it is probably safe to rely on the central stars to give a good reduction. This appears to be the case. Having decided to reject altogether the stars outside the 20′ square, I chose some additional standard stars, and got their places as before, measured them on the Algiers plates, and so had always at least six stars for the reduction, except in a few cases where scarcely any central stars appeared on the plates; these were rejected.

The new places of the planet differed from the old by amounts up to 0″·65 and averaging 0″·48. They seemed to fit the ephemeris very well. But it must be admitted that this is our only ground for assuming that the error for whose existence we can assign no reasonable cause has been altogether eliminated.

C. *Cambridge Plates. Search for Réseau Errors.*—On the Cambridge plates the planet is not always referred to precisely the same réseau line, whose error, if any, might appear to give a uniform correction to the place; nor is it referred to so many different lines that accidental errors might be expected to go out in the long run. The réseau must, therefore, be examined. Several published investigations of Gautier réseaux show that the errors of the réseaux themselves are very small, of the order of the division errors in the best divided scales. But it has recently been shown by Ludendorf (*A. N.* 3746) and Bohlin (*Astr. Jakktagelser*, Stockholm, Bd. 6, No. 5) that a réseau perfect in itself may produce imperfect copies through what is known as projection error. It seems that something—probably the fine cut in the glass made by the diamond point that cuts away the silver—may deviate the light just as it passes through the réseau, and produce errors in the copy much larger than in the original. If we determine the errors of the original we must also determine the projection errors, and both are difficult and troublesome matters. We may, on the other hand, achieve our end, which is to find the errors of the photographic copy (apart from gelatine distortion), by examining plates impressed with a réseau to a number sufficiently large to eliminate the gelatine distortion, provided that this may be treated as accidental, and vanishing in the mean of a large number of plates. The supposition is

most likely correct for the central portions, and incorrect f
edges of the plate ; it will serve our present purpose.

I am indebted to Mr. Russell for the use of a strip of
graphed réseau, of which he has determined the errors, s
it can be used as a standard scale. This was superpo
several plates, parallel to the X axis, so as to almost co
with the réseau lines at the points from which the x coord
of the planet are measured. The distances between eight
réseau lines on these plates and adjacent lines of the sta
scale were measured, the corrections for errors of the scal
applied, and the resulting apparent errors of the photogr
réseau in sixteen strips altogether were plotted. The r
are briefly as follows :

The apparent errors of successive lines deviate from a s
and nearly straight line by quantities averaging about 0·001
equivalent to $0''{\cdot}035$ on my plates ; a considerable part o
deviation must be accidental error of measurement. The
of two points of a line not more than 1 mm. apart are freq
of opposite sign—that is, the line images are a little rough.
means are taken the deviations of the apparent mean
from uniformity are almost within the limits of probable
dental error. The conclusion is that there is practical
evidence of sensible error in the photographed copies o
réseau, apart from the general smooth distortions of the ge
film which the use of the réseau eliminates. In one case
did an error amount to 0·005 mm., and at that point the r
line was obviously defective ; the defect had been noted
the plate was measured, and special precautions taken to av

Search for Hour Angle Error.—It is clear that errors o
category can produce no resulting error in the place o
planet unless they are differential, varying from star to st
the plate. If they are due to dispersion they will bear a de
relation to the projected position of the zenith on the plate
they are due to variable distortion they will probably at
change continuously with the hour angle. In looking for
I plotted the apparent correction to the adopted place of
comparison star deduced from thirteen groups of four expo
each ; and alongside them the relative positions of the proj
zenith. There was no relation to the zenith ; nor was
continuity of any kind in the group displacements. I con
that there is no sign of dispersion or of any differential disto
in the stars on the Cambridge plates. That does not prove
there is no dispersion effect on the planet, but it renders it
unlikely, since there is some experimental evidence to show
the effective wave-length of the planet's light is in no
abnormal (Prosper Henry, *Paris Circular*, No. 8, p. 41).

Guiding Error.—In searching for guiding error one natu
looks for large residuals for the few bright stars, balance
smaller residuals of opposite sign for the more numerous
stars. It must, however, be borne in mind that an uneven

tribution of the bright stars may to a large extent mask this effect. If we have two or three bright stars at one edge of the group, and no accompanying faint stars, the linear reduction may strain the fit to suit these bright stars at the edge, and leave them with small residuals. The existence of large residuals for certain bright stars, especially if near the centre, is therefore an indication of guiding error which is not necessarily contradicted by the fact that certain other bright stars have not large residuals of the same sign.

On examination of the residuals for the Cambridge plates I was struck with the fact that the average residual of two bright central stars was large on twelve consecutive plates (1900 November 10), while the residuals for two bright stars at the edge of the group were small. This looked like a case of guiding error, and I was fortunately able to make a more than usually conclusive test. My earlier experimental reduction of this plate depended on faint stars. A comparison of the numerical terms in the equations of condition of this first reduction with the corresponding terms in the present solution ought to show, perhaps, a steadily increasing difference, but no irregularities. This was so, except for the twelve exposures under suspicion, which showed marked divergences of about $0''{\cdot}2$, and strengthened the idea that the bright stars were in error and spoiling the reduction. I therefore rejected these four stars and used four fainter, making a new solution which gave decidedly different values of the constants, and altered the place of the planet by about $0''{\cdot}1$ on an average.

No other plates show signs of guiding error comparable with this found on the bad set of twelve consecutive plates, which may perhaps be attributed to some temporary derangement of the automatic control. But the possibility of so large an error affecting a series of exposures with large parallax factors of the same sign is one that must be reckoned with, and seems to me a most powerful argument for the necessity of examining the residuals of all the comparison stars on every plate.

D. *San Fernando Plates.*—The result of the reduction of the San Fernando plates is disappointing. Admiral Viniegra very kindly had measured for me the places of all my selected comparison stars, in order to give additional material for the formation of my standard system; but when the reductions were made the results differed so largely and irregularly from the accordant results of Paris and Cambridge that it seemed unwise to use them in the formation of the standard system. On inquiry I learned that the plates had been measured in only one orientation. This fact compels me, very regretfully, to exclude the San Fernando results from my solution.

E. *Northfield Plates.*—A good many of the Northfield plates were taken on a plan which, one may venture to say, is certainly unsatisfactory. The guiding star was placed successively on the four corners of a square reticle, and then once more on the first

corner. The motion of *Eros* in the interval was sufficient to clear the images of the planet, but the stars in the first corner exposures are superposed and measured as one set. Apart from the fact that the fit of the two sets of star images cannot be perfect, owing to changes in the differential refraction, orientation due to maladjustment of polar axis, &c., there are grave reasons for fearing guiding error when two exposures are superposed with an accuracy limited by the accuracy of pointing with a not very powerful guiding telescope. I think that there can be no doubt that the comparative roughness of the Northfield results is partly due to this procedure. An examination of the comparison star residuals does not, however, show any signs of systematic error on the plates.

F. *Minneapolis Plates.*—An examination of the residuals in the reduction of the comparison stars to standard showed an error which is probably due to some kind of optical distortion. The measurers' notes contain references to elongated and winged images, and it is clear from the Lick results that these must be treated with caution. I should have tried to treat these plates in the same way that the Lick plates were treated had it not been for the fact that on many Minneapolis plates the images of the planet are far from the centre of the plate, and the group of comparison stars is in consequence very unsymmetrically placed upon it. As an experiment I have made new solutions after rejecting the outer stars of the originally selected groups, and have obtained new places of the planet differing systematically from the old. But, owing to the want of symmetry, there does not seem to be any guarantee that these new places are clear of error ; and the danger of using them is aggravated by the fact that the parallax factors for the Minneapolis plates are nearly all of one sign. On taking all these things into account I was very reluctantly compelled to decide that it would be safer to leave out the Minneapolis results from my solution.

The Minneapolis plates also suffer from the fact that a good many of the exposures on the stars are multiple, made at intervals great enough to allow the images of the moving planet to clear one another. The roughness of the accordance between the resulting places of the planet and the ephemeris confirms the impression derived from the Northfield plates that this is a method of procedure which should not be employed again.

G. *Oxford Plates.*—On one of the Oxford plates there is a clear case of guiding error running almost uniformly through half a dozen exposures. It has been treated in the same way as the like Cambridge case, by rejecting the bright stars and solving afresh.

H. *Paris Plates.*—No case of abnormality or suspicion of systematic error has arisen in the examination of the Paris star residuals.

§ 12. *Adopted Weights of the Different Series.*—We have now to adopt a system of weights for the different series of plates,

taking into account both the tendency towards systematic error revealed in some of them and also the general character of the plates as indicated by the average size of the comparison star residuals upon them. The average residuals are :—

	In X.	In Y.	Adopted Weight.
	"	"	
Algiers	0·16	0·12	¾
Cambridge	0·11	0·10	1
Lick	0·10	0·12	1
Minneapolis	0·21	0·20	0
Northfield	0·18	0·17	½
Oxford	0·17	0·17	¾
Paris	0·10	0·11	1
Tacubaya	0·28	0·19	⅛

The eleven Cambridge plates on which guiding error was found have been given weight ⅔.

Part of the average residual is due to errors in the adopted places of the stars, whose probable error is less than 0″·04. And it will be shown later that the connection between the average star residual on the plate and the final residual in the equations of condition is not so close as might have been expected. After consideration of all the circumstances I have adopted the weights given in the last column of the above table.

§ 13. *Summary of Observations used.*—

Algiers.

40 plates were measured and communicated.
1 was rejected for large discordance, evident in original measures.
8 were rejected for want of central stars.
31 were used.

Cambridge.

110 plates were measured.
5 were rejected for various reasons—wrong time record, clouds during part of exposure, or large discordances for images marked "probably defective" during measurement.
1 was omitted by mistake.
104 were used.

Lick.

28 were measured, and all were used.

Northfield.

23 were measured.
2 were rejected for clouds during exposure.
21 were used.

There seems to be no doubt that this ephemeris is free from accidental roughness to a nicety well within what our observations will demand; and I must repeat the expression of my acknowledgments to M. Lœwy for his kindness in providing the separate heliocentric ephemerides with which it was constructed.

It may be noted here that there was an error in the computation of the plate ephemeris used in my second paper. The terms expressing the small heliocentric latitude of the Earth were taken with the wrong sign, which accounts for a peculiarity of the solution given in *Monthly Notices*, lxii. p. 559.

§ 15. *Interpolation from the Plate Ephemeris and Calculation of Parallax Factors.*—It is a decided advantage of the plate ephemeris that the third differences multiplied by the interpolation coefficients are small and the fourth negligible. It is also most convenient to have the ephemeris expressed in terms of a single quantity, the adopted unit, instead of in hours, minutes, and seconds of time, and degrees, minutes, and seconds of arc; the labour of computation is thereby much reduced.

The geocentric coordinates of the observatories at the epochs of observation were calculated according to the formulæ given in *Monthly Notices*, lxii. p. 30. (But see below.) But it should have been remarked there that it is proper to use the *apparent* R.A. and Dec. of the standard centre, viz. $1^h\ 57^m\ 15^s$, $+54^\circ\ 22'{\cdot}5$, in calculating those coordinates.

In computing the Lick coordinates the altitude of the observatory was taken into account.

The parallactic displacements of the planet were calculated with an assumed value of the solar parallax $\pi = 8''{\cdot}800$.

[It may be noted here that there is an error in my first experimental paper (*Monthly Notices*, lxii. pp. 30 and 31). The equations at the bottom of p. 30 should be printed

$$\xi = \frac{X}{Z} = \frac{I}{N}\left\{L - \left(a - \frac{c\,L}{N}\right)\pi\right\} = \nu\left\{L - (a - c\xi_0)\pi\right\}$$

$$\eta = \frac{Y}{Z} = \frac{I}{N}\left\{M - \left(b - \frac{c\,M}{N}\right)\pi\right\} = \nu\left\{M - (b - c\eta_0)\pi\right\}$$

and throughout the following page the quantities printed $\nu(a - c\nu)$ and $\nu(b - c\nu)$ should read $\nu(a - c\xi_0)$ and $\nu(b - c\eta_0)$. The mistake arose in preparing the paper for press; the right formulæ were used in the calculations.]

§ 16. *Formation of the Equations of Condition.*—On comparing the observed places of the planet with the ephemeris it was evident that the correction to the x ephemeris was nearly constant throughout, and that the correction to the y ephemeris varied slightly with the time. There were no indications of terms depending on squares and higher powers of the time. Of course it will be necessary to make a careful search for such terms; but what would be, from the parallax point of view, a good deal more important would be the existence of a short

period variation in the place of the planet related to the light variation. This could be detected most conveniently in the residuals from a preliminary solution. I therefore adopted as my first equations of condition a simple form with only three unknowns in each, $\Delta_1\xi_0$ or $\Delta_1\eta_0$, constant corrections to the plate ephemeris, $\Delta_2\xi_0$ or $\Delta_2\eta_0$ the variation of these corrections per day, and $\Delta\pi$ the correction to the assumed parallax.

The equations expressing the comparison between observation and ephemeris have the form

$$\xi_0+\Delta_1\xi_0+t\,.\,\Delta_2\xi_0\ -(\pi+\Delta\pi)\,(a-c\xi_0)/\mathrm{N}=x$$
$$\eta_0+\Delta_1\eta_0+t\,.\,\Delta_2\eta_0\ -(\pi+\Delta\pi)\,(b-c\eta_0)/\mathrm{N}=y.$$

Hence, putting

$$\xi_0-\pi(a-c\xi_0)/\mathrm{N}-x=m'$$
$$\eta_0-\pi(b-c\eta_0)/\mathrm{N}-\eta=n',$$

we have as the form of our equations of condition

$$\Delta_1\xi_0+t\,.\,\Delta_2\xi_0-(a-c\xi_0)/\mathrm{N}\,.\,\Delta\pi+m'=0$$
$$\Delta_1\eta_0+t\,.\,\Delta_2\eta_0=(b-c\eta_0)/\mathrm{N}\,.\,\Delta\pi+n'=0.$$

Working in units of the fifth place of decimals (that is, in hundred-millionths of the radius of projection), we have on the average

$$m'=+1090$$
$$n'=-\ 120,$$

and we shall take as the numerical terms in our equations of condition

$$m=m'-1090$$
$$n=n'+\ 120.$$

§ 17. *Normal Equations.*—The normal equations derived from the measures of each observatory are as follows: the coefficients of $\Delta_1\eta_0$ and $\Delta_2\eta_0$ are the same as those of $\Delta_1\xi_0$ and $\Delta_2\xi_0$, and are not repeated:—

Algiers (with weight $\frac{2}{3}$).

$20{\cdot}67\Delta_1\xi_0$	$+\ 48{\cdot}00\Delta_2\xi_0$	$+\ 1{\cdot}05\Delta\pi$	$-\ 23=0$	$-\ 1{\cdot}80\Delta\pi$	$+\ 191=0$
	$+\ 144{\cdot}9460$	$+\ 1{\cdot}1289$	$+\ 24{\cdot}00$	$-\ 1{\cdot}9751$	$+1541{\cdot}67$
		$+\ 64{\cdot}8469$	$+\ 502{\cdot}17$	$+\ 7{\cdot}2655$	$-\ 138{\cdot}14$

Cambridge (209–252).

39	$-\ 86{\cdot}62$	$-\ 14{\cdot}77$	$+\ 143$	$-22{\cdot}73$	-3554
	$+\ 203{\cdot}9652$	$+\ 34{\cdot}6727$	$-\ 648{\cdot}74$	$+52{\cdot}9410$	$+8341{\cdot}17$
		$+\ 69{\cdot}5633$	$-\ 265{\cdot}00$	$+19{\cdot}5289$	$+2189{\cdot}68$

Cambridge (254–265, *with weight* $\frac{3}{4}$).

7·33	− 8·68	+ 10·13	− 86	+ 4·09	− 162
	+ 10·2813	− 12·0257	+ 106·15	+ 4·8973	+ 212·92
		+ 14·1997	− 134·35	+ 2·6597	+ 135·04

Cambridge (266–335).

54	+ 64·36	− 15·03	− 1294	− 21·64	+ 896
	+ 320·3484	+ 52·4368	− 2041·52	− 27·6246	+ 7266·53
		+ 78·3565	− 482·45	+ 16·6724	− 347·92

Lick.

28	+ 8·60	+ 25·63	+ 614	+ 5·40	− 553
	+ 91·7948	+ 5·7855	+ 676·45	+ 6·1198	+ 3376·39
		+ 63·2493	+ 1047·91	+ 7·5570	+ 15·22

Northfield (*with weight* $\frac{1}{4}$).

5·25	+ 4·69	+ 5·09	− 180	− 0·47	+ 114
	+ 53·8704	+ 7·4601	− 283·77	− 4·7879	+ 1550·90
		+ 9·6121	− 238·79	+ 1·1716	− 26·30

Oxford (*with weight* $\frac{3}{4}$).

20	+ 1·19	− 4·25	− 691	− 20·14	− 377
	+ 111·9618	+ 41·7571	− 353·11	+ 11·2237	+ 2924·95
		+ 46·1206	− 375·12	+ 24·8782	+ 805·81

Paris.

21	− 25·19	+ 7·28	+ 299	− 3·73	− 561
	+ 202·3461	− 46·8666	+ 666·11	+ 0·1252	+ 3416·05
		+ 24·7698	+ 84·97	+ 7·1409	+ 166·38

Tacubaya (*with weight* $\frac{1}{10}$).

1·40	+ 2·44	+ 0·35	+ 11	+ 1·35	+ 107
	+ 13·7509	− 2·2404	+ 541·80	+ 1·8865	+ 674·97
		+ 3·7134	− 94·33	+ 1·8564	+ 134·49

The normal equations for the combined solution are, therefore :—

+ 196·65$\Delta_1\xi_0$	+ 8·79$\Delta_2\xi_0$	+ 15·48$\Delta\pi$	− 1207·00 = 0	− 67·75$\Delta\pi$	− 3899 = 0
	+ 1153·26	+ 82·11	− 1312·63	+ 42·81	− 29305·55
		+ 374·43	+ 45·01	+ 88·73	− 2934·26

And the solution is : From the x equations :—

	Weight.
$\Delta_1\xi_0 = +6''\cdot136$	196
$\Delta_2\xi_0 = +1''\cdot136$	1135
$\Delta\pi = -0''\cdot623$	368

From the y equations :—

$$\Delta_1\eta_0 = +18{\cdot}688 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 143$$
$$\Delta_2\eta_0 = -25{\cdot}309 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 1121$$
$$\Delta\pi \;\; = -\;6{\cdot}589 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 64$$

§ 18. *Relative Weights from the Residuals.*—The following table gives the average residual in an equation of condition weighted according to the scheme given above.

	Weight Adopted.	Average Residual. X Equations.	Y Equations.	No. of Equations.
		"	"	
Algiers	$\frac{2}{3}$	0·109	0·111	31
Cambridge (209–252) ...	1	·152	·129	39
„ (254–265) ...	$\frac{2}{3}$	·132	·080	11
„ (266–335) ...	1	·105	·087	54
Lick	1	·101	·093	28
Northfield	$\frac{1}{4}$	·111	·134	21
Oxford	$\frac{2}{3}$	·134	·105	30
Paris	1	·093	·099	21
Tacubaya	$\frac{1}{10}$	·128	·103	14

The result shows that some improvement might perhaps be effected by further weighting. The first Cambridge series is not good, and should have less weight; so also should the Oxford series; while Paris should have had more weight. This is on the assumption that the residuals are accidentally distributed, which I shall show is not the case. It follows that the above average residuals are not necessarily proportional to the true weights, but they may be taken as showing that the system of weights adopted did not differ widely from the truth. We shall not improve matters by making a fresh solution with revised weights before we have considered the systematic deviations of each series from the mean.

§ 19. *Systematic Deviations of each Series from the Mean.*—In the first column of the following table we have the mean residual (taking account of signs) in the equations of condition from each series of plates :—

	Mean Residual. In X.	In Y.	Parallax Factors in X. Numerical Sum.	Algebraic Sum.
	"	"		
Algiers	+0·013	−0·050	41	+ 1
Cambridge (209–252) ...	+ ·016		50	−15
„ (254–265) ...	− ·013	+ ·004	12	+12
„ (266–335) ...	− ·033		58	−14
Lick	+ ·057	− ·021	34	+26
Northfield	− ·028	+ ·019	13	+10
Oxford	− ·047	+ ·009	37	− 5
Paris	+ ·039	+ ·048	18	+ 7
Tacubaya	+ ·011	+ ·029	6	+ 1

Taking the probable error of an X equation of condition as $\pm 0''\cdot10$, we see that the third Cambridge series, the Lick and Oxford, and perhaps the Paris series have mean residuals considerably larger than might be expected if they were fortuitous; there is an appearance of systematic divergence from the general mean. It becomes important to see whether this may have affected prejudicially the value found for the parallax. The fourth and fifth columns contain the numerical and algebraical sums of the parallax factors in the X equations of condition. They show whether in any series parallax factors of one sign have predominated. Further, since similarity of sign in the corresponding quantities of the second and fifth columns shows a tendency to diminish the deduced value of the parallax, we are able to judge from this table in what direction a systematic error will have acted.

The most conspicuous case of want of balance in the parallax factors is found in the Lick series (where evening plates were deliberately selected to coincide in time with morning plates of the eastern hemisphere). If the Lick residuals are really systematic this series will have tended to reduce the value of the parallax. A similar, though very much smaller, effect may have been produced by systematic error in the third Cambridge series, the Oxford and Paris series.

We may conclude that if these systematic discordances are real, and have produced any sensible effect, they have probably tended to make the parallax too small rather than too large. I am disposed myself to think that there is some reality in them, and that we cannot say that the results of a series of photographs made with different instruments are really homogeneous, though they have been reduced to the same system of carefully determined stars. If this is really the case it is hardly necessary to point out the futility of supposing that the most accurate results can be obtained by combining simultaneous observations at widely separated stations, especially if somewhat different systems of star places have been used.

When the values of the unknowns derived from this solution are substituted in the equations of condition given by the Minneapolis results we have the following results :—

	Mean Residual. in X.	in Y.	Average Residual. in X.	in Y.
	″	″	″	″
For the 10 single exposures	+0·212	+0·006	0·216	0·165
„ 11 multiple „	−0·247	+0·035	0·389	0·173

These show that the weight of one of the components of a multiple exposure is only about one-quarter of that of a single exposure. But, further, we have the curious result that the single exposures give uniformly large positive residuals, while those from the multiple exposures are strongly negative. I think that this anomaly adds weight to the reasons which compelled us

from observations whose epochs fall into corresponding eighths, we have the following result :—

X *Equations. No. of Residuals and Mean for Each Group.*

I.		II.		III.		IV.	
31	+0″·030	26	+0″·023	55	−0″·025	24	−0″·005
V.		**VI.**		**VII.**		**VIII.**	
25	+0″·048	25	+0″·012	21	−0″·060	28	−0″·030

These figures show no periodicity in $5\frac{1}{4}$ hours. But if we add together groups I. and V., and so on, we have

I. V.		II. VI.		III. VII.		IV. VIII.	
56	+0″·038	51	+0″·017	76	−0″·034	52	−0″·020,

which seems to show an oscillation of the planet in the half-period of $2\frac{3}{4}$ hours.

Before discussing the probability that this is real let us examine the Y residuals in the same way.

Y *Equations.*

I. V.	II. VI.	III. VII.	IV. VIII.
−0″·006	−0″·002	+0″·001	+0″·026
−0″·062	+0″·007	+0″·011	+0″·001

Again, there is no evidence of a $5\frac{1}{4}$-hour period. Taking means of I. and V., &c., as before, we have

I. V.	II. VI.	III. VII.	IV. VIII.
−0″·034	+0″·002	+0″·004	+0″·013

which does not suggest any periodicity of the Y residual in the half-period.

If these means are accidental their probable values are about ±0″·013 in X and ±0″·012 in Y. It seems to me that we may consider the Y means accidental ; only one is larger than might be expected, and that depends upon an abnormally large result in group V., quite unsupported by group I.

The case of the X residuals is different. All four are greater than their probable values if accidental ; two are more than twice as great ; the two half-series give very much the same effect independently, and the balance of groups in opposite phases of the apparent periodicity is remarkably good.

The reality of the period is, moreover, confirmed by the discussion of residuals from an earlier solution, from somewhat different material, differently weighted, and starting from a different zero of time. These give an apparent periodicity of very nearly the same amplitude and phase.

might affect very seriously a set of plates taken at nearly the same time on several successive days.

§ 23. *Results of the Solution.*—Having shown that there is no sensible term depending on t^2 to be included in our equations of condition, that the assumed weights were fairly correct, that there are no very serious discordances between the results of different observatories, and that the periodic inequality in X which seems to exist cannot affect the parallax to any extent, we may take our first solution as definitive and set out the following results :—

P.E. of one equation of condition of weight unity :—

In X $\pm 0''\cdot100$. In Y $\pm 0''\cdot090$.

From the X equations :—

$\Delta_1\xi_0$, constant correction to the ephemeris

$= [-2''\cdot245] + 0''\cdot0126 \pm 0''\cdot0071$

$\Delta_2\xi_0$, correction varying with the time per day

$= +0''\cdot0023 \pm 0''\cdot0030$

$\Delta\pi$, correction to the adopted value of the solar parallax

$= -0''\cdot0013 \pm 0''\cdot0052$.

From the Y equations :—

$\Delta_1\eta_0$, constant correction to the ephemeris

$= [+0''\cdot247] + 0''\cdot0385 \pm 0''\cdot0077$

$\Delta_2\eta_0$, correction varying with the time per day

$= -0''\cdot0522 \pm 0''\cdot0028$

$\Delta\pi$, correction to the adopted value of the solar parallax

$= -0''\cdot0136 \pm 0''\cdot0116$.

Combining the two values of $\Delta\pi$ according to their weights we have

From the X equations	$368\Delta\pi = -0''\cdot4784$
„ Y „	$77\Delta\pi = -1\cdot0472$
	$445\Delta\pi = -1\cdot5256$

whence $\Delta\pi = -0''\cdot0034 \pm 0''\cdot0047$.

And since the value of the solar parallax assumed in the computations was $8''\cdot80$, we have as the result of this discussion

$$\pi = 8''\cdot7966 \pm 0''\cdot0047.$$

Conclusion.

§ 24. This value of the solar parallax has no kind of claim to be called definitive. But it has just succeeded in fulfilling the

might well be entangled with the parallax displacements in a quite considerable series of observations made at a single observatory; it is completely separated from the parallax when a general solution is made; and the search for it throughout the period of observation of the planet will make a beautiful test of the real delicacy of our results.

It seems that we may draw, from the experience gained in the work of the present paper, the conclusion that future work would be greatly facilitated by the adoption of a close system of standard stars. The formation of the system that I have used made a considerable part of the whole labour. But in future the task will be very much lightened, because it will be possible to make the star system depend upon the very extensive series of star places derived from the work of the four French observatories. If we have a standard system whose relative places are known with a probable error of a few hundredths of a second we can get as much accuracy as an individual plate is capable of giving by measuring the planet and eight or ten of these stars well distributed around it. With such a standard system we can discover systematic errors, provided that the residuals in the reduction of the stars are open to inspection; but if any such error is found, it is of the greatest advantage to have at hand the original measured coordinates. It is doubtful whether those observatories whose aim is to contribute their results in the form most convenient for a general reduction of all the material could do better than publish simply the original measures. It will probably pay better to give the man who undertakes a general solution the means to carry out reduction *ab initio*, rather than to do any part of it before publication, for the discovery of some systematic error when the observations are combined with others will often necessitate a new reduction.

Finally, if we are able to admit that these conclusions are sound, we are led to the proposition that any observatory with photographs of *Eros* still unmeasured can make its contribution to the definitive determination of the solar parallax of greatest effect by agreeing to select its stars from a close standard system, and doing as speedily as possible the absolute minimum of work. It is the hope of the writer that he may be allowed to submit, in the near future, a selection of standard stars for consideration.

§ 26. I must acknowledge with gratitude the most efficient help which has been given me by Miss Julia Bell, of Girton College, and Miss Anne Malden, of Newnham College, and must thank the Committee administering the Government Grant Fund of the Royal Society for the means of securing this help, without which the work could not have been done at Cambridge.

Cambridge Observatory:
1904 June 21.

The Observations.

The observations that we are dealing with were made between 1901 June 24 and September 27, and between 1902 June 10 and October 4, that is, within two months of the oppositions of 1901 and 1902; only one or two opportunities of securing observations within those periods were missed. In the following table the total number of observations of each pair of satellites is shown; the figures under the letter "s" denote the number of observations in distance, those under "p" the number in position-angle.

Pair of Satellites	I.-II.		I.-III.		I.-IV.		II.-III.		II.-IV.		III.-IV.		Total	
	s	p	s	p	s	p	s	p	s	p	s	p	s	p
1901	29	30	27	28	28	28	27	28	30	30	27	27	168	171
1902	35	35	38	38	32	33	37	37	38	37	42	42	222	222
Total ...	64	65	65	66	60	61	64	65	68	67	69	69	390	393

Each observation gives rise to an equation of condition containing as a rule thirteen unknowns; we have therefore to deal with 783 such equations. But a few more words concerning the observations themselves must first be said.

It is of the greatest importance to have means of controlling the scale value and the equatorial adjustments of the heliometer.

This was effected by observing, together with the satellites, a pair of standard stars whose distance and position-angle were accurately known from carefully determined meridian places. There was one pair of standards for 1901 and another for 1902, so chosen that they were very near to *Jupiter* throughout the period of observation, that their position-angle only differed by a few degrees from the average position-angle of a pair of satellites, and that their distance was as great as possible. All measures of distance were then referred to the adopted distance of these standards, and the zero of position-angle depended upon their adopted position-angle. The standards for 1901 were 24 and 26 *Sagittarii*; their distance and position-angle were

$$6591''{\cdot}100 \text{ and } 84^\circ\ 20'\ 36'' \ (1901{\cdot}0)$$

those for 1902 were 21 and 23 (θ) *Capricorni*, their adopted distance and position-angle being

$$4486''{\cdot}930 \text{ and } 76^\circ\ 31'\ 9'' \ (1902{\cdot}0).$$

Any correction that it may be found desirable to apply to the adopted places of these stars can be allowed for at the end of the work; for a change in the adopted distance affects only the mass of *Jupiter*, and a change in position-angle is equivalent to a change in the position of the fundamental plane to which the orbits of the satellites are referred.

A complete observation of either standards or satellites

planes for the mean epochs of the two sets of observations are defined by

	Node (Ω)	Inclination
1901·61	315° 30′ 35″·2	3° 4′ 6″·6
1902·60	315° 31′ 29″·1	3° 4′ 7″·0

the node being reckoned from the Earth's mean equinox. The longitudes and nodes of the satellites are reckoned from the descending node ($\mathfrak{S}$) of this plane.

As was pointed out by Professor Sampson in the *Observatory*, 1901 August, the values of the major axes of the satellites' orbits given by Marth are not quite correctly associated with the mass of *Jupiter* owing to his having neglected all the non-periodic perturbations of the axes, excepting that due to the compression of *Jupiter*. The correct values of log a'', a'' being the semi-major axis in seconds of arc at unit distance, are

2·764534 2·966232 3·169012 3·414255

which according to Marth would correspond, not to a mass of 1/1047·7, but to

1/1047·68 1/1047·56 1/1047·65 1/1047·45

Since in the equations of condition the corrections to the axes are eliminated by expressing them in terms of corrections to the mass of *Jupiter*, and to the constant of compression Jb^2, this is a point of importance.

The coordinates of the satellites were calculated from the expressions given by Marth (*Monthly Notices*, vol. li. p. 506) (with the necessary corrections), and from them were found the tabular distance and position-angle of a pair of satellites, which were then compared with the observed quantities. The differences between them were next expressed in terms of corrections to the constants of the theory.

The Unknowns.

Each of the six elements of a satellite's orbit was regarded as subject to correction. As there are four satellites this would give twenty-four unknowns. But as mentioned before the corrections to the axes were expressed in terms of two other unknowns, viz. the mass and the compression of *Jupiter*; moreover, following de Sitter, three other unknowns were introduced, namely, corrections to the coefficients of the large inequalities in the longitudes of the first three satellites. This leaves us with twenty-five unknowns, which it will be convenient to enumerate here. Let

M = reciprocal of the mass of *Jupiter*.
Jb^2 = constant of compression of *Jupiter*.
ϵ = longitude of epoch
e = eccentricity
ϖ = longitude of perijove
Ω = longitude of node
ι = inclination
} of a satellite.

			I.-II.	I.-III.	I.-IV.	II.-III.	II.-IV.	III.-IV.	General Solution.
1901									
Mean	...	...	1048·36	1047·39	1047·94	1047·45	1048·02	1047·65	1047·68
$\Delta\omega_1$	...	...	$+0\overset{\circ}{.}168 \pm 0\overset{\circ}{.}042$	$+0\overset{\circ}{.}141 \pm 0\overset{\circ}{.}014$	$+0\overset{\circ}{.}126 \pm 0\overset{\circ}{.}017$	...	...	...	$+0\overset{\circ}{.}136 \pm 0\overset{\circ}{.}010$
$\Delta\omega_2$	...	...	$+0\overset{\circ}{.}021 \pm 0\overset{\circ}{.}009$	...	...	$-0\overset{\circ}{.}081 \pm 0\overset{\circ}{.}027$	$+0\overset{\circ}{.}028 \pm 0\overset{\circ}{.}010$	...	$+0\overset{\circ}{.}020 \pm 0\overset{\circ}{.}006$
$\Delta\omega_3$	...	...	...	$-0\overset{\circ}{.}062 \pm 0\overset{\circ}{.}008$	...	$-0\overset{\circ}{.}001 \pm 0\overset{\circ}{.}010$	...	$-0\overset{\circ}{.}041 \pm 0\overset{\circ}{.}005$	$-0\overset{\circ}{.}038 \pm 0\overset{\circ}{.}005$
$\Delta\omega_4$	...	...	...	...	$-0\overset{\circ}{.}041 \pm 0\overset{\circ}{.}004$	...	$+0\overset{\circ}{.}017 \pm 0\overset{\circ}{.}003$	$-0\overset{\circ}{.}035 \pm 0\overset{\circ}{.}002$	$-0\overset{\circ}{.}036 \pm 0\overset{\circ}{.}002$
Prob. Error		...	$\pm 0\overset{\circ}{.}071$	$\pm 0\overset{\circ}{.}079$	$0\overset{\circ}{.}104$	$0\overset{\circ}{.}100$	$0\overset{\circ}{.}089$	$0\overset{\circ}{.}060$	
1902									
Mean	...	...	1046·66	1046·89	1046·85	1047·33	1048·03	1047·92	1047·65
$\Delta\omega_1$	...	...	$+0\overset{\circ}{.}035 \pm 0\overset{\circ}{.}022$	$+0\overset{\circ}{.}131 \pm 0\overset{\circ}{.}010$	$+0\overset{\circ}{.}153 \pm 0\overset{\circ}{.}013$	...	...	...	$+0\overset{\circ}{.}134 \pm 0\overset{\circ}{.}008$
$\Delta\omega_2$	...	...	$-0\overset{\circ}{.}017 \pm 0\overset{\circ}{.}007$	...	...	$-0\overset{\circ}{.}024 \pm 0\overset{\circ}{.}014$	$-0\overset{\circ}{.}045 \pm 0\overset{\circ}{.}009$	...	$-0\overset{\circ}{.}025 \pm 0\overset{\circ}{.}005$
$\Delta\omega_3$	...	..	...	$-0\overset{\circ}{.}025 \pm 0\overset{\circ}{.}004$	...	$-0\overset{\circ}{.}012 \pm 0\overset{\circ}{.}005$	...	$-0\overset{\circ}{.}028 \pm 0\overset{\circ}{.}005$	$-0\overset{\circ}{.}023 \pm 0\overset{\circ}{.}003$
$\Delta\omega_4$	...	...	...	...	$-0\overset{\circ}{.}030 \pm 0\overset{\circ}{.}004$	...	$-0\overset{\circ}{.}029 \pm 0\overset{\circ}{.}004$	$-0\overset{\circ}{.}039 \pm 0\overset{\circ}{.}003$	$-0\overset{\circ}{.}034 \pm 0\overset{\circ}{.}002$
Prob. Error		...	$\pm 0\overset{\circ}{.}085$	$\pm 0\overset{\circ}{.}083$	$\pm 0\overset{\circ}{.}104$	$\pm 0\overset{\circ}{.}091$	$\pm 0\overset{\circ}{.}115$	$\pm 0\overset{\circ}{.}097$	

The solution involved twenty-four and not twenty-five normals, because one of the unknowns, ΔJ, was transferred to the right-hand side of the equations, and all the other unknowns were expressed in terms of it, its value being left to be determined by other methods. The reason for doing this was that ΔJ could not be determined with sufficient weight from such a set of observations as the present, but was probably by no means a negligible quantity.

Let Δ denote the determinant formed by the coefficients in the normals, *i.e.* :—

$$\begin{vmatrix} [aa] & [ab] & [ac] & . & . \\ [ba] & [bb] & [bc] & . & . \\ [ca] & [cb] & [cc] & . & . \\ . & . & . & . & . \end{vmatrix}$$

and let Δ_{ik} be the first minor of $[ik]$; then the solution was made in the form,

$$x = \frac{\Delta_{aa}}{\Delta} X + \frac{\Delta_{ab}}{\Delta} Y + \frac{\Delta_{ac}}{\Delta} Z + \ldots$$

$$y = \frac{\Delta_{ba}}{\Delta} X + \frac{\Delta_{bb}}{\Delta} Y + \frac{\Delta_{bc}}{\Delta} Z + \ldots$$

. . . . . .

where X is the absolute term in the normal for x and so on. Since $[ik] = [ki]$, it follows that the solution of the original normals is also a set of normals. The weight of an unknown k is Δ/Δ_{kk}, *e.g.* of z it is Δ/Δ_{cc}. The best account of the practical application of this form of solution is given by de Sitter in the appendix to his discussion of Sir D. Gill's observations. As Dr. de Sitter there remarks, this method deserves to be better known than it appears to be.

In this way values of the unknowns were found which exactly satisfied the combined normals.

In the case of the 1902 normals, owing to the better separation of the distance and position-angle unknowns, a good first approximation to the values of Δp, Δq was obtained from a solution of the position-angle normals, neglecting the other unknowns; similarly a good approximation to those other unknowns was found from the distance normals, neglecting the Δp's and Δq's. Then by an obvious process of approximation it was easy to find the values which would satisfy the combined normals from distance and position-angle observations together. These values of all the unknowns from the two sets of observations are given below. It should be remarked that owing to the approximate method of solution the weights of the 1902 unknowns are not rigorously correct, but can differ very little from the rigorous weights which would be those derived from the solution of the combined normals.

A table of residuals grouped according to distance shows no dependence of the magnitude or sign of a residual on the distance observed, and it is concluded that the observations of distance are free from systematic error.

When we come, however, to inspect the residuals in the position-angle equations, we meet with a curious phenomenon, for which no plausible explanation can at present be found.

It was noticed that the residuals in the pairs I.–IV., II.–IV., and III.–IV. were markedly larger than in the other pairs, and that large residuals from the three pairs occurred on the same day; moreover the same change in the y coordinate of IV. would cause all these residuals practically to vanish. Thus the y, i.e. the latitude of IV. seemed to be in error. The mean of the residuals from I.–IV., II.–IV., and III.–IV. was therefore plotted for each day by observation, and it then appeared that a period of eight days would fit the graph of residuals. The mean of the residuals for IV. was therefore plotted with twice the mean longitude of IV. as abscissæ—this corresponds to a period of 8·23 days. The result was that for both years of observation the residuals obviously lay about a sine curve, that is, the mean residual of I.–IV., II.–IV., and III.–IV. could be approximately represented by the expressions

$$\text{Residual} \times \operatorname{cosec}(p-\mathrm{P}) = 0''\cdot050 + 0''\cdot150 \sin(2l_4 + 10^\circ) \text{ in } 1901$$
$$= 0''\cdot100 + 0''\cdot150 \sin(2l_4 - 100^\circ) \text{ in } 1902$$

Here p is the position-angle of the pair of satellites, and P is the position-angle of *Jupiter's* axis. If we apply these empirical corrections to the mean residual for IV., we get a much better representation of the observations; thus in 1901 the sum of the squares of the mean residuals is $0''\cdot858$, whilst of the mean residuals corrected as above it is $0''\cdot628$—a reduction of 27 per cent. In 1902 it was desired to see the effect of the periodic term alone; therefore $0''\cdot100$ was subtracted from each mean residual $\times$ cosec $(p-\mathrm{P})$ and the sum of the squares found: it amounted to $0''\cdot587$. But on taking out the periodic as well as the constant term, the sum of the squares was reduced to $0''\cdot356$—a reduction of 39 per cent.

It seems clear, then, that the observations are much better represented by an empirical term in the latitude of IV., which is

$$\text{in } 1901 \quad \Delta\lambda_4 = 17'' + 50'' \sin(2l_4 + 10^\circ)$$
$$\text{in } 1902 \quad \Delta\lambda_4 = 32'' + 48'' \sin(2l_4 - 100^\circ)$$

To make the phases for the two years the same, it would be necessary to change the period from $8^d\cdot23$ to $8^d\cdot08$, which would have practically no effect on the representation of the observations by the curve; thus we may say that the same periodic term with a period of $8^d\cdot08$ represents the observations of both years.

should be said that the nodes are reckoned from the Earth's moving equinox.

Now, in Souillart's notation the theoretical expressions of the above quantities are

$$p = N_i' \sin(b_i t+\gamma_i) + 102''{\cdot}3 \sin(b_2 t+\gamma_2) + 15''{\cdot}7 \sin(b_3 t+\gamma_3) + 2''{\cdot}8 \sin(2L-b_4 t-\gamma_4)$$

q = same with cosines.

Whence N_i' and $b_i t+\gamma_i$, on the assumption that the other coefficients and angles are correct. In this way we find

	N_i'	$b_i t+\gamma_i$
1879·00	0°4733	179°38 ± 2°40
1891·75	0·4956	25·45 ± 0·37
1901·61	0·5010	278·65 ± 0·65
1902·60	0·4793	267·48 ± 0·52

The probable errors given after $b_i t+\gamma_i$ are those of the node of the orbit on the planet's equator, not of $b_i t+\gamma_i$, and the relative weights of the different determinations may be taken as inversely proportional to the squares of these probable errors. Comparing with Souillart's values we get the following equations of condition from which to find corrections to his values of b_i and γ_i (1891·75) :—

$-4657\Delta b_i + \Delta\gamma_i = 13°21$	*wt* 0·16	residual $-5°35$
$\Delta\gamma_i = 12·84$	7·29	$-1·78$
$3605\Delta b_i + \Delta\gamma_i = 24·78$	2·25	$+1·01$
$3963\Delta b_i + \Delta\gamma_i = 25·44$	3·61	$+0·52$

whence

$$\Delta b_i = 0°00294 \pm 0°00037 \text{ and } b_i = -0°03003 \pm 0°00037$$
$$\Delta\gamma_i = +13·51 \quad \pm 0·96$$

It may be doubtful whether the first equation should be included or not. No details were given by Schur as to his method of observing position-angles ; he found, moreover, that his observations were better represented by assuming the existence of a large empirical term depending on whether the satellite followed or preceded the planet. If we assign weight zero to the first equation we get

$$\Delta b_i = 0°00322 \pm 0°00012 \text{ and } b_i = -0°02975 \pm 0°00012$$
$$\Delta\gamma_i = +12·85 \quad \pm 0·30$$

These probable errors are calculated in the usual way from the sum of the squares of the residuals ; but a truer estimate of the probable error of b_i can be found from the consideration

It is interesting to see what confirmation of this value can be found from the node of III. Treating it in the same way, we find from Schur's re-reduction of Bessel's observations that

$$\begin{aligned} \text{in } 1836 \qquad & b_2 t + \gamma_2 = 120{\cdot}3 \\ \text{in } 1902 \qquad & = 321{\cdot}1 \end{aligned}$$

giving a motion of 159·2 in sixty-six years against Souillart's 168·3—*i.e.* a reduction of 1/18·49. The theoretical expression for Δb_2 is

$$-\frac{\Delta b_2}{\boxed{2}} = 0{\cdot}752\frac{\Delta J}{J} + 0{\cdot}126\frac{\Delta m'}{m'} + 0{\cdot}088\frac{\Delta m''}{m''} + 0{\cdot}019\frac{\Delta m}{m}$$

neglecting the correction to the Sun's mass. Now, as will be seen further on, the mass of II. is certainly not much in error; hence we may safely neglect $\Delta m'$. And the other two terms may be neglected on account of the smallness of their coefficients. With sufficient accuracy we have therefore

$$0{\cdot}752\Delta J/J = -1/18{\cdot}49$$

whence

$$Jb^2 = 0{\cdot}02062 \pm 0{\cdot}00051$$

The probable error is only roughly determined from the fact that the above difference of 159° has a probable error amounting to about 1/40 of itself.

These two determinations of Jb^2 agree nearly within the limits of their probable errors, and the conclusion is that the value

$$J = 0{\cdot}01987\left(\frac{39''{\cdot}00}{d}\right)^2$$

is the value which best satisfies the existing heliometer observations of the satellites. This would give 863° for the annual motion of the perijove of V., and it will be of extreme interest to see how nearly a comparison of the latest observations of V. with the earliest reproduces this value. It must be remembered that the expression for the motion of the perijove is

$$\frac{\partial P}{\partial t} = n\left(\frac{Jb^2}{a^2} + \frac{Kb^4}{a^4} + \ldots\right)$$

and that, as de Sitter pointed out, the second term in the case of V. amounts to 1/50 of the first, and therefore cannot be neglected. The inclusion of this term raises the value of the motion of the perijove to 880°, which does not differ much from the value Miss Dobbin finds from observation, viz. 887°. Part of the difference between the value of Jb^2 as derived from II. and that from V. is thus due to the term containing the constant

But on the whole the evidence goes to show that x_3 is in need of a position correction of about 0°·05. This would bring the adopted value of Souillart, *i.e.* 3693″, into near agreement with Damoiseau's value, *i.e.* 3867″. It would also appear that Adams' value of the mass of I. is nearer the truth than Damoiseau's.

Jupiter's Equator.—Owing to the great equatorial protuberance of the planet the determination of what may be called its dynamical equator is of importance. To determine its position we have referred the orbits of I., III., and IV. as given by observation, to Souillart's fixed plane, *i.e.* the planet's orbit of 1850. Then in Souillart's notation we have

$$N_4 \sin(b_4 t+\gamma_4) = \text{observed } p - N \sin(bt+\gamma) - N_1 \sin(b_1 t+\gamma_1) - \&c.$$

$$N_4 \cos(b_4 t+\gamma_4) = \text{observed } q - \text{same with cosines.}$$

Hence assuming his values of the coefficients and angles, and using his values of σ_4'' σ_4''', which, be it remembered, depend on the adopted masses, we get the following values of N_4 and of $b_4 t+\gamma_4$:—

	N_4			$b_4 t+\gamma_4$			
	I.	III.	IV.	I.	III.	IV.	Souillart.
1891·75	3°·0928	3°·1486	3°·0637	316°·07	315°·46	316°·68	315°·32
1901·61	3·1355	3·0997	3·0888	315·65	315·22	316·47	315·45
1902·60	3·1417	3·0974	3·0573	315·59	315·37	316·91	315·46

The values for 1891·75 have been deduced from de Sitter's paper. If weights are assigned inversely proportional to the squares of the probable errors, we have

From	I. $N_4 = 3°·1190$ at	6·8	Weighted mean ... $3°·0806 \pm 0°·0094$
,,	III. $= 3·1207$,,	45·2	
,,	IV. $= 3·0666$,,	147·8	Souillart ... 3°·0658

It follows that the observations of III. give for the inclination of *Jupiter's* equator to his orbit

$$\omega = 3°\ 7'\ 22''$$

whereas the fourth satellite gives

$$\omega = 3°\ 4'\ 8''$$

the value adopted by Souillart from Damoiseau being

$$\omega = 3°\ 4'\ 5''$$

It is a significant fact that the value from IV. is 3′ 14″ less than that from III., Damoiseau having also found a difference of 2′ 47″ in the same sense. This difference is probably due to errors in the adopted masses and in Jb^5.

If we adopt Laplace's theoretical period, viz. 6·215 years, and reckon the time from 1901·61, we find that these values are represented by

$$\theta = 180° - 0°·156 \sin (bt + 193°)$$

which gives for the epoch 1891·75

$$\theta = 180°·049$$

Dr. de Sitter found from the observations at this epoch a value practically in perfect agreement, viz.

$$\theta = 180°·053 \pm 0°·014$$

The period 6·215 years depends on the adopted values of the masses. If we take Adams' values, the period is reduced to 5·36 years, which would give

$$\theta = 180° - 0°·151 \sin (bt + 193°)$$
$$\theta \text{ at epoch } 1891·75 = 180°·141$$

This does not agree with the observed value, and though it may require a small correction on account of ΔJ, yet such correction cannot be sufficient to bring the two into agreement.

In view of the uncertainty as to the true values of the masses, and consequently of the period of the libration, we cannot determine its coefficient and phase by any method involving an accurate knowledge of its period.

The libration is distributed amongst the longitudes as follows, according as Damoiseau's or Adams' masses are adopted :—

I.	$\delta l = -0°·274\, D \sin (bt + E)$	or	$-0°·178\, D \sin (bt + E)$
II.	$\delta l' = +0°·231\, D \sin (bt + E)$		$+0°·262\, D \sin (bt + E)$
III.	$\delta l'' = -0°·016\, D \sin (bt + E)$		$-0°·019\, D \sin (bt + E)$

The substitution of the above values of D gives for the coefficients of $\sin (bt + E)$:—

$-0°·043$	$+0°·036$	$-0°·002$	Damoiseau.
$-0°·026$	$+0°·040$	$-0°·003$	Adams.

The corrections to the mean longitudes as found from observation are :—

	1891.	1901.	1911.
$\Delta\epsilon_1$	$+0°·102 \pm 0°·006$	$+0°·151 \pm 0°·010$	$+0°·132 \pm 0°·007$
$\Delta\epsilon_2$	$-0°·001 \pm 0°·004$	$+0°·015 \pm 0°·006$	$-0°·020 \pm 0°·004$
$\Delta\epsilon_3$	$-0°·026 \pm 0°·002$	$-0°·035 \pm 0°·005$	$-0°·021 \pm 0°·003$

Damoiseau's revised value, viz. 65″·073, of the second eccentricity of III. instead of his earlier value, viz. 116″·73. From this table it appears that the mass of II. is the only one that is known with any degree of certainty: the mass of III. is doubtful to the extent of not more than 10 per cent., but the masses of I. and IV. are at present so uncertain that they may almost be looked upon as unknown. We may, however, confidently expect that much of this uncertainty will be removed by the publication of Professor Sampson's discussion of the Harvard eclipses, which is therefore anxiously awaited.

In conclusion I should like to say how grateful I am to Sir David Gill for having given me an opportunity of carrying out this piece of work: for this and many other kindnesses, too numerous to mention, I wish to offer him my most sincere thanks. It is a pleasure also to acknowledge the unfailing courtesy and kindness I received from Mr. S. S. Hough and all the staff of the Cape Observatory during the two years I spent with them.

Note on the Distribution of Sun-spots in Heliographic Latitude, 1874 to 1902. By E. Walter Maunder.

Rather more than a year ago the Astronomer Royal communicated to this Society a paper which he had desired me to prepare on the "Mean Daily Areas of Sun-spots for each Degree of Solar Latitude for each Year from 1874 to 1902 as Measured on Photographs at the Royal Observatory, Greenwich." The paper was communicated at the Meeting of 1903 May 8, and printed in vol. lxiii. No. 8 of the *Monthly Notices*. It summarised in both tabular and graphical forms the results of thirty years' work in the measurement and reduction of about 9000 photographs of the Sun, and of the latitudes and areas of about 5000 separate groups of spots, involving upwards of a quarter of a million measures made in duplicate upon those photographs.

It appeared to me, when preparing the above paper, that the results given in it could be exhibited in somewhat different aspects, and I therefore prepared other diagrams at the same time, which were not communicated to this Society, but were exhibited at the Official Visitation of the Royal Observatory on 1903 June 6. These diagrams, altered only as to scale, so as to render them suitable for reproduction in the *Monthly Notices*, I would now submit to the consideration of the Royal Astronomical Society.

At the same time I would wish to supplement them by other diagrams which the appearance of a paper by Dr. W. J. S. Lockyer in the Appendix to the last number of the *Monthly Notices* has seemed to me to render necessary. A considerable

areas of the northern hemisphere are joined by a continuous line, those of the southern hemisphere by a broken line. The chief facts to be gathered from this diagram are :

(*a*) The two hemispheres do not, on the whole period, differ very greatly in total spotted area ; but the southern is the more prolific of the two, the northern claiming 43½ per cent. of all spots, reckoning by area ; the southern 56½ per cent.

(*b*) The critical points in the progress of the solar cycle are marked earlier, in both the two cycles shown, by the northern hemisphere than by the southern ; and this on both the ascending and descending parts of the curve.

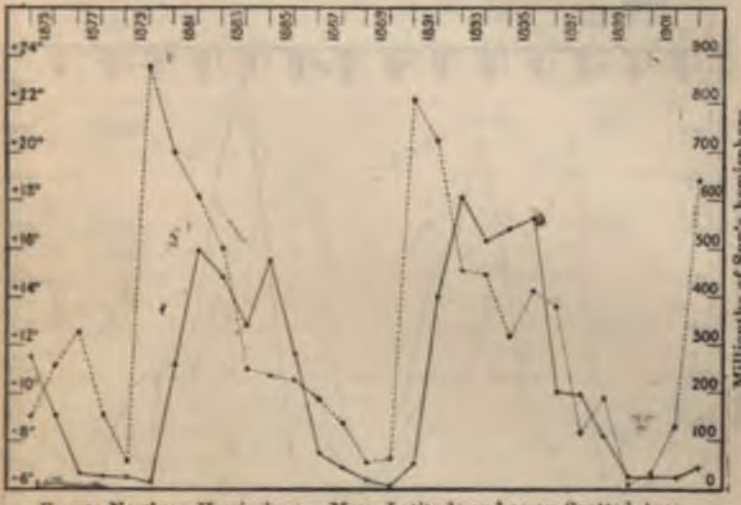


Fig. 2. Northern Hemisphere. Mean Latitude and mean Spotted Area, 1874-1902.
Latitude ..... Area ——

(*c*) The northern hemisphere shows a double maximum in both cycles, the first falling about three years before the second. The general form of the northern spot-curve, therefore, shows a long-continued but not very pronounced maximum.

(*d*) The southern hemisphere, on the contrary, shows but one very sharply marked maximum, synchronous with the slight depression between the two maxima of the northern curve.

Fig. 2 shows the mean daily spotted area for the northern hemisphere of the Sun for each year from 1874 to 1902, precisely as in fig. 1. But the mean heliographic latitude, year by year, of the spots of the northern hemisphere has also been indicated ;

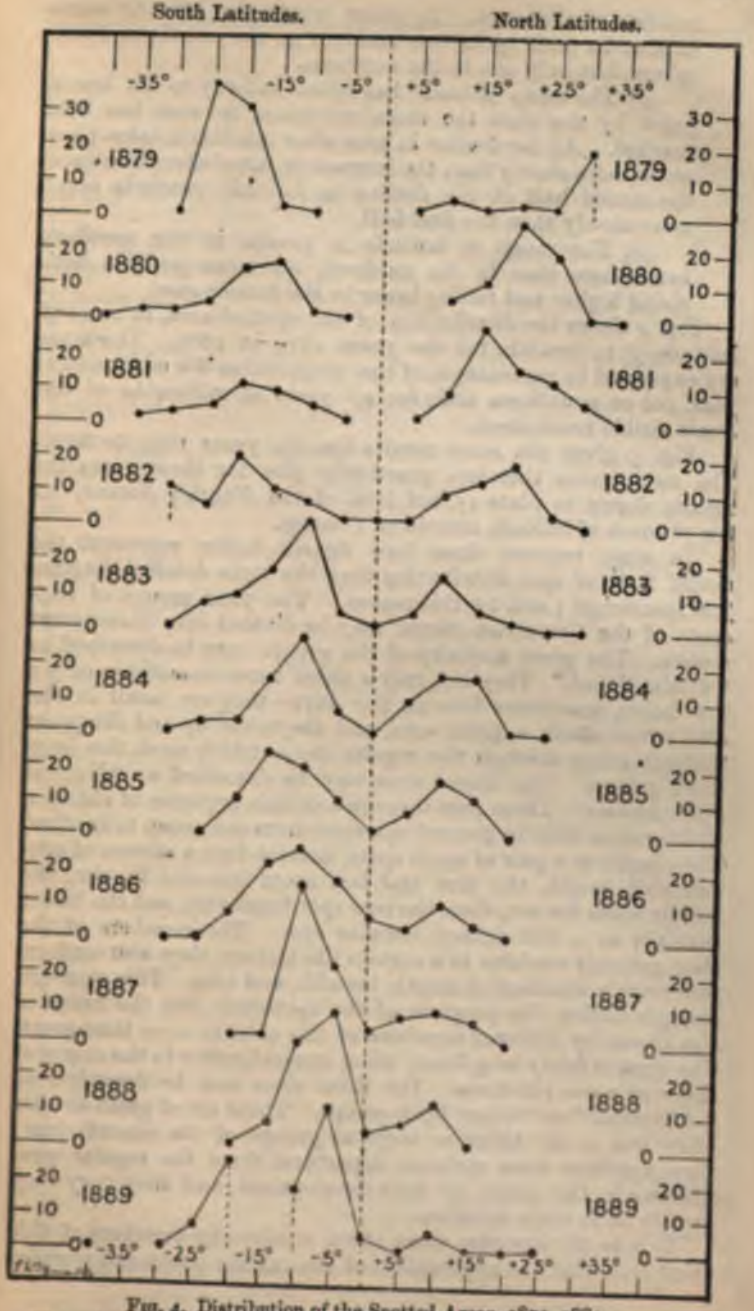


Fig. 4. Distribution of the Spotted Areas, 1879–1889.
The ordinates are percentages of the mean daily area for each several year

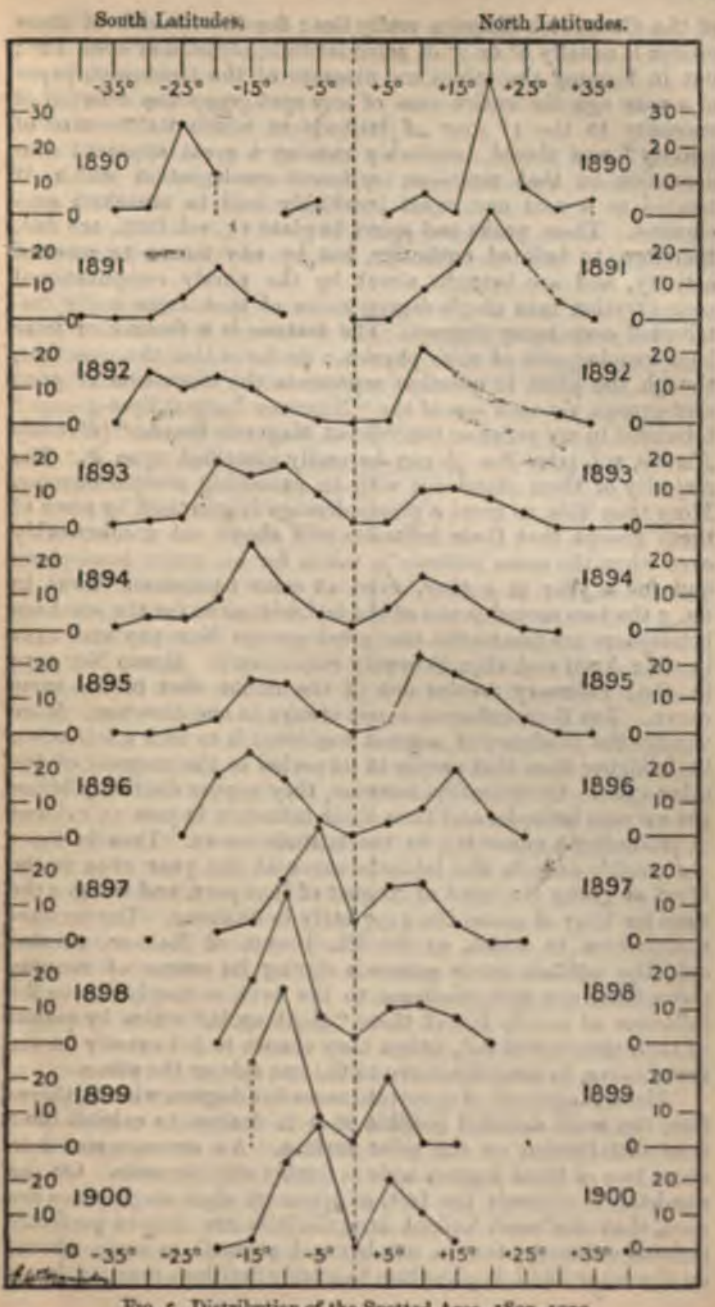


FIG. 5. Distribution of the Spotted Area, 1890–1900.
The ordinates are percentages of the mean daily area for each several year.

concentrates into zones of two or three degrees spot-areas that on the Sun are actually spread over five, six, seven, or it may be even ten degrees of latitude. The maxima, therefore, obtained by Dr. Lockyer in the paper referred to above by taking 3° zones are not real maxima but merely apparent.

Turning to the curves shown in figs. 4 and 5, it will be seen that with 5° zones certain salient features of spot-distribution in latitude are clearly brought out.

(*i*) There is one and only one maximum, and that sharply defined, in each hemisphere in each year. This is the general rule.

Two classes of apparent exceptions are to be noted. In four instances, even taking the spots of a year, and grouping them in 5° zones, a single spot-group still succeeds in standing out. These are all in the southern hemisphere and are the following :—

Group	No. 729, 1882 April,	Lat.	−28°·5
„	„ 2421, 1892 February	„	−28·3
„	„ 3106, 1893 August	„	−17·6
„	„ 3281, 1893 November	„	− 5·1

The other instances are just after minimum, and are illustrations of "Spoerer's Law of Spot-zones"; for, as Spoerer pointed out, at this stage of the spot-cycle the first members of the new spot-cycle begin to appear in high latitudes, whilst the last members of the expiring spot-cycle have not yet wholly died out in the low latitudes to which they are confined. Thus in 1879 and in 1889 two clearly defined maxima are seen in both hemispheres, separated, as the more detailed diagram in plate 15 of the former Greenwich paper shows, by a broad zone absolutely free from any spots at all. In 1880 the old cycle-spots in low latitude have completely disappeared, and only a single maximum, and that sharply defined, is seen in each hemisphere. In 1890 the process of elimination of the old cycle spots is not quite complete, and they still show feebly; but the new spots in high latitudes are in a most overwhelming preponderance. It is possible to consider two or three very small spots of the following year as being survivors of the old cycle, but there is no possible question of a double maximum.

(*j*) The real exceptions to (*i*) occur, therefore, only immediately after the sun-spot minimum, precisely in accordance with "Spoerer's Law."

Some time ago Dr. Lockyer stated to this Society that "the curves of Spoerer are very misleading, for by taking the mean position of several spot-zones you arrive at a latitude in which spots may not exist at all" (*Observatory*, 1903 June, p. 236). This statement is true only respecting the commencement of the new cycle. Then, and only then, are there definite and distinct zones of spots in either hemisphere. But so far from

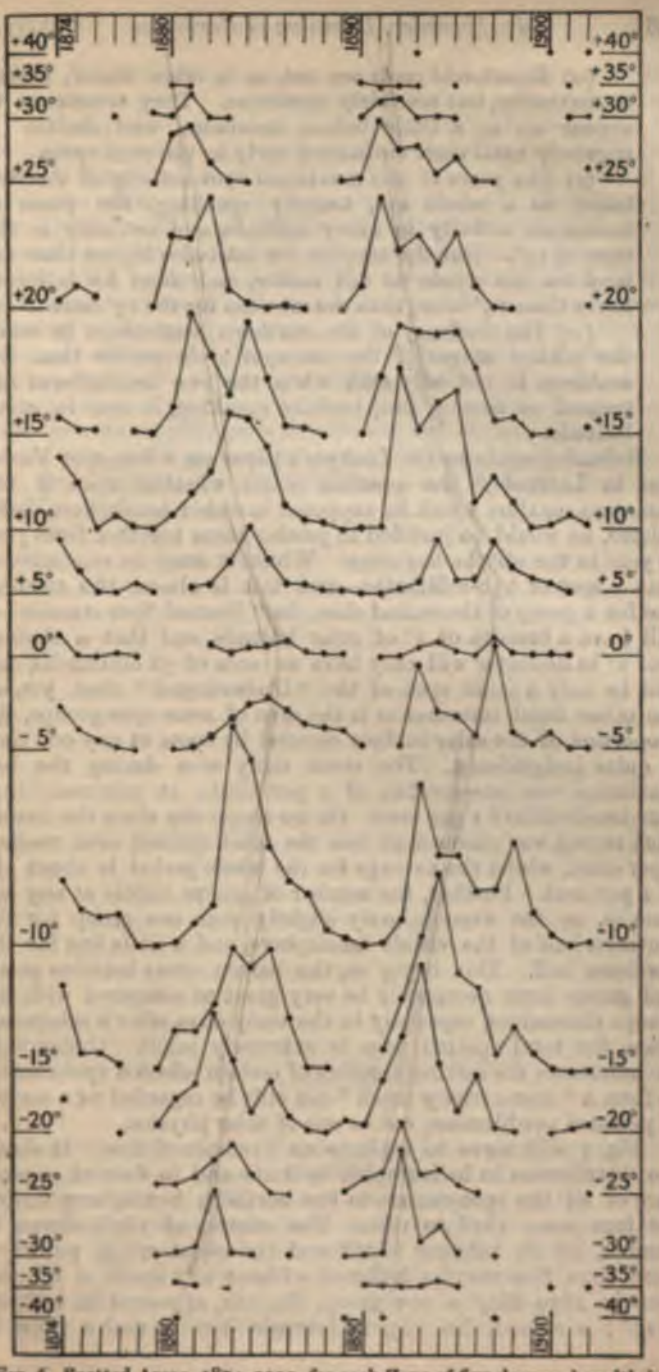


Fig. 6. Spotted Areas, 1874–1902, for each Zone of five degrees breadth in Latitude.

latitude +6°, broke out in July ; and nine others followed before the end of the year. Following Spoerer's enunciation, the group at +28° was clearly the first of the new cycle, whilst that at 6° was as clearly a survivor of the cycle that had just passed ; but in which category should the first group, at +14°, be classed ?

Of the other groups of the year one at +18°, No. 289, may be mentioned. This and the group already referred to at +28° are considered by Dr. Lockyer as commencing two "spot-activity tracks." He writes : "Considering the curves relating to the Sun's northern hemisphere, it will be seen that in 1879, the year following a sun-spot minimum, when the spots were ending a cycle near the equator, two new outbreaks occurred in latitudes about 20° and 30°. These two centres of activity moved towards the equator next year, and by 1881 the former had disappeared, while the other rapidly grew in intensity and reached latitude 15°. During this year a new outbreak in latitude 30° made its appearance, and this in the two following years had an equatoreal trend."

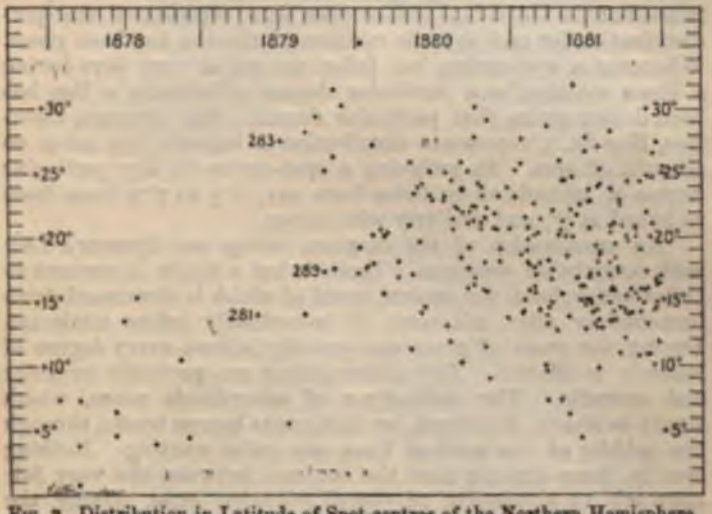
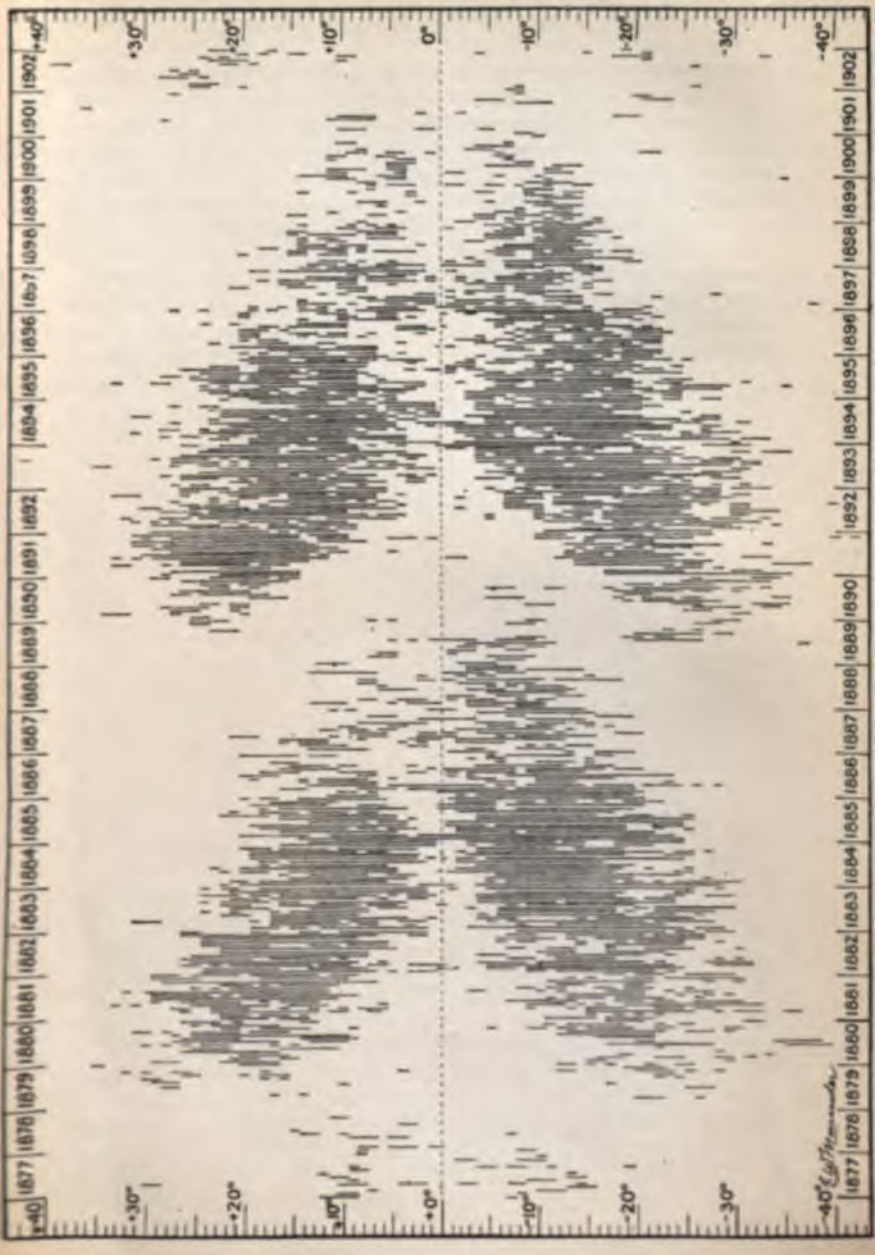


FIG. 7. Distribution in Latitude of Spot-centres of the Northern Hemisphere, 1878–1881.

An examination of the diagram (fig. 7) shows that this association of groups Nos. 283 and 289 with a downward moving current is purely arbitrary. The groups succeeding group No. 283 in or near the same latitude tend very markedly on the whole towards a higher latitude, not towards a lower. So with group No. 289. The first ten groups succeeding it in order of date are all either at the same or at higher latitudes, and it is not until five months after its appearance that another spot forms in


+40°
+30°
+20°
+10°
0°
-10°
-20°
-30°
-40°
1877 1878 1879 1880 1881 1882 1883 1884 1885 1886 1887 1888 1889 1890 1891 1892 1893 1894 1895 1896 1897 1898 1899 1900 1901 1902

spot-distribution is steadily and continuously mounting. Fig. 7 shows this in detail for the northern half of the first cycle, group No. 283 and its successors moving steadily upwards. Inasmuch as the general increase in spot activity is eventually seen in all zones there must be a broadening of the general area of disturbance, and therefore it might be expected there would always be a few spots which could be selected in order to favour the idea of short-lived downward currents. In 1889 in the southern hemisphere, however, the current moved upwards, and upwards only. There were no spots for a year to offer any excuse for the idea of a downward moving "spot-activity track." In the previous cycle a similar feature was presented, but was not quite so strongly marked. The limit, therefore, of spot-distribution in latitude varies in precise analogy with the variation in area—that is to say, it shows a swift run up to maximum, a slow and undulating movement down to minimum.

Though the diagram shows clearly that there is but a single spot-zone in either hemisphere in each of these two cycles, a zone which moves in general accordance with Spoerer's curves, it reveals a striking and unexpected fact—namely, that the southern current not only reaches the equator, but crosses it. The limit which bounds spot-distribution in the southern hemisphere on the equatorial side can be traced not only as far as the equator, but beyond it.

Fig. 8 therefore shows :

(*q*) That Spoerer's Law is completely true within the limits of its enunciation.

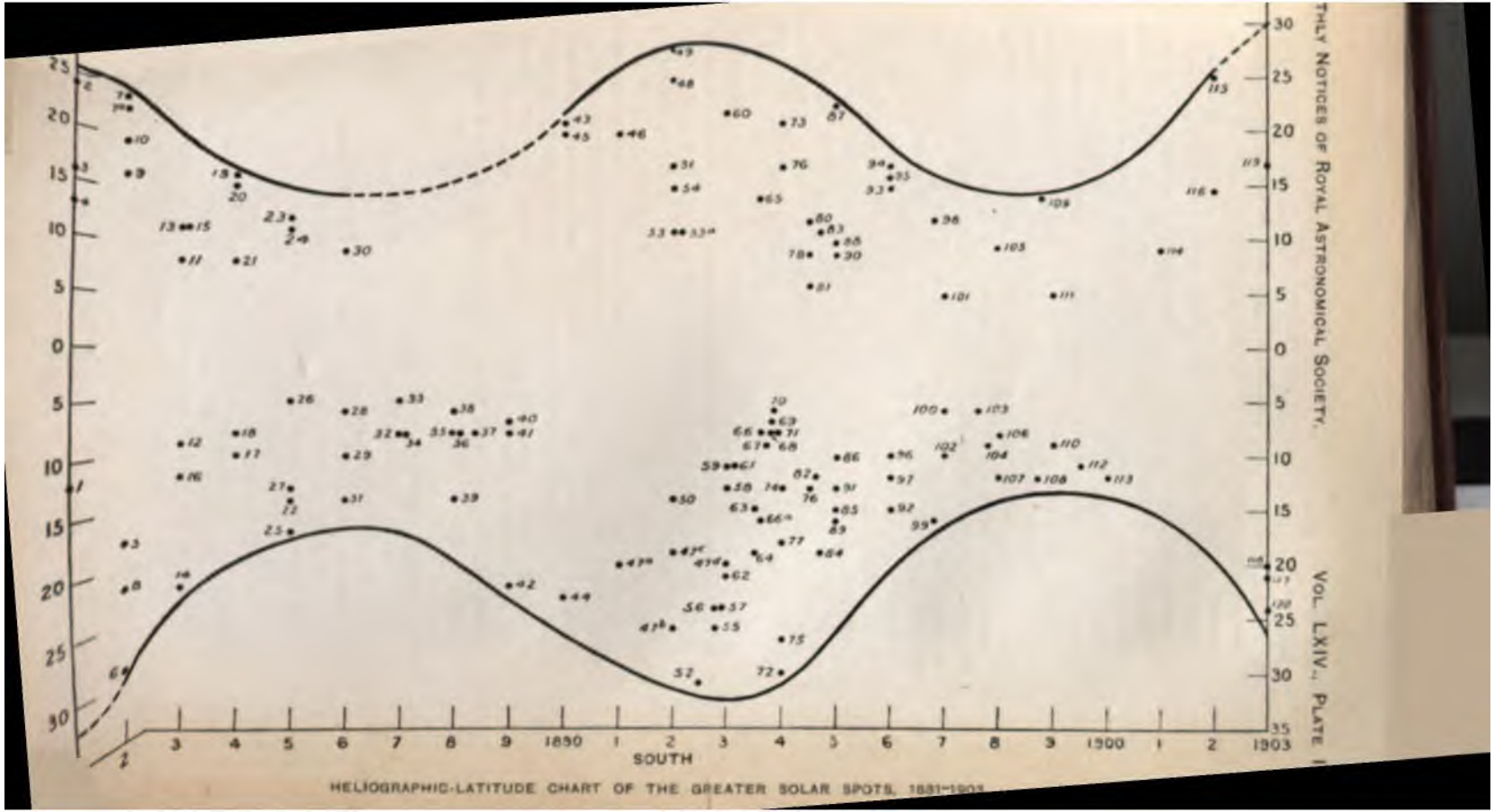
(*r*) That there is only one spot-zone—a very broad one—in either hemisphere except during the brief period just after minimum, when two cycles overlap.

(*s*) That Spoerer's Law is defective in so far as it does not take note of the short period during which the limit of spot-distribution moves upward at the beginning of a new cycle.

(*t*) That it is also defective in not taking note that the downward movement of the limits of spot-distribution may, as in 1886 *et seq.*, lead one of the two spot-zones right across the equator.

On the whole the results of the Greenwich record for the twenty-nine years 1874 to 1902, briefly summarised on pp. 452 and 453 of the *Monthly Notices*, vol. lxiii., fully confirm and bear out "Spoerer's Law of Spot-zones," whilst indicating some important additions to it.

86 *Tyrwhitt Road, St. John's, S.E.:*
1904 *June* 9.



HELIOGRAPHIC-LATITUDE CHART OF THE GREATER SOLAR SPOTS, 1881–1903

MONTHLY NOTICES OF ROYAL ASTRONOMICAL SOCIETY. VOL. LXIV., PLATE I.

mum, les taches, qui avaient depuis longtemps déserté les hautes latitudes, s'y montrent brusquement vers $\pm 30°$" (*Comptes Rendus*, cviii. 486). The process, therefore, is that of a succession of overlapping waves of activity descending from higher to lower latitudes. All solar observers will be agreed as to the general correctness of the first proposition. An inspection of Spörer's own charts, published in the Potsdam Observatory volumes, is sufficiently convincing on this point. At the same time all will equally admit that a mean latitude curve of sun-spots drawn for each solar hemisphere north and south of the equator is not a continuous curve, but that while on the whole the decline in latitude is gradual and continual, it proceeds in a series of waves. This is well shown in the curve drawn by Father Braun from observations taken at Kalócsa during the period 1880–1884, and reproduced in Dr. Lockyer's paper on this subject (*Proc. R.S.*, vol. lxxiii., and *Monthly Notices*, Appendix to vol. lxxiii.). In this paper, however, Dr. Lockyer comes to the conclusion that Spörer's law gives only a very general idea of sun-spot circulation, and that the curves are but the integrated result of a series of well-defined spot-tracks which are analogous to atmospheric storm-tracks on our globe. These "spot-activity tracks," according to Dr. Lockyer, commence successively in higher latitudes as the maximum is approached, and then fall nearly continuously in latitude. Instead, therefore, of two large overlapping waves, the action would be a succession of smaller waves or tracks; and whereas, according to Spörer, a large mean latitude wave falls continuously from higher latitudes at maximum to low latitudes at minimum, the starting points of Dr. Lockyer's tracks increase in latitude as maximum is approached. Still they all tend downwards, their deviations being from higher to lower latitudes. But it would appear from a study of the mean heliographic latitudes of the greater disturbances for the period under discussion, that the wave of sun-spot activity is highest at maximum, descends as minimum is approached, and after minimum that the direction is polewards until a second maximum is attained. This hypothesis would substitute for Spörer's large overlapping high to low latitude waves and Dr. Lockyer's series of "spot-activity tracks" a wave which falls from maximum to minimum and then rises. In so far as Dr. Lockyer places the successive waves with their starting points in higher and higher latitudes as maximum is approached, this suggested wave of "limiting sun-spot latitude" would pass through his successive points, but would disagree in making the trend of activity polewards from minimum to maximum, instead of towards the equator. It would contravene, too, the second main proposition of Spörer's law. But it would bring the variation of latitude in the case of greater spot disturbances into accord with the general poleward tendencies of the centres of prominence action from minimum to maximum, and the gradual opening out of the great coronal streamers as maximum is approached. At times of

years 1889–90, disturbances about the same latitude continued to occur until the end of 1895, though they ceased in the belt −25°–30° for the southern hemisphere in 1894 May and for the northern hemisphere after 1892 March. Spörer has already indicated the mean latitudes ±20° as the seats of great disturbances about the times of maximum. At the times of maximum too not only are greater disturbances found in high latitude zones, but in all zones. For instance, the belt at mean latitude −8° S., which had shown many signs of activity during the minimum years 1887–1889, again became active in 1893 August, a maximum year, no fewer than six greater disturbances occurring in a belt between 1893 August and 1894 February. The same belt or zone was not again active until 1897–1899, after which the greater disturbances began to go polewards once more. Very remarkable too is the persistence of a region of disturbance between the limits −20°–26° from 1891 September to 1893 March, the groups being numbered on the chart 47*a*, 47*b*, 47*c*, and 47*d*, their life-histories showing that they were all linked by means of smaller disturbances in the same region. The great magnetic storm of 1892 February was connected with this persistent activity, as indicated by the spot-groups. Two smoothed curves have been drawn on the chart to indicate the nature of the variation in latitude, as shown by the greater disturbances, and these have been called curves of "limiting latitude." The dotted portion of the curve for the northern hemisphere denotes a dearth of points to settle the trend of the curve for the period 1886–1890, though the fall from 1882 to 1886 and the rise subsequently to 1890 is very evident, and similarly for the southern hemisphere in 1900–2.

In Dr. Lockyer's paper (*loc. cit.*) the "spot-activity tracks" are formed by joining together successive centres of sun-spot activity, such as are tabulated and shown diagrammatically in the Greenwich results already referred to, the observations being grouped in strips of latitude 3° in width. Assuming a general trend of sun-spot activity towards the equator from the poles, such as is demanded by Carrington and Spörer, and given a number of centres of activity, it is evident that such tracks can be formed by joining together the different centres of activity. But the validity of this interpretation of the variation of the spots in latitude may be questioned when it is observed that various centres of activity that ascend from lower to higher latitudes, from minimum to maximum, are taken as the starting-points of tracks which are descending. Would it not be at least equally legitimate to make a series of ascending tracks at such epochs, which would be more consonant with the trend of the centres of prominence activity? Again, although there is distinct evidence of various latitude zones becoming intermittently active at different times in a spot cycle, and prescinding from the fact that in order to form "spot-activity tracks" it is necessary to join together spots that differ widely in longitude

be due to the overlapping of cycles of activity, as demanded by Spörer's law.

6. Nor is it apparently due to the existence of a series of "spot-activity tracks," the subsidence of activity from maximum to minimum being zonal and the foci of activity in the zones occurring in different longitudes.

The Rotation Period of Saturn. *By* W. F. Denning.

The conspicuous spots visible on *Saturn* in 1903 offered an opportunity for accurately determining the rotation period of the planet which it was most desirable to utilise to the best possible advantage. I regretted, therefore, to see that Professor Hough had apparently misidentified certain of the markings, and deduced a rotation period in excess of the correct value (*Monthly Notices*, 1904 December).

Professor Hough places an exclusive reliance on the micrometric method of taking transits, and gives the times to tenths of a minute, but the number of the observations he employs are so limited that at least very grave doubts must exist as to his identifications. He regards all eye-estimated transits of markings on *Jupiter* and *Saturn* as useless, though Professor Barnard, Mr. Stanley Williams (*Monthly Notices*, 1904 March), and others have clearly shown that the latter are comparable with the former in regard to accuracy.

It is indeed quite fair to say that if all the micrometrically measured transits of *Jupiter's* spots were put on one side, and only the eye-estimated transits retained, our knowledge of Jovian phenomena would not suffer in the least!

The conclusions I arrived at respecting the identification and rotation period of the spots on *Saturn* (*Monthly Notices*, 1904 March) are, I believe, perfectly sound, and this is also the opinion of several well-qualified observers who have not only obtained transits, but have pretty thoroughly compared and investigated the details.

As I have previously stated, the identification of the individual objects is a point of greater importance than the method of taking transits; and to be certain of the correctness of the identifications a fair number of observations must be accumulated. The materials adduced by Professor Hough in his paper (*Monthly Notices*, 1903 December) are obviously too scanty for the individual spots to be followed with certainty. The spots were quite numerous, some of them pretty close together, and liable to variation in brightness, shape, and motion. Professor Hough relies upon an observation, after an interval of three weeks, of a "faint, difficult" spot on August 19 as being the same as Barnard's, though it was more than 5,000 miles nearer the

Observations at sea differ from those on land in this respect: while on land the utmost accuracy is but a question of meteorological conditions, at sea considerations of a mechanical nature, even with the most favourable weather, prevent the obtainment of more than an approximation. Indeed, a greater exactitude would not be greatly beneficial; one cannot be certain of the speed of a tide current more closely than within half a mile or so—even a good deal of experience is needed to ascertain it as closely—and no refinement in navigating methods would justify the captain of a ship to pass a rock at a distance of but a hundred yards, on pretence that he knows his position with mathematical accuracy. Where rocks are to be found tidal streams generally prevail, and in the best conditions a quarter of a mile is the closest berth the ablest navigator dares to give them.

Taking into consideration the various factors that enter into the determination of the position of a ship at sea, it may safely be said that to know that this position is within a circle on the chart of half a mile radius, or, to be more correct, and keeping in view the coordinate nature of the data, within a square the side of which is a nautical mile, is all that is required, and it is rare to obtain with certainty a smaller area of position, except in the neighbourhood of land, in calm, clear weather, by station-pointer observations of some selected landmark.

The sea horizon gives the above approximation in fine weather, when there is no abnormal refraction—a fact of which one is not generally aware when taking the observation. It fails entirely when the reflection of the Sun falls on the sea line, in misty weather, and at night, except in few special cases, it is at best undefined. Although whenever it is available it answers all the requirements of navigation, in many cases the opportunity of a good "fix" allowed by a clear day sky or a starry night is lost or gives but a very rough indication of position, because of the want of a reliable horizon.

I shall not refer to the various appliances that can be used to obtain this line of reference, of paramount importance to the navigator. I have tried several of them, and found them to be either unreliable or to need such preparations and favourable state of the weather as to be unpractical. Amongst the unsuccessful attempts to solve the problem I will, however, mention two, which have a special interest with regard to the Gyroscopic Collimator. One, invented, I believe, some sixty years ago, consisted in a horizontal mirror fixed on the top of a revolving gyroscope. The very nature of the support was the cause of failure, as the axis of such a revolving body is rarely vertical. The other was a pendulum, altitudes being taken with regard to a horizontal line marked on the bob when the latter was at three consecutive positions—at the beginning, middle, and end of a double oscillation. The altitude was then calculated for the mean position of the mark. This may be realised with a fixed point of suspension, but at sea such a point is nowhere to be

by the inflow of air from the apertures A in communication with the atmosphere at E, when air is pumped out of the case B by the tube D.

On the top of the gyroscope is the optical apparatus giving the sensible horizon. This consists in a plano-convex lens L and a field grating S, placed at the opposite extremities of a same diameter, the grating, which is a succession of equidistant transparent lines perpendicular to the axis of rotation, on a black background (fig. 3), being at the focus of the lens.

The axis of this optical system is in the same horizontal plane as the line of sight of the observing telescope, the case B having two windows W facing each other along the line of sight, so that the light of an electric lamp, or of the Sun—reflected, in the latter case, in a small mirror fixed on the case—can pass through the latter and be received by the telescope whenever the axis of the optical system fixed on the gyroscope is coincident with the vertical plane through the collimation line of the telescope; this occurs at every half-revolution. As the normal speed is about 100 revolutions per second a persistent image of the grating is observed in bright lines on a dark background, on which the image of the observed body can be brought down.

When a top so supported is made to revolve round its axis, this axis describes a cone of decreasing amplitude, and a point on it will describe a closing spiral. If the eye is at about the level of the pivot, and in a fixed vertical plane through it, the axis will be seen to oscillate on each side of the vertical, exactly as a pendulum, the amplitude of the oscillations decreasing gradually. By decreasing the distance of the centre of gravity from the pivot, or increasing the speed of rotation, the time of a cycle of precession can be increased, so as to minimise the influence of the motion of the vessel. We have thus a short pendulum with the time of oscillation of a very long one. This motion of precession is really made up of a succession of minute loops, similar to the loops described by the Earth in its nutation, but, with a sufficient speed of rotation, these loops are too small to be detected by the eye, as it is the point of the pivot, and not the centre of gravity, which describes them.

When the axis of the gyroscope is vertical the middle of the dark space of the grating is the trace of the sensible horizon in the field, a constant instrumental error $\pm e$, made up of two parts, (*a*) the instrumental error of the sextant itself, and (*b*) the collimation error of the gyroscope, being allowed for. The axis, however, is rarely vertical, and its spiral motion will give a balancing motion to the lines of the grating. When the axis of rotation is in the plane of the sextant the lines are horizontal and parallel to a spider thread in the telescope, perpendicular to the plane of the sextant, the centre line being below or above the plane of the sensible horizon, according to the position of the axis, inclined towards or away from the observer. As the gyroscope rotates the lines become slightly inclined to the horizontal,

From these three readings and the time of a cycle of precession, given by the interval between them, a correction I shall consider later on, due to the motion of the Earth, is calculated, together with the altitude corresponding to the motion of the gyroscope when the axis is vertical. In other words, the effect of the rotation of the Earth and of the erecting tendency of the gyroscope must be ascertained.

To observe well-defined maxima and minima the precession must have a certain minimum value, especially for the observation of a celestial body out of the meridian, the movement in altitude of which may attain 15″ per second of time when on the prime vertical. Another method is to take a succession of readings as near each other as possible, either when the precession has almost ceased or with the full precession, as for the maxima and minima method. These, plotted with the time, will give in the first case almost a straight line from which the readings corresponding to any time may be taken; and, in the second, a wavy line, on which the maxima and minima can be selected and the corresponding times taken from the curve.

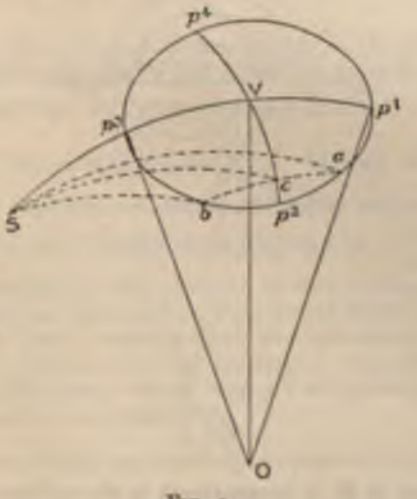


Fig 4.

When the celestial body is motionless, as it is approximately in a meridian observation, we can either take the mean of the three readings, and then we have Obs. Alt. $A = Ai + \frac{1}{4}(a' + 2a'' + a''')$, where Ai is the reading on the limb corrected for instrumental error—that is, $Ai = Ao \pm e$, Ao being the reading on the limb—or we can apply to any of the three readings a correction to bring it to what it would have been had the axis of the gyroscope been vertical by calculating the coefficient k of tendency to

$\frac{Vp2}{p2p3} = k =$ constant (as long as the pivot does not alter in shape). In this expression $k = \cdot5 + E$, E being proportional to $\frac{Cr}{d^3}$, where C = constant dependent upon the state of surfaces, r = radius of the spherical point, d = distance of the point from the centre of gravity (fig. 5).

The greater the radius of curvature of the point the greater k, the greater the speed with which the axis becomes vertical; k is therefore a coefficient of tendency to verticality. With a mathematically pointed pivot $k = \cdot5$ and the loxodrome becomes a circumference. All is therefore as if the positions Op, $Op1$, $Op2$ . . . were the directions of a simple pendulum oscillating in the plane OVS.

But $SV = \frac{1}{2}(Sp + Sp1)$ is not accurate; we have

$$SV = Sp + pV = Sp + kpp1,$$
$$SV = Sp1 - Vp1 = Sp1 - kp1p2,$$
$$SV = Sp2 + Vp2 = Sp2 + kp2p3,$$
. . . . . .

Meridian Observation.—If the body is on the meridian three observations are sufficient to find k, for we have from the three equations above $Sp1 - Sp = k(pp1 + p1p2)$, and $k = \frac{pp1}{pp1 + p1p2}$

Let $pp1 = M$, $p1p2 = N$, then $k = \frac{M}{N+M}$, M and N being the amplitudes of the first and second oscillations respectively. For the three positions of the star shown (fig. 3) $k = \frac{139}{139+114} = \cdot55$.

If Ai is the reading on the limb, $s1$, $s2$ two successive readings on the grating, sx the reading that would have been made if the axis had been vertical, then the calculated altitude is $Ac = Ai + sx$. When the first reading $s1$ was taken S was $s1$ from the central line, which was $pV = kpp1 = Mk$ from the horizon, and so $sx = s1 \pm Mk$, then

$$Ac = Ai + s1 \pm Mk, \text{ and similarly } Ac = Ai + s2 \pm Nk \ldots .$$

But sx is between $s1$ and $s2$; no error on the sign of M or N is possible, it is always the opposite sign to $s1$ for M or $s2$ for N.

The reading on the limb Ai has, of course, been corrected for the instrumental error: this is complex, being partly due to want of parallelism between the reflecting mirrors of the sextant, to want of perpendicularity of these with regard to the plane of the instrument, and to the collimation error of the gyroscope itself, the latter error being due to the fact that the centre of the grating is not always exactly in the plane of the sensible horizon, even when the axis is vertical. Another correction in

is $s''_2 = s2 \pm Nk$. The mean will become $s_2 = \frac{1}{2}(s'_2 + s''_2) = \frac{1}{2}(s1 \pm Mk + s2 \pm Nk) = \frac{1}{2}(s1 - s2) + k\frac{M-N}{2} = \frac{s1+s2}{2} + \frac{M(M-N)}{2(M+N)}$, but $\frac{1}{2}M = \frac{1}{2}pp1 = \frac{1}{2}(Sp1 - Sp2) = \frac{1}{2}(s2 - s1)$, so that

$$s_2 = \tfrac{1}{2}(s1+s2) - (M-N)\frac{\frac{1}{2}(s2-s1)}{M+N}.$$

Let $\frac{M-N}{M+N} = R$, then $s_2 = \frac{1}{2}(s1+s2) + \frac{1}{2}R(s2-s1)$; the correction is much smaller. The mean of the means is $\frac{1}{2}\left(\frac{s1+s2}{2} + \frac{s2+s3}{2}\right)$; then

$$s_2 = \tfrac{1}{2}\left(\frac{s1+s2}{2} + \frac{s2+s3}{2}\right) + \tfrac{1}{2}R\left(\frac{s2-s1}{2} - \frac{s2-s3}{2}\right).$$

The correction is smaller still; it may be written

$$\frac{M-N}{2(M+N)}\left(\frac{pp1}{2} - \frac{p1p2}{2}\right) = \frac{M-N}{2(M+N)}(\tfrac{1}{2}M - \tfrac{1}{2}N), \text{ and we have}$$

$$s_2 = \tfrac{1}{2}\left(\frac{s1+s2}{2} + \frac{s2+s3}{2}\right) + \frac{(M-N)^2}{4(M+N)}.$$

The correction being very small, a set of curves gives its value with a sufficient exactitude. (See *Annales Hydrographiques* for 1901.)

Observations out of the Meridian.—It is preferable to reduce all the observations to the same instant by correcting for the motion in altitude of the celestial body: $dh = \frac{1}{4}\sin Z \cos L dT$, and the case is simplified into the preceding one, where the body has no motion along its vertical. The calculation is long, but is simplified by two tables due to Enseigne de Vaisseau Cretin (*Annales Hydrographiques*, 1901, pp. 96 and 98). Table I. gives in seconds of arc the motion of a body in altitude for each second of time: $dh = 15 \sin Z \cos L$, argument: Z and L. Table II. gives, in function of the result found in Table I., the movement in altitude for an interval of 30 to 74 seconds, argument: T and dh from Table I.

Influence of the Motion of the Earth.—As may be expected the motion of the gyroscope is perceptibly influenced by the motion of the Earth. The direction of the motion of precession depends on the direction of rotation of the gyroscope and on the position of the pivot with regard to the centre of gravity. The rotation being clockwise, looking from above the instrument, the precession is counter-clockwise.

The effect of the rotation of the Earth is to make the axis of the gyroscope precess round a deviated vertical, the true zenith being then between the apparent zenith and the celestial pole. For our hemisphere the altitudes observed, facing north, will be always too small, those facing south always too large, the sign of the correction depending of course on the direction of rotation

The number of turns can only be ascertained by careful stroboscopic measurements, and this being a delicate operation it would be impossible to perform it as an auxiliary observation when taking the altitudes. A lengthy series of observations was therefore made in 1902 and 1903 by M. L. Favé, at the Service Hydrographique, to ascertain how far the theory was able to give formulæ from which the number of turns could be deduced.

The mechanical constants necessary for these investigations were obtained by oscillating the gyroscope at rest. The gyroscope is really a compound pendulum, and its oscillations are of a somewhat complicated nature, being plane but for a very short time and soon becoming conical. The axis then describes an elliptic cone, the major axis of which rotates in azimuth, the eccentricity decreasing till, after the major axis has turned 45°, the ellipse has become a circle; after this it becomes an ellipse again. After a while the oscillation is again in a plane perpendicular to the primitive plane, and the same series of motions reproduces itself in reverse order till the oscillation is again in the primitive plane. This is due to the fact that the body is not entirely of revolution; moreover the pivot is not theoretically sharp. The determination of the time of oscillation is therefore a delicate operation.

We have then

$$\theta = \pi\sqrt{\frac{d+\frac{B}{md}}{g}}$$

where

θ is the time of a simple oscillation in seconds,

d is the distance of the point of the pivot to the c. g.,

B is the moment of inertia with regard to a perpendicular to the axis of figure passing through the c. g.;

$m = w/g$ is the mass in grams.

Two values of θ were sufficient to obtain B and d. From B the value of the radius of gyration r was obtained; the value of c could therefore be calculated for any value of N.

Measurements were also taken with a stroboscope at different times, so as to give the decrease of N corresponding to the time elapsed since the launching of the gyroscope. This was done for various degrees of vacuum, and a set of curves was obtained (see *Annales Hydrographiques* for 1904). The time of half a cycle of precession being ascertained at the beginning, when the precession can be easily observed, the time elapsed between this observation and the astronomical observation gives, on the curve corresponding to the vacuum indicated by the gauge on the instrument, the number of turns at the time the altitude was observed, or, by means of a special scale, c directly.

If the motion of the ship was uniform the radius Vp would describe around the vertical 3° in 1 sec. about, if 2T is equal to two minutes. The rolling of the ship will bring the pole successively in T, V, T′ . . . V being a point on the true vertical and T, T′ points on the deviated vertical (fig. 8).

Suppose a rolling period of six secs. for a single oscillation, let the acceleration be in the direction VT, the pole will precess round T for two seconds and will reach 1, the next two seconds (period of uniform motion) it will precess round V and reach 2, then round T′ and reach 3 . . &c.

FIG. 8. Influence of the Motion of the Vessel.
(Precession 2ᵐ 18ˢ, clockwise.)

We see that in the direction TT′ the path of the pole differs little from the circle it would follow if the ship were steady. In the direction T″T‴ the deviation is considerable. In this direction, if the ship were motionless, the central line of the image of the grating would rise above the sensible horizon and descend below it in a quantity corresponding to VO. When the ship is

Cometary Observations at the Liverpool Observatory, 1902–3. By W. E. Plummer.

Comet 1903 *II.* (1902 *d*), *Giacobini.*

Greenwich Mean Time of Observation 1902.				φ − ■ R.A.	No. of Comp.	Apparent R.A. of φ	φ − ■ Decl.	No. of Comp.	App. Decl. of φ	Log. Factor of Parallax in *a*.	in *d*.	Star of Comp.
1902	h	m	s	m s		h m s	′ °		° ′ ″			
Dec. 3	12	10	6·2	−1 20·38	Ret.	7 17 28·19	+ 1 24·7	Ret.	− 1 51 47·0	−0·4801	0·8564	1
6	11	43	16·0	+ 54·33	...	7 16 36·35	− 1 31·0	...	− 1 26 32·9	−0·5161	0·8543	2
11	12	22	29·4	−1 6·31	...	7 14 44·97	+ 1 51·1	...	− 0 38 14·6	−0·3188	0·8513	3
11	12	22	29·4	+1 44·01	...	7 14 44·83	− 2 45·4	...	− 0 38 15·5	−0·3188	0·8513	4
23	9	40	22·9	− 19·74	...	7 8 32·20	+ 6 0·3	...	+ 1 46 13·9	−0·6097	0·8396	5
1903												
Jan. 16	8	40	5·2	− 34·35	...	6 51 37·31	− 16·5	...	+ 8 34 6·0	−0·5139	0·8028	6
21	10	2	5·0	− 55·25	...	6 48 7·57	+ 3 33·2	...	+10 13 24·4	−0·0253	0·7796	7
23	9	21	6·3	+ 14·49	...	6 46 50·64	− 2 1·3	...	+10 52 38·3	−0·2510	0·7771	8
Feb. 17	9	0	13·2	+ 9·69	...	6 35 44·40	+ 6 48·8	...	+18 59 21·9	+0·3472	0·6948	9
20	9	11	16·5	+ 55·28	...	6 36 31·62	− 9 56·6	...	+19 53 17·2	+0·9086	0·6861	10

Comet 1903 *I.* (1903 *a*), *Giacobini.*

Jan. 21	6	33	14·0	+ 44·80	10	22 59 52·57	− 1 36·5	5	+ 2 44 45·5	0·6070	0·8385	11
23	6	35	40·3	− 39·96	10	23 2 2·94	+ 0 20·8	5	+ 3 13 44·6	0·6191	0·8360	12
28	6	10	59·7	+1 34·40	10	23 7 52·42	+ 1 53·1	5	+ 4 30 34·6	0·6029	0·8298	13
28	6	10	59·7	− 10·74	10	23 7 52·37	− 5 35·9	5	+ 4 30 35·3	0·6029	0·8298	14
29	6	20	26·2	+ 11·99	Ret.	23 9 7·74	+ 0 57·7	Ret.	+ 4 46 55·0	0·6274	0·8303	15

Greenwich Mean Time of Observation 1903.				φ − ✱ R.A.		No. of Comp.	Apparent R.A. of φ			φ − ✱ Decl.		No. of Comp.	App. Decl. of φ			Log. Factor of Parallax in α.	Log. Factor of Parallax in δ.	Star of Comp.
1903		h	m	s	m	s	h	m	s	′	″		°	′	″			
July	10	10	40	16·3	−	10·69	8	20	58	15·93	+ 2	22·3	4	+29	9	18·0	−0·6642	0·6534	33
	10	10	40	16·3	+	35·63	8	20	58	15·96	− 5	50·1	4	+29	9	18·4	−0·6642	0·6534	34
	13	10	30	22·5	−1	25·65	10	20	23	31·32	+ 3	55·1	5	+43	9	12·8	−0·6733	0·3822	35
	15	9	40	14·2	−1	37·32	8	19	45	10·23	+10	22·3	4	+52	58	55·7	−0·7655	0·0432	36
	16	9	32	11·0	−1	46·15	10	19	17	56·67	+ 4	25·2	5	+57	39	26·2	−0·7624	9·2314	37
	22	10	5	20·8	−9	1·83	10	14	37	39·60	− 4	4·0	5	+67	44	59·9	1·0170	−9·8115	38
	24	9	42	16·0	+3	40·88	8	13	30	17·97	− 2	20·7	4	+64	38	42·0	1·0334	9·8676	39
	26	9	40	46·4	+1	13·14	10	12	47	8·17	−	24·1	5	+60	57	12·0	1·0177	0·3291	40
	26	9	40	46·4	+2	42·90	10	12	47	8·50	+ 6	9·5	5	+60	57	13·6	1·0177	0·3291	41
	28	9	32	19·0	+2	16·76	12	12	18	54·25	− 5	21·4	4	+57	25	47·2	0·9847	0·4871	42
	28	9	31	26·3	−1	30·69	10	12	18	53·97	+ 6	48·4	4	+57	25	49·3	0·9847	0·4871	43
	29	9	21	11·2	+5	1·86	8	12	8	17·40	−11	45·4	4	+55	48	32·9	0·9674	0·5207	44
Aug.	3	9	16	15·0	+2	55·35	10	11	32	59·56	+	55·3	5	+49	0	36·3	0·9039	0·6903	45
	4	9	29	20·9	−	19·99	10	11	27	59·62	− 9	11·6	5	+47	52	7·8	0·8897	0·7290	46
	4	9	29	20·9	−	2·03	10	11	27	59·54	+ 5	22·1	5	+47	52	7·9	0·8897	0·7290	47
	6	9	18	26·3	−3	17·13	12	11	19	15·50	+ 4	1·0	5	+45	47	9·7	0·8689	0·7520	48
	9	9	19	59·0	−2	29·21	10	11	7	59·71	−	50·6	5	+42	57	48·9	0·8377	0·7934	49
	10	9	24	30·4	+3	49·79	8	11	4	32·96	− 4	14·2	4	+42	4	44·0	0·8239	0·8092	50
	12	9	16	28·3	−2	42·64	10	10	58	0·62	+ 3	3·4	5	+40	22	7·4	0·8080	0·8217	51

Stars of Comparison. Comet 1903 *II.*

Date. 1902.	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year.	Correction to Mean R.A.	Mean Declination Equinox of Year.	Correction to Mean Declination.	No. of Reference.
		h m s	s	° ′ ″		
Dec. 3	A.G.Z. Nicolajew, No. 2150	7 18 44·33	+4·24	− 1 52 59·3	−12·4	1
6	A.G.Z. „ No. 2132	7 15 37·70	+4·32	− 1 24 49·1	−12·8	2
11	A.G.Z. „ No. 2134	7 15 46·84	+4·44	− 0 39 51·9	−13·8	3
11	Gottingen$_2$, No. 2436–7	7 12 56·37	+4·45	− 0 40 47·2	−13·7	4
23	A.G.Z. Albany No. 2688	7 8 47·19	+4·75	+ 1 40 29·0	−15·7	5
1903. Jan. 16	A.G.Z. Leipzig II., No. 3378	6 52 9·74	+1·92	+ 8 34 35·1	−12·6	6
21	A.G.Z. Leipzig I., No. 2571	6 49 0·89	+1·93	+10 10 3·9	−12·7	7
23	A.G.Z. Leipzig I., No. 2542	6 46 34·21	+1·94	+10 54 52·3	−12·7	8
Feb. 17	A.G.Z. Berlin A., No. 2328	6 36 32·87	+1·84	+18 52 44·4	−11·3	9
20	A.G.Z. Berlin A., No. 2318	6 35 34·54	+1·80	+20 3 24·7	−10·9	10
	Comet 1903 *a.*					
Jan. 21	A.G.Z. Albany, No. 7960	22 59 8·05	−0·28	+ 2 46 19·2	+ 2·8	11
23	A.G.Z. „ No. 7979	23 2 43·18	−0·28	+ 3 13 21·2	+ 2·6	12
28	A.G.Z. „ No. 8002	23 6 18·30	−0·28	+ 4 28 39·1	+ 2·4	13
28	A.G.Z. „ No. 8010	23 8 3·39	−0·28	+ 4 36 8·8	+ 2·4	14
29	A.G.Z. „ No. 8019	23 8 56·03	−0·28	+ 4 45 55·0	+ 2·3	15
Feb. 3	A.G.Z. Leipzig II., No. 11610	23 15 23·53	−0·27	+ 6 20 26·2	+ 2·0	16

Date. 1903.	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year. h m s	Correction to Mean R.A. s	Mean Declination Equinox of Year. ° ′ ″	Correction to Mean Declination.	No. of Reference.
Feb. 12	A.G.Z. Leipzig II., No. 11703	23 31 22·99	−0·24	+ 9 9 1·8	+ 1·3	17
17	Rumker, No. 11503	23 38 31·33	−0·22	+11 3 50·2	+ 1·1	18
17	A.G.Z. Leipzig I., No. 9407	23 37 33·55	−0·23	+10 42 20·7	+ 1·1	19
24	A.G.Z. „ No. 9476	23 48 47·11	−0·20	+13 24 53·5	+ 0·6	20
24	A.G.Z. „ No. 9482	23 49 19·93	−0·20	+13 37 24·2	+ 0·6	21
Mar. 6	A.G.Z. Berlin A, No. 47...	0 10 59·26	−0·15	+17 1 28·9	− 0·2	22
7	A.G.Z. „ No. 65...	0 14 5·78	−0·15	+17 2 2·2	− 0·3	23
10	A.G.Z. „ No. 100	0 21 57·38	−0·14	+17 21 59·6	− 0·6	24
11	„ „ „	...	−0·13	...	− 0·8	24
12	A.G.Z. „ No. 109	0 22 59·40	−0·13	+17 21 20·9	− 0·9	25

Stars of Comparison. Comet 1903 IV.

Date. 1903.	Star's Designation or Authority for Place.	Mean R.A. Equinox 1903. h m s	Correction to Mean R.A. s	Mean Declination Equinox, 1903. ° ′ ″	Correction to Mean Declination.	No. of Reference.
June 22	A.G.Z. Ottak., 85, 166	21 51 6·90	+2·40	− 7 26 24·6	+16·7	26
21	A.G.Z. Nicolajew, No. 5517	21 45 15·46	+2·60	− 1 3 26·8	+15·9	27
21	„ „ No. 5512	21 42 15·59	+2·60	− 0 58 32·5	+15·9	28
29	„ „ No. 5510	21 41 56·42	+2·64	+ 0 24 14·8	+15·9	29
July 1	A.G.Z. Albany, No. 7590	21 39 24·41	+2·71	+ 3 46 52·9	+15·2	30
7	A.G.Z. Berlin A, No. 8737	21 20 38·35	+2·96	+18 2 40·3	+13·5	31

Results of Micrometric Measures of Double Stars made with the 28-inch Refractor at the Royal Observatory, Greenwich, in the year 1903.

(*Communicated by the Astronomer-Royal.*)

The measures were made with a bifilar position micrometer on the 28-inch refractor, focal length 28 feet. The power generally used was 670, but when the definition permitted a power of 1120 was employed for observing very close pairs. A blue glass shade was employed to diminish the light and irradiation when bright stars were observed. The observations were made in variously coloured fields or in a dark field with illuminated wires. The initials in the last column are those of the observers, viz. :—

L. Mr. Lewis. W. B. Mr. Bowyer.
B. Mr. Bryant. H. F. Mr. Furner.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1903.	Obs.
	h m	° ′	°	″				
Σ 3061 ...	0 1	72 43	148·4	7·90	1	8·0 8·0	·854	L.
Σ 3060 ...	0 1	72 28	121·0	3·47	1	8·5 8·7	·854	L.
Ho. 1.	0 6	60 57	170·6	1·14	1	8·5 8·8	·991	L.
OΣ 2 A.C.	0 8	63 34	225·1	17·42	1	6·5 9·6	·917	L.
A.B.			43·2	0·48	3	6·5 8·0	·901	W.B.
			43·6	0·76	1	...	·923	B.
Σ 19	0 12	53 55	135·0	2·29	1	7·0 9·5	·824	L.
B.D. + 20°, 18...	0 12	68 47	135·8	1·24	1	8·9 9·4	·991	L.
Σ 24	0 13	64 23	248·4	4·94	1	7·2 8·0	·879	B.
			247·2	4·94	1	...	·895	W.B.
Σ 25 ...	0 14	74 34	192·0	1·31	1	8·5 8·5	·854	L.
β 779	0 23	66 58	253·8	0·84	1	8·9 9·0	·991	L.
OΣ 12	0 26	36 2	156·4	0·41	1	5·6 5·9	·802	B.
			160·3	0·39	1	...	·953	W.B.
Σ 46	0 35	69 7	191·7	6·81	1	8·0 8·2	·895	W.B.
β 257	0 35	43 17	241·9	0·54	1	8·1 8·5	·824	L.
Σ 60 η Cass. ...	0 43	32 43	227·4	5·63	1	4·0 7·6	·953	W.B.
β 495	0 43	71 51	226·1	0·60	3	7·5 7·7	·838	W.B.
			218·3	0·71	2	...	·896	H.F.
			224·0	0·76	1	...	·923	B.
			219·6	0·59	1	...	·931	L.
OΣ 20... ...	0 49	71 22	321·7	0·48	1	5·9 7·0	·923	B.
			314·6	0·37	1	...	·931	L.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1903.	Obs.
β 524 A.B.	2 47	52 5	201·9	0·18	1	5·5 6·4	·192	L.
			180·5	0·17	1	...	·802	B.
			219·9	0·17	1	...	·923	B.
$\frac{A.B.}{2}$ C.	...	...	236·8	14·43	2	5·5 9·2	·126	L.
			234·7	13·86	1	...	·088	W.B.
β 1173 ...	2 53	66 16	332·0	0·09	2	7·7 8·8	·038	L.
			339·3	0·20	2	...	·109	W.B.
β 525	2 53	68 47	129·0	0·28	3	7·5 7·5	·336	L.
			128·3	0·17	1	...	·129	W.B.
			132·0	0·25	1	...	·923	B.
Σ 333	2 53	69 4	201·6	1·38	2	5·7 6·0	·474	L.
			202·9	1·27	2	...	·470	W.B.
			201·5	1·19	1	...	·099	H.F.
Σ 346...A.B.	3 0	65 8	267·1	0·43	1	6·0 6·0	·060	L.
			269·4	0·48	3	...	·624	W.B.
$\frac{A.B.}{2}$ C.	...	...	352·0	5·19	1	6·0 10·8	·060	L.
			352·6	5·02	1	...	·088	W.B.
β 530	3 9	67 25	194·3	1·58	1	9·7 10·1	·041	W.B.
Oz 53	3 11	51 44	228·4	0·65	1	7·2 8·0	·192	L.
			230·8	0·52	1	...	·934	B.
			229·3	0·66	2	...	·938	W.B.
Σ 412...A.B.	3 29	65 52	4·6	0·25	2	6·6 6·7	·471	W.B.
			6·1	...	1	...	·072	L.
			18·6	0·21	1	...	·923	B.
β 533	3 29	58 39	47·2	0·51	4	8·5 8·5	·692	W.B.
			48·7	0·55	1	...	·934	B.
Σ 425	3 34	56 10	90·6	2·44	5	7·3 7·3	·748	W.B.
β 535	3 38	58 2	51·6	0·81	1	4·0 8·5	·923	B.
β 880	3 38	58 9	355·1	0·71	1	8·5 8·5	·923	B.
Hussey 103 ...	3 38	40 27	209·5	0·89	1	8·1 8·4	·115	L.
β 536 $\frac{A.B.}{2}$ C.	3 40	66 6	292·9	38·30	1	8·2 8·2	·060	L.
A.B.	...	...	292·9	...	1	8·2 8·7	·060	L.*
β 1184... ...	3 42	67 56	274·1	0·50	4	8·1 8·3	·472	W.B.
Oz 66	3 45	49 30	139·6	0·72	1	7·5 8·0	·192	L.
Σ 483	3 57	50 45	229·8	0·46	1	8·0 9·5	·129	W.B.

* Elongated.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1903.	Obs.
	h m	° ′	°	″				
Σ 749	5 31	63 8	169·4	0·96	2	7·1 7·2	·027	L.
			172·1	0·83	2	...	·086	W.B.
			170·4	0·83	1	...	·118	H.F.
β 1240...A.H.	5 32	59 34	319·1	0·20	2	5·6 6·0	·140	B.
			332·5	0·21	1	...	·129	W.B.
Σ 753...A.C.	...	...	267·6	12·42	2	5·6 8·7	·567	W.B.
OΣ 112 ...	5 33	52 6	69·6	0·60	2	7·5 7·5	·162	W.B.
β 1032 ...	5 34	92 39	337·3	0·23	2	4·0 5·0	·140	B.
β 1007 ...	5 36	73 31	253·7	0·21	2	6·0 6·2	·140	B.
OΣ 118 ...	5 42	69 10	311·6	0·64	4	6·2 7·7	·114	W.B.
			308·7	0·74	1	... ...	·118	H.F.
OΣ 119 ...	5 43	82 4	318·3	0·45	1	7·5 8·3	·189	W.B.
β 560	5 43	60 18	156·7	0·73	1	8·0 8·5	·016	L.
* near β 560 ...	5 43	60 20	165·0	3·93	2	8·8 9·8	·188	L.
Σ 799	5 45	51 28	184·1	1·02	2	7·2 8·3	·162	W.B.
Aitken 54 ...	6 3	60 45	351·7	0·46	1	7·6 8·5	·189	W.B.
β 1241 A.B.	6 4	66 52	339·0	...	1	5·9 10·0	·208	L.
A.C.	...		63·1	17·85	1	5·9 14·0	·208	L.
β 1058 ...	6 4	66 59	287·5	0·32	1	6·4 6·5	·167	B.
			273·7	0·29	1	...	·189	W.B.
Aitken 55 ...	6 5	61 13	288·1	0·58	1	8·7 9·3	·189	W.B.
Aitken 56 ...	6 7	60 56	51·8	1·46	1	8·1 11·8	·189	W.B.
β 895 A.B.	6 14	61 31	143·4	0·24	1	7·5 7·5	·167	B.
			170·4	0·19	1	...	·189	W.B.
Σ 888 $\frac{A.B.}{2}$ C.	6 14	61 31	253·0	2·50	1	7·5 9·5	·038	L.
			250·9	2·88	1	...	·189	W.B.
Ho. 25... ...	6 16	64 43	250·9	0·89	1	8·8 9·0	·189	W.B.
β 1020 ...	6 17	61 11	155·9	1·12	1	8·2 10·0	·038	L.
Σ 899	6 17	72 22	19·8	2·29	3	7·0 8·0	·096	W.B.
			19·6	2·19	2	...	·109	H.F.
β 1021 ...	6 25	61 32	90·8	0·77	2	8·0 9·0	·169	L.
			83·7	0·74	1	...	·934	B.
Σ 932	6 29	75 10	326·6	2·07	3	8·2 8·3	·094	W.B.
			331·3	1·94	2	...	·109	H.F.
OΣ 149 ...	6 30	62 38	279·3	0·94	1	6·5 9·0	·110	W.B.
			276·9	0·64	2	...	·169	L.
			274·6	0·73	1	...	·923	B.
Σ 945	6 33	48 56	276·2	0·89	1	7·1 8·0	·934	B.

Star's Name.		R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1903.	Obs.
				110·6	5·38	1	...	·208	L.
				109·1	5·25	2	...	·211	W.B.
	B.C.	...	...	126·0	5·86	2	5·7 6·5	·256	W.B.
				124·8	6·03	1	...	·175	H.F.
				123·4	5·99	1	...	·208	L.
β 1244	...	8 7	89 43	37·4	0·69	1	7·9 8·1	·167	B.
β 1243	...	8 8	72 2	346·5	1·03	1	7·1 13·0	·208	L.
Σ 1202	...	8 8	78 51	317·0	2·59	1	7·7 9·8	·145	L.
				314·7	2·58	1	...	·170	W.B.
				322·0	2·41	1	...	·175	H.F.
Σ 1216	...	8 16	91 17	199·0	0·37	1	7·5 8·2	·167	B.
β 585 ...	...	8 36	69 10	106·4	0·59	1	7·7 9·0	·170	W.B.
				108·4	0·49	1	...	·208	L.
Σ 1273	A.B.	8 41	83 13	26·6	0·34	1	3·8 6·0	·282	W.B.
	$\frac{A.B.}{2}$C.	...	...	230·3	3·44	1	3·8 3·7	·145	L.
				235·8	3·34	3	...	·246	W.B.
				235·1	2·90	2	...	·225	H.F.
β 1068	A.B.	8 44	80 46	189·0	0·30	2	7·7 8·8	·236	W.B.
	A.C.	...	...	310·9	17·58	1	7·7 13·0	·282	W.B.
Perrotin	...	8 46	81 17	347·7	0·69	1	7·5 8·7	·145	L.
				350·2	0·65	2	...	·249	W.B.
Σ 3121	...	9 12	61 0	24·8	0·79	3	7·5 7·8	·189	B.
				26·9	0·68	2	...	·184	W.B.
				20·1	0·73	1	...	·230	L.
Aitken 221	...	9 13	59 50	304·3	0·44	1	8·7 8·8	·304	L.
				301·6	0·43	1	...	·304	W.B.
Ho. 43...	...	9 13	68 47	297·5	0·47	2	8·0 8·5	·229	W.B.
				291·2	0·49	1	...	·208	L.
OΣ 201	...	9 18	61 39	221·2	1·53	4	7·5 9·0	·176	W.B
β 1070...	...	9 18	63 20	82·8	0·70	1	9·1 10·2	·309	W.B.
Σ 1348	...	9 19	83 13	319·2	1·73	2	7·5 7·6	·320	W.B.
Aitken...	...	9 20	58 25	43·5	0·84	1	8·7 10·2	·304	L.
				44·2	1·11	1	...	·304	W.B.
Aitken 222	...	9 20	60 55	324·3	0·34	1	8·3 8·5	·309	W.B.
Σ 1356	...	9 23	80 31	111·0	0·92	1	6·2 7·0	·167	B.
				117·6	0·82	2	...	·223	W.B.
				115·6	0·80	1	...	·285	L.
				117·5	0·72	1	...	·290	H.F.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1903.	Obs.
			110·7	1·35	1	...	·309	W.B.
Ho. 48... ...	10 59	66 18	6·5	1·58	1	8·3 11·0	·290	H.F.
			9·9	2·17	1	...	·304	W.B.
Σ 1517 ...	11 8	69 19	271·2	0·42	4	7·3 7·3	·285	W.B.
			268·5	0·33	1	...	·288	B.
Σ 1523 ...	11 13	57 54	142·8	2·47	2	4·0 4·9	·178	W.B.
			142·4	2·15	1	...	·290	H.F.
Σ 1527 ...	11 14	75 11	18·0	3·31	4	6·9 8·1	·254	W.B.
			15·2	3·34	1	...	·290	H.F.
Σ 1534 ...	11 17	71 15	325·6	5·44	2	8·0 11·2	·286	W.B.
Σ 1536 ...	11 19	78 55	53·8	2·24	3	3·9 7·1	·241	W.B.
Lalande 21846	11 24	58 59	8·8	1·00	1	7·0 11·5	·304	L.
			8·8	1·09	1	...	·304	W.B.
OΣ 234 ...	11 25	48 10	142·2	0·34	1	7·0 7·4	·419	B.
Σ 1554 ...	11 31	76 36	257·0	0·91	3	8·9 9·1	·294	W.B.
			260·0	0·84	1	...	·285	L.
Σ 1555 ...	11 31	61 40	357·0	0·35	3	6·4 6·8	·272	W.B.
			357·3	0·31	1	...	·285	L.
			351·2	0·27	1	...	·288	B.
β 603	11 44	75 10	313·4	0·83	3	6·4 10·2	·333	L.
Hn. 570 ...	12 9	68 9	108·4	2·04	1	9·0 13·0	·394	L.
Σ 1621 ...	12 11	83 49	138·0	2·71	1	8·8 10·3	·318	L.
			135·2	2·58	1	...	·359	W.B.
Ho. 52 ...	12 16	71 39	46·0	...	1	5·0 13·0	·318	L.
Σ 1639... ...	12 20	63 52	357·7	0·28	1	6·7 7·9	·288	B.
			357·3	0·35	3	...	·344	L.
			360·9	0·27	1	...	·359	W.B.
Σ 1643 ...	12 22	62 15	37·7	1·95	2	8·4 8·7	·196	W.B.
			39·4	1·94	1	...	·290	H.F.
Σ 1647 ...	12 25	79 44	42·0	1·53	2	7·5 7·8	·271	W.B.
Hn. 571 ...	12 27	69 27	84·4	0·37	1	8·8 8·8	·416	W.B.
β 797	12 29	83 39	169·2	0·61	2	8·5 8·5	·328	L.
			160·2	0·59	1	...	·359	W.B.
Σ 1658 ...	12 30	82 0	358·2	2·35	2	8·0 9·8	·334	W.B.
			361·0	2·33	2	...	·328	L.
Σ 1661 ...	12 30	78 3	239·4	2·48	2	8·5 8·5	·277	W.B.
			240·2	2·05	1	...	·290	H.F.
Σ 1663 ...	12 32	68 15	100·7	0·54	2	7·8 8·7	·277	W.B.

Star's Name.		R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1903.	Obs.
				243·8	0·20	2	...	·428	W.B.
				252·5	0·22	1	...	·419	B.
Σ 1777	...	13 38	85 57	230·0	3·37	2	5·8 8·2	·358	L.
Σ 1781	...	13 41	84 23	277·7	0·92	2	7·8 8·2	·358	L.
				283·1	1·09	1	...	·375	W.B.
β 801 ...	...	13 42	78 40	327·0	2·41	1	8·2 10·2	·337	L.
Σ 1785	...	13 45	62 31	290·7	1·43	1	7·2 7·5	·318	L.
				287·8	1·21	1	...	·329	H.F.
				291·5	1·43	2	...	·367	W.B.
β 613 ...	...	13 47	54 50	149·6	0·94	1	9·0 9·0	·476	W.B.
Σ 1788	...	13 50	97 34	81·2	2·32	1	6·7 7·9	·389	H.F.
β 30 ...	...	13 53	70 5	200·1	8·57	3	8·1 10·7	·370	L.
β 1270	...	13 59	81 2		Round		8·2 8·3	·345	L.
				22·6	0·19	1	...	·378	L.
				23·5	0·15	1	...	·416	W.B.
Σ 1808	...	14 6	62 56	74·3	2·80	1	8·0 9·0	·318	L.
				73·6	2·80	1	...	·375	W.B.
Σ 1819	...	14 10	86 24	361·1	1·23	2	7·9 8·0	·401	L.
				360·0	1·21	1	...	·389	H.F.
				359·3	1·51	1	...	·416	W.B.
* with Σ	1819	14 11	86 12	198·5	1·53	1	7·5 9·5	·416	W.B.
Σ 1832	A.B.	14 14	85 39	135·4	0·61	2	9·0 9·0	·400	L.
				129·1	0·65	1	...	·416	W.B.
	A.C.			72·3	20·18	2	9·0 10·5	·400	L.
Σ 1835	A.B.	14 18	81 6	188·5	6·70	1	5·8 6·8	·422	L.
β 1111	B.C.			36·9	0·25	1	8·2 8·4	·422	L.
Hu. 574	...	14 31	70 19	100·2	0·24	1	8·5 8·8	·416	W.B.
				111·2	0·24	1	...	·422	L.
Σ 3087	...	14 32	70 10	221·9	2·04	1	9·5 9·5	·403	L.
Σ 1865	...	14 36	75 51	153·5	0·33	2	3·5 3·9	·354	B.
				145·8	0·45	2	...	·370	L.
				139·2	0·25	2	...	·378	W.B.
Hu. 575	...	14 38	70 5	168·0	0·80	2	9·0 9·5	·413	L.
Hu. 576	...	14 41	69 25	190·7	4·80	1	8·2 13·0	·403	L.
Σ 1877	...	14 41	62 30	329·8	2·75	2	3·0 6·3	·378	W.B.
				325·5	2·81	1	...	·403	L.
Σ 1879	...	14 41	79 55	131·7	0·52	1	7·8 8·8	·337	L.
				140·2	0·53	1	...	·419	B.

Star's Name.		R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1903.	Obs.
Σ 1967	...	15 39	62 23	118·7	0·58	1	4·0 7·0	·381	W.B.
				116·2	0·75	1	...	·419	B.
β 621 ...	...	15 47	45 8	51·7	0·42	1	7·5 8·0	·345	L.
β 810 ...	...	15 48	47 14	91·4	1·09	1	8·7 11·5	·345	L.
OΣ 303	...	15 56	76 27	144·2	0·80	2	7·4 7·9	·399	W.B.
β 355 ...	...	16 5	44 21	274·4	0·44	1	7·8 8·9	·345	L.
Σ 2021	...	16 8	76 12	334·0	3·67	1	6·7 6·9	·416	W.B.
				334·7	4·11	2	...	·482	H.F.
Hu. 481	...	16 17	66 16	224·6	0·42	1	7·3 9·2	·471	L.
				229·2	0·40	1	...	·512	W.B.
Hu. 482	...	16 18	67 28	148·4	1·58	1	9·0 13·8	·496	W.B.
β 951 ...	...	16 20	56 23	61·5	0·98	1	8·2 8·7	·345	L.
OΣ 312	...	16 22	28 16	142·1	5·30	1	2·1 8·1	·383	L.
Hu. 483	A.B.	16 23	68 53	296·2	1·84	1	8·2 13·0	·496	W.B.
OΣ 311	A.C.	16 23	68 53	203·5	6·56	1	8·2 10·3	·496	W.B.
				200·9	6·44	1	...	·518	L.
Σ 2054	...	16 24	28 4	9·7	0·96	1	5·7 6·9	·383	L.
Σ 2052	...	16 25	71 23	94·1	1·42	1	7·5 7·5	·490	L.
				90·7	1·59	1	...	·559	W.B.
Σ 2055	...	16 26	87 48	61·6	1·15	1	4·0 6·1	·490	L.
				58·5	1·46	1	...	·559	W.B.
Hu. 486	...	16 35	67 4	138·3	1·19	2	9·0 10·0	·504	W.B.
Σ 2084 (ζ Herculis)	...	16 36	58 13	200·6	1·14	1	3·0 6·5	·383	L.
				201·0	0·95	1	...	·419	B.
				197·5	0·78	1	...	·676	W.B
De. 15 ...	...	16 41	46 20	310·5	0·42	1	8·2 8·6	·383	L.
OΣ 315	...	16 46	88 37	157·6	0·86	1	6·2 8·1	·444	H.F.
				155·0	0·75	1	...	·559	W.B.
β 627 ...	...	16 46	43 51	321·5	1·48	1	5·0 9·5	·383	L.
Σ 3107	...	16 54	85 53	95·5	1·39	2	8·5 8·5	·482	H.F.
Σ 2118	...	16 56	24 49	204 ±	...	1	6·4 6·9	·383	L.*
				89·1	0·13	1	...	·419	B.
Σ 2114	...	16 57	81 24	161·8	1·23	1	6·2 7·4	·444	L.
Σ 2130	A.B.	17 3	35 24	135·9	2·23	1	5·0 5·1	·383	L.
β 1088	B.C.			189·0	12·37	1	5·1 13·5	·383	L.
Σ 2140	...	17 10	75 30	110·5	4·68	2	3·0 6·1	·482	H.F.

* Suspected.

Star's Name.		R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.		Epoch 1903.	Obs.
β 641 ...	...	18 18	68 32	345·6	0·90	2	7·5	9·0	·623	L.
Σ 2314	...	18 19	66 36	322·8	2·22	1	8·4	9·6	·671	L.
Σ 2315	...	18 21	62 39	193·0	0·23	1	7·0	8·0	·419	B.
				197·6	0·18	1	...		·575	L.
Σ 2319	...	18 23	70 46	192·6	5·53	1	7·2	7·6	·671	L.
Σ 2336	...	18 28	76 15	8·3	6·25	1	8·7	9·8	·671	L.
Σ 2339	...	18 29	72 21	274·7	2·11	1	7·2	8·0	·671	L.
Σ 2345	...	18 31	69 0	204·9	8·65	1	8·4	10·1	·635	W.B.
OΣ 357	...	18 31	78 21	240·8	0·43	3	7·5	7·6	·567	W.B.
OΣ 358	...	18 31	73 6	12·0	1·82	3	6·8	7·2	·503	W.B.
Σ 2356	...	18 34	61 23	55·7	1·15	1	8·0	9·0	·635	W.B.
Σ 2358	...	18 35	59 22	221·3	2·82	1	8·8	9·0	·635	W.B.
Σ 2360	...	18 35	69 9	360·8	2·45	1	7·5	8·7	·635	W.B.
Σ 2364	...	18 36	65 23	178·2	7·50	1	8·0	10·2	·635	W.B.
Σ 2367	A.B.	18 37	59 48	69·3	0·13	1	7·0	7·5	·419	B.
				78·5	...	1	...		·575	L.
				70·2	0·19	4	...		·655	W.B.
	A.C.	18 37	59 48	193·3	14·22	1	7·0	8·2	·635	W.B.
Σ 2371	...	18 38	62 28	235·9	9·53	1	8·5	8·5	·635	W.B.
Σ 2369	...	18 39	87 29	94·6	1·04	1	7·5	8·0	·575	L.
Σ 2375	...	18 41	84 36	113·0	2·12	2	6·2	6·6	·589	H.F.
				113·6	2·15	1	...		·594	W.B.
Σ 2400	A.D.	18 44	73 52	184·8	2·27	1	8·1	10·6	·575	L.
	A.C.	...	...	186·3	3·68	1	8·1	11·0	·575	L.
	B.C.	...	...	183·2	0·85	1	10·6	11·0	·575	L.
Σ 2402	...	18 45	79 25	207·3	0·89	1	8·0	8·4	·594	W.B.
Σ 2422	...	18 53	64 2	92·4	0·81	3	7·6	7·7	·524	W.B.
				95·9	0·88	1	...		·657	H.F.
Σ 2437	...	18 58	70 58	59·7	0·76	3	7·8	8·0	·549	W.B.
Σ 2454	...	19 2	59 43	239·3	0·84	1	8·0	9·2	·676	W.B.
Σ 2455	...	19 3	67 59	77·3	3·42	3	7·2	8·3	·579	W.B.
				77·0	3·45	3	...		·638	H.F.
Σ 2488	...	19 11	70 9	327·6	1·71	1	8·5	9·7	·559	W.B.
OΣ 368	...	19 12	74 1	217·3	0·82	3	7·3	8·5	·581	W.B.
OΣ 371	...	19 12	62 42	153·6	0·74	4	6·8	6·9	·561	W.B.
Σ 2525	...	19 23	62 52	308·8	0·61	2	7·4	7·6	·659	W.B.
OΣ 375	...	19 30	72 7	145·3	0·64	1	7·2	8·4	·676	W.B.
Σ 2587	...	19 46	86 10	101·9	3·84	3	6·5	9·2	·699	H.F.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1903.	Obs.
	h m	° ′	°	″				
Σ 2767 ...	21 6	70 27	29·8	2·30	1	7·8 8·2	·701	L.
Σ 2769 ...	21 6	67 58	209·2	18·16	1	6·5 7·5	·701	L.
Σ 2765 ...	21 6	80 51	265·9	2·73	2	7·8 8·0	·772	L.
			266·3	2·80	1	...	·879	B.
OΣ 535 A.B. (δ Equulei)	21 10	80 24	23·8	0·34	6	4·1 4·1	·775	L.
			25·1	0·29	6	...	·812	W.B.
			26·4	0·30	4	...	·838	B.
			27·4	0·35	2	...	·863	H.F.
Σ 2785 ...	21 14	50 40	242·7	2·97	1	8·1 10·0	·882	L.
Σ 2797 ...	21 22	76 45	211·7	2·94	1	6·7 8·2	·701	L.
Σ 2799 ...	21 24	79 21	295·5	1·42	2	7·0 7·0	·758	W.B.
			297·5	1·26	2	...	·852	H.F.
Σ 2804 ...	21 28	69 44	333·8	2·84	4	7·3 8·0	·681	W.B.
			336·9	2·51	1	...	·827	H.F.
Σ 2808 ...	21 31	59 23	173·4	...	1	...	·682	L.
Ho. 163 ...	21 32	58 50	42·6	6·64	1	8·0 13·0	·682	L.
β 167 ...	21 32	60 24	87·3	1·94	1	7·0 12·0	·682	L.
β 1212 ...	21 34	90 30	271·7	0·47	2	6·5 6·9	·823	B.
Σ 2814 ...	21 35	54 5	162·6	7·82	1	8·3 9·8	·663	L.
			158·1	7·96	1	...	·665	W.B.
Ho. 166 ...	21 40	62 38	85·5	0·30	2	7·5 7·5	·773	L.
			83·5	0·34	2	...	·759	W.B.
Σ 2822 ...	21 40	61 42	123·0	2·37	3	4·0 5·0	·753	W.B.
			125·8	2·38	1	...	·663	L.
			125·5	2·29	1	...	·827	H.F.
β 989 A.B. (κ Pegasi)	21 40	64 49	137·6	0·18	5	3·9 4·4	·740	L.
			131·4	0·25	5	...	·833	W.B.
			125·1	0·18	3	...	·850	B.
Σ 2824 A.C.			297·4	12·85	4	3·9 10·8	·701	L.
Σ 2825 ...	21 42	89 36	115·1	0·97	1	8·0 8·2	·808	L.
Σ 2829 ...	21 45	59 43	195·2	17·41	1	8·2 9·2	·882	L.
Σ 2834 ...	21 47	71 10	300·6	3·77	1	7·3 10·6	·882	L.
Ho. 171 ...	21 48	62 41	354·5	0·76	2	8·2 8·2	·773	L.
Σ 2846 ...	21 51	44 42	274·3	3·24	1	8·5 10·3	·882	L.
β 75 ...	21 51	79 35	43·7	0·90	1	8·1 8·3	·876	H.F.
Σ 2866 ...	22 5	49 50	55·0	8·91	1	8·3 11·3	·665	W.B.
Σ 2878 ...	22 10	82 31	125·1	1·31	1	6·5 9·0	·808	L.
Σ 2882 ...	22 10	52 45	327·4	3·17	1	9·2 9·2	·665	W.B.

Star's Name.		R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1903.	Obs.
		h m	° ′	°	″				
β 858	A.B.	23 36	58 0	252·3	0·92	1	7·7 8·2	·950	L.
				256·2	0·57	1	...	·953	W.B.
β 389	A.C.	...	...	255·4	23·37	1	7·7 10·8	·950	L.
A.G.C. 14	...	23 39	61 12	193·8	1·81	2	5·0 9·5	·926	W.B.
				195·4	1·67	1	...	·950	L.
Barnard	...	23 42	85 19	171·3	0·54	1	8·6 8·6	·846	B.
Σ 3050	...	23 54	56 0	215·5	2·46	3	6·0 6·0	·893	L.
				216·4	2·30	1	...	·827	H.P.
				215·1	2·25	3	...	·932	W.B.
Σ 3056	A.B.	23 59	56 17	156·3	0·50	1	7·4 7·4	·824	L.
				152·6	0·66	3	...	·922	W.B.
	A.C.	...	...	357·4	22·71	1	7·4 9·0	·824	L.
	$\frac{A.B.}{2}$ C.	...	...	359·7	22·50	1	7·4 9·0	·739	L.

Report on the Work of the Argentine National Observatory, 1904.
By the Director, John M. Thome.

(*Communicated by the Astronomer Royal.*)

In the twelve months ending with March, after deducting the losses occasioned by the Moon and unfavourable weather, there remained 93 nights, long and short, available for photographic and Durchmusterung work; for general meridian work, in which a moonless and cloudless sky is not essential, 180 nights were recorded.

During this limited period every effort was made to secure the maximum of results, and we obtained 298 plates, 108 of which were for the atlas and contained three exposures of thirty minutes', or longer, duration, according to the steadiness of the images and the transparency of the atmosphere at the time. Long-exposure plates are taken whenever the time disposable and the atmospheric conditions warrant the risk.

Both the Repsold-Gill and the Gautier machines have been in constant use, and we have measured the coordinates of 34,100 stars, in both positions of the plates, with an approximation to thousandths of a millimetre; and also the diameters of the images, according to Gill's method. Over two-thirds of these measures correspond to the Gill machine. Two observers take part in the measurement of every plate, reading alternate 5′ zones; each observer, again, measuring the same 5′ zone when the plate is reversed. The images of the reference stars, which have been previously marked upon the plate, are read by both observers. Nine hours, including an interval of two hours at noon, represent a day's work.

which is not sufficient to ensure readings accurate to the hundredth-part of an interval. Fortunately our clockmaker was able to give a speed to the barrel corresponding to twice that interval, or sixteen millimetres to a second, which is sufficient for all purposes. The Hipp chronograph enables one also to detect any error in the alignment of the pens at any moment by simply moving the carriage laterally and noting the trace or traces.

The illumination of the reticule and microscopes of our meridian circle had always been unsatisfactory; at times the transits being noted with difficulty, until we tried a lamp in which the illuminant is vapour of carburetted alcohol impinging on a gauze wick. This was still not entirely satisfactory for the microscopes, the illuminating mirrors for which had become tarnished, and I then decided to fix a small electric lamp, enclosed in a wooden box with an opening in the side directly facing the reflecting surface of each microscope, and this arrangement has given perfect satisfaction. The lamps are operated by the ordinary key, and are employed only during the few moments required in reading the microscopes. With the enlargement, besides, of the meridian room to four times its original length, of which I gave an account in my former report, provided throughout with sliding shutters and doors which can be opened in sections, or altogether, so as to maintain a proper ventilation and a uniform temperature with the outside air, everything has been done that I can think of to increase the accuracy of the results from this notable instrument, which has probably furnished a larger number of stellar positions (over 400,000 in both coordinates) than any other meridian circle in the world. It has suffered no injury during the thirty-two years that I have known it, except the unavoidable friction on the pivots, which has been guaranteed against by counterpoises as much as possible.

Under the original conditions indicated (of instrument and observing-room) the reobservations of the Dm. stars to 9·3 between 22° and 37°, giving nearly 156,000 positions, have been made, excepting that the heat escaping from the tube of the illuminating lamp was carried out of the room by a pipe with a funnel-shaped mouth, which is fixed directly above the lamp in a niche in the wall, and that the meridian shutters in the north and south walls of the room remained open, full or half-length, as the conditions—wind, dust, &c.—during the course of work permitted.

The method followed in these observations is not as complete and accurate as when all four microscopes are used, and to obtain the best results two observers and two microscope-readers, carefully trained and in hearty sympathy with the work, should perform the whole, alternating nightly. That condition I have found it impossible to maintain here.

This great mass of results obtained from the collaboration of many observers requires great care and much time in the treatment, and so long as there remain any errors to be detected

seems necessary to again call attention to it. See also the preface to vol. xiii. p. 13.

"The publication of these annual catalogues, beginning with 1874, has occupied a large part of our time, and has been a very considerable drain upon our steadily diminishing resources. The effort to maintain the activity and honourable record of the institution under the smaller salaries consequent upon a vanishing budget, which cut off from me all hope of obtaining trustworthy help abroad, has been attended with painful experiences, and I have repeatedly been compelled to stop and train new men, and to repeat operations performed by inefficient, negligent, or designing assistants."

And in the introduction to vol. xviii., published in 1900 March 15, I stated that the observations were completed in 1897, but that the Minister of Public Instruction had considered it necessary, in view of the possibility of a war over the Chilian boundary question, to reduce by two-thirds our already minimum appropriation, and that I had only recently succeeded in having the original appropriation restored.

At the time I was appointed to the directorship the value of the gold dollar was 0·82 cent; from then until 1889 the average value was 0·72; from 1889 to 1897 this was 0·34, being at one time as low as 0·22, and averaging only 0·28 from 1891 to 1895. For the whole period from March 1885 to the end of 1900 the average was 0·41, and it is now fixed at 0·44. We have always been paid in paper!

During the first administration there were published the *Uranometria Argentina*, the *Zone Catalogue*, and the *General Catalogue*. The observing books show that there was an average of 270 working nights per year during this period, and that the number of stellar positions determined was as follows:—

For the *Zone Catalogue* . .	105,252	observations.
„ „ *General Catalogue* .	160,506	„
Total . . .	265,758	„

During the present administration there have been produced four volumes of the *Cordoba Dm.*, with a total of 630,000 stellar positions and magnitudes, resulting from over 1,800,000 observations; and there are ready eighteen charts, containing 550,000 stars.

The number of meridian observations obtained is 198,000, all reduced to the mean epoch of 1900.

Now, the actual time and labour required to reduce, collect, and prepare for the press that number of Dm. observations, and to publish the results as given in our volumes, is very considerable—apart from the preparation of the atlas—being, in fact, considerably greater than that given to the Uranometry.

There remains, then, the discrepancy between the previous 265,000 observations and the present 198,000; in explanation of

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No. 3.

CONTENTS.

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Sir N. and Dr. W. J. S. Lockyer, The Behaviour of the Short-Period Atmospheric Pressure Variation over the Earth's Surface . . . . [36]

Professor S. Arrhenius, On the Electric Equilibrium of the Sun . . [50]

which have a regular and pronounced annual pressure variation, such as India, and where the yearly barometric range is of far greater magnitude than any other aperiodic fluctuation.

In those regions where the mean yearly curve is more misleading than otherwise, the division, according to the two seasons included in the two groups of months, April to September and October to March, is best adapted.*

The system adopted in the present investigation was to take the pressure variations over India and Cordoba as the chief types of each region, denoting those of the former by the symbol (+), and those of the latter by (−). The pressure curve of any other place was then taken and compared with each. If, for example, it was found that the curve extending over several years exhibited an excess pressure at those epochs when the Indian pressure curve was in excess, then it was classified as being similar to the Indian type and represented by a (+). If it was seen that although it was more like the Indian curve than that of Cordoba, but yet not quite the exact counterpart of India, then it was denoted by (+ ?). In a similar way pressure curves like Cordoba were classified as (−), and those more like Cordoba than India as (− ?).

In some regions the pressure variation curves were distinctly a mixture of both the Indian and Cordoba types, and it was difficult to classify them satisfactorily by the above method. The symbol adopted for these cases was (± ?). Again, there were further some curves in which even this mixed type of symbol was not sufficient to exhibit the relationship of their variations to the other curves, so a special symbol (?) denoting ambiguity was used.

In the present investigation of this similarity or dissimilarity of atmospheric pressure changes over large areas, it was found that the special types were apparent sometimes in the yearly curves, sometimes in those for one or other of the high or low pressure groups of months, or sometimes in both of these. It did not, however, appear to follow that, because the type was distinguishable in the yearly curves, it was necessarily apparent in both the curves of the high and low pressure months.

The accompanying table, although yet somewhat incomplete, gives a tabulated statement of the data employed in the present survey.

The table explains itself, but it may be remarked that in Columns 6

* To show the misleading nature of the mean annual pressure-variation curve over, for example, the British Isles, it is only necessary to plot the actual monthly values of pressure for any one year on this mean curve and draw a curved line through them, when it will be seen that there is practically no relationship whatever between the two curves. If, on the other hand, the actual monthly pressure values during any one year be plotted on the mean annual pressure-variation curve for India, the former follow very closely the swing and amplitude of the latter.

ressure Types.

Months in which types are most conspicuous.		Source of data.	General remarks.
pr.—Sept.	Low press.	Indian Monthly Weather Reviews	Type equally prominent in curve for year.
,,	,,	,, ,,	,, ,,
,,	,,	,, ,,	,, ,,
,,	,,	,, ,,	Yearly curve only examined.
,,	,,	,, ,,	,, ,,
pr.—Sept.	Low press.	,, ,,	Type equally prominent in curve for year.
ct.—Mar.	High press.	,, ,,	Record very short.
pr.—Sept.	Low press.	,, ,,	Equally prominent in yearly and Oct.—Mar. curves.
,,	,,	Mauritius Met. Obsns.	Yearly curve alone examined.
,,	,,	,, ,, ...	Short and uncertain record. Yearly curve examined.
ay—Oct.	High press.	,, ,, ...	Prominent in yearly curve.
une—Oct.	,,	Meteorological Office	,, ,,
,,	,,	Report of Philippine Commission	
,,	,,	Met. Obsns. at Straits Settlements	,, ,,
pr.—Sept.	High press.	Met. Obsns. at Perth and district	
,,	,,	Met. Obsns. at Adelaide.....	Very prominent in yearly curve.
,,	,,	Met. Obsns. in New South Wales	,, ,,
,,	,,	New Zealand Statistics (Met.)	1877 pulse absent.
,,	,,	Met. of New Zealand	
ov.—Apr.	Low press.	Indian Monthly Weather Reviews	Record very broken and short.
ct.—Mar.	,,	Report of Govt. Astronomer, Natal	
pr.—Sept.	High press.	Meteorological Office	Broken record, 1871—1877.
,,	Low press.	Met. Report of Abassia Obsy., Cairo	
,,	,,	Met. Zeitschrift, 1897	
ct.—Feb.	High press.	Kong. Sv. Vet. Ak. Handlingar, vol. 29, No. 3	Record too short.
,,	,,	Army Medical Dept. Reports	Yearly curve alone examined.
ov.—May	Low press.	Lisbon Met. Obsns.	Record too short.
,,	,,	,, ,, ,,	Yearly curve alone examined.
,,	,,	Supplement to Annals of Lisbon Observatory and Loanda Met. Obsns.	,, ,,
pr.—Sept.	High press.	Anales de l'Oficina Met. Argentina, vol. xiii	
,,	,,	Anales de l'Oficina Met. Argentina	Broken record.
,,	,,	,, ,,	Continuous good record.
ay—Oct.	,,	Kong Sv. Vet. Ak. Handlingar, vol. 29, No. 3	
ay—Sept.	,,	Meteorological Office	
,,	,,	Army Medical Dept. Reports	Yearly curve alone examined. Two short breaks record.
,,	,,	,, ,,	,,

Types—continued.

Months in which types are most conspicuous.		Source of data.	General remarks.
Nov.—Feb.	High press.	Report of Chief of Weather Bureau, U.S.A.	
Oct.—Mar.	,,	,, ,,	
Nov. Mar.	,,	,, ,,	
Sept.—Feb.	,,	,, ,,	
Nov.—Apr.	,,	,, ,,	
(Mar.—Aug.)	(Low press.)	,, ,,	
Apr.—Sept.	Low press.	,, ,,	
,,	,,	,, ,,	Short record.
Oct.—Mar.	High press.	,, ,,	,,
May—Oct.	Low press.	,, ,,	
Dec.—May	,,	,, ,,	Record broken, 1882—1885.
June—Nov.	High press.	,, ,,	
,,	,,	,, ,,	Yearly mean curve alone examined.
Oct.—Mar.	High press.	,, ,,	
,,	,,	,, ,,	,, ,,
,,	,,	,, ,,	,, ,,
,,	,,	,, ,,	Short record.
,,	,,	Kong. Sv. Vet. Ak. Handlingar, vol. 29, No. 3	Yearly curve alone examined. Broken record.
Dec.—Mar.	High press.	Army Medical Dept. Reports	Record somewhat broken.
,,	,,	Report of Met. Service of Dom. of Canada	Yearly curve alone examined.
,,	,,	,, ,,	,, ,,
Aug.—Nov.	High press.	,, ,,	Record broken after 1891.
Aug.—Feb.	Low press.	Met. Obsns., Honolulu.....	Record twice broken.
,,	,,	Met., Zeitschrift, 1892	Yearly mean curve examined.
,,	,,	Danish Met. Aarbog.....	Record broken, 1883—1887.
Oct.—Mar.	Low press.	,, ,,	,, 1874—1875.
Apr.—Sept.	High press.	,, ,,	
,,	,,	,, ,,	
Sept.—Mar	Low press.	Met. Aarbog Norsk	
,,	,,	,, ,,	
Apr.—July	High press.	,, ,,	
,,	,,	MSS. Met. Office Records	Yearly curve alone examined.
Apr.—Sept.	High press.	,, ,,	
,,	,,	,, ,,	
,,	,,	Results of met. and mag. observations, Stonyhurst	,, ,,
Apr.—Sept	High press.	MSS. Met. Office Records	
,,	,,	Kong. Sv. Vet. Ak. Handlingar, vol. 29, No. 3	
Mar.—Aug.	Low press.	Madrid Met. Obsns	
	,,	Army Medical Dept. Reports.	,, ,,
,,	,,	Lisbon Met. Obsns.....	
,,	,,	Denkschriften Kaiserlichen Ak. de Wissenschaften	
Apr.—Sept.	Low press.	Kong. Sv. Vet. Ak. Handlingar, vol. 29, No. 3.	Short record of definite

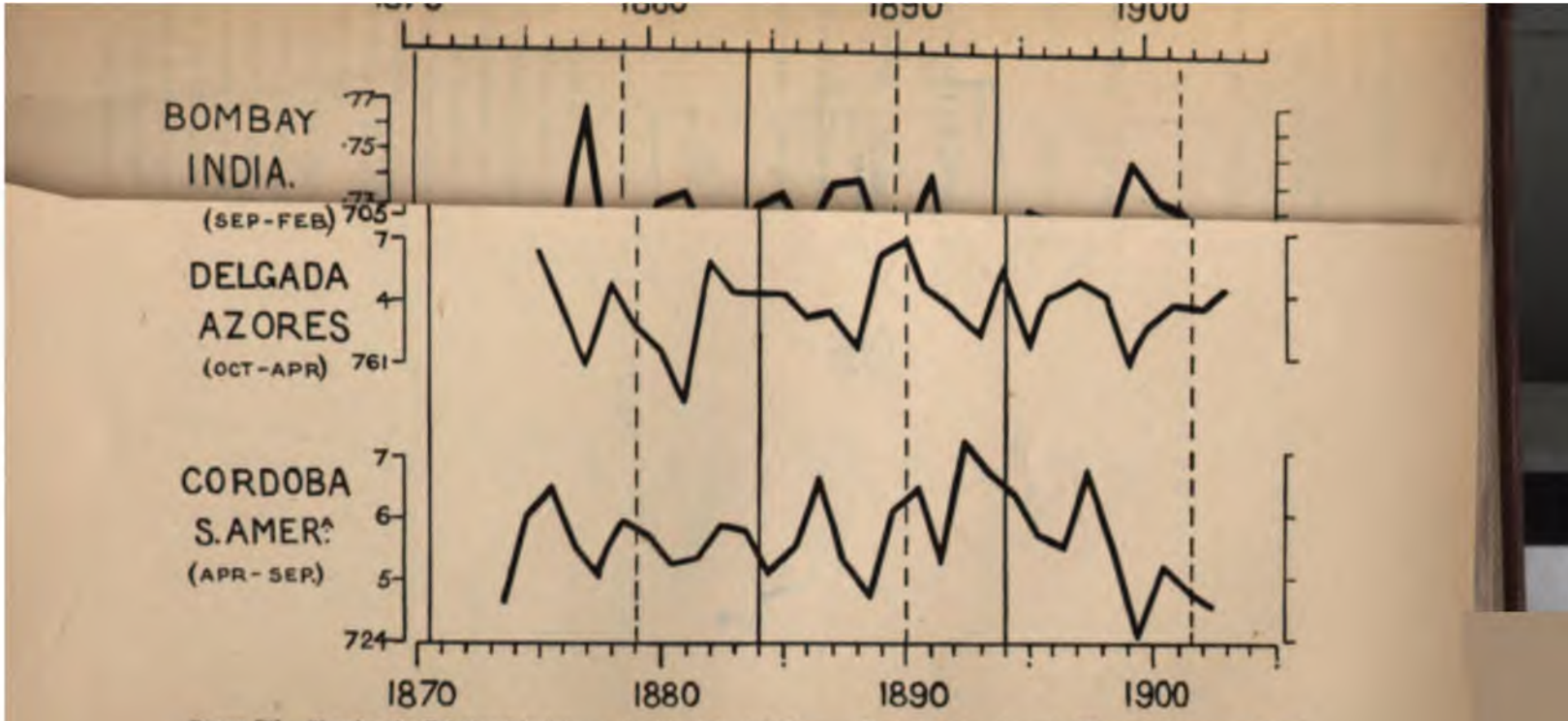

PLATE [7].—Shewing the Relationship between the Short Period Pressure Variations for places in Western Europe, and the two Main Types of Pressure Variations, namely: India (+) and Cordoba (−). This Plate should be studied in relation to the map in fig. 1. *Note.*—The continuous and broken vertical lines denote the epochs of sunspot maxima and minima as determined from the mean daily areas of both hemispheres of the sun.

ypes—continued.

Months in which types are most conspicuous.		Source of data.	General remarks.
Oct.—Apr.	Low press.	Annales de l'Obsy. do Infante D. Luiz	Prominent in yearly curve.
..	..	Résumé de las Obs. Met. de Provinces (Spanish)	1886 no record.
..	..	Annales de l'Obsy. Cent. Phys. de Russe	Yearly curve alone examined.
Sept.—Feb.	High press.	" "	
..	..	Kong. Sv. Vet. Ak. Handlingar, vol. 29, No. 3	
Oct.—Mar.	High press.	Meteorological Office	
..	..	Annales de l'Obsy. Cent. Phys. de Russe	Break in record 1875—1887. Yearly curve alone examined.
..	..	" "	Yearly curve alone examined.
Jan.—June	High press.	Meteorological Office	
Oct.—Mar.	"	" "	Prominent in yearly curve.
"	"	" "	" "
Apr.—Sept.	Low press.	" "	
..	..	Annales de l'Obs. Cent. Phys. de Russe	
Oct.—Mar.	High press.	Kong. Sv. Vet. Ak. Handlingar, vol. 29, No. 3	Short record. Prominent in yearly curve.
Apr.—Sept.	Low press.	Met. Zeitschrift, 1886, and Observations made at Hong-Kong Observatory	Visible in yearly curve.
"	"	Met. Zeitschrift, 1899.....	
..	..	Report of Cent. Met. Obsy. of Japan.	All the curves for the half-yearly and yearly values show similar variations.

It will be seen that although practically the same groups of months have been taken in each case, pressure in excess of the mean value in Greenland or Iceland corresponds to a deficiency of pressure over the area covered by Great Britain, Austria and Spain, the curves being in the main the reverse of each other. Again, the pressure curve for the Azores follows more nearly the (−) type, as will be seen by comparing it with the Cordoba curve, but it has a certain similarity to those of Madrid, Vienna, etc., to which it must therefore be closely connected.

While the western portion of Europe is of this (± ?) type, the eastern portion gradually assumes the (− ?) type, and this region extends not only probably to Norway and Sweden, but right across European and Asiatic Russia. The European Russian type of curve has an undoubted similarity to those of more Western Europe, but there are variations which indicate that the type is more like that of Cordoba than India.

Again, another region in which rather mixed types of pressures are

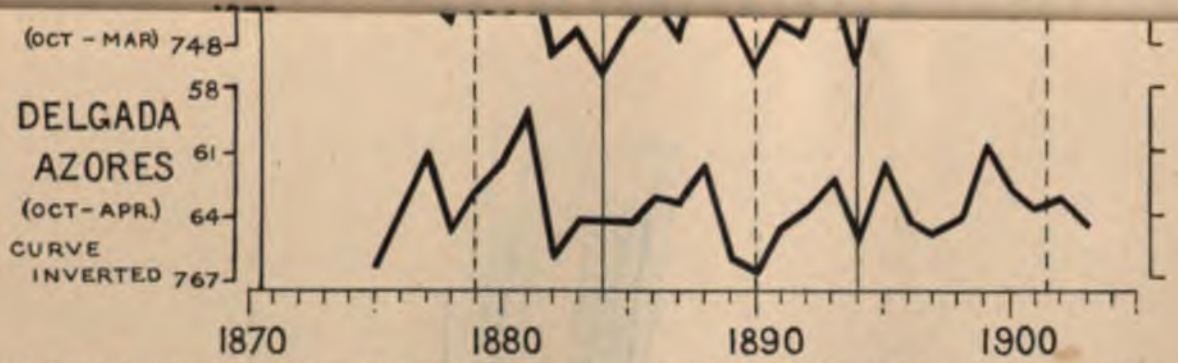

PLATE [8].—Three pairs of Curves to illustrate Reverse Pressure Conditions at stations widely separated geographically, such as India and Cordoba (S. America), Adelaide (S. Australia) and Sydney (Nova Scotia), and at two stations near each other, as Berufjórd (Iceland) and Delgada (Azores). In each case the second curve has been reversed. *Note.*—Vertical lines same as in Plate [7].

was covered by the continuous excessive high pressure that prevailed over the Indian region from the end of 1876 to about the middle of 1878.

On fig. 1 is given a map of the world on which are marked the types of pressure variations in each region which is included in this barometric survey.

An attempt has been made by means of a neutral line to show approximately the mean lines of separation of these two chief pressure types, although it must be remembered that this line is liable to a probable small oscillation about its mean position.

As far as can at present be determined, one line commencing to the west of Alaska, separating this region from Siberia, passes easterly along about the 60° parallel of latitude and runs in a south-easterly direction between South-west Greenland and North-east Canada. It then crosses the North Atlantic, passing to the north of the Azores, and skirts the south-western portion of Portugal. It then strikes down towards the Equator, cutting North-west Africa, as far as can be judged from the scant pressure values available, through the middle of the Sahara. It leaves Africa near the Gold Coast, passes into the South Atlantic, where it cannot be traced further owing to lack of observations in this southern ocean.

The other boundary or neutral line passes to the north-east of Greenland and north of Iceland, crosses the southern portion of Norway and Sweden, and traverses Southern European Russia. It then takes a course somewhat more easterly, skirting the northern part of the Caspian Sea and Turkestan, passes between Tibet and Mongolia, and through China. It then leaves the Continent a little to the south of the Yellow Sea, and passes into the North Pacific Ocean. Here its path cannot be traced, but it evidently passes well to the east of the Philippine Islands and Solomon Islands, takes a new south-westerly course, skirting the eastern side of Australia and passing between Tasmania and New Zealand. Its track is then again lost in the Southern Pacific Ocean.

Although too much weight must not at present be given to the positions of these neutral lines throughout their whole length, it is interesting to note that they are fairly symmetrical to one another, although no attempt has been made to make them so.

Both lines apparently cross the equator at about antipodal points, and both appear to have a similar trend in northern and southern latitudes.

We seem then to be in presence of a general law relating to the pressures which occur simultaneously in two different regions of the globe, separated and defined more or less by a neutral line, this neutral line forming a fulcrum about which see-saws of pressure from one region to another take place. Special cases of such reverse pressure variations have been previously detected.

Thus as long ago as 1879 Blanford,* from a discussion of the secular variations of barometric pressure over the wide area covering Siberia, Indo-Malaysia and Australia, pointed out that there existed a kind of long-period see-saw of a character that, while the pressure at the tropical stations was low, that in Siberia was high, and *vice versâ*. This fact, it will be seen, is quite in harmony with the pressure-type distribution, as shown in the accompanying map (fig. 1).

Hildebrandsson† has discussed the relation between the pressure variations of numerous places mainly situated in the chief centres of action of the atmosphere widely distributed on the earth's surface for the period 1874—1884. In this valuable communication, some of the chief results which he was led to deduce were that there were several regions which exhibited opposite types of pressure variations.

The following places are those to which he calls attention, and for comparison we give the types in brackets which have been allotted according to the method adopted in the present paper; where no type is added the region has not been examined :—

The Azores (− ?) and Iceland (+ ?); Siberia (− ?) and Alaska (+ ?), especially in winter; Tahiti (± ?) and Tierra del Fuego; India (+) and Siberia (− ?); Greenland (+ ?) and Key West (Florida) (−); Buenos Ayres (− ?) and Sydney (Australia) (+).

It is interesting to note that these results agree well in the main with the present distribution of the regions which have been examined.

Again Hann‡ has recently drawn attention to the fact that there exists a see-saw between the Azores and Iceland, and he showed that in 80 per cent. of cases the largest positive pressure variations at Stykkisholm (Iceland), corresponded to negative pressure variations at Ponta Delgada (Azores), and that the largest negative pressure variations at Stykkisholm were in 87 per cent. of cases positive variations at Ponta Delgada.

This result obtained from the observations extending from 1846—1900 endorses Hildebrandsson's previous conclusion deduced from observations over the period 1874—1884, and confirms the position of the neutral line shown on fig. 1, dividing the two large types of pressure areas.

Quite recently Professor Bigelow§ has published a map of the world on which he has indicated the distribution of the pressure types according as they follow the Indian (or direct type, as he calls it) or the Cordoba (indirect) pressure variations.

Professor Bigelow has also found that there are many regions in

* 'Report of the Meteorology of India in 1878,' pp. 2—35.

† "Quelques Recherches sur les Centres d'Action de l'Atmosphère," 'Kongl. Svenska Vetenskaps-Akad. Handlingar,' vol. 29, No. 3.

‡ 'Kaiserliche Akademie der Wiss. in Wien,' January 7, 1904.

§ 'Monthly Weather Review,' p. 509, November, 1903.

meteorologists to form a more definite idea of the prospects of approaching seasons.

We wish to express our thanks to Dr. W. N. Shaw, F.R.S., who has kindly assisted the work by permitting us to utilise the valuable collection of pressure data deposited in the archives of the Meteorological Office.

We also owe a debt of gratitude to Messrs. W. Moss and T. F. Connolly, who have shown great zeal in completing the necessary computations and drawing the numerous curves which were required for the different stations that have been investigated.

will be driven away with the greatest force from the sun if its circumference is just as great as the wave-length of the radiation. The wave-length of the maximal radiation of the sun is about 0·5 μ. Therefore the optimal dimension for a drop, that is driven away by the pressure of radiation, will be about 0·06 μ. If we suppose a specific weight of the drop like that of water, the repulsive force for a perfectly reflecting drop amounts to about ten times its weight. For a perfectly black drop it is half as great. Now, most drops are neither perfectly reflecting nor perfectly black. Most fluids absorb nearly completely the non-luminous radiation, and reflect a part of the other. An appreciation of these two factors leads to the estimation that the effect for the translucent fluids will be about half as great as for a perfectly black body, *i.e.*, about 2·5 times greater than the gravity against the sun. Such a particle will move away from the sun with 1·5 times greater speed than that calculated above, *i.e.*, it will reach the earth in about 46 hours.

Of course, there may be represented speeds that are more than the double this for drops of low specific weight (compounds of carbon and hydrogen). On the other hand, the speed may be extremely little (or negative), for drops of high specific weight (*e.g.*, gold). This will also be the case for great or very small drops, as Schwarzschild has shown.

These figures have recently acquired a great interest through the discussion by Ellis, Maunder, and Riccò of the connection between sunspots and magnetic storms. Riccò had already, in 1892, stated that in six cases of very strong magnetic storms, these appeared in mean 45·5 hours after the passage of a great sunspot over the central meridian of the sun. In one case the difference of time was only 20 hours.

From the researches of Ellis and Maunder it appears that the magnetic storms *commence* in mean 26 hours after the great groups of sunspots, which probably caused them, had passed the central meridian of the sun. Riccò applies a correction to these figures. He says that in mean the great magnetic storms, quoted by Ellis and Maunder, lasted for 33 hours, and therefore it is natural to assume that the maximum of the magnetic storm, which will probably fall near its middle, arrives 16·5 hours after its commencement. It will, therefore, be nearly true that the maximum of the magnetic storms observed by Ellis, came $26 + 16{\cdot}5 = 42{\cdot}5$ hours after the passage of the corresponding spot through the central meridian of the sun. The figure very nearly coincides with those of Riccò and also with that calculated above. Riccò also makes the observation that the velocity of the small particles, which in my opinion cause the auroras and the magnetic storms, is of the same order of magnitude as the observed velocity with which the cause of these perturbations moves from the sun.

as assumed above, we find that electrons moving with the velocity of light are caught by the sun, if the asymptote of their hyperbolic path is less distant from the sun than 2420 times the mean distance of the earth from the sun (or one twenty-fourth of a light-year). This limit distance is inversely proportional to the velocity of the electrons, and nearly proportional to the square-root of the charge of the sun. As now, according to the researches of Lenard, the electrons from a negatively electrified body possess a much less velocity than light, this distance is really much greater than that just calculated.

If, for instance, the velocity of the electrons is the thirtieth part of that of light, a number that is in good agreement with Lenard's measurements, all electrons from space which came along a path that is less distant than 1·25 light-year from the sun, will be caught by the sun. Of course the electrons move with different velocity, so that the said distance may only be regarded as a mean, or as representing the order of magnitude.

Now our nearest star (α Centauri) is distant from us by about 4 light-years, and other stars lie within less than 10 light-years. Thus it is evident that the negative electrons, which are sent off from aggregates of negatively charged drops (these aggregates are probably identical with what we call cosmic dust or meteorites), can in general not pass by many suns without being caught by them. And on the other hand the suns recover in mean from space as much negative electricity as they lose. The electric charges of the suns are in this respect very effective regulators. If the charge is quadrupled the mean distance of the caught electrons is doubled, or, in other words, as they are uniformly disseminated in space, their quantity is quadrupled. Therefore the supply of negative electricity to the suns is proportional to their defect thereof.

From these considerations we see that a very effective balance of gains and losses of negative electricity is maintained. Evidently this balance depends upon the supposition that for the particles that drive away from the sun, other forces than the electric, viz., the pressure of radiation, are preponderating, whilst for the negative electrons caught by the sun, other forces than the electric are wholly insignificant compared with these.

If one supposed, as some authors do, that the negative electricity was carried away from the sun by means of cathode rays, an effective circulation like that described above would be wholly impossible.

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MONTHLY NOTICES

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ROYAL ASTRONOMICAL SOCIETY.

VOL. LXIV. SUPPLEMENTARY NUMBER, 1904. No. 9

On the Eclipses of the Satellites of Saturn in the Years 1904 *and* 1905. By H. Struve.

The cycle of eclipses and other phenomena of the satellites of *Saturn*, which begins in the present year and will extend over the next five years, gives an opportunity for a series of important observations to which the attention of astronomers may be directed. In the first place, the dimensions of the planet can be determined by these observations with considerable precision. At the same time they will lead to interesting conclusions as to the diameters of the satellites and the atmosphere of the planet and the penumbra. Further they will serve to correct the elements of the orbits of the satellites.

In previous periods very few observations of these phenomena have been made. During the last period, beginning with the year 1889, more attention has been directed to them, thanks to the predictions of Mr. Marth. Some observations of eclipses of the brighter satellites which have been published and discussed in the *Monthly Notices*, vol. liv., show the great accuracy of these observations, and the desirability of a more extensive repetition at the next opportunity. For the success of these observations it is important that astronomers should be acquainted with the conditions of the phenomena, particularly the times of the eclipses and the geocentric places of the satellites. During the next years the reappearances taking place after the opposition can be observed with more facility and accuracy than the disappearances before the opposition, as they occur at greater distances from the planet's limb. On the contrary, after 1907, when the Earth

Date. 1904.	Heliocentric Conj. G.M.T. h m	Geocentric Coord. x	Geocentric Coord. y	Semi-diam. b	Semi-duration. h m
		"	"	"	
Nov. 23	17 6				
25	2 0				
26	10 53				
27	19 47				
29	4 40	+2·8	+8·7	7·6	0 11
30	13 34				
Dec. 1	22 27				
3	7 21				
4	16 14				
6	1 8				
7	10 1				
8	18 55				
10	3 48	+2·5	+8·4	7·5	0 16
11	12 42				
12	21 35				
14	6 29				
15	15 22				
17	0 16				
18	9 9				
19	18 3	+2·2	+8·1	7·4	0 19
21	2 56				
22	11 50				
23	20 43				
25	5 37				
26	14 30				
27	23 24				
29	8 17	+1·9	+7·8	7·3	0 22
30	17 10				

As can be seen the distance of the satellite from the limb at the heliocentric conjunction on November 10 is nearly 2″, on December 19 still nearly 1″, distances in which with powerful instruments and in the most favourable conditions it will be possible to recognise *Enceladus*.

In the next year, 1905, attention may be drawn to the two eclipses of *Japetus*, the first on May 15, when *Japetus* will pass also through the shadow of the ring-system; the second, August 3, on which *Japetus* is only eclipsed by the planet. In both instances the satellite will be seen from the Earth at considerable distance from the disc of the planet and the rings. The following predictions are based on the elements of *Japetus*

deduced formerly by myself and, s
mean distance, on the suppositions

Semi-axis of the outer ring	...
Semi-axis of Cassini's division	...
Semi-axis of the inner ring	...
Major semi-axis of the planet	...
Minor „ „ „	...

The uncertainty of these values minutes and more in the predicte centric coordinates x, y are referre

1. *Eclipse of Japetus*

Ingress into the shadow of the outer ring	
Helioc. transit through Cassini's division	
Ingress into the shadow of the planet	...
Egress out of the shadow of the planet	...
Ingress into the shadow of the inner ring	
Helioc. transit through Cassini's division	
Egress out of the shadow of the outer ring	

In the first part of the phenom of *Japetus* through the interval interior ring remains invisible, shadow of the planet before comi interior ring. In the second part a nearly central passage through ring takes place, like the transit, November 1–2, which formed one vations of Barnard (*Monthly Notice* tion might be given to the appearar transit through Cassini's division satellites is out of question, for all nearer the line of the ring. As th southern declination, and passes t the morning, this eclipse will be southern observatories, while in nc position of the planet and early twi tion difficult.

2. *Eclipse of Japetus* 1

Ingress into the shadow of the planet (disa	
Egress out of the „ „ „ (reapp.	

The beginning of this eclipse can only be observed in countries east of Greenwich. For orientation it may be remarked that *Enceladus*, *Tethys*, *Dione*, and *Rhea* will be on the eastern side of the planet near the axis of the ring, and *Titan* not far from the western elongation.

Concerning the other satellites only *Tethys* will be eclipsed in the year 1905, besides *Mimas* and *Enceladus*. The reappearances of *Tethys* after the opposition, taking place at a greater distance from the planet's limb, may be well observed. The opposition of 1906 is still more favourable for these observations, as the eclipses of all the interior satellites, *Tethys*, *Dione*, and *Rhea*, still occur at sufficient distances from the ring. In the year 1907 the interior satellites move very near the axis of the ring; in the same year the cycle of eclipses of *Titan* begins. In these years also the transits of the satellites and their shadows before the planet's disc, and the occultations and the conjunctions with the ends of the ansæ, will be worthy of particular attention.

The following tables give the times of the eclipses for the opposition of 1905 and also, for intervals of about fifteen days, the geocentric distance s of the satellite from the limb and the geocentric position-angle p, counted from the north pole of the planet (to the west for disappearance, to the east for reappearance). In computing the tables the latest elements of *Mimas* and *Tethys* (*Astron. Nachr.* vol. clxii.) are used, and the semi-axes $a_0 = 8''{\cdot}735$, $b_0 = 7''{\cdot}825$. The possible error in the predicted times should not exceed a few minutes. In the approximate indications of the eclipses published in the *Berliner Jahrbuch* for 1905 the improved elements could not be considered. *Before* the opposition only the disappearances, *after* the opposition only the reappearances are visible from the Earth.

Eclipses of Mimas, 1905.

Disappearance.

Date. 1905.	G.M.T. h m	s.	p. West. °	Date. 1905.	G.M.T. h m	s.	p. West. °
May 25	4 24	1·0	64	June 4	13 12		
26	3 1			5	11 49		
27	1 38			6	10 26		
28	0 15			7	9 3		
28	22 53			8	7 41		
29	21 30			9	6 18	1·0	63
30	20 7			10	4 55		
31	18 44			11	3 32		
June 1	17 21			12	2 9		
2	15 58			13	0 46		
3	14 35			13	23 23		

Eclipses of Mimas, 1905.

Reappearance.

Date. 1905.	G.M.T. h m	s. ″	p. East. °	Date. 1905.	G.M.T. h m	s. ″	p. East. °
Aug. 23	17 39	0·0	57	Sept. 26	16 6		
24	16 16			27	14 44		
25	14 54			28	13 21		
26	13 31			29	11 59		
27	12 8			30	10 36		
28	10 46			Oct. 1	9 14		
29	9 23			2	7 51		
30	8 0			3	6 28		
31	6 38			4	5 6		
Sept. 1	5 16			5	3 43		
2	3 53			6	2 21		
3	2 30			7	0 58		
4	1 8			7	23 36		
4	23 45			8	22 13	1·9	64
5	22 23			9	20 51		
6	21 0			10	19 28		
7	19 38	0·8	60	11	18 5		
8	18 15			12	16 43		
9	16 53			13	15 20		
10	15 30			14	13 58		
11	14 7			15	12 35		
12	12 45			16	11 13		
13	11 22			17	9 50		
14	10 0			18	8 27		
15	8 37			19	7 5		
16	7 15			20	5 42		
17	5 52			21	4 20		
18	4 29			22	2 57		
19	3 7			23	1 35		
20	1 44			24	0 12	2·3	66
21	0 22			24	22 49		
21	22 59			25	21 27		
22	21 37	1·4	62	26	20 4		
23	20 14			27	18 42		
24	18 52			28	17 19		
25	17 29			29	15 56		

Date. 1905.		G.M.T. h m	s. ″	p. West. °
July	21	11 39		
	22	20 32	0·5	53
	24	5 25		
	25	14 18		
	26	23 11		
	28	8 4		
	29	16 57		
	31	1 51		
Aug.	1	10 44		
	2	19 37		
	4	4 30		
	5	13 23		
	6	22 16		
Aug.	8	7 9	0·3	49
	9	16 2		
	11	0 55		
	12	9 49		
	13	18 42		
	15	3 35		
	16	12 28		
	17	21 21		
	19	6 14		
	20	15 7		
	22	0 1		
	23	8 54	0·1	44

Reappearance.

Date. 1905.		G.M.T. h m	s. ″	p. East. °
Aug.	23	10 40	0·1	44
	24	19 34		
	26	4 27		
	27	13 21		
	28	22 14		
	30	7 8		
	31	16 1		
Sept.	2	0 55		
	3	9 48		
	4	18 42		
	6	3 35		
	7	12 29	1·0	47
	8	21 22		
	10	6 16		
	11	15 9		
	13	0 2		
	14	8 56		
	15	17 49		
	17	2 43		
	18	11 36		
	19	20 30		
	21	5 23		
Sept.	22	14 17	1·8	50
	23	23 11		
	25	8 4		
	26	16 58		
	28	1 52		
	29	10 45		
	30	19 39		
Oct.	2	4 32		
	3	13 26		
	4	22 19		
	6	7 13		
	7	16 6		
	9	1 0	2·5	52
	10	9 53		
	11	18 47		
	13	3 41		
	14	12 34		
	15	21 28		
	17	6 21		
	18	15 15		
	20	0 8		
	21	9 2		

Date. 1905.	G.M.T. h m	s.	p. East. °
Oct. 22	17 56		
24	2 49	3·0	54
25	11 43		
26	20 36		
28	5 30		
29	14 24		
30	23 17		
Nov. 1	8 11		
2	17 5		
4	1 58		
5	10 52		
6	19 45		
8	4 39		
Nov. 9	13 32	3·2	55
10	22 26		
12	7 20		
13	16 13		
15	1 7		
16	10 1		
17	18 54		
19	3 48		
20	12 41		
21	21 35		
23	6 29		
24	15 23	3·2	56

Eclipses of Tethys, 1905.

Disappearance.

Date. 1905.	G.M.T. h m	s.	p. West. °
May 24	18 36	0·1	48
26	15 54		
28	13 12		
30	10 30		
June 1	7 48		
3	5 6		
5	2 24		
6	23 42		
8	21 0	0·2	50
10	18 19		
12	15 37		
14	12 55		
16	10 13		
18	7 31		
20	4 49		
22	2 7		
23	23 25	0·3	49
25	20 43		
27	18 1		
29	15 20		
July 1	12 38		
July 3	9 56		
5	7 14		
7	4 32		
9	1 51	0·3	47
10	23 9		
12	20 27		
14	17 46		
16	15 4		
18	12 22		
20	9 41		
22	6 59		
24	4 17	0·3	43
26	1 36		
27	22 54		
29	20 12		
31	17 31		
Aug. 2	14 49		
4	12 8		
6	9 26		
8	6 45		
10	4 3	0·2	37

Date. 1905.	G.M.T. h m	s. "	p. West. °
Aug. 12	1 22		
13	22 40		
15	19 59		
17	17 17		
Aug. 19	14 36		
21	11 54		
23	9 13	0·1	31

Reappearance.

Date. 1905.	G.M.T. h m	s. "	p. East. °
Aug. 23	10 35	0·1	28
25	7 54		
27	5 13		
29	2 32		
30	23 52		
Sept. 1	21 11		
3	18 30		
5	15 49		
7	13 8	1·2	34
9	10 27		
11	7 47		
13	5 6		
15	2 25		
16	23 44		
18	21 4		
20	18 23		
22	15 42	2·1	39
24	13 1		
26	10 21		
28	7 40		
30	4 59		
Oct. 2	2 18		
3	23 38		
5	20 57		
7	18 16		
9	15 36	2·9	42
Oct. 11	12 55	2·9	42
13	10 14		
15	7 33		
17	4 53		
19	2 12		
20	23 31		
22	20 51		
24	18 10	3·4	45
26	15 29		
28	12 49		
30	10 8		
Nov. 1	7 27		
3	4 47		
5	2 6		
6	23 25		
8	20 45	3·7	46
10	18 4		
12	15 23		
14	12 43		
16	10 2		
18	7 21		
20	4 41		
22	2 0	3·6	47
23	23 19		
25	20 39		

Königsberg: 1904 *August* 20.

Schmidt, from a discussion of all his observations on the Red Spot, 1879 to 1881, found a variable error amounting in the maximum to 8 minutes, which he thought was dependent on the hour angle of the planet. He also compared the observations made by twenty other observers, and found a range of 9 minutes in the mean personal equation. These facts were apparently established, but the causes were somewhat obscure. Schmidt thought the variable error might be due to the inclination of the belts as seen by the observer; but from his remarks on the uncertainty of the date it appears that he was not certain that this was the true explanation. It has occurred to me that it might be due to an optical illusion with which every one is familiar. If we look at the figure 8 inverted the two parts are no longer symmetrical. Now, if such variable error really exists, any discussion which aims to determine the magnitude of error is absolutely worthless if this phase of the subject is ignored.

Mr. Williams, without investigation, asserts that a variable error as indicated by Schmidt is not found in the work of other observers, "most of whom, it is believed, take the precaution of keeping the eyes parallel to the belts when observing the transits." It would appear from the above statement that Mr. Williams has never examined his own observations.

Lohse, *Potsdam Pub.* No. 9, found from his own observations mean errors $\pm 2^m.0$ to $\pm 6^m.0$. Hall, observations of *Saturn*, found a mean error $\pm 3^m.0$. All these results are from the comparison of observations *inter se*, and do not necessarily represent the true error. Barnard, without any investigation, *thought* some of his observations would have a mean error $\pm 0^m.7$. The mean error was about $\pm 4^m$. Some years ago I showed that the small residual $\pm 0^m.7$ imagined by Barnard was a mistake, but Mr. Williams has not hesitated to incorporate it in his list of data.

Schmidt, *Ast. Nach.* 2410, gives the following for the Red Spot, 1879–81:

	Reduced without Weight.	Empirical Corrections applied.	
	m	m	
Mean error ...	± 3·00	± 1·90	97 obs.
Probable error ...	± 2·02	± 1·28	

Mr. Williams, by a strange oversight, has assigned to these observations a mean error $\pm 1^m.5$. Then with this fictitious value annexed he brings them forward in proof of the accuracy of eye estimates. If Schmidt's corrections were valid, which is extremely doubtful, as will be seen later, the corrected observations can no longer be regarded as eye estimates and have no place in his discussion.

It seems to me that Mr. Williams is on the horns of a dilemma with regard to Schmidt. If he admits the validity of a variable error, it requires no argument to show the untrustworthy nature of eye estimates. If, on the contrary, he denies

The longitude differences are almost identical, a good proof that the personal equation was zero.

These observations are a fair average of the kind of work done by all observers when the time interval is considerable. The table is worthy of serious study by any one who imagines the eye-estimate method is capable of yielding results of precision, such as are demanded in other lines of astronomical work.

Hough.				Denning-Williams.				
1887.	T. h m	Long. m	O−E. m	1886.	Long. m		O−E. m	Actual Error. m
Jan. 28	18 02·3	− 9·3	−1·3	Nov. 23	− 2·5	D.	+ 9·0	+ 5·5
Mar. 8	15 14·9	− 6·3	+1·7	Dec. 17	− 4·9	D.	+ 6·6	+ 3·1
Apr. 7	9 54·0	− 9·8	−1·8	20	−11·0	W.	+ 0·5	− 3·0
24	8 56·4	− 6·3	+1·7	1887.				
29	8 00·7	− 9·2	−1·2	Jan. 1	− 6·0	,,	+ 5·5	+ 2·0
May 13	9 31·0	− 9·9	−1·9	Feb. 26	− 4·8	,,	+ 6·7	+ 3·2
18	8 46·6	− 1·8	+6·2	Mar. 20	− 3·9	,,	+ 7·6	+ 4·1
28	6 57·1	− 7·1	+0·9	27	− 6·9	,,	+ 4·6	+ 1·1
June 8	11 00·4	− 7·2	+0·8	29	−15·8	,,	− 4·3	− 7·8
11	8 29·1	− 8·3	−0·3	Apr. 3	− 9·9	,,	+ 1·6	− 1·9
13	10 09·1	− 6·9	+1·1	10	−21·8	,,	−10·3	−13·8
16	7 38·7	− 7·3	+0·7	17	−12·7	,,	− 1·2	− 4·7
23	8 23·6	− 9·7	−1·7	20	−13·8	,,	− 2·3	− 5·8
25	10 02·9	− 8·2	−0·2	24	−15·2	,,	− 3·7	− 7·2
28	7 29·9	−12·2	−4·2	27	−14·9	,,	− 3·4	− 6·9
July 7	10 00·7	− 7·9	+0·1	29	−17·9	,,	− 6·4	− 9·9
12	9 11·3	− 6·5	+1·5	May 2	−15·6	,,	− 4·1	− 7·6
19	9 55·5	−10·5	−2·5	7	−15·5	D.	− 4·0	− 7·5
22	7 25·7	−10·9	−2·9	9	−15·5	W.D.	− 4·0	− 7·5
29	8 13·9	− 6·0	+2·0	10	−13·8	D.	− 2·3	− 5·8
Mean Long.		− 8·0	± 1·7	14	−15·1	W.	− 3·6	− 7·1
				26	−14·8	D.	− 3·3	− 6·8
				June 10	−11·3	D.	+ 0·2	− 3·3
				19	−21·0	W.	− 9·5	−13·0
				22	− 6·1	D.	+ 5·4	+ 1·9
				July 16	− 5·8	D.	+ 5·7	+ 2·2
				Aug. 6	− 4·2	D.	+ 7·3	+ 3·8
				Mean Long.	−11·5		± 4·7	± 5·6

The column "Long." is the longitude in time from Marth's Meridian II. The column "Actual Error" is the difference between the observed longitude and the true longitude resulting from the micrometer measures. The residuals O—E, for both micrometer

Object.	Time Interval. Days.	No. Obs.	O−E.	Mean Personal Equation.	Observer.
			m	m	
Long Red 1891 ...	85	13	±1·7	−2·5	Barnard
Small Black 1891 (*a*)	119	10	±1·5	...	Ho.
„ (*a*)	92	7	±5·3	−5·8	Barnard
„ (*b*)	85	13	±4·2	0	Barnard
Red Spot 1899 ...	225	27	±1·6	...	Ho.
„ ...	95	9	±2·3	−7·7	Williams
Red Spot 1900 ...	199	17	±1·3	...	Ho.
„ ...	113	4	±2·8	−7·3	Williams
Black Spot 1898 ...	191	18	±2·2	...	Ho.
„ ...	144	24	±4·1	−2·4	Gledhill
Red Spot 1887 ...	182	20	±1·7	...	Ho.
„ ...	252	10	±4·7	−1·4	Denning
„ ...	181	17	±4·9	−4·4	Williams

The variable personal equation was :

Barnard ...	Red Spot	Beginning −2m	End −7m
„ ...	Long Red	Not well indicated	
„ ...	Small Black (*a*)	Beginning −15m	„ 0
„ ...	„ (*b*)	„ −4m	„ +3m
Williams...	Red Spot 1899	Not well indicated	
„ ...	„ 1900	Beginning −3	„ +3
Gledhill ...	Black Spot 1898	„ −4	„ +6
Denning ...	Red Spot 1887	Beginning and end +4	Middle −5
Williams...	„ „	Beginning +3	Middle and end −7

The numbers indicating variable personal equation are the average of a number of contiguous observations, and are necessarily approximate.

The spots *a* and *b* observed by Barnard in 1891 present a curious phase in eye-estimate observations. Barnard observed both spots on the same night at the beginning and end of the series. The other observations alternate on different nights. The two spots were in the same latitude and in close proximity, the difference of longitude by the micrometer being $24^m{\cdot}8 \pm 0^m{\cdot}6$. Barnard's observations give $30^m{\cdot}8 \pm 0^m{\cdot}9$, or a constant error of six minutes in the distance between them. The mean personal equation, as already given, indicates the same constant error. In other words, his central meridian differed $5^m{\cdot}8$ for the two spots.

The following inferences may be drawn from the above data :

1st. In all cases the "mean" personal equation was negative,

by assuming a uniform rotation period. Occasionally, however, it is necessary to use a variable rotation period to bring the residuals within the probable errors of observation. Examples of this kind may be found in *Monthly Notices*, lii. 412, and *Ast. Nach.* 3354.

The personal equation problem is one of so much importance that I have made a few additional comparisons, to ascertain its amount in different years. For convenience of reference all the data are arranged in the following table:

		Mean Pers. Eq.	Variable.
	Denning.	m	m
Red Spot	1880	−4·6	6
,, ...	1883	−7·7	6
,, ...	1885	−3·2	
,, ...	1887	−1·4	6
,, ...	1901	−2·5	
White Spot	1881	0	3
	Williams.	m	m
Red Spot	1887	−4·4	10
,, ...	1899	−7·7	
,, ...	1900	−7·3	5
,, ...	1902	−2·9	6
White Spot	1881	+0·8	7
Spot (*a*)	1898	−5·9	3
	Schmidt.	m	m
Red Spot	1880 Sept.	−8·3	±0·7
,, ...	Nov.	−0·6	±1·5
,, ...	1881 Jan.	−4·0	±1·0

Denning, 1880.—The personal equation was derived from five consecutive observations at two dates, October 11 and December 20, giving respectively $-7^m\cdot7$ and $-1^m\cdot5$ with a probable mean error less than $\pm1^m$.

Schmidt's observations were treated in the same manner for the dates given. It is readily seen that there is no constancy in the mean personal equation from year to year.

During the opposition the variation is usually in one direction only. In view of the difficulty in making an observation it is possible that these abnormal results are simply due to accidental error.

As an illustration of the uncertainty involved in eye estimates

that the errors derived in my report, 1882, were determined in the usual way and are correct. The residuals when observations are compared with an ephemeris are made up of mean error, constant error, and variable motion.

In *Monthly Notices*, lii. 412, for the Red Spot the mean residual for uniform motion was $\pm 1^m\cdot 7$, for variable motion $\pm 1^m\cdot 2$. In my report for 1882 the observations of the Red Spot from 1879 September 25 to 1882 March 29, a period of 916 days, are compared with an ephemeris in which the rotation period increased with the time. Mr. Williams, perhaps inadvertently, stated that Schmidt's observations from 1879 November 10 to 1881 March 20 gave a mean error of $\pm 1^m\cdot 5$, "exactly the same as mine." Schmidt's mean error was $\pm 3^m\cdot 0$.

Identification of Spots.

I am at a loss to know what Mr. Williams requires for the identification of a spot on *Jupiter*. Here are four observations which he says belong to three different spots:

Days.	O−E.	App. Lat.
	m	″
0	+0·0	−0·93
37	+3·3	...
78	+0·8	−1·74
80	+0·1	−1·61

This spot is identified both by latitude and longitude. The longitudes are represented by a mean error of $\pm 1^m\cdot 0$, and the latitudes are well within the limit of error, the first observation differing only $0''\cdot 50$ from the mean. I venture to say that neither Mr. Williams nor anybody else has ever made a more certain identification.

Three observations of a spot at unequal intervals of time are sufficient to definitely fix the rotation period. The spots I observe are generally identified by both longitude and latitude, whereas in eye estimates one must rely on longitude only.

Mr. Williams remarks that about 40,000 transits have been observed, "and that these constitute the foundation of much of our knowledge of the rotation of *Jupiter* and the various surface currents known to exist," &c. He has also remarked on another occasion that eye estimates could accomplish all that can be done with the micrometer. I have grave doubts regarding both statements. I have repeatedly urged that the element of latitude is of equal value with that of longitude in arriving at a knowledge of the conditions of the surface of the planet.

The foregoing discussion proves conclusively the existence of a variable error, even beyond the limits assigned by Schmidt. All the observers, in the examples cited, without exception, either gradually or suddenly changed the zero of reference in

Observations of the Satellite of Neptune from Photographs taken at the Royal Observatory, Greenwich, between 1903 *December* 4 *and* 1904 *April* 18.

(*Communicated by the Astronomer Royal.*)

The following measures of position angle and distance of *Neptune's* satellite were made from photographs taken with the 26-inch refractor of the Thompson equatorial. The occulting shutter was used, as in previous years, the photographs being taken by Messrs. Davidson, Edney, or Melotte. The photographs were measured in a position micrometer in direct and reversed positions by Mr. Dyson and Mr. Edney. The tabular positions with which comparison is made were computed from the data given in the *Connaissance des Temps*, based on Dr. H. Struve's elements, the eccentricity of the orbit being neglected on account of the uncertainty in the present position of peri-astron. For fuller particulars as to the photographs and their measurement reference may be made to a previous paper in the *Monthly Notices*, vol. lxii. pp. 622–626.

NEPTUNE AND SATELLITE.

Position Angle and Distance, from Photographs taken with the 26-inch Refractor.

Date and G.M.T.					Position Angle.			Distance.			Remarks.
	d	h	m	s	Observed.	Tabular.	T−O.	Observed.	Tabular.	T−O.	
					°	°	°	″	″	″	
ept.	4	11	20	34	97·25	97·70	+0·45	16·46	16·56	+0·10	
	8	12	32	26	225·60	226·72	+1·12	13·28	13·40	+ ·12	
	9	11	25	46	145·03	146·41	+1·38	12·32	11·98	− ·34	
	9	12	10	35	142·94	143·97	+1·03	12·11	12·15	+ ·04	
	10	11	8	18	91·03	92·95	+1·92	16·76	16·80	+ ·04	
	10	11	47	50	91·53	91·86	+0·33	17·02	16·81	− ·21	
	14	10	43	25	223·17	223·66	+0·49	13·03	13·13	+ ·10	
	15	11	40	23	135·15	136·47	+1·32	12·68	12·72	+ ·04	
	17	10	26	21	40·08	40·32	+0·24	13·05	12·84	− ·21	
	30	11	14	11	297·34	298·44	+1·10	14·36	14·49	+ ·13	
	30	11	51	15	296·50	297·11	+0·61	14·54	14·62	+ ·08	
	31	11	11	28	254·55	255·18	+0·63	16·71	16·44	− ·27	Definition very poor.
	31	11	42	41	252·84	254·26	+1·42	16·28	16·37	+ ·09	Definition very poor.
ec.											
an.	6	10	55	54	249·35	250·28	+0·93	15·83	16·04	+ ·21	
	13	9	1	24	177·16	179·21	+2·05	11·15	11·00	− ·15	
	13	9	33	25	176·68	177·04	+0·36	11·10	10·99	− ·11	
	14	10	4	22	105·00	105·53	+0·53	16·09	15·79	− ·30	
	14	10	39	39	103·04	104·42	+1·38	15·98	15·90	− ·08	
	15	10	0	32	62·79	63·48	+0·69	15·25	15·34	+ ·09	

New Variable Stars found on the Astrographic Plates at the Royal Observatory, Greenwich.

(Communicated by the Astronomer Royal.)

In the course of the work for the *Astrographic Catalogue* under Mr. Hollis's direction a star whose approximate position for 1900 is R.A. $8^h\ 14^m\ 5^s$, Dec. $+73°\ 25'{\cdot}6$, has been found to be variable to the extent of about three magnitudes. The following table gives the measured diameter (unit $= 0''{\cdot}15$) and that of neighbouring comparison stars whose magnitudes have been inferred from the formulæ

$$m = 16{\cdot}1 - 0{\cdot}84\sqrt{d} \text{ for exposure 40 minutes}$$
$$m = 13{\cdot}7 - 0{\cdot}77\sqrt{d} \quad \text{,,} \quad 6 \quad \text{,,}$$

given in vol. i. of the Greenwich Section of the *Astrographic Catalogue*, p. xxx. The magnitude of the variable on the different plates is readily obtained by comparison with these standards.

No. of Plate.	Date.	Exp.	Measured Diameters.						Deduced Mag. of Variable.
			Var.	α.	β.	γ.	δ.	ε.	ζ.
				m 13·0	m 12·1	m 11·5	m 10·8	m 10·0	m 9·6
		min.							m
3782	1897 Dec. 20	40	20	—*	20	30	42	54	62	12·1
4288	1898 Feb. 21	40	16	12	22	26	42	66	64	12·6
4320	1899 ,, 27	40	38	6	16	24	34	38	50	10·0
4361	1899 Mar. 11	40	16	14	22	32	42	56	64	12·8
4761	1900 Jan. 2	6	5	6	12	19	26	38	44	13·1
4793	1900 ,, 24	6	22	3	8	11	20	26	34	10·5
6004	1902 Mar. 25	40	22	10	16	20	30	40	42	11·4
6010	1902 ,, 27	40	28	12	18	24	36	46	58	11·3

The positions of the variable and reference stars are as follows :

	Zone and No. in *Astrogr. Catalogue.*	Zone and No. in B.D.	Approximate Position, 1900. R.A.	Dec.
			h m s	° ′
Variable	73. 3544		8 14 5	73 26
α	73. 3547		8 17 12	73 28
β	73. 3514		8 16 34	73 15
γ	73. 3525		8 16 18	73 15
δ	73. 3545		8 16 21	73 29
ε	73. 3509	73° 417	8 21 8	73 7
ζ	73. 3523	73. 411	8 13 56	73 19

A star, whose approximate position for 1900·0 is R.A. $19^h\ 40^m\ 38^s$, Dec. $+80°\ 42'$, is suspected of being variable, and further photographs are being taken for comparison.

* Not shown, owing to fog on plate.

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DETACHED NEBULA IN CYGNUS.

The great nebula north of *Ancharen*.
" " *ρ Sagittarii*.
" " *η Carinæ*.
" " *S Monocerotis*.
" " *ζ Persei*.
" " *β Cassiopeiæ*.
" near *γ Cygni*, &c.

All these examples show the above-mentioned relation, and also the fact that, though partially or wholly surrounded by void zones, the nebulæ are generally placed at the end of a longer extended lacuna, so that we are led to the impression that we here see the result of some cosmic movement, the end of the lacuna showing the place where this unknown event began.

I had already found something similar in the Triple Cave of *Aquila*, but not typical enough for discussion. The photograph accompanying the present paper shows an extremely curious example of these relations.

This nebula is situated about 2° south-east of *γ Cygni*, and involves in its centre the 9·5 mag. star B.D. +46°, 3474, the coordinates of which are given by Argelander for 1855·0, viz.:

$$\text{R.A.} = 21^h\ 47^m\ 54^s{\cdot}4 \quad \text{Decl.} = +46°\ 34'{\cdot}9.$$

A second *Durchmusterung* star is enclosed by the same nebula, somewhat more south; it is the 9·3 mag. star B.D. +46°, 3475, which has the position:

$$\text{R.A.} = 21^h\ 47^m\ 55^s{\cdot}3 \quad \text{Decl.} = +46°\ 30'{\cdot}9.$$

This star is situated near the southern edge of the nebula. The object was photographed by myself for the first time on 1894 July 28 with a 6-inch camera. The last picture obtained, which is reproduced here, was taken 1904 July 10 with the 16-inch camera, with 240 minutes' exposure. The reproduction (Plate 18) is made from a part of one of the original plates on the scale of 1° = 60 mm.

The nebula is somewhat round and is about 10′ in diameter. It is of a very complicated structure, somewhat resembling the trifid nebula in *Sagittarius*. It is placed centrally in a very fine lacuna, void of faint stars, which surrounds the luminous cloud like a trench. The most striking feature with regard to this object is that the star-void halo encircling the nebula forms the end of a long channel, running eastward from the western nebulous clouds and their lacunæ to a length of more than two degrees.

The channel is somewhat similar to one of the three arms of the *Aquila* Triple Cave (reproduced in *Knowledge*, xxv. p. 203).

The nebulous clouds to the west, partly visible at the right edge of the picture, are placed near a region likewise poor in faint

of about three years. The orbit is probably, however, a parabola, the latest elements received at this writing, as issued from the

FIG. 1.—Telescopic Discovery Field of Brooks's Comet, a 1904.

Lick Observatory by Messrs. Atkin and Maddrill, from observations on April 17, May 8, and May 24, being as follows:

$$T = 1904 \text{ March } 6{\cdot}9049 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 53^\circ\ 27'\ 13''{\cdot}8 \\ \Omega &= 275\ 46\ 05{\cdot}5 \\ i &= 125\ 07\ 33{\cdot}1 \end{aligned}\right\} \text{M. Eq. 1904.}$$

$$\log. q = 0{\cdot}432475.$$

The comet's position being most favourable for observation, and being visible the entire night, a large number of observations have been secured, the latest being last night, July 11, $11^h\ 5^m$ standard M.T., when its approximate place was R.A. $12^h\ 27^m\ 30^s$, Decl. $+51^\circ\ 14'$. The most northerly position observed here was on June 10, when it equalled $+56^\circ\ 57'$.

Although theoretically the comet should have grown fainter subsequent to its discovery the decline was so slow as to be imperceptible for some weeks; indeed there were times when it appeared considerably brighter than it was at discovery. I am inclined to attribute this appearance to actual fluctuations in brightness (as noted in other instances) as well as to changes in

the clearness of the sky—a very variable quality indeed, even when the sky is quite free from clouds, in this very delightful lake region of America. I select a few of many observations:

April 22. 9^h 50^m.

Position R.A. 16^h 40^m 45^s, Decl. $+47°$ $50'$.

Easily seen in presence of half-moon.

May 2. 8^h 30^m.

R.A. 16^h 3^m 50^s, Decl. $+52°$ $59'$.

FIG. 2.

FIG. 3.

Tail not quite so conspicuous but the nucleus bright and sparkling. Comet was also well seen after moonrise, three nights after full.

May 11. 9^h 0^m.

R.A. 15^h 29^m 10^s, Decl. $+56°$ $9'$.

Very clear, and comet looked brighter. More of the tail seen and the nucleus very sparkling.

June 10. 9^h 35^m.

R.A. 13^h 21^m 0^s, Decl. $+56°$ $57'$.

FIG. 4.

FIG. 5.

First observation after long period of clouds and rain. Comet fainter, but easily seen in the 3-inch finder. Tail traced nearly to edge of half-degree field, and the nucleus almost as bright as at any previous observation.

I trust that my fellow-workers in England and on the Continent have been interested in following this celestial visitant in its favourable pathway across the sky.

May I be allowed to put on record here that this is the twenty-fourth cometary discovery I have been permitted to make? Eleven of these were found with reflectors at the old Red House Observatory, Phelps, N.Y., two with the same instrument soon after my removal here, the 9-inch reflector being set up in the garden pending the installation of the observatory equipment. The remaining eleven comets have been discovered with the 10-inch refractor of this observatory. Comet-seeking is, however, only carried on in the few intervals between other duties, among which is the entertainment of visitors, the Observatory being freely open to the public on every clear night. This explains why most of my Geneva comets have been discovered in the morning sky.

Smith Observatory, Geneva, N.Y., U.S.A.:
1904 *July* 12.

Errata.

Mean Daily Area of Sun-spots for Each Degree of Solar Latitude for each Year from 1874 *to* 1902 *as Measured on Photographs at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

The following corrections should be made in the paper bearing the above title, and appearing in the *Monthly Notices*, vol. lxiii. No. 8 :

Page.	Year.	Latitude.	*For*	*Read*
457	1896	+20°	2·2	21·5
460	1892	−25	10·8	17·8
,,	,,	−35	0·01	...
,,	,,	−40	...	0·01

Erratum in Mr. Maunder's Paper on the Distribution of Spots in Heliographic Latitude.

Vol. lxiv. p. 758, line 32, *for* $\frac{1}{250}$ *read* $\frac{1}{25}$.

[redacted]

[redacted]

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

APPENDIX TO VOL. LXIV.

[*From Proceedings of the Royal Society, Vol. LXXIV.*]

With indication of the original pagination.

No. 4.

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b

In comparing, for this purpose, the spectra taken with the 6-inch Henry prismatic camera it was noticed that certain lines were relatively intensified in passing from the spectrum of Capella to that of Arcturus.

Similar comparisons of the Fraunhoferic spectrum with the spectra of Capella and Arcturus respectively were next made. This work led to the following conclusions:—(1) That the line absorptions of Capella and the sun are practically identical; (2) that although, speaking generally, the same lines occur in the spectra of the sun and Arcturus, yet in the latter many lines are relatively more intense than in the former. Moreover, in the great majority of such cases *the lines so intensified are probably due to Vanadium and Titanium.*

Thus we see that whilst the temperature classification mentioned above certainly places Arcturus on a lower temperature level than Capella and, therefore, the sun, the evidence obtained from a study of the line absorptions of Arcturus and of sunspots indicates very clearly that the temperature of the Arcturian absorbing atmosphere is about the same as that of the sunspot nuclei during the above-mentioned period.

This conclusion justifies the ideas formulated by De la Rue, Stewart, and Loewy that the spots are produced by the downrush of cooler material.

In a recent publication,* which has been received here since the above-mentioned comparisons were completed, Professor Hale suggests that because the lines which are widened in sunspots appear as strong dark lines in Piscian stars, the effect may be produced because sunspots are more numerous in such stars. From the evidence adduced above it seems a far more probable explanation to suppose that these lines are intensified in sunspots, and strengthened in those stars which have been placed on lower temperature levels than the sun, because the general temperature conditions are similar. That is to say, the fall of temperature experienced by the metallic vapours in passing from the photosphere to the spot nucleus is of the same order as that to which an absorbing atmosphere is subjected in passing from the temperature conditions of Capella or the sun to that of Arcturus or the lower temperature stars.

* "The Spectra of Stars of Secchi's Fourth Type" ('The Decennial Publications,' Chicago University, 1903).

"A Probable Cause of the Yearly Variation of Magnetic Storms and Auroræ." By Sir NORMAN LOCKYER, K.C.B., LL.D., F.R.S., and WILLIAM J. S. LOCKYER, M.A. (Camb.), Ph.D. (Gött.), F.R.A.S., Chief Assistant Solar Physics Observatory. Received June 3,—Read June 16, 1904.

The ordinary meteorological elements, such as atmospheric pressure, temperature, etc., have a yearly change satisfactorily explained as due to changes of the position of the earth's axis in relation to the sun, or, in other words, the variation of the sun's declination. There are, however, other phenomena, such as magnetic disturbances and auroræ, which have been explained differently.

Thus, in regard to this seasonal variation Mr. Ellis* has written, "The related physical circumstance is that at the equinoxes, when disturbance is more frequent, the whole surface of the earth comes under the influence of the sun, whilst at the solstices, when magnetic disturbance is less frequent, a portion of the surface remains for a considerable period in shadow."

The object of the present communication is to put forward another possible cause.

It has been previously pointed out† that a very close relationship exists between the epochs of occurrence of prominences in the polar regions of the sun and Ellis's "great" magnetic disturbances. This synchronism showed that either the polar prominences themselves, or the disturbances thus indicated in these polar regions, were the origin of these "great" magnetic storms, or that they were caused by a more general stirring-up of a greater extent in latitude of the solar atmosphere.

A further investigation‡ indicated, however, that in all probability it was either the actual polar prominences themselves, or the activity in the solar polar regions, that initiated these magnetic disturbances, for it was there pointed out that the presence of polar prominence activity-tracks synchronised with the appearances of large "polar" coronal streamers. Here we have an indication of a local cause and effect.

It will be gathered, then, that, even as regards terrestrial magnetic phenomena, considerable importance must be attached to action taking place in the regions about the solar poles.

Since the axis on which the sun rotates is inclined to the plane of the ecliptic, there will be times throughout the course of a year when the solar polar regions will be exposed most and least to the earth.

* 'Monthly Notices,' vol. 61, p. 540.

† 'Roy. Soc. Proc.,' vol. 71, p. 244; also 'Monthly Notices, R.A.S.,' vol. 63, Appendix I, p. 6.

‡ 'Monthly Notices, R.A.S.,' vol. 63, p. 481.

It should be expected, then, that if the polar regions of the sun have any action, as above suggested, the effects of the action on the earth should vary according to the positions of the solar poles relative to the earth.

The actual inclination of the sun's axis being 82° 45′, and the longitude of the ascending node being 74° 25′, or the tilt of the axis being in the direction of about 19 hours in right ascension, it follows that, in each year, the south pole of the sun is most turned towards the earth in the beginning of March (about the 6th), and the north pole most towards the earth in the beginning of September (about the 5th). At the two intermediate epochs, in June (about 5th) and December (about 6th), neither pole is turned towards or away from the earth, but occupies an intermediate position. Hence we see that the equinoxes occur in the same months as those in which one or other of the solar poles is turned towards the earth, while the neutral positions of the solar poles in relation to the earth occur in the same months as the solstices.

The accompanying diagram shows graphically the relation between

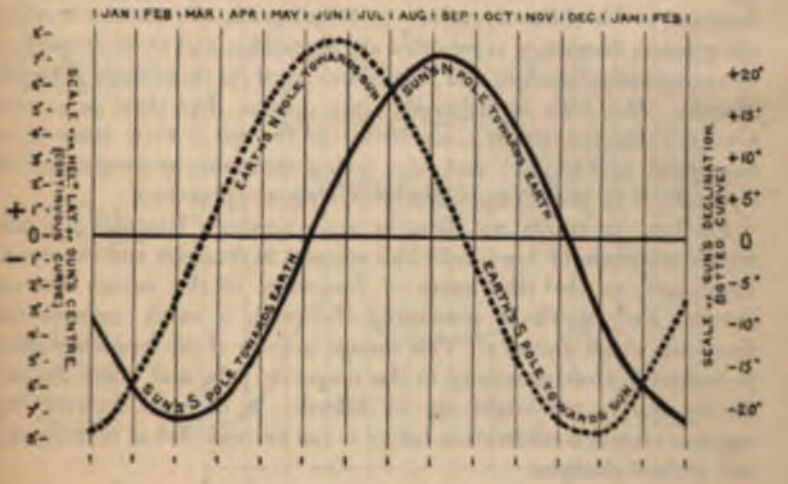

FIG. 1.—Curves showing the relation between the change of declination of the Sun (broken curve) and the positions of the Sun's north and south poles with regard to the earth (continuous curve) throughout a year.

the two curves representing the variation of the sun's declination and the change of the latitude of the sun's centre or the variation of the amount of the tilt of the solar poles, in relation to the earth throughout a year.

It will be seen that the curve representing the tilt of the solar axis

is nearly (a little less than) a quarter of a phase in advance of that indicating the declination change, so that the maximum or minimum point of the latter curve is only slightly in advance of the *mean* points respectively of the former curve.

If, therefore, these solar polar regions are capable of disturbing the magnetic and electric conditions on the earth, as has been above suggested, then, when they are most directed to her at the equinoxes, the greatest effects during a year should be recorded, and when they are least directed the effects should be at a minimum.

It will not be necessary here to refer at any great length to statistics relating to the annual inequality of magnetic disturbances and auroræ, for these have been very efficiently worked out and the results published by Mr. William Ellis.*

Mr. Ellis has shown that the curves of frequency of magnetic disturbances at Greenwich and Paris are very similar, "showing maxima at or near the equinoxes, and minima at or near the solstices." These also, he further points out, are similar, with regard to the epochs of maxima, to the curve representing the frequency of the aurora at London. In the case of auroræ observed in Edinburgh, North-East Scotland and in different regions in Scandinavia, the months in which the greatest frequency is recorded are September and October (perhaps more generally October) and March and April (perhaps more generally March). Mr. Ellis is inclined to the opinion that there is a small tendency for the autumn maximum to become a little later (from September to October) and the spring maximum somewhat earlier (from April to March) as higher latitudes are approached.

Further, he points out that in more northern latitudes the mid-winter minimum of lower latitudes appears to diminish and eventually disappears, so that the curve of frequency of the aurora between October and March is practically flat with a small intermediate maximum about January. This change in form of the frequency curve in regions in close proximity to the magnetic pole, and where the conditions of day and night are so different, is of great interest, but requires careful consideration before it can be regarded as representing real auroral changes.

The accompanying curves, fig. 2, illustrate the relation throughout a year between the positions of the earth's poles with reference to the sun; the positions of the sun's poles as regards the earth; the frequency of magnetic storms at Greenwich and Paris; and lastly, the frequency of the aurora as observed at Edinburgh and at stations in Scandinavia below latitude 65° N. The first two curves are those that have already been given in fig. 1, but plotted differently. They have here been so arranged that the maxima points represent the epochs when each of the poles is most inclined to the sun or earth as the case

* 'Monthly Notices, R.A.S.,' vol. 60 p. 142; vol. 61, p. 537; vol. 64, p. 229.

may be. Both the magnetic and auroral curves represent four of the set of curves which Mr. Ellis* has recently published.

It need scarcely be pointed out that the low minima of the auroral

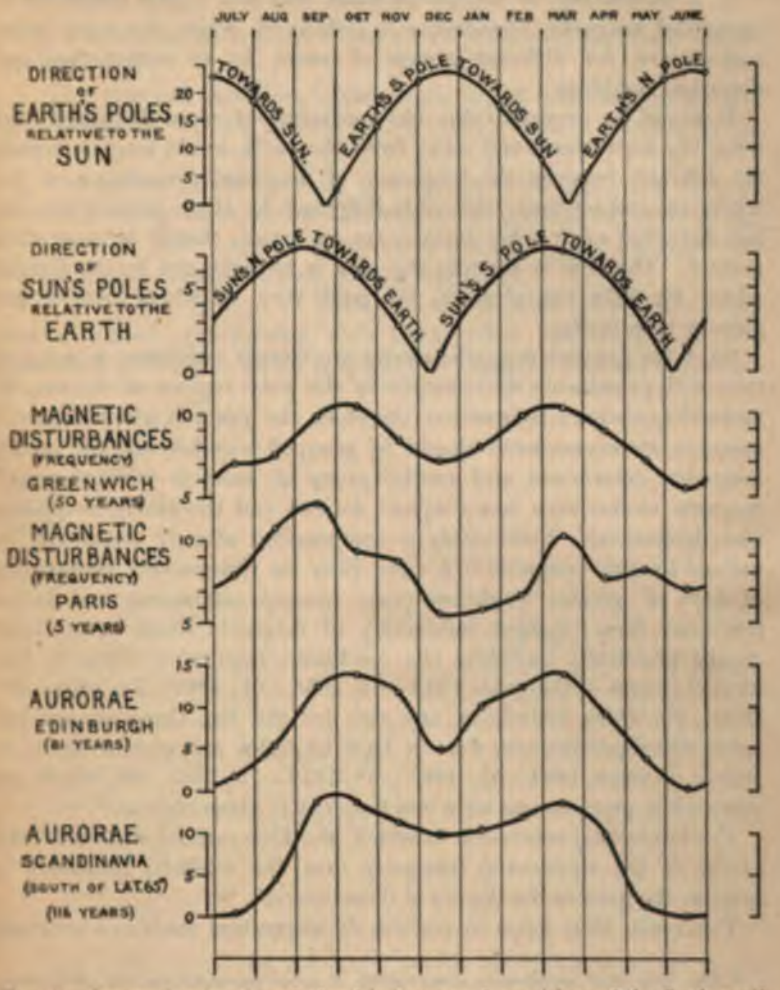
JULY AUG SEP OCT NOV DEC JAN FEB MAR APR MAY JUNE
DIRECTION OF EARTH'S POLES RELATIVE TO THE SUN
TOWARDS SUN
EARTH'S S. POLE TOWARDS SUN
EARTH'S N. POLE
DIRECTION OF SUN'S POLES RELATIVE TO THE EARTH
SUN'S N. POLE TOWARDS EARTH
SUN'S S. POLE TOWARDS EARTH
MAGNETIC DISTURBANCES (FREQUENCY) GREENWICH (50 YEARS)
MAGNETIC DISTURBANCES (FREQUENCY) PARIS (5 YEARS)
AURORAE EDINBURGH (81 YEARS)
AURORAE SCANDINAVIA (SOUTH OF LAT. 65°) (116 YEARS)

FIG. 2.—Curves showing the relationship between the positions of the Sun's north and south poles with regard to the earth and the frequency of magnetic disturbances and auroræ throughout a year.

curves during the summer months are due in great part to the shortness of the nights, and therefore to the restriction of the time available for aurora observations.

The coincidence in time between the epochs of the maxima of the

* 'Monthly Notices, R.A.S.,' vol. 64, p. 229.

paragraph that the greater the disturbed solar polar regions, the greater the difference between the magnetic frequency at the equinoxes and solstices.

Conclusions.

The conclusions arrived at in the above paper may be briefly stated as follows :—

1. The seasonal variation in the frequency of magnetic storms and aurorae depends on the positions of the sun's axis in relation to the earth.

2. The epochs of the greatest inclinations of the sun's axis towards or away from the earth, or in other words the greatest exposure of the N. or S. solar polar regions to the earth during a year, correspond to those of greatest magnetic and auroral frequency.

3. The epochs (groups of years), when the solar polar regions are most disturbed, synchronise with those when the excess of the equinoctial over the solstitial frequency of magnetic storms is greatest.

HARRISON and SONS, Printers in Ordinary to His Majesty, St. Martin's Lane.

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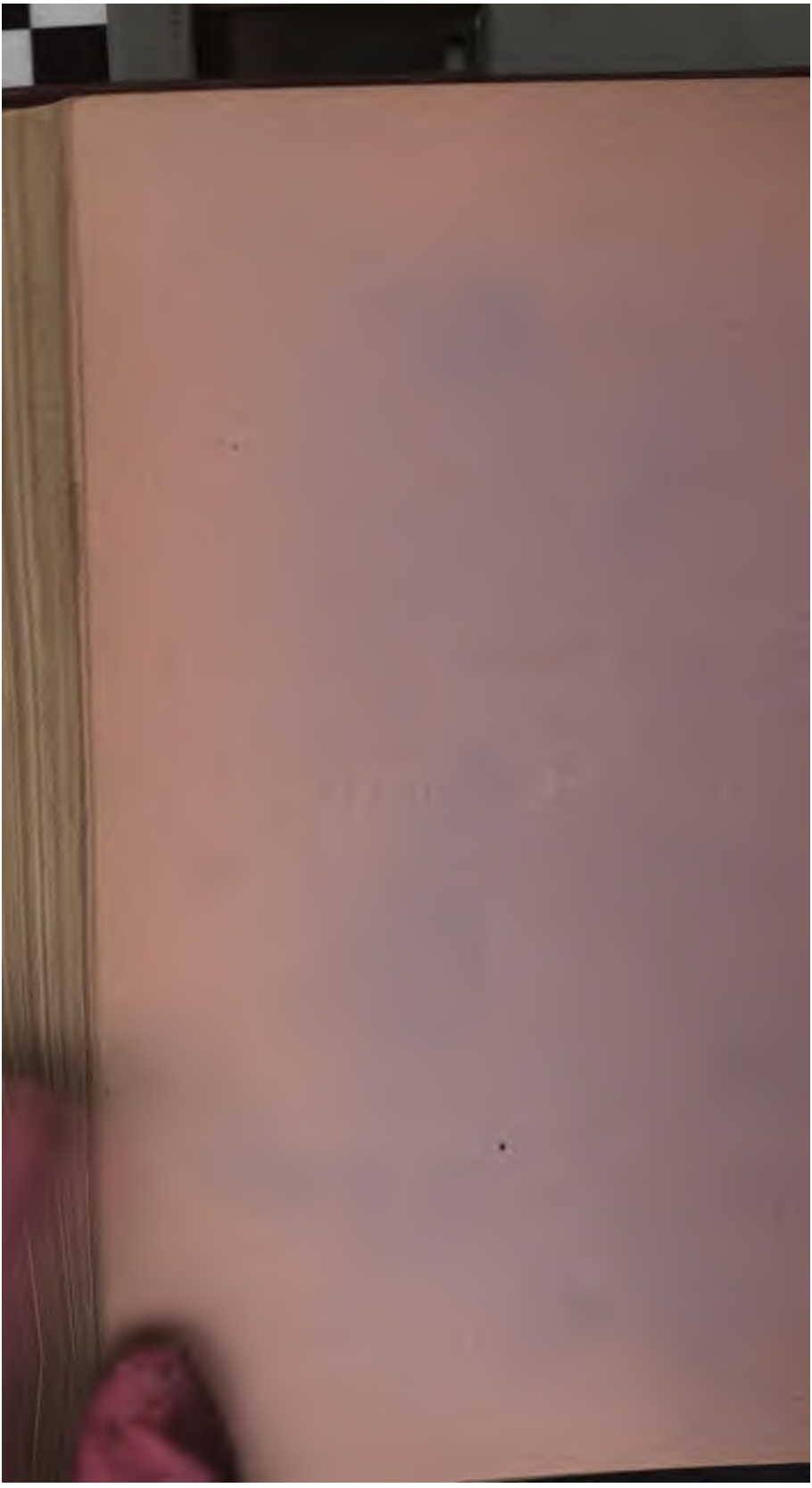

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